YITP molecule-type workshop Cosmological Perturbation and CMB,
2011/03/22

# Constraints on a f(R) gravity dark energy model with early scaling evolution

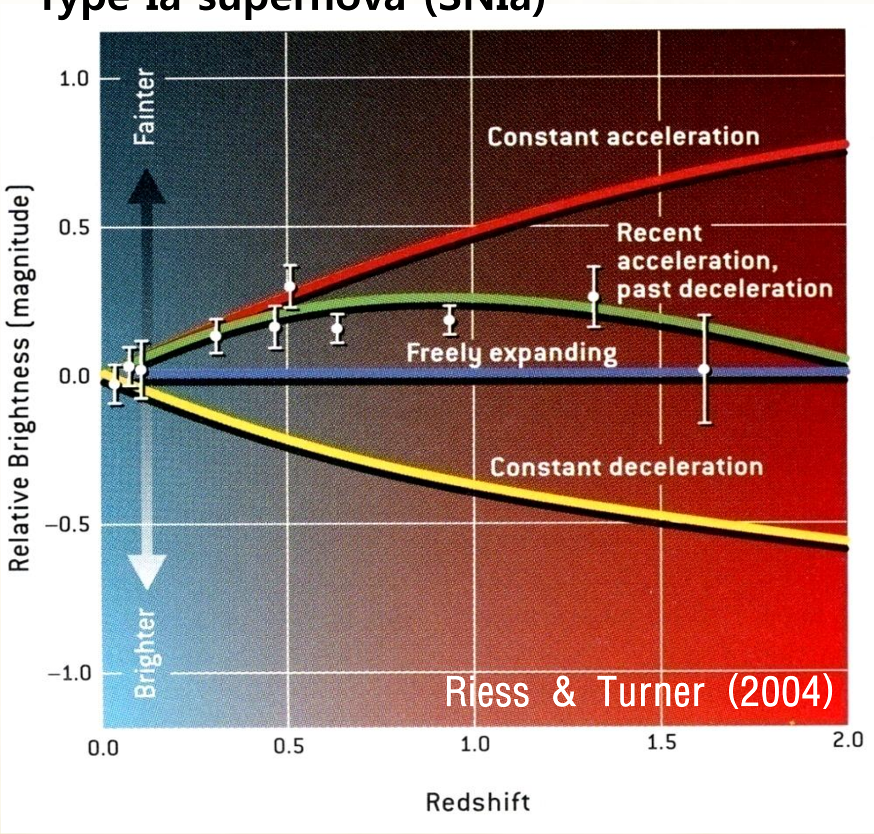
Chan-Gyung Park
Kyungpook National University

Based on arXiv:1012.1662 [with Jai-chan Hwang (KNU) and Hyerim Noh (KASI)]

# Evidence of cosmic acceleration

The high-z type Ia supernova (SNIa) luminosity-distance relation, large-scale structures and CMB observations suggest that the expansion rate of our universe is currently under acceleration.

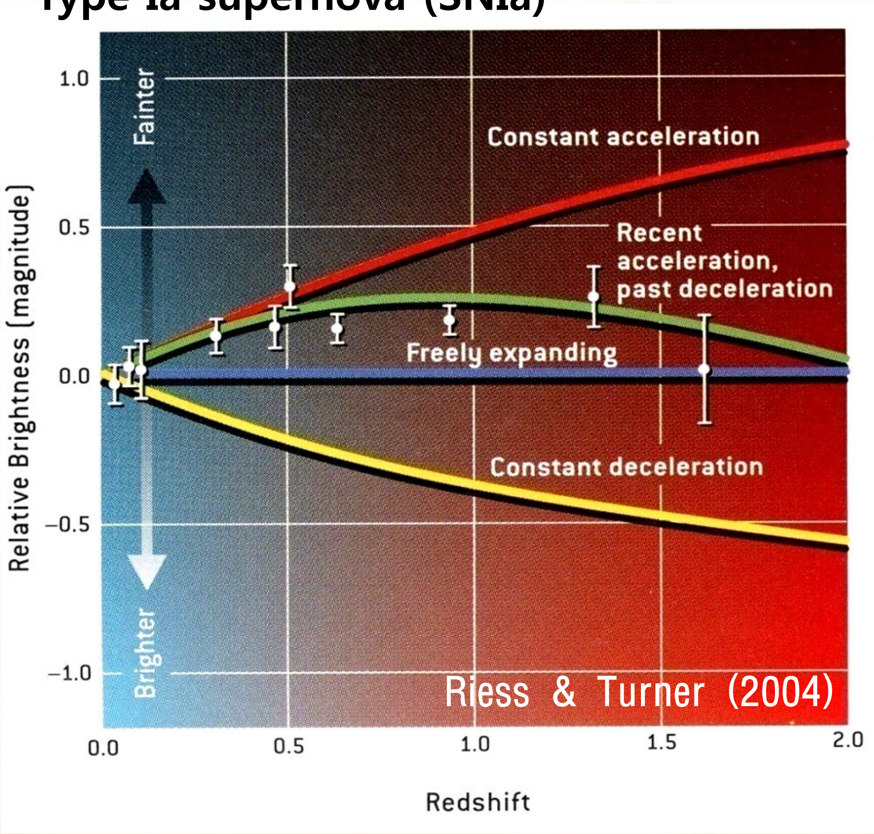
**Type Ia supernova (SNIa)**



# Evidence of cosmic acceleration

The high-z type Ia supernova (SNIa) luminosity-distance relation, large-scale structures and CMB observations suggest that the expansion rate of our universe is currently under acceleration.

**Type Ia supernova (SNIa)**




Introduction of cosmological constant Λ to explain the late-time acceleration:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\Big(\underbrace{\mu_m + 3p_m}_{\substack{\propto a^{-4}(R) \\ a^{-3}(M)}}\Big) + \underbrace{\frac{\Lambda}{3}}_{=\text{const.}}$$

Λ equation of state:

$$w = p_\Lambda / \mu_\Lambda = -1$$

The nature of the agent causing the acceleration is still unknown, and it is one of the fundamental mysteries in the present day theoretical cosmology.

→ dark energy

# Many Dark Energy Models

Shinji Tsujikawa
arXiv:1004.1493v1

**I. Cosmological constant**

**II. Modified matter models**
- A. Quintessence
- B. k-essence
- C. Coupled dark energy
  1. The coupling between dark energy and dark matter
  2. Coupled dark energy and coincidence problem
  3. Chameleon mechanism
  4. Varying $\alpha$
- D. Unified models of dark energy and dark matter

**III. Modified gravity models**
- A. $f(R)$ gravity
  1. Viable $f(R)$ dark energy models
  2. Observational signatures of $f(R)$ dark energy models
  3. Local gravity constraints
  4. Palatini $f(R)$ gravity
- B. Gauss-Bonnet dark energy models
- C. Scalar-tensor theories
- D. DGP model

**IV. Cosmic acceleration without dark energy**
- A. Inhomogeneous Lemaître-Tolman-Bondi model
- B. Backreaction of cosmological perturbations

# f(R) gravity

Reviews:
de Felice & Tsujikawa (2010)
Sotiriou & Faraoni (2010)
Nojiri & Odinsov (2010)

Action:

$$S = \int \left[\frac{1}{2} f(R) + L_m\right] \sqrt{-g} d^4x$$

(We use the Planck unit with $8\pi G \equiv 1 \equiv c$)

Modified Einstein equations:

$$F(R)R_{ik} - \frac{1}{2} g_{ik} f(R) + (g_{ik}\Box - \nabla_i \nabla_k) F(R) = T_{ik}$$

$[F(R) \equiv df / dR]$

Trace: $$FR - 2f + 3\Box F = -\mu_m + 3p_m$$

→ Differential equations for dynamics of modified gravity sector

# Background evolutions

$$\ddot{F} + 3H\dot{F} + \frac{1}{3}(2f - FR) = \frac{1}{3}(\mu_m - 3p_m)$$

or

$$\ddot{R} + \frac{F_{,RR}}{F_{,R}}\dot{R}^2 + 3H\dot{R} + \frac{2f - RF}{3F_{,R}} = \frac{1}{3F_{,R}}(\mu_m - 3p_m)$$

$$\boxed{\begin{aligned} F &\equiv df/dR \\ \dot{F} &= F_{,R}\dot{R} \\ F_{,R} &\equiv dF/dR \end{aligned}}$$

Collective energy density and pressure:

$$\mu_m \equiv \mu_R + \mu_M \qquad p_m \equiv p_R + p_M$$

# Background evolutions

$$\ddot{F} + 3H\dot{F} + \frac{1}{3}(2f - FR) = \frac{1}{3}(\mu_m - 3p_m)$$

or

$$\ddot{R} + \frac{F_{,RR}}{F_{,R}}\dot{R}^2 + 3H\dot{R} + \frac{2f - RF}{3F_{,R}} = \frac{1}{3F_{,R}}(\mu_m - 3p_m)$$

$$\boxed{\begin{aligned} F &\equiv df/dR \\ \dot{F} &= F_{,R}\dot{R} \\ F_{,R} &\equiv dF/dR \end{aligned}}$$

