Cosmological Perturbation and CMB @YITP

Effect of Kurtosis-type of Primordial Non-Gaussianity on Halo Mass Function

Shuichiro Yokoyama (Nagoya Univ.)

in collaboration with N. Sugiyama(Nagoya U.),
S. Zaroubi(U. of Groningen) and J. Silk (Oxford U.)

arXiv:1103.2586

Primordial non-Gaussianity

f_{NL} g_{NL}

Primordial non-Gaussianity

- A new probe of the physics of the early Universe
  - David Langlois; CMB non-Gaussianities from primordial isocurvature perturbations
  - Masahide Yamaguchi; G-inflation
  - Tomo Takahashi; Scale dependence of non-Gaussianity
  - Osamu Seto; Curvaton with a double well potential
  - Kouji Nakamura; Construction of gauge-invariant variables for linear metric perturbation …
  - Shuntaro Mizuno; Primordial non-Gaussianity from the multi-field DBI Galileon
  - Antonio De Felice; Primordial non-Gaussianities in general modified gravitational models of inflation
  - Gianmassimo Tasinato; Scale dependent local non-Gaussianity
  - Atsushi Naruko; Conservation of the nonlinear curvature perturbation in generic single-field inflation
  - Ivonne Zavala; Multifield Cosmology in Large Volume Scenarios
  - Dominic C. Galliano; Developing trispectrum estimators to measure non-Gaussianity in …
  - Soo A Kim; Non-gaussianity in axion Nflation models
  - Teruaki Suyama; Temporal enhancement of super-horizon curvature perturbation from decays …
  - Guido Walter Pettinari; Quantifying nonlinear contributions to the CMB bispectrum in …
  - Frederico Arroja ;On the role of the boundary terms for the non-Gaussianity in k-inflation
  - Maresuke Shiraishi CMB bispectrum generated from primordial magnetic fields

Primordial non-Gaussianities have a potential to discriminate models

τ_{NL}

e.g., … canonical field or non-canonical ??

single field or multi-field ??

How to parameterize ?

- Local type non-Gaussianities

Komatsu & Spergel (2001), ...

$$\zeta = \zeta_{\rm G} + \frac{3}{5} f_{\rm NL} (\zeta_{\rm G}^2 - \langle \zeta_{\rm G}^2 \rangle) + \frac{9}{25} g_{\rm NL} \zeta_{\rm G}^3 + \cdots$$

non-linear parameters

⇒ Non-zero higher order spectra

(higher order correlation functions)

Leadingly, ...

- Bispectrum (3-point corr. func.) ⇔ $f_{\rm NL}$
- Trispectrum (4-point corr. func.) ⇒ ...

fNL vs tauNL

- Trispectrum ("local-type")

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle = (2\pi)^3 T(k_1,k_2,k_3,k_4)\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)$$

$$T(k_1,k_2,k_3,k_4) = \tau_{\rm NL}(P(k_1)P(k_2)P(k_{13}) + 11 \text{ perms.})$$
$$+\frac{54}{25}g_{\rm NL}(P(k_1)P(k_2)P(k_3)+3\text{ perms.})$$

← 2 parameters

Byrnes, et al, arXiv:0705.4096
SY, T.Suyama and T.Tanaka, arXiv:0810.3053

cubic term → gNL
quadratic term x quadratic term → tauNL

$$\zeta(\mathbf{x}) = \zeta_{\rm G}(\mathbf{x}) + \frac{3}{5}f_{\rm NL}\left(\zeta_{\rm G}^2(\mathbf{x}) - \langle\zeta_{\rm G}^2(\mathbf{x})\rangle\right) + \frac{9}{25}g_{\rm NL}\zeta_{\rm G}^3(\mathbf{x}) + \cdots$$

$$\tau_{\rm NL} = \left(\frac{6}{5}f_{\rm NL}\right)^2$$

Consistency relation

fNL vs tauNL

- "Local-type" inequality

In general, for local-type non-Gaussianity we have

$$\tau_{\rm NL} \geq \left(\frac{6}{5} f_{\rm NL}\right)^2$$

T. Suyama and M. Yamaguchi, arXiv:0709.2545

e.g.

$$\zeta = \phi_G + \psi_G + \frac{3}{5} f_{\rm NL} \left(\phi_G^2 - \langle \phi_G^2 \rangle\right)$$

$$\langle \phi_G \psi_G \rangle = 0$$

$$P_\zeta = (1+R) P_\psi$$

(mixed inflaton and curvaton case)

$$\tau_{\rm NL} = \left(\frac{1+R}{R}\right) \left(\frac{6}{5} f_{\rm NL}\right)^2$$

$$R \equiv P_\phi / P_\psi$$

Note that it is important to consider tauNL independently of fNL !!