Collective energy density and pressure:

$$\mu_m \equiv \mu_R + \mu_M \qquad p_m \equiv p_R + p_M$$

<u>Hubble parameter:</u>

$$H^2 = \frac{1}{3F}(\mu_m + F\mu_X)$$

<u>Ricci scalar:</u>

$$R = 6(2H^2 + \dot{H}) \qquad \text{(K=0)}$$

<u>Energy density and pressure of modified gravity sector:</u> $\mu_X$ and $P_X$

$$F\mu_X \equiv \frac{1}{2}(FR - f) - 3H\dot{F}$$

$$Fp_X \equiv -\frac{1}{2}(FR - f) + \ddot{F} + 2H\dot{F}$$

<u>Density parameters:</u>

$$\Omega_i = \mu_i/(3H^2)\ (i = R,\ M,\ X) \qquad (\Omega_R + \Omega_M)/F + \Omega_X = 1$$

# Motivations

Popular f(R) gravity models

$$f(R) = R + f_{\rm DE}(R)$$

**Ricci scalar**

**a correction term that drives the late-time acceleration**

$f_{\rm DE} = qR^{-n}$ [simple power-law form]

$- \mu R_c \frac{(R/R_c)^{2n}}{(R/R_c)^{2n} + 1}$ (Hu & Sawicki 2007)

$- \mu R_c \left[1 - \left(1 + R^2/R_c^2\right)^{-n}\right]$ (Starobinsky 2007)

and so on (Appleby & Battye 2007; Tsujikawa 2008).

In all models, the second term $f_{DE}$ becomes extremely subdominant compared with R in the early radiation dominated era (RDE) so that the f(R) gravity effectively goes over into the Einstein gravity.

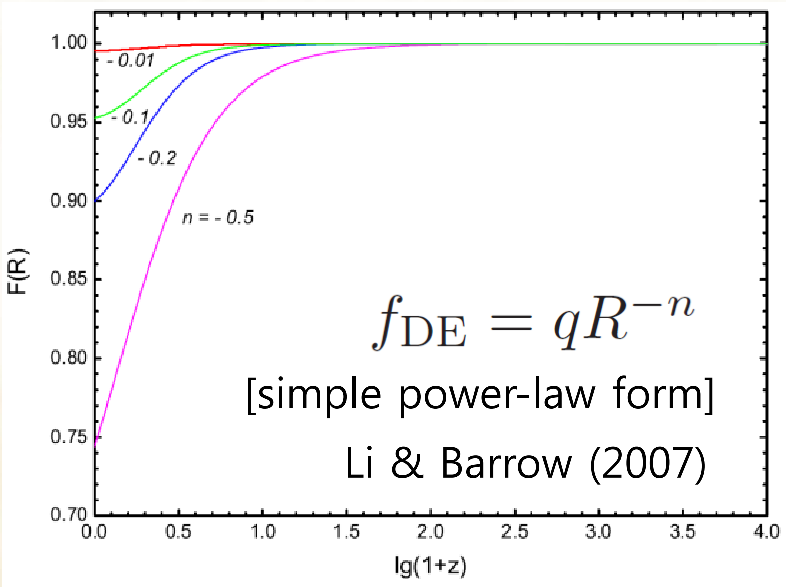
# Motivations

Popular f(R) gravity models

$$f(R) = R + f_{\rm DE}(R)$$

**Ricci scalar**

**a correction term that drives the late-time acceleration**




Since the quantity F≡df/dR becomes extremely close to unity in RDE, evolving a differential equation is sometimes not numerically feasible.

# Our f(R) gravity model

Double power-law f(R) gravity model:

$$f(R) = R^{1+\varepsilon} + qR^{-n} \qquad (\varepsilon > 0,\ \ -1 < n \leq 0)$$

**exact scaling during the radiation and matter dominated eras**

**late time acceleration**

It is known that the first term $R^{1+\varepsilon}$ which is dominant in the early epoch allows the density of gravity sector to follow that of dominant fluid **(scaling evolution)**. (Amendola et al. 2007; Tsujikawa 2007)

# Our f(R) gravity model

Double power-law f(R) gravity model:

$$f(R) = R^{1+\varepsilon} + qR^{-n} \qquad (\varepsilon > 0,\ \ -1 < n \leq 0)$$

**exact scaling during the radiation and matter dominated eras**

**late time acceleration**

Adopting the modified form with small positive ε together with appropriate initial conditions we can evade the numerical problem.

Considering this scaling evolution, we can investigate the effect of early subdominant modified gravity sector on the evolution of perturbation in baryon and photon densities.

# Our f(R) gravity model

Double power-law f(R) gravity model:

$$f(R) = R^{1+\varepsilon} + qR^{-n} \qquad (\varepsilon > 0,\ \ -1 < n \le 0)$$

**exact scaling during the radiation and matter dominated eras**

**late time acceleration**

Scalar field analogy: $$V(\phi) = V_1 e^{-\lambda_1 \phi} + V_2 e^{-\lambda_2 \phi}$$

(Bassett et al. 2008; Park et al. 2009)

# Deriving initial conditions for scaling evolution

In the early era, let us consider a functional form

$$f(R) = \hat{\alpha} R^{1+\epsilon} \qquad \text{(positive } \varepsilon\text{)}$$

The gravity of pure power-law form allows scaling evolution in which the density of X-component follows that of the dominant fluid, and the corresponding potential in the Einstein frame is a pure exponential potential.

# Deriving initial conditions for scaling evolution

In the early era, let us consider a functional form

$$f(R) = \hat{\alpha} R^{1+\epsilon} \qquad \text{(positive } \varepsilon\text{)}$$

The gravity of pure power-law form allows scaling evolution in which the density of X-component follows that of the dominant fluid, and the corresponding potential in the Einstein frame is a pure exponential potential.

Here we put an ansatz that $F\mu_X$ evolves as the dominant ideal fluid with constant equation of state ($w = p_w/\mu_w = \delta p_w/\delta\mu_w$) as

$$\boxed{F\mu_X = \frac{1}{2}(FR - f) - 3H\dot{F} \equiv A\mu_w}$$

We can derive

$$3FR - f = 2\mu_w[(1-3w) + (2-3w)A],$$

$$\frac{F'}{F} = \frac{(1-3w)[(1+A)f - FR] + A(1+3w)FR}{(1+A)(f-3FR)} \qquad F' \equiv dF/d\ln a \quad (K \equiv 0)$$

By specifying the form of $f(R)=\hat{\alpha}R^{1+\epsilon}$, we obtain

$$R = H_0^2 \left[ \frac{6\Omega_{w0}[(1-3w)+(2-3w)A]}{\hat{\alpha}(2+3\epsilon)(a/a_0)^{3(1+w)}} \right]^{\frac{1}{1+\epsilon}}$$

$$R' = -3\frac{1+w}{1+\epsilon}R$$

$$A = \frac{\epsilon(7+10\epsilon)+3\epsilon(1+2\epsilon)w}{2-3\epsilon-8\epsilon^2-3\epsilon(1+2\epsilon)w}$$

$$\Omega_{w0} = \mu_{w0}/(3H_0^2)$$

- density parameter of w-fluid at the present epoch

The scaling behavior is possible for general constant w value of the dominant fluid.

The density of X-component follows (scales) the dominant fluid component even for changing w value; for example, from radiation dominated era (w=1/3) to the matter dominated era (w=0).