Current observational limits

- CMB observations

(temperature bi-,tri-spectra (WMAP 7yr))

$$-10 < f_{\rm NL} < 74 \quad (95\% \ {\rm CL})$$

also,

Komatsu et al.(2010)

$$g_{\rm NL} = (1.6 \pm 7.0) \times 10^5$$

$$t_{\rm NL} = (-1.33 \pm 3.62) \times 10^6$$

$$t_{\rm NL} = 1.5\tau_{\rm NL} + 1.08 g_{\rm NL}$$

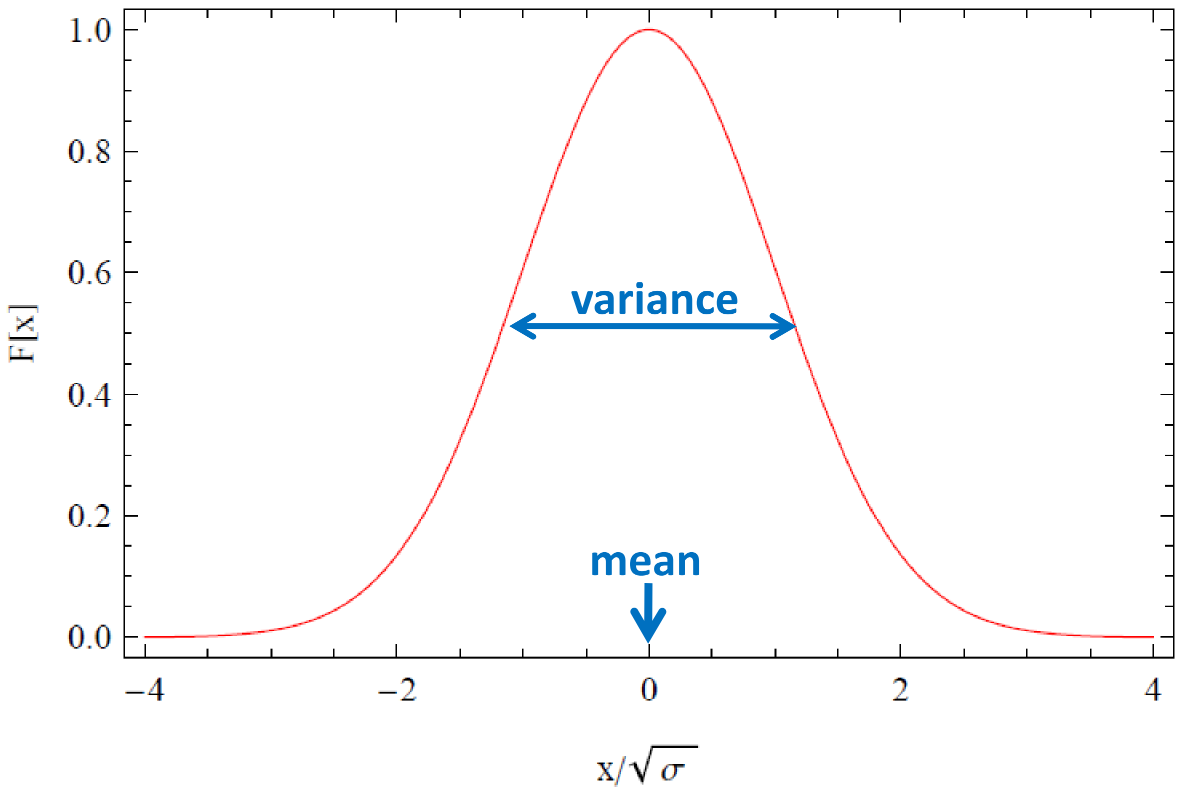
Fergusson Regan and Shellard (2010)
Dominic's talk

Effect on the structure formation

How NG affect the LSS formation?

- Probability Density Function (PDF)

Gaussian fluctuation



characterized by mean and variance

How NG affect the LSS formation?

- Moments for the given distribution function

Gaussian

Fourier space

mean; $\langle\zeta\rangle = \int \zeta f(\zeta) d\zeta$

variance; $\int (\zeta - \langle\zeta\rangle)^2 f(\zeta) d\zeta = \int \frac{d^3k}{(2\pi)^3} P_\zeta(k)$

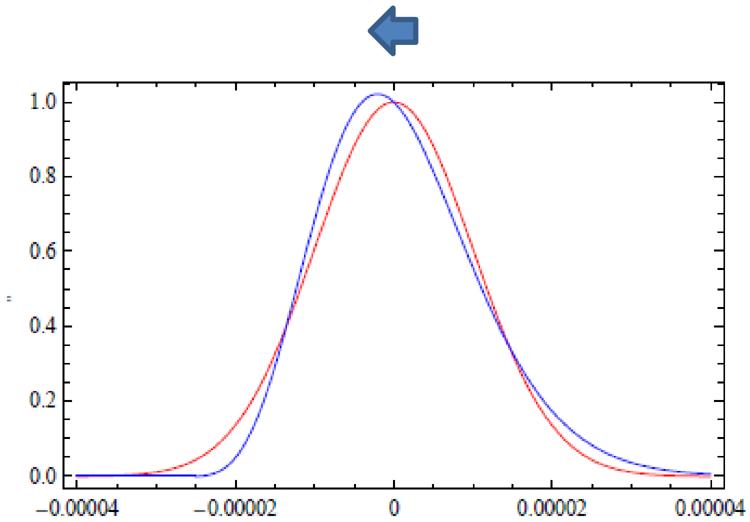
skewness; $\int (\zeta - \langle\zeta\rangle)^3 f(\zeta) d\zeta = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} B_\zeta(k_1, k_2)$ fNL

kurtosis; $\int (\zeta - \langle\zeta\rangle)^4 f(\zeta) d\zeta = \int \frac{d^3k_1 d^3k_2 d^3k_3}{(2\pi)^9} T_\zeta(k_1, k_2, k_3)$ gNL, τNL

These parameters characterize the non-Gaussianities !!

• PDF of ζ

skewness

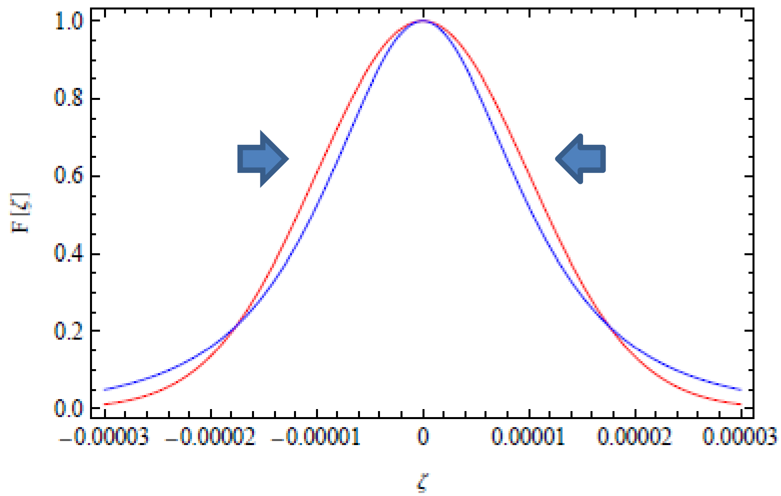


Red; Gaussian

Blue ; non-zero skewness

➔ Peak shift

Kurtosis



Red; Gaussian

Blue ; non-zero kurtosis

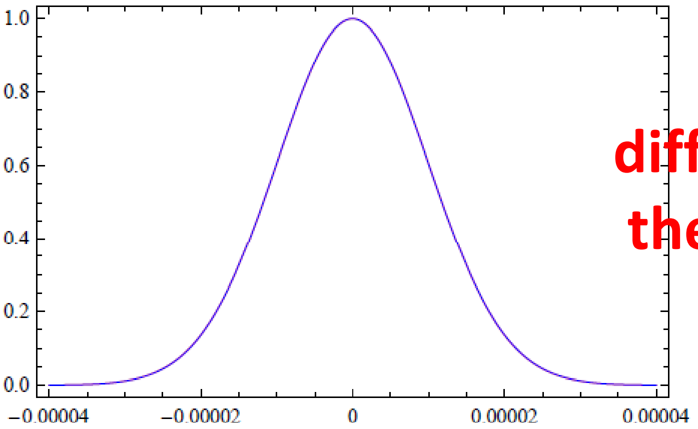
➔ Sharp peak / smooth peak

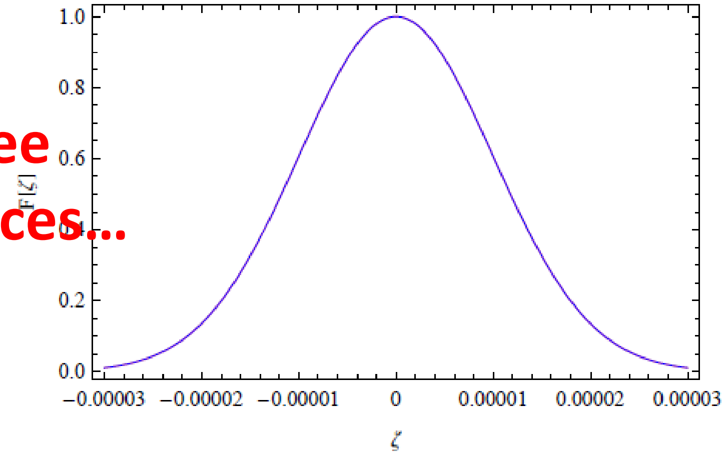
However, … if we consider …

- PDF of ζ

Skewness (fNL = 100)

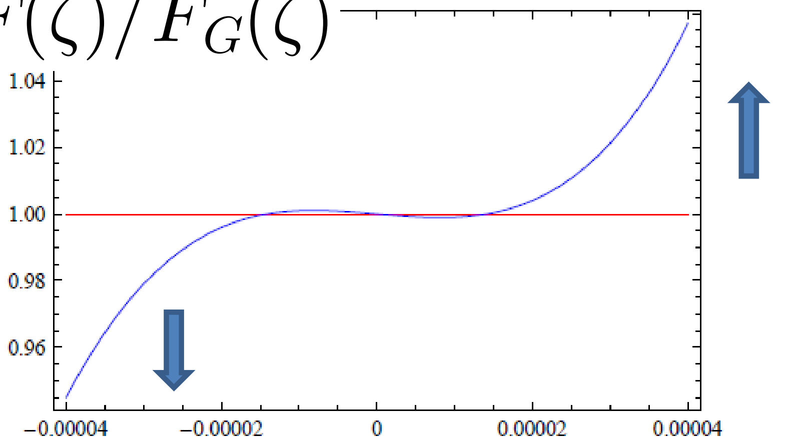
Kurtosis (gNL = 10^6)