# A system of equations for scalar-type perturbations

## Metric

$$ds^2 = -(1+2\alpha)dt^2 - 2a\beta_{,\alpha}dtdx^\alpha + a^2[g^{(3)}_{\alpha\beta}(1+2\varphi) + 2\gamma_{,\alpha|\beta}]dx^\alpha dx^\beta$$

## Energy-momentum tensor

$$T_{(eff)}{}^0_0 = -(\mu + \delta\mu)$$

$$T_{(eff)}{}^0_\alpha = -\frac{1}{k}(\mu + p)v_{,\alpha}$$

$$T_{(eff)}{}^\alpha_\beta = (p + \delta p)\delta^\alpha_\beta + \Pi^\alpha_\beta$$

## Einstein equations

In gauge-ready form (Bardeen 1988, Hwang 1991)

$$\chi \equiv a(\beta + a\dot{\gamma}), \qquad \kappa \equiv 3(-\dot{\varphi} + H\alpha) + \frac{k^2}{a^2}\chi$$

$$-\frac{k^2 - 3K}{a^2}\varphi + H\kappa = -\frac{1}{2}\delta\mu$$

$$\kappa - \frac{k^2 - 3K}{a^2}\chi = \frac{3}{2}(\mu + p)\frac{av}{k}$$

$$\dot{\chi} + H\chi - \alpha - \varphi = \Pi$$

$$\dot{\kappa} + 2H\kappa + \left(3\dot{H} - \frac{k^2}{a^2}\right)\alpha = \frac{1}{2}(\delta\mu + 3\delta p)$$

Perturbed density, pressure, velocity, and anisotropic stress:

$$\delta\mu = \frac{1}{F}\delta\mu_m + \delta\mu_X - \mu_m\frac{\delta F}{F^2},$$
$$\delta p = \frac{1}{F}\delta p_m + \delta p_X - p_m\frac{\delta F}{F^2},$$
$$(\mu + p)v = \frac{1}{F}(\mu_m + p_m)v_m + (\mu_X + p_X)v_X,$$
$$\Pi = \frac{1}{F}\Pi_m + \Pi_X,$$

Contribution of modified gravity sector (X) to perturbed quantities:

$$\delta\mu_X = \frac{1}{F}\left[-3H\delta\dot{F} + \left(\frac{1}{2}f + 3H\dot{F} - F\frac{k^2}{a^2}\right) + \dot{F}\kappa + 3H\dot{F}\alpha\right]$$

$$\delta p_X = \frac{1}{F}\left[\delta\ddot{F} + 2H\delta\dot{F} - \left(\frac{1}{2}f + \ddot{F} + 2H\dot{F} - \frac{2}{3}F\frac{k^2}{a^2}\right)\frac{\delta F}{F} - \frac{2}{3}\dot{F}\kappa - \dot{F}\dot{\alpha} - 2(\ddot{F} + H\dot{F})\alpha\right]$$

$$(\mu_X + p_X)v_X = -\frac{k}{aF}(-\delta\dot{F} + H\delta F + \dot{F}\alpha)$$

$$\Pi_X = \frac{1}{F}(\delta F - \dot{F}\chi)$$

## Energy-momentum conservation equations (i=γ,ν,b,c)

$$\delta\dot{\mu}_i + 3H(\delta\mu_i + \delta p_i) = (\mu_i + p_i)\left(\kappa - 3H\alpha - \frac{k}{a}v_i\right)$$

$$\frac{[a^4(\mu_i + p_i)v_i]^\bullet}{a^4(\mu_i + p_i)} = \frac{k}{a}\left(\alpha + \frac{\delta p_i}{\mu_i + p_i} - \frac{2}{3}\frac{k^2 - 3K}{a^2}\frac{\Pi_i}{\mu_i + p_i}\right)$$

For dynamics of modified gravity sector:

$$\boxed{\begin{aligned}&\delta\ddot{F} + 3H\delta\dot{F} + \left(\frac{k^2}{a^2} - \frac{R}{3}\right)\delta F\\ &\quad = \dot{F}\dot{\alpha} + (2\ddot{F} + 3H\dot{F})\alpha + \dot{F}\kappa - \frac{1}{3}F\delta R + \frac{1}{3}(\delta\mu_m - 3\delta p_m)\end{aligned}}$$

$$\delta R = \delta\mu - 3\delta p = 2\left[-2\dot{\kappa} - 4H\kappa + \left(\frac{k^2}{a^2} - 3\dot{H}\right)\alpha + 2\frac{k^2 - 3K}{a^2}\varphi\right]$$

※ Radiation is handled by using the Boltzmann hierarchy equations.

(Hwang & Noh 2001)

# Perturbation equations

We choose the CDM-comoving gauge (CCG) as the temporal gauge (hypersurface) condition.

$$v_{\mathrm{CDM}} \equiv 0 \quad \Rightarrow \alpha = 0$$

(equivalent to synchronous gauge without gauge modes)

# Perturbation equations

We choose the CDM-comoving gauge (CCG) as the temporal gauge (hypersurface) condition.

$$v_{\rm CDM} \equiv 0 \quad \Rightarrow \alpha = 0$$

(equivalent to synchronous gauge without gauge modes)

$$\left(\frac{\delta R}{R}\right)'' + \left(3 + \frac{\dot{H}}{H^2} + 2\frac{F_{,RR}}{F_{,R}}R' + 2\frac{R'}{R}\right)\left(\frac{\delta R}{R}\right)' + \left\{\frac{F}{3H^2F_{,R}} - \frac{R}{3H^2} + \frac{F_{,RR}}{F_{,R}}\left[R'' + \left(3 + \frac{\dot{H}}{H^2}\right)R'\right]\right.$$
$$\left. + \frac{F_{,RRR}}{F_{,R}}R'^2 + \frac{R''}{R} + \frac{R'}{R}\left(3 + \frac{\dot{H}}{H^2} + 2\frac{F_{,RR}}{F_{,R}}R'\right) + \frac{k^2}{a^2H^2}\right\}\left(\frac{\delta R}{R}\right) = \frac{R'}{R}\left(\frac{\kappa}{H}\right) + \frac{1}{3H^2F_{,R}R}\left(\delta\mu_m - 3\delta p_m\right),$$

$$\left(\frac{\kappa}{H}\right)' + \left(2 + \frac{\dot{H}}{H^2} - \frac{F_{,R}}{F}R'\right)\left(\frac{\kappa}{H}\right) = -3\frac{F_{,R}}{F}R\left[\left(\frac{\delta R}{R}\right)'\right.$$
$$\left. + \left(-\frac{f}{6H^2F} + \frac{\mu_m}{3H^2F} + \frac{F}{6H^2F_{,R}} - \frac{F_{,R}}{F}R' + \frac{F_{,RR}}{F_{,R}}R' + \frac{R'}{R} + \frac{1}{3}\frac{k^2}{a^2H^2}\right)\left(\frac{\delta R}{R}\right)\right] + \frac{1}{H^2F}\delta\mu_m,$$

$$\delta_c' = \frac{\kappa}{H},$$
$$\delta_w' = (1+w)\left(\frac{\kappa}{H} - \frac{k}{aH}v_w\right),$$
$$v_w' + (1-3w)\,v_w = \frac{w}{1+w}\frac{k}{aH}\delta_w$$

$$w \equiv p_w/\mu_w, \quad \delta_w \equiv \delta\mu_w/\mu_w$$
$$F' \equiv dF/d\ln a$$

k : the comoving wave number

# Large-scale limit during early scaling era

Background-related coefficients:

$$\frac{\dot{H}}{H} = -\frac{3}{2}\left(\frac{1+w}{1+\epsilon}\right), \quad \frac{R'}{R} = -3\left(\frac{1+w}{1+\epsilon}\right),$$

$$\frac{R''}{R} = 9\left(\frac{1+w}{1+\epsilon}\right)^2, \quad \frac{F_{,RR}}{F_{,R}}R' = \frac{3(1-\epsilon)(1+w)}{1+\epsilon},$$

$$\frac{F}{3F_{,R}H^2} = \frac{1+4\epsilon-3w}{\epsilon(1+\epsilon)}, \quad \frac{R}{3H^2} = \frac{1+4\epsilon-3w}{1+\epsilon},$$

$$\frac{F_{,RRR}}{F_{,R}}(R')^2 = \frac{9(\epsilon-1)(\epsilon-2)(1+w)^2}{(1+\epsilon)^2},$$

$$\frac{1}{F_{,R}R}\left(\frac{\delta\mu_m}{3H^2} - \frac{\delta p_m}{H^2}\right)$$

$$= \frac{2-3\epsilon-8\epsilon^2-3\epsilon(1+2\epsilon)w}{2\epsilon(1+\epsilon)^2}(1-3w)\delta_w,$$

$$\frac{F_{,R}}{F}R' = -3\frac{\epsilon(1+w)}{1+\epsilon}, \quad \frac{F_{,R}}{F}R = \epsilon,$$

$$\frac{f}{6FH^2} = \frac{1+4\epsilon-3w}{2(1+\epsilon)^2},$$

$$\frac{\mu_m}{3FH^2} = \frac{2-3\epsilon-8\epsilon^2-3\epsilon(1+2\epsilon)w}{2(1+\epsilon)^2}$$