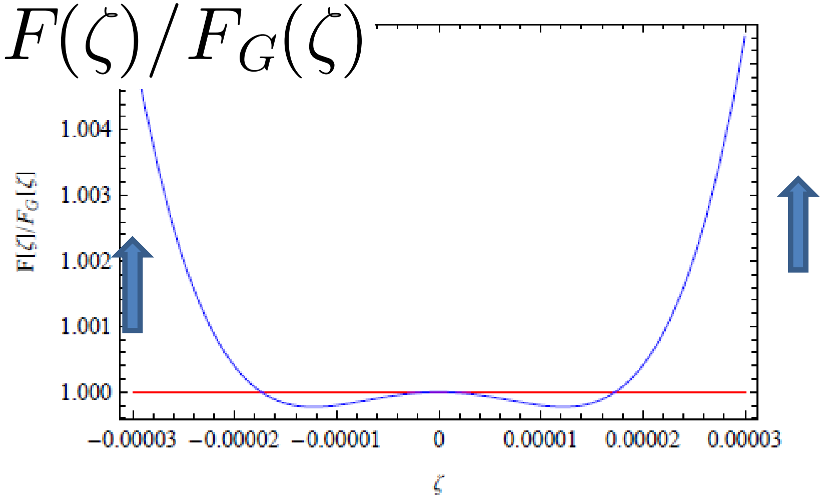


difficult to see the differences...

$F(\zeta)/F_G(\zeta)$



$F(\zeta)/F_G(\zeta)$



large effect on the tails of distribution !!!

How NG affect the LSS formation?

- Primordial non-Gaussianity

  ➔ large effect on the tails of PDF

  primordial curvature fluctuations ➔ density fluctuations

- In the context of LSS formation,…

⬇

Large effect on the rare event!!

e.g., massive clusters, large voids,

high-redshift objects, …

How NG affect the LSS formation?

- The effect of fNL (skewness)

  - halo mass function
    (analytically , N-body simulation)
  - scale-dependent bias
  - matter power spectrum, bispectrum, …

There are a lot of works … **Reviews; Verde (2010), Desjacques and Seljak (2010), …**

We focus on the kurtosis-type especially, non-zero (large) τNL case.

Formulation for the halo mass function

Formula for halo mass function

- number density of collapsed structures (halos) with the mass between M and M + dM

Based on the spirit of Press-Schechter formula,

$$\frac{dn}{dM}(M,z)dM = -dM\frac{2\bar{\rho}}{M}\frac{d}{dM}\int_{\delta_c/\sigma_M}^{\infty} d\nu F(\nu)$$

PDF of the density field $F(\nu)d\nu$ (including non-Gaussian features)

$\nu \equiv \delta_M/\sigma_M$; smoothed density field on a mass scale M

σ_M ; variance of δ_M

$\bar{\rho}$; background energy density of matter

Collapsed structures are formed in the overdensity region (>δ_c)

δ_c ; critical density (= 1.69 for spherical collapse)

Formula for halo mass function

- Non-Gaussian PDF of the density field δ_M

ζ —Poisson eq.→ δ —smoothed on scale M→ δ_M

Primordial curvature perturbations with non-Gaussianity

matter density fluctuations

Including the information of primordial non-Gaussianity

Based on Edgeworth expansion (Hermite polynomials expansion),

$$F(\nu)d\nu = \frac{d\nu}{\sqrt{2\pi}} \exp\left(-\nu^2/2\right) \left[1 + \frac{S_3(M)\sigma_M}{6} H_3(\nu) + \frac{S_4(M)\sigma_{\bar{M}}^2}{24} H_4(\nu) + \right.$$

non-Gaussian corrections

Hermite polynomials;

$$H_3(\nu) = \nu^3 - 3\nu \ ,$$

$$H_4(\nu) = \nu^4 - 6\nu^2 + 3$$

$S_3(M)$; skewness

$S_4(M)$; kurtosis

Non-Gaussian Halo mass function

$$\frac{dn}{dM}(M,z)dM = -dM\frac{2\bar{\rho}}{M}\frac{d}{dM}\int_{\delta_c/\sigma_M}^{\infty} d\nu F(\nu)$$

$$= -dM\sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M}\exp\left[-\frac{\nu_c^2}{2}\right]\left\{\frac{d\ln\sigma_M}{dM}\nu_c\left[1 + \frac{S_3(M)\sigma_M}{6}H_3(\nu_c) + \frac{S_4(M)\sigma_M^2}{24}H_4(\nu_c)\right] + \frac{d}{dM}\left(\frac{S_3(M)\sigma_M}{6}\right)H_2(\nu_c) + \frac{d}{dM}\left(\frac{S_4(M)\sigma_M^2}{24}\right)H_3(\nu_c)\right\} + \cdots$$

non-Gaussian corrections

In general, skewness and kurtosis include the multiple integrations. ..
(skewness → 3, kurtosis → 6) ➔ some simple formulae

For local type non-Gaussianities (in the squeezed limit), we obtain

$$\sigma_M S_3(M) = 4.3\times 10^{-4} f_{\rm NL}\times\sigma_M^{0.13} \quad (10^{12}h^{-1}M_\odot < M < 2\times 10^{15}h^{-1}M_\odot)$$

$$\sigma_M^2 S_4(M) = \underline{1.9\times 10^{-7}\tau_{\rm NL}\times\sigma_M^{0.25}} + 9.4\times 10^{-8} g_{\rm NL}\times\sigma_M^{0.27}$$

new term

De Simone et al.(2010), Enqvist et al(2010), Chongchitnan and Silk(2010)