# Large-scale limit during early scaling era

Background-related coefficients:

$$\frac{\dot{H}}{H} = -\frac{3}{2}\left(\frac{1+w}{1+\epsilon}\right), \quad \frac{R'}{R} = -3\left(\frac{1+w}{1+\epsilon}\right),$$

$$\frac{R''}{R} = 9\left(\frac{1+w}{1+\epsilon}\right)^2, \quad \frac{F_{,RR}}{F_{,R}}R' = \frac{3(1-\epsilon)(1+w)}{1+\epsilon},$$

$$\frac{F}{3F_{,R}H^2} = \frac{1+4\epsilon-3w}{\epsilon(1+\epsilon)}, \quad \frac{R}{3H^2} = \frac{1+4\epsilon-3w}{1+\epsilon},$$

$$\frac{F_{,RRR}}{F_{,R}}(R')^2 = \frac{9(\epsilon-1)(\epsilon-2)(1+w)^2}{(1+\epsilon)^2},$$

$$\frac{1}{F_{,R}R}\left(\frac{\delta\mu_m}{3H^2} - \frac{\delta p_m}{H^2}\right) = \frac{2-3\epsilon-8\epsilon^2-3\epsilon(1+2\epsilon)w}{2\epsilon(1+\epsilon)^2}(1-3w)\delta_w,$$

$$\frac{F_{,R}}{F}R' = -3\frac{\epsilon(1+w)}{1+\epsilon}, \quad \frac{F_{,R}}{F}R = \epsilon,$$

$$\frac{f}{6FH^2} = \frac{1+4\epsilon-3w}{2(1+\epsilon)^2},$$

$$\frac{\mu_m}{3FH^2} = \frac{2-3\epsilon-8\epsilon^2-3\epsilon(1+2\epsilon)w}{2(1+\epsilon)^2}$$

In the large-scale limit $\left(\frac{k}{aH} \to 0\right)$ the perturbation equations become

$$\left(\frac{\delta R}{R}\right)'' + \frac{3[(1-2\epsilon)-(1+4\epsilon)w]}{2(1+\epsilon)}\left(\frac{\delta R}{R}\right)' + \left[-\frac{9\epsilon(1+w)[1-(1+2\epsilon)w]}{2(1+\epsilon)^2} + \frac{(1-\epsilon)(1+4\epsilon-3w)}{\epsilon(1+\epsilon)}\right]\left(\frac{\delta R}{R}\right) = -\frac{3(1+w)}{1+\epsilon}\left(\frac{\kappa}{H}\right) + \frac{2-3\epsilon-8\epsilon^2-3\epsilon(1+2\epsilon)w}{2\epsilon(1+\epsilon)^2}(1-3w)\delta_w$$

$$\left(\frac{\kappa}{H}\right)' + \frac{1+10\epsilon+3(2\epsilon-1)w}{2(1+\epsilon)}\left(\frac{\kappa}{H}\right) = -3\epsilon\left(\frac{\delta R}{R}\right)' - \frac{3[1+5\epsilon-8\epsilon^2-3(1-\epsilon+2\epsilon^2)w]}{2(1+\epsilon)}\left(\frac{\delta R}{R}\right) + \frac{3[2-3\epsilon-8\epsilon^2-3\epsilon(1+2\epsilon)w]}{2(1+\epsilon)^2}\delta_w$$

$$\delta_w' = (1+w)\left(\frac{\kappa}{H}\right)$$

$$v_w' = -(1-3w)v_w$$

# Initial conditions of perturbed variables during scaling era

Assuming the form of solution as $\delta_w \propto \frac{\delta R}{R} \propto e^{n \ln a}$

we obtain quartic equation for n:

$$\left(n^2 + \frac{3[(1-2\epsilon)-(1+4\epsilon)w]}{2(1+\epsilon)}n - \frac{9\epsilon(1+w)[1-(1+2\epsilon)w]}{2(1+\epsilon)^2} + \frac{(1-\epsilon)(1+4\epsilon-3w)}{\epsilon(1+\epsilon)}\right)$$

$$\times\left(\frac{1}{1+w}n^2 + \frac{1+10\epsilon+3(2\epsilon-1)w}{2(1+\epsilon)(1+w)}n - \frac{3[2-3\epsilon-8\epsilon^2-3\epsilon(1+2\epsilon)w]}{2(1+\epsilon)^2}\right)$$

$$= \left(-\frac{3}{1+\epsilon}n + \frac{[2-3\epsilon-8\epsilon^2-3\epsilon(1+2\epsilon)w](1-3w)}{2\epsilon(1+\epsilon)^2}\right)\left(-3\epsilon n - \frac{3[1+5\epsilon-8\epsilon^2-3(1-\epsilon+2\epsilon^2)w]}{2(1+\epsilon)}\right)$$

# Initial conditions of perturbed variables during scaling era

Assuming the form of solution as $\delta_w \propto \frac{\delta R}{R} \propto e^{n \ln a}$

we obtain quartic equation for n:

$$\left(n^2 + \frac{3[(1-2\epsilon)-(1+4\epsilon)w]}{2(1+\epsilon)}n - \frac{9\epsilon(1+w)[1-(1+2\epsilon)w]}{2(1+\epsilon)^2} + \frac{(1-\epsilon)(1+4\epsilon-3w)}{\epsilon(1+\epsilon)}\right)$$
$$\times\left(\frac{1}{1+w}n^2 + \frac{1+10\epsilon+3(2\epsilon-1)w}{2(1+\epsilon)(1+w)}n - \frac{3[2-3\epsilon-8\epsilon^2-3\epsilon(1+2\epsilon)w]}{2(1+\epsilon)^2}\right)$$
$$= \left(-\frac{3}{1+\epsilon}n + \frac{[2-3\epsilon-8\epsilon^2-3\epsilon(1+2\epsilon)w](1-3w)}{2\epsilon(1+\epsilon)^2}\right)\left(-3\epsilon n - \frac{3[1+5\epsilon-8\epsilon^2-3(1-\epsilon+2\epsilon^2)w]}{2(1+\epsilon)}\right)$$

Solutions obtained by MAPLE:

$$\boxed{n = \frac{1-2\epsilon+3w}{1+\epsilon}}, \quad -\frac{3}{2}\left(\frac{1+w}{1+\epsilon}\right),$$
$$\frac{1}{4\epsilon(1+\epsilon)}\Big\{-3\epsilon+3\epsilon(1+2\epsilon)w \pm \Big[-16\epsilon - 31\epsilon^2+160\epsilon^3+256\epsilon^4+3\epsilon(16-22\epsilon - 28\epsilon^2+64\epsilon^3)w - 9\epsilon^2(7+12\epsilon-4\epsilon^2)w^2\Big]^{\frac{1}{2}}\Big\}$$

growing
mode

Finally we obtain the initial conditions of perturbed variables in the CDM-comoving gauge condition,

$$\delta_w = Ce^{\frac{1-2\epsilon+3w}{1+\epsilon}\ln a}, \quad \left(\frac{\kappa}{H}\right) = \frac{1-2\epsilon+3w}{(1+\epsilon)(1+w)}\delta_w,$$

$$\frac{\delta R}{R} = \frac{1-4\epsilon-3(1+2\epsilon)w}{1+7\epsilon-12\epsilon^2-3(1-\epsilon)(1-2\epsilon)w}\delta_w,$$

$$\left(\frac{\delta R}{R}\right)' = \frac{1-2\epsilon+3w}{1+\epsilon}\left(\frac{\delta R}{R}\right),$$

where C is the initial amplitude.