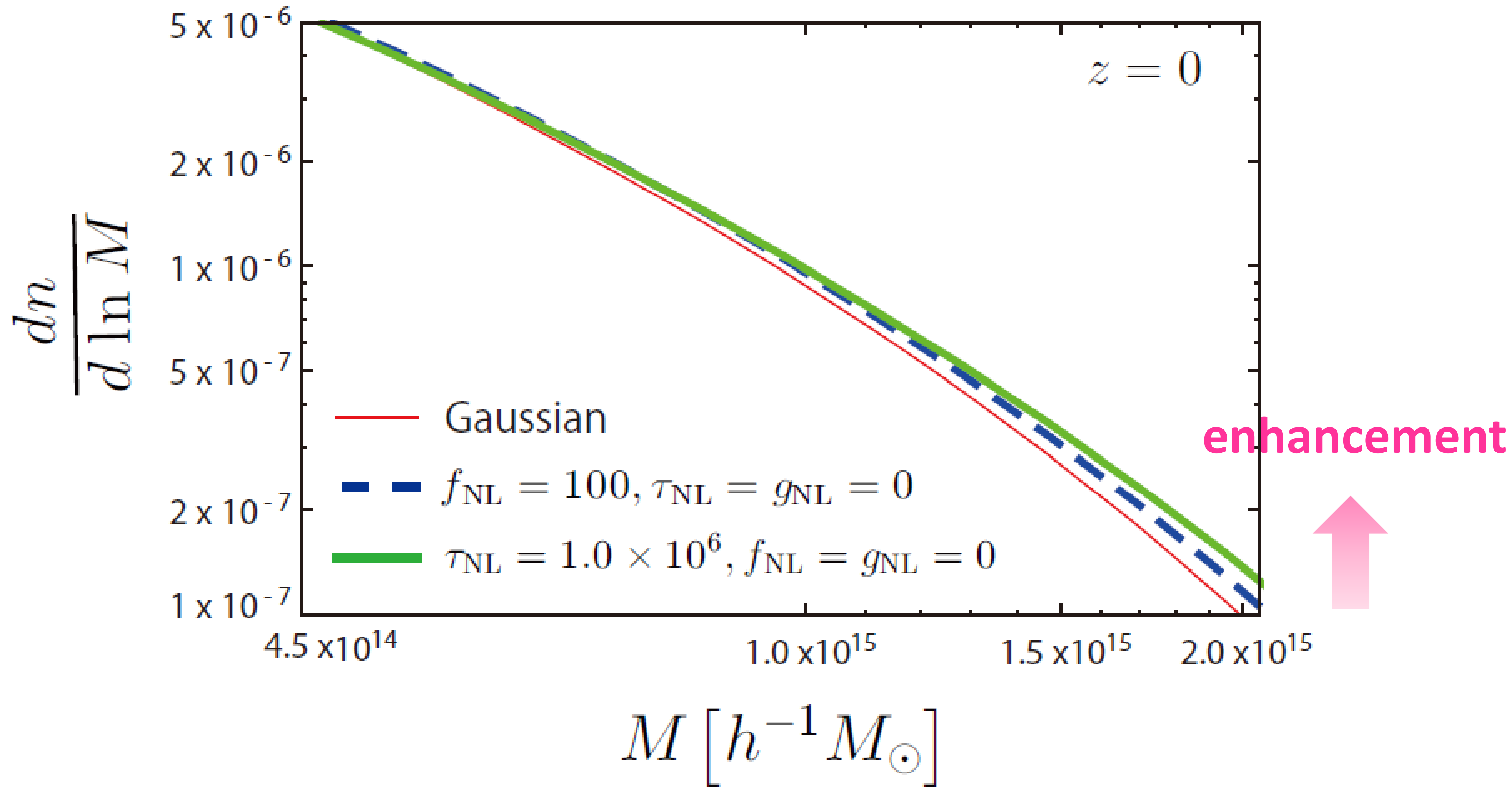
Results

Following discussion, we mainly consider fNL = 100 case and τNL = 10^6 case,
which are large value but roughly consistent with current observational constraints and based on ...

$$\sigma_M S_3(M) = 4.3 \times 10^{-4} f_{\rm NL} \times \sigma_R^{0.13} \quad (10^{12} h^{-1} M_\odot < M < 2 \times 10^{15} h^{-1} M_\odot)$$