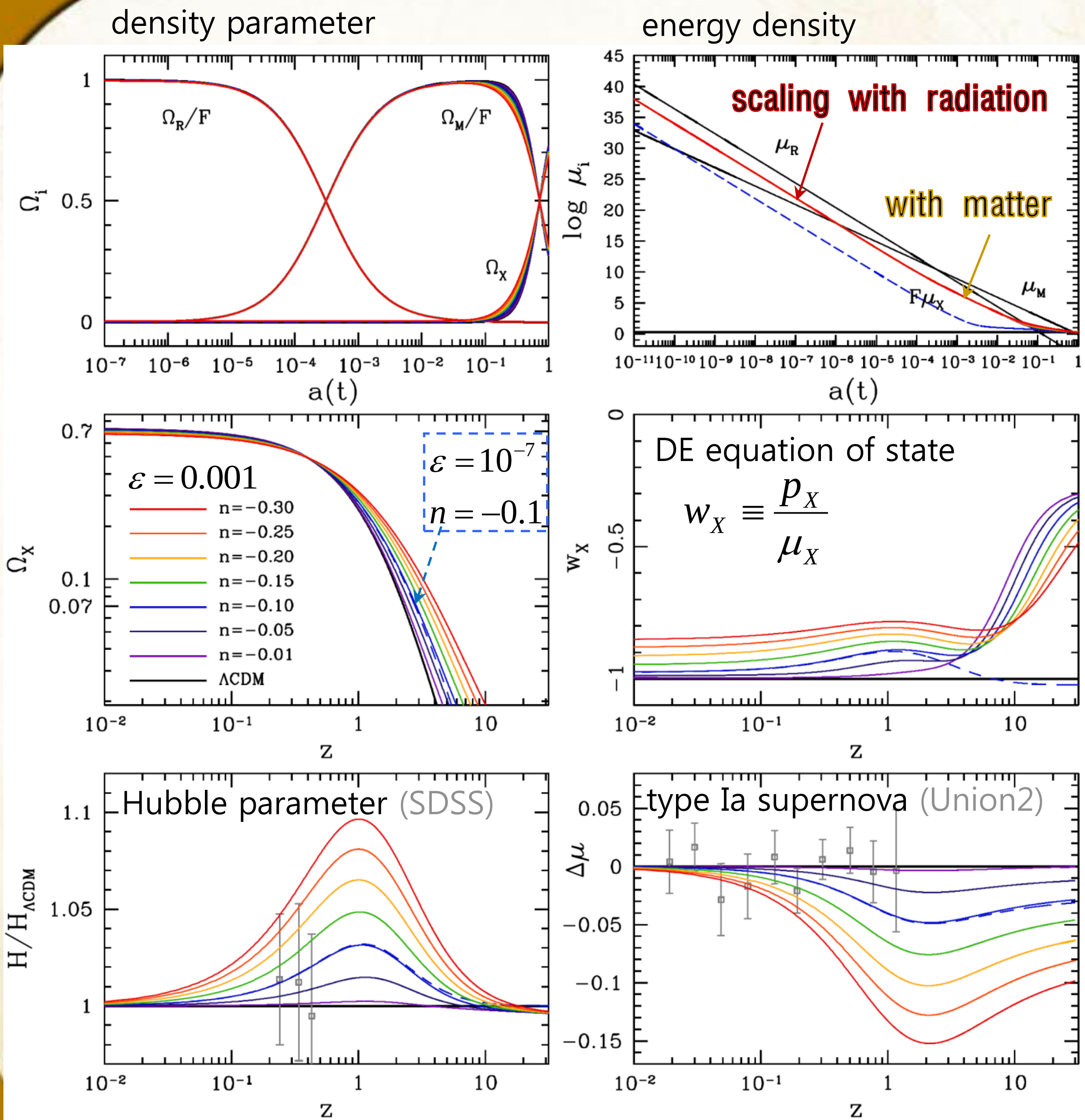
Similar solutions have been already found by Carloni et al. (2008, PRD) in a different gauge condition.

Background evolution of our gravity models

$$f(R) = R^{1+\varepsilon} + qR^{-n}$$

For fiducial ΛCDM model, we use cosmological parameters consistent with WMAP 7-year observations (WMAP7+BAO+$H_0$).

$$\Omega_{M0} = 0.274 \quad \Omega_{\Lambda 0} = 0.7278$$
$$(\Omega_{c0} = 0.2284, \quad \Omega_{b0} = 0.0456)$$
$$h = 0.704 \quad n_s = 0.963$$
$$T_0 = 2.725 \text{ K}, \quad Y_{\mathrm{He}} = 0.24$$
$$N_\nu = 3.04, \quad \tau = 0.087$$




Background evolution of our gravity models

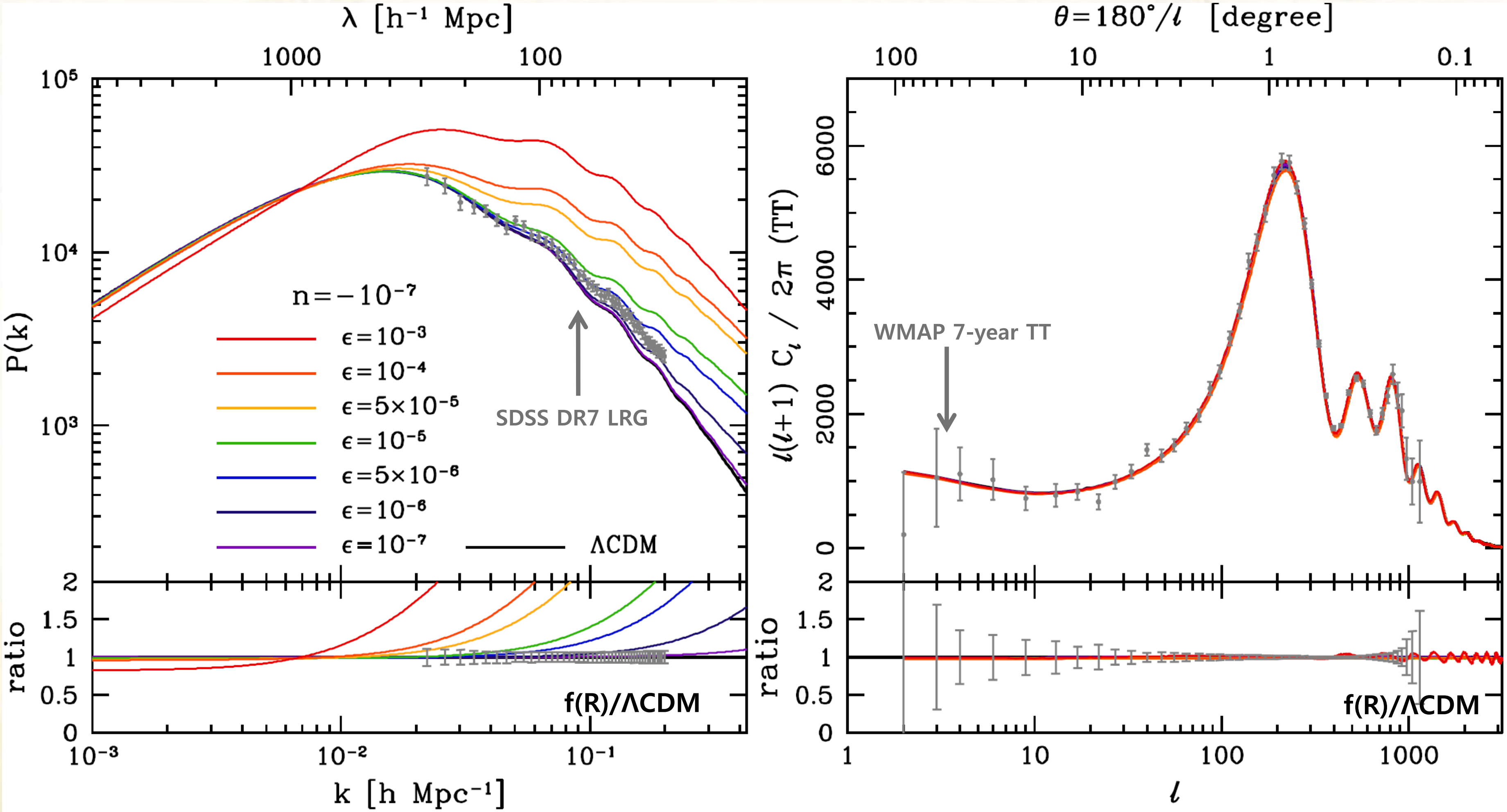
$$f(R) = R^{1+\varepsilon} + qR^{-n}$$

In all models, we set

$$\Omega_{X0} \equiv \Omega_{\Lambda 0}$$

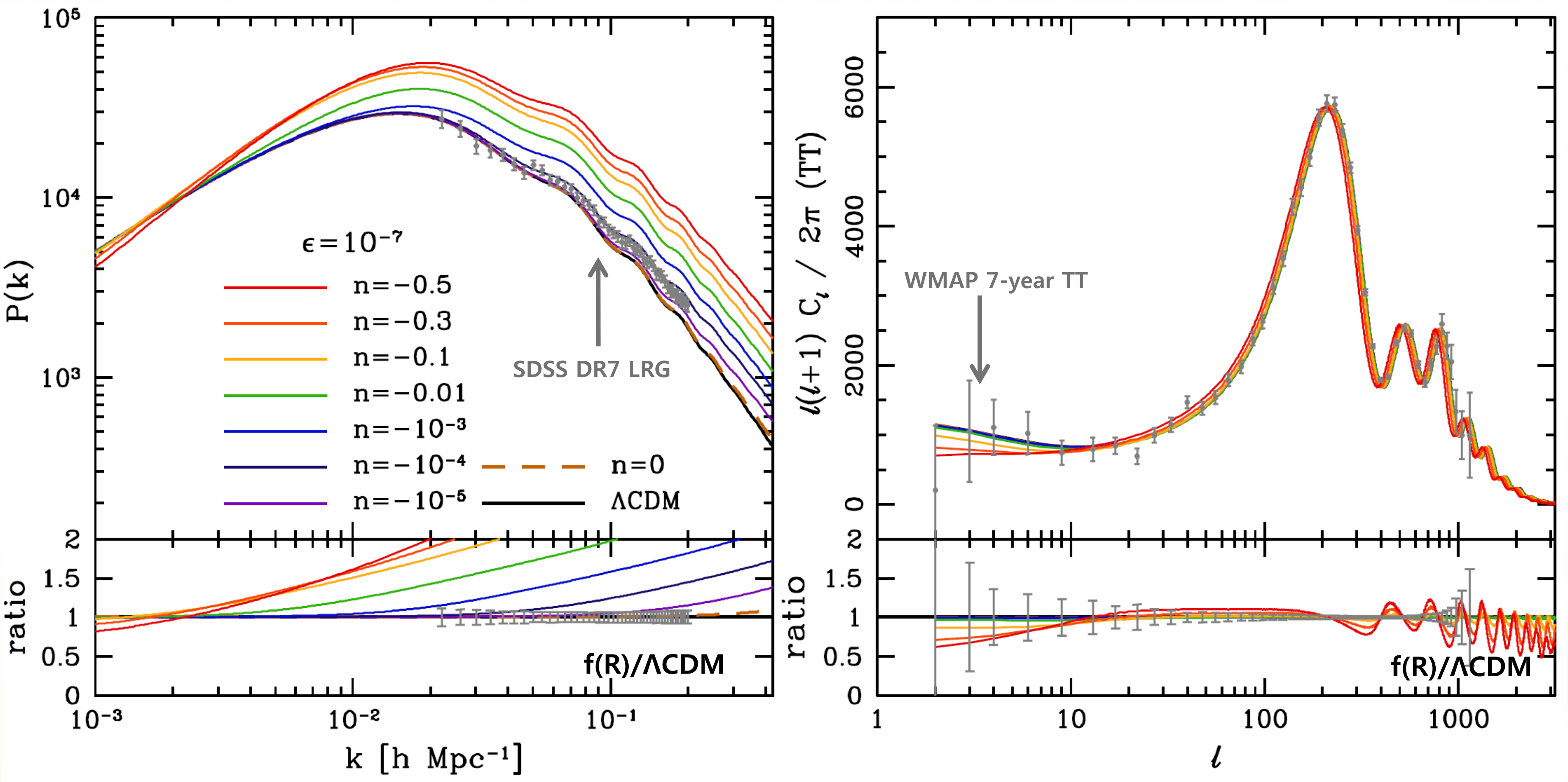
(by adjusting q for given set of initial conditions of R, R')

# Power spectra of f(R) gravity models for varying ε (with $n=-10^{-7}$)



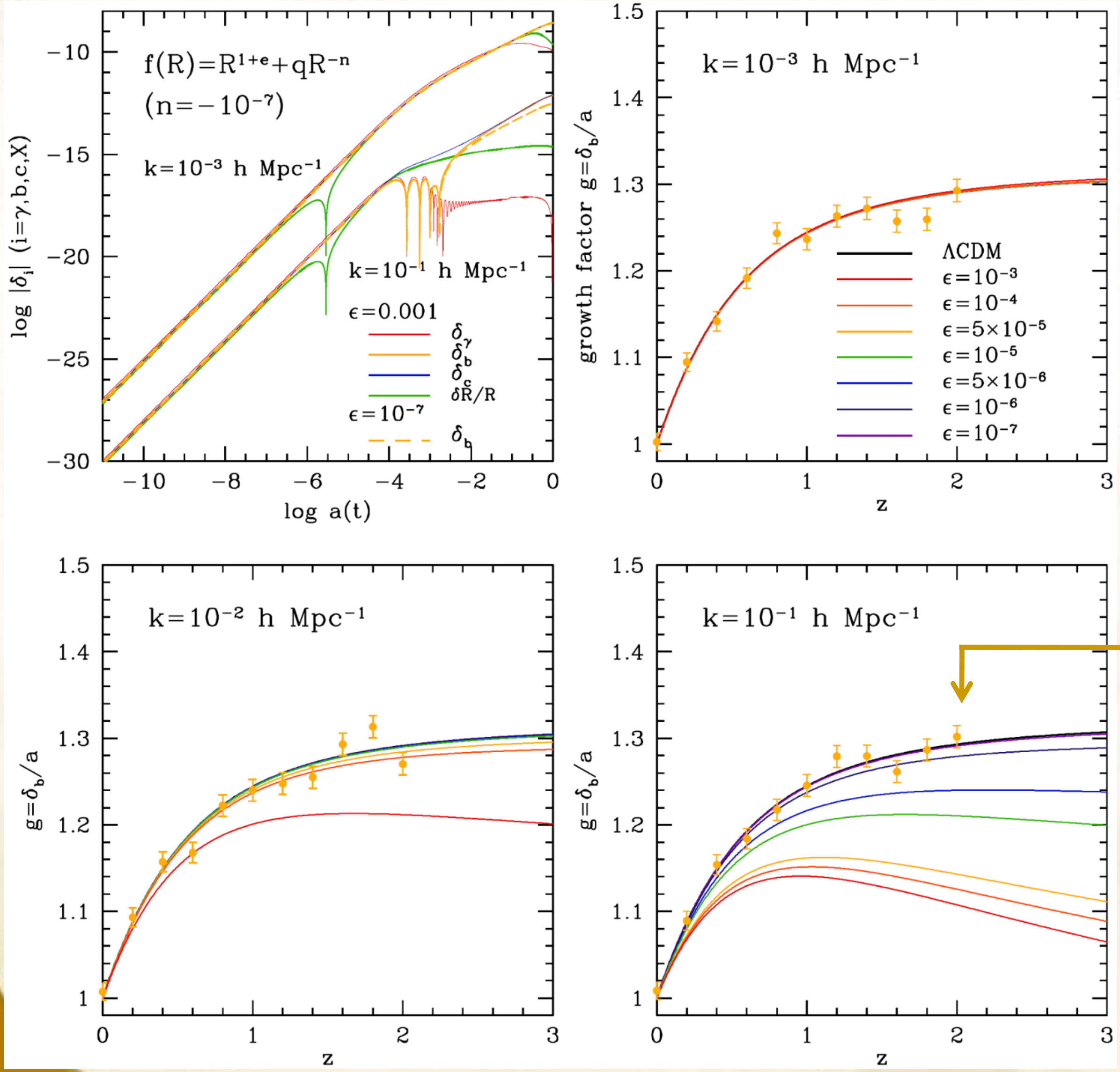

Unlike baryonic matter power spectrum (PS), the CMB PS is not sensitive to ε.

Power spectra of f(R) gravity models for varying n (with $\varepsilon=10^{-7}$)




The sensitivity of CMB PS to parameter n is weak compared to baryonic matter PS.