$$\sigma_M^2 S_4(M) = 1.9 \times 10^{-7} \tau_{\rm NL} \times \sigma_R^{0.25}$$

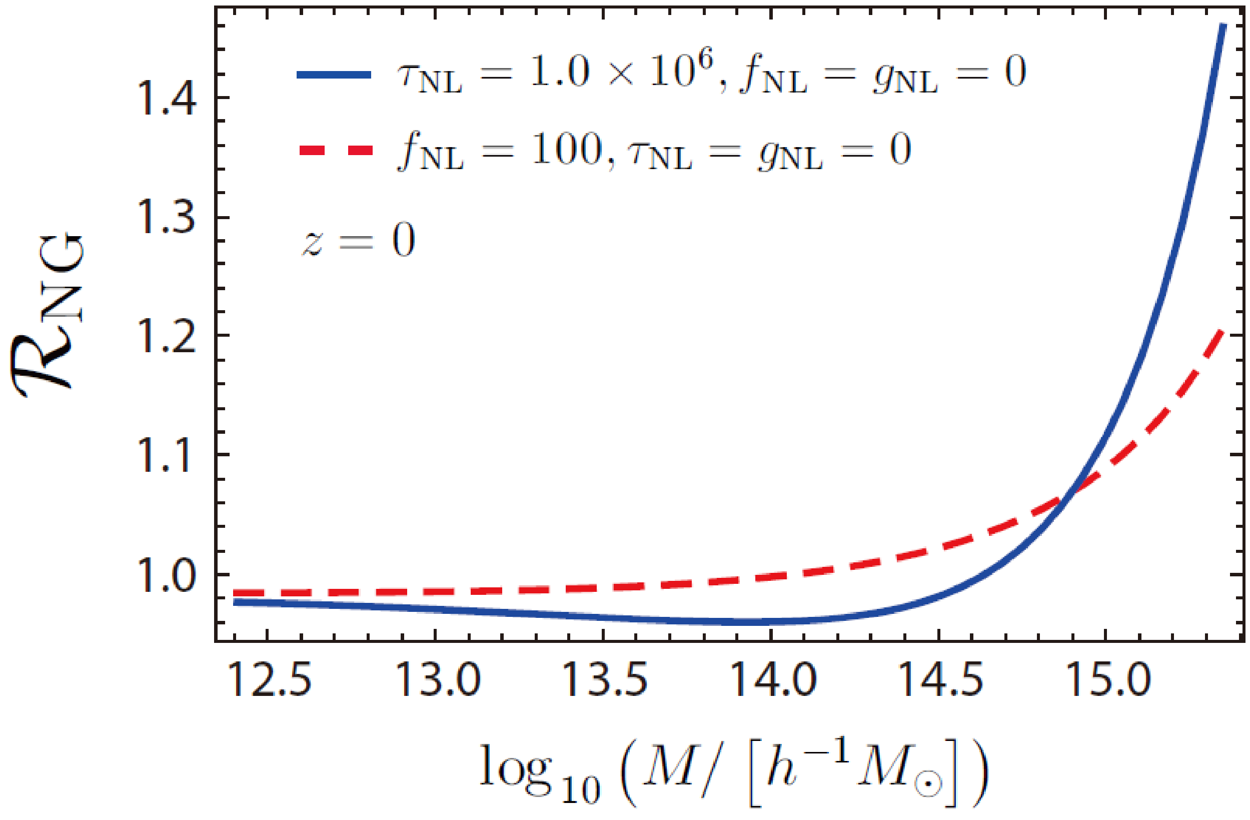
halo mass function



Due to the positive primordial non-Gaussianities, we can see the enhancement of the halo mass function for more massive objects.

fNL vs tauNL

$$\mathcal{R}_{\rm NG} = \frac{dn_{\rm NG}/d\ln M}{dn_{\rm G}/d\ln M}$$; ratio between non-Gaussian mass func. and Gaussian one



form of correction terms;

Skewness

$$H_3\left(\frac{\delta_c}{\sigma_M}\right) \propto \left(\frac{\delta_c}{\sigma_M}\right)^3 \nearrow$$

Kurtosis

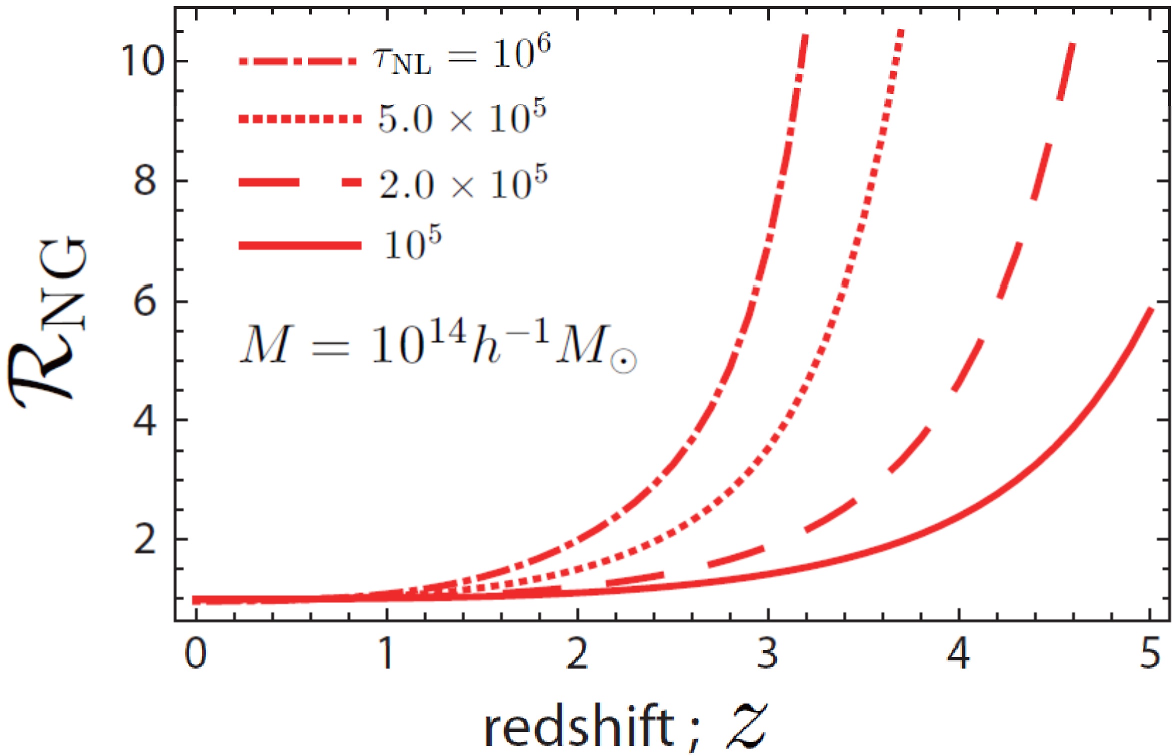
$$H_4\left(\frac{\delta_c}{\sigma_M}\right) \propto \left(\frac{\delta_c}{\sigma_M}\right)^4 \nearrow$$