# Perturbation growth in f(R) gravity models for varying ε (with $n=-10^{-7}$)




**Perturbation growth factor:**

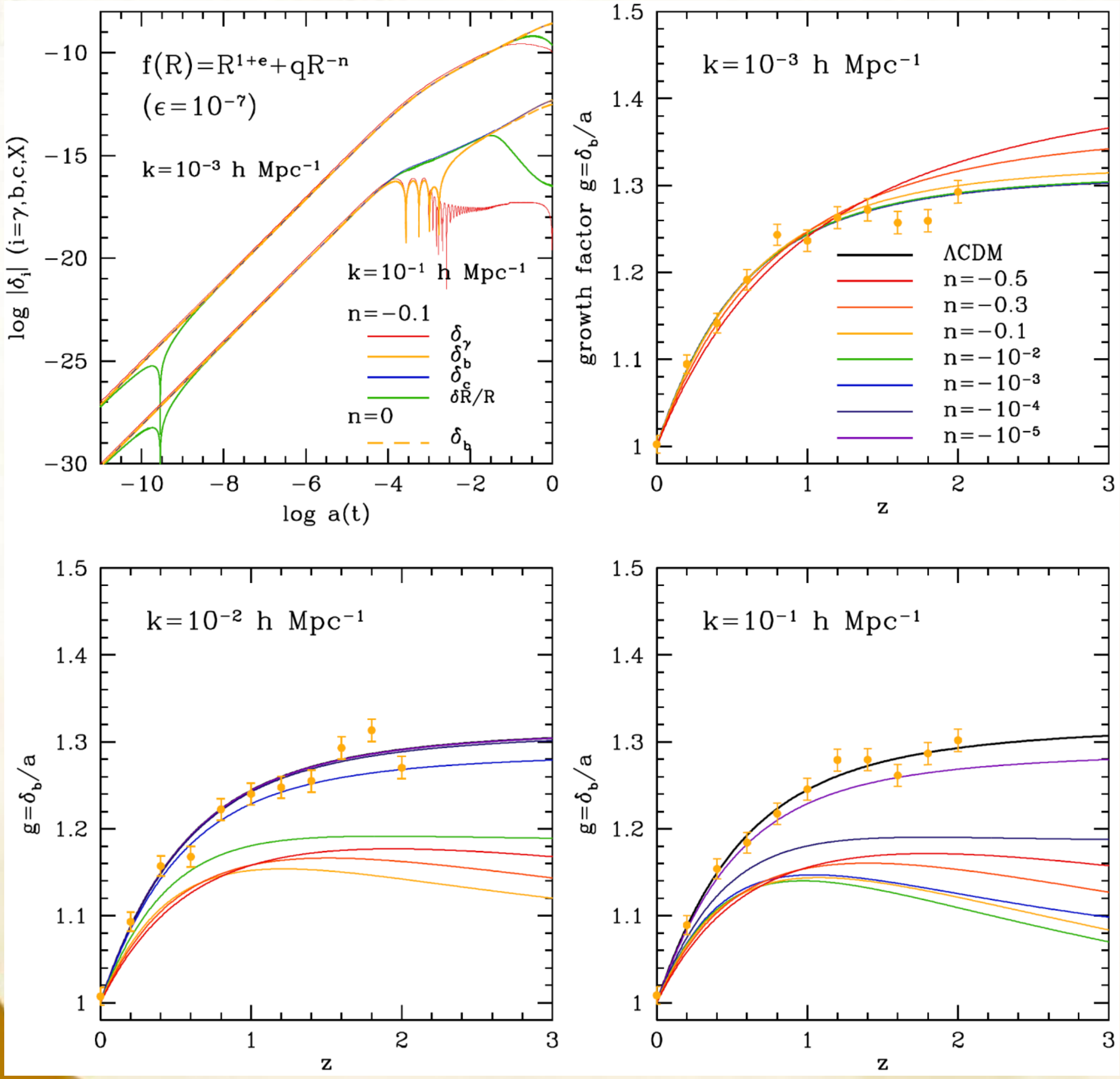
$$g \equiv \delta_b / a$$

(normalized to unity at present)

ΛCDM-motivated mock growth factor expected in the future X-ray and weak-lensing observations (1% precision; 11 data points between z=0-2.

Growth factor deviations from ΛCDM are particularly significant at small scale.

# Perturbation growth in f(R) gravity models for varying n (with ε=$10^{-7}$)




**Perturbation growth factor:**

$$g \equiv \delta_b / a$$

(normalized to unity at present)

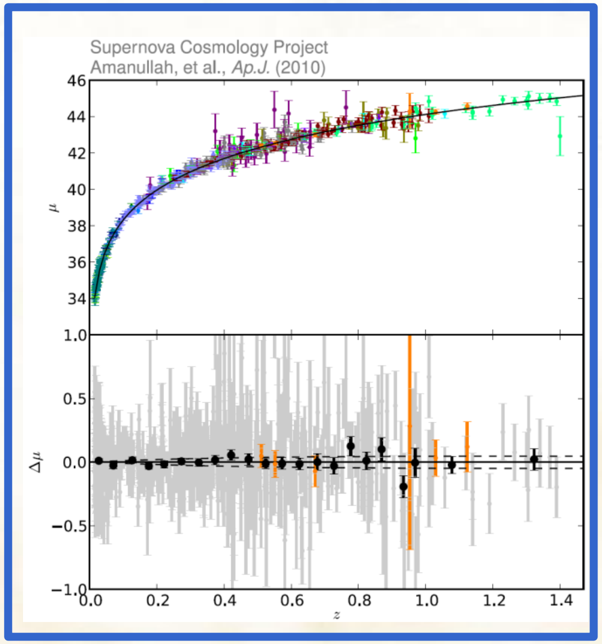
Growth factor deviations from ΛCDM are noticeable at all scales for smaller n.

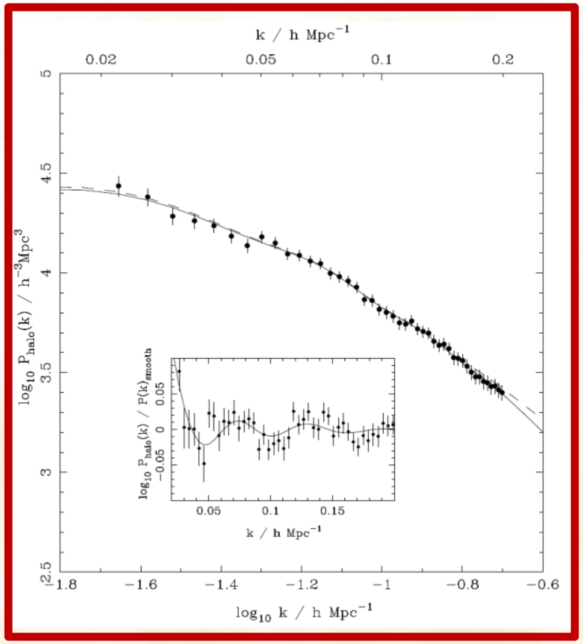
For all cases, the deviation is significant at small scale.

# Likelihood distribution of f(R) gravity parameters

We explore the (ε,n)-parameter space to estimate the likelihood using SNIa, matter PS, and perturbation growth factor data.

(Other cosmological parameters are fixed with WMAP 7-yr best-fit values.)




**Data used:**

SNIa (Union2)

Amanullah et al. 2010

SDSS DR7 LRG PS (k<0.1h/Mpc) (window-convolved)

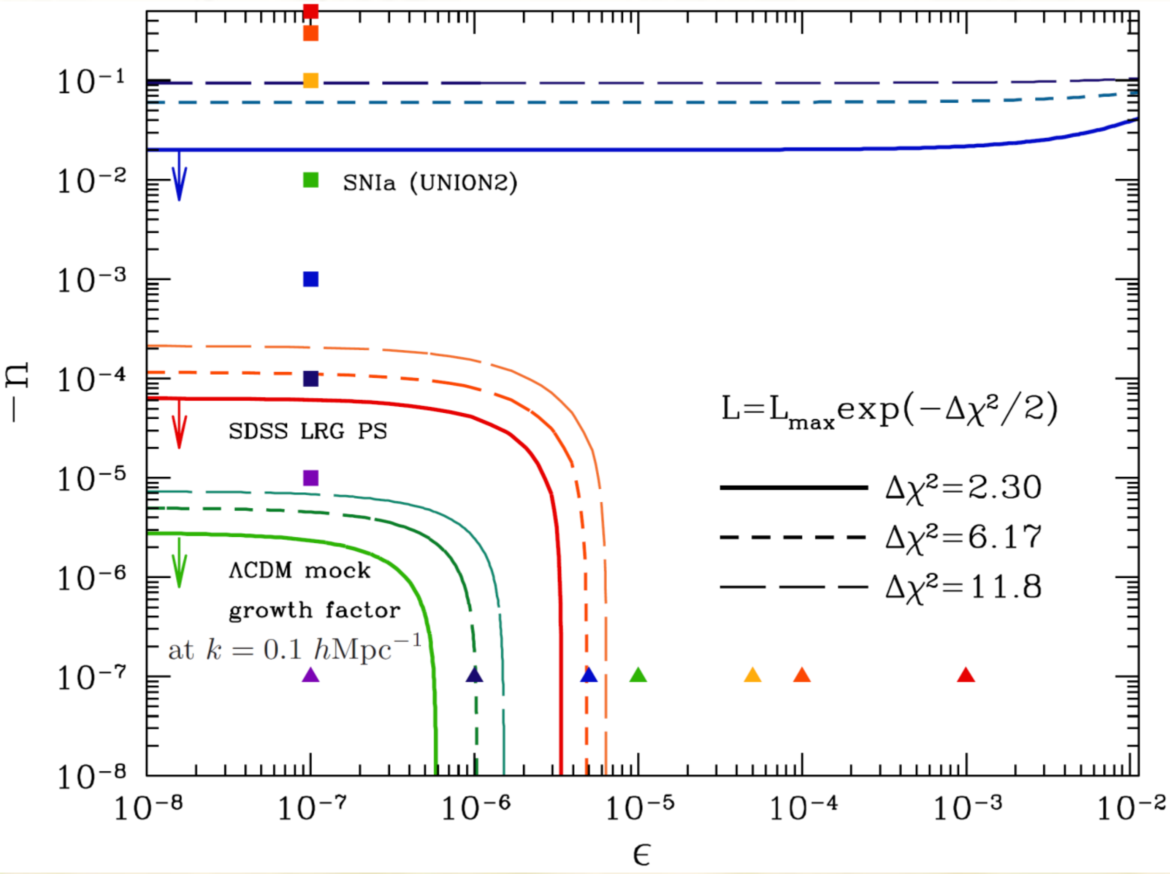
Reid et al. arXiv:0907.1659v2

ΛCDM-motivated mock growth factor at small-scale k=0.1 h/Mpc

# Likelihood distribution of f(R) gravity parameters

We explore the (ε,n)-parameter space to estimate the likelihood using SNIa, matter PS, and perturbation growth factor data.

(Other cosmological parameters are fixed with WMAP 7-yr best-fit values.)




**Data used:**

SNIa (Union2)

Amanullah et al. 2010

SDSS DR7 LRG PS (window-convolved)

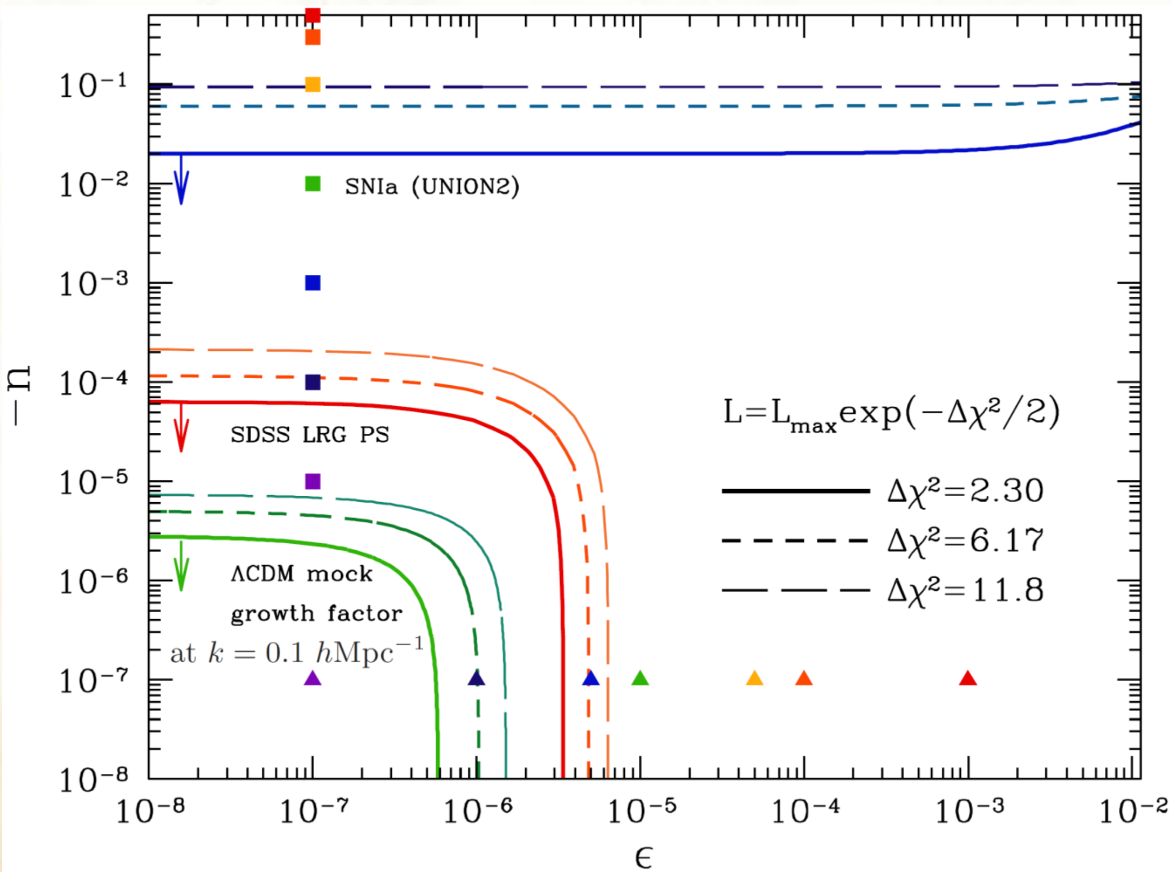
Reid et al. arXiv:0907.1659v2

ΛCDM-motivated mock growth factor at small-scale k=0.1 h/Mpc

# Likelihood distribution of f(R) gravity parameters

We explore the (ε,n)-parameter space to estimate the likelihood using SNIa, matter PS, and perturbation growth factor data.

(Other cosmological parameters are fixed with WMAP 7-yr best-fit values.)




f(R) gravity parameters, ε and n, are very sensitive to the growth factor at small scales, and are already tightly constrained by the current measurement of galaxy power spectrum.

# Summary and Discussion

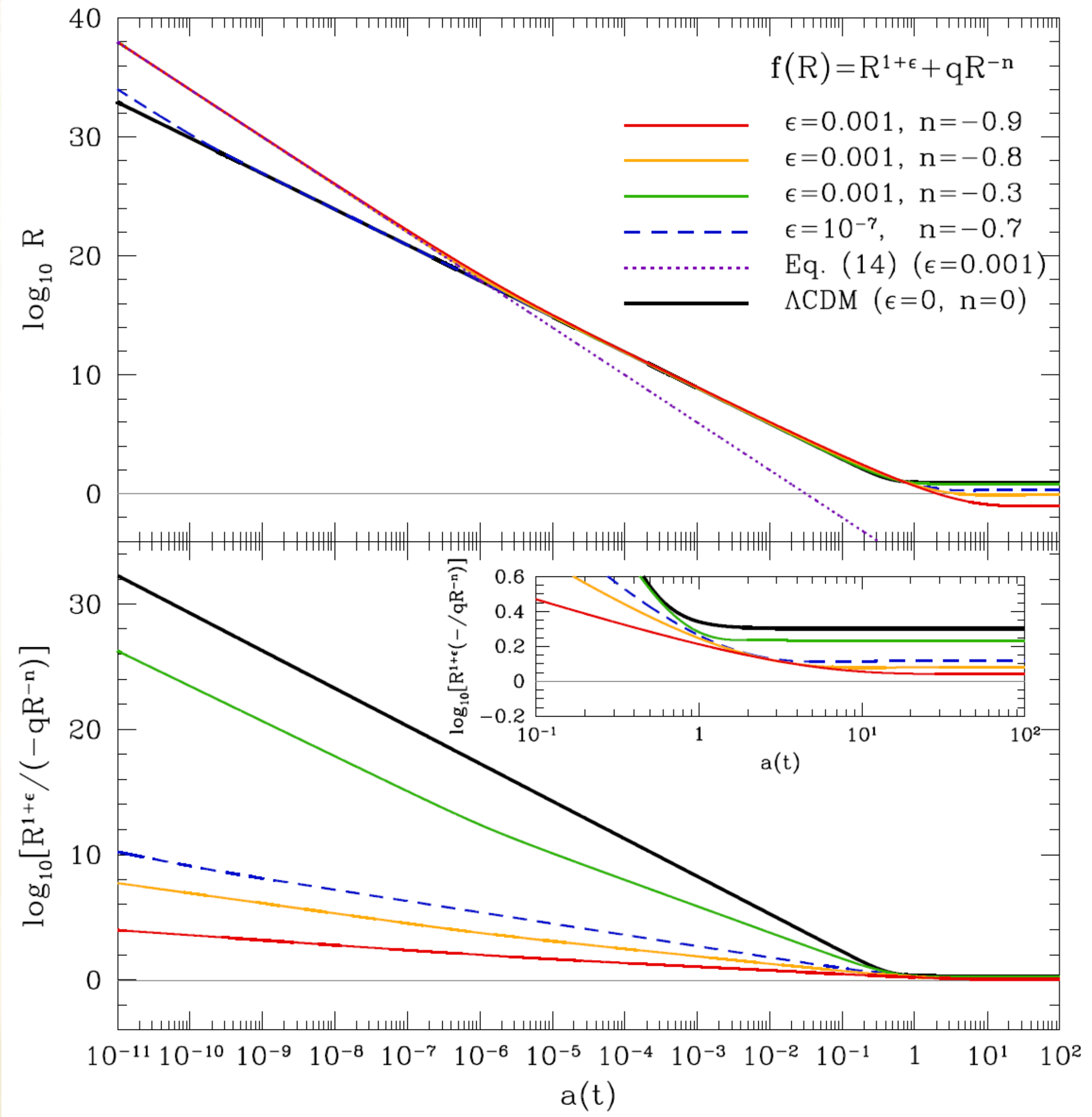
We have studied a f(R)-gravity based dark energy model with early scaling era (mainly to avoid numerical problem in popular models).

We have presented initial conditions of background and perturbed variables during the early scaling evolution regime in the modified gravity with a pure power-law form $f(R) = R^{1+\varepsilon}$ in the early era.

Our f(R) gravity parameters are very sensitive to the baryon perturbation growth and baryon density power spectrum. Our analysis suggests that only the parameter space extremely close to the ΛCDM model is allowed.

The value of ε can be more tightly constrained by the solar system test. For example, according to the criterion given by Lin, Gu, and Chen [arXiv:1009.3488], we expect $\epsilon \lesssim 10^{-17}$ for $R/H_0^2 = 10^5$ .

# Thank You



f(R)=R^{1+ϵ}+qR^{-n}
ϵ=0.001, n=−0.9
ϵ=0.001, n=−0.8
ϵ=0.001, n=−0.3
ϵ=10^{-7}, n=−0.7
Eq. (14) (ϵ=0.001)
ΛCDM (ϵ=0, n=0)
log10 R
log10[R^{1+ϵ}/(−qR^{-n})]
a(t)