$\sigma_M \searrow$ for larger mass

some difference of the enhancement behavior ?? ➔ can we distinguish ?

Redshift dependence

Here, we change the value of τNL with fixing mass.



form of correction terms;

Kurtosis

$$\propto \left(\frac{\delta_c}{\sigma_M}\right)^4 \nearrow$$

$$\sigma_M \propto D(z) \searrow$$

with increasing z

D(z);growth function

during matter-dominant era,

$D(z) \propto 1/(1+z)$

Due to the positive τNL (also fNL), we can see the enhancement of the halo mass function at higher redshift.

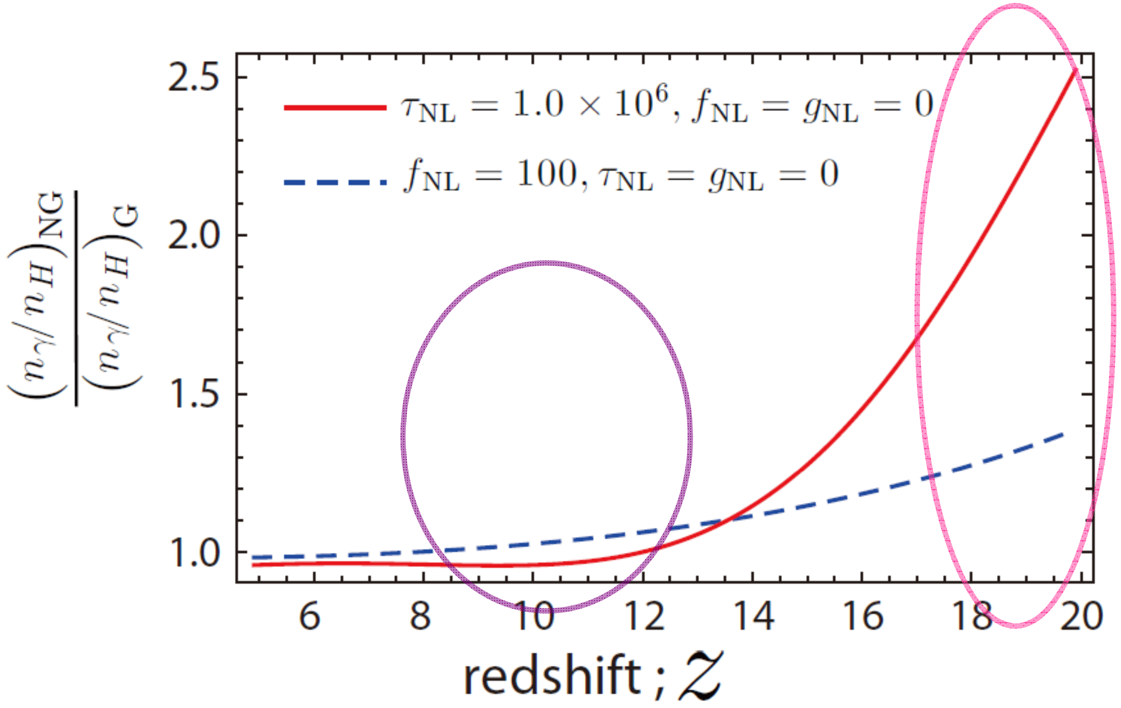
Massive and high redshift objects !!

- Effects on reionization history of the Universe

(z > 10)

n_γ / n_H ; Cumulative photon number density emitted from the pop III stars per neutral hydrogen density

Ref.) Somerville et al (2003)



Around z ~ 10, the primordial NG is not so effective.

In the early stage (z ~ 20), the NG effect becomes large.

Massive and high redshift objects !!

- High redshift massive clusters

Weak lensing analysis of the galaxy cluster XMMU J2235-2557

presented by Jee, et al(2009) and Rosati, et al(2009)

$$z \approx 1.4$$

$$M \approx 6.4 \times 10^{14} M_{\odot}$$ (~ 0.4 Mpc^-1)

In ΛCDM (+ Gaussian) universe, such a massive cluster at this redshift would be a rare event (at least 3σ).

In order to explain the existence of such a cluster naturally (at least 2σ), Cayon, et al.(2010) found $f_{\rm NL} \approx 450$

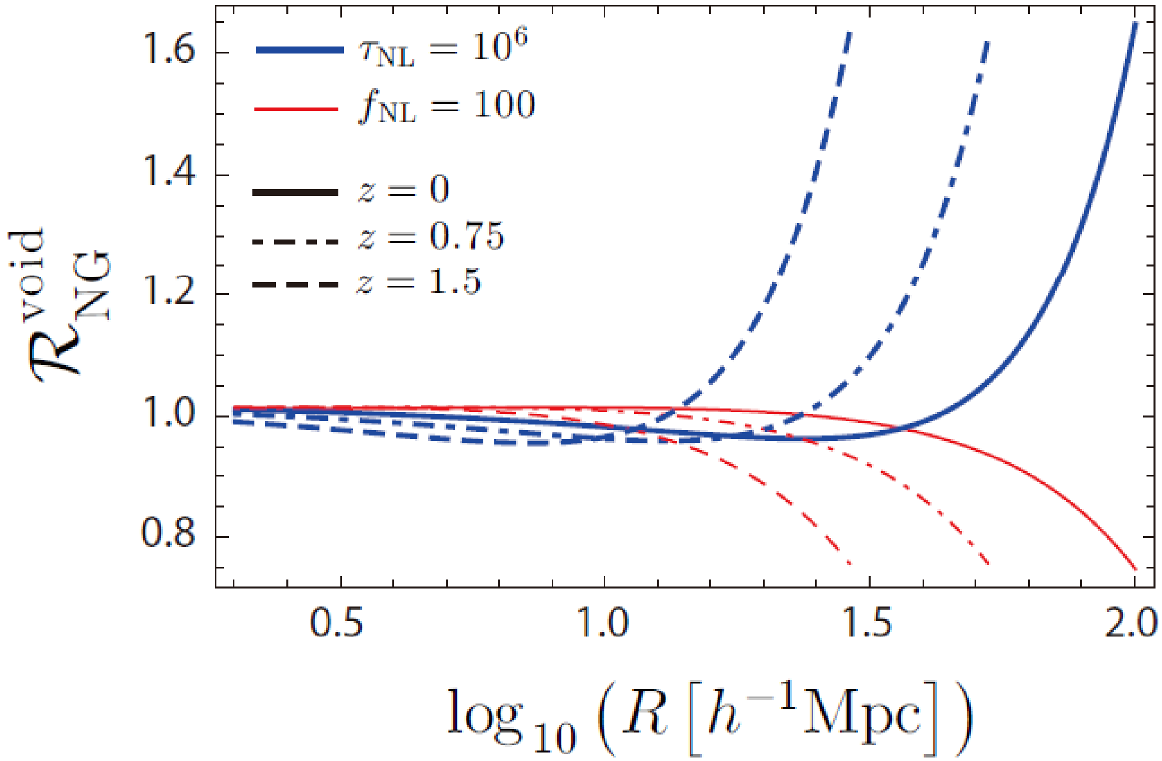
Scale-dependent fNL ?? (Ref.) Takahashi-san's talk and Tasinato-san's talk)

On the other hand, we find $\tau_{\rm NL} \approx 1.5 \times 10^6$

For gNL, Enqvist et al.(2010)

Massive objects ←→ large scale voids?

- Abundance of voids (underdensity region (< δv))



Positive τNL
➔ enhancement !!
(same in cluster abundance)

Positive fNL
➔ damping
(opposite to in cluster abundance)

ref. Kamionkowski et al(2009)

By comparing the observations of clusters and that of void abundance, we could distinguish skewness-type and kurtosis-type ??

Summary and Discussion

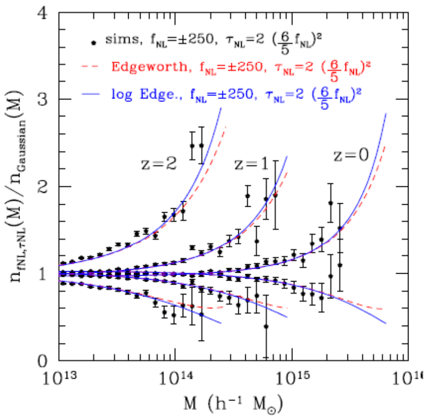
- We consider the effect of the primordial non-Gaussianity, (especially, kurtosis-type) on the large scale structure formation.
- We obtain a formula of the halo mass function with the primordial non-Gaussianities (including fNL, gNL τNL).
- We find the enhancement of the formation of the massive and high redshift objects.

  - early phase of reionization of the Universe
  - massive clusters at high redshift
  - abundance of voids

    (has a potential to distinguish between skewness- and kurtosis-type.)

Summary and Discussion

- How to relate our results with observables ??
  Ref. Hoyle et al.(2010)
- Can we distinguish the effects of fNL, gNL, τNL ??
- Related interesting issues
  - N-body simulation (Ref.) LoVerde & Smith (2011), …)
  - scale-dependent bias (Ref.). Tseliakhovich, et al.(2010), …)
  - other shapes of primordial non-Gaussianity
    (Ref.) Wagner et al.(2010), …)



LoVerde & Smith (2011)