Change in the Quasi-particle Picture in Association with QCD Phase Transitions

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Korea-Japan Joint workshop of Nuclear Physics
at
Korea Physics Society Meeting

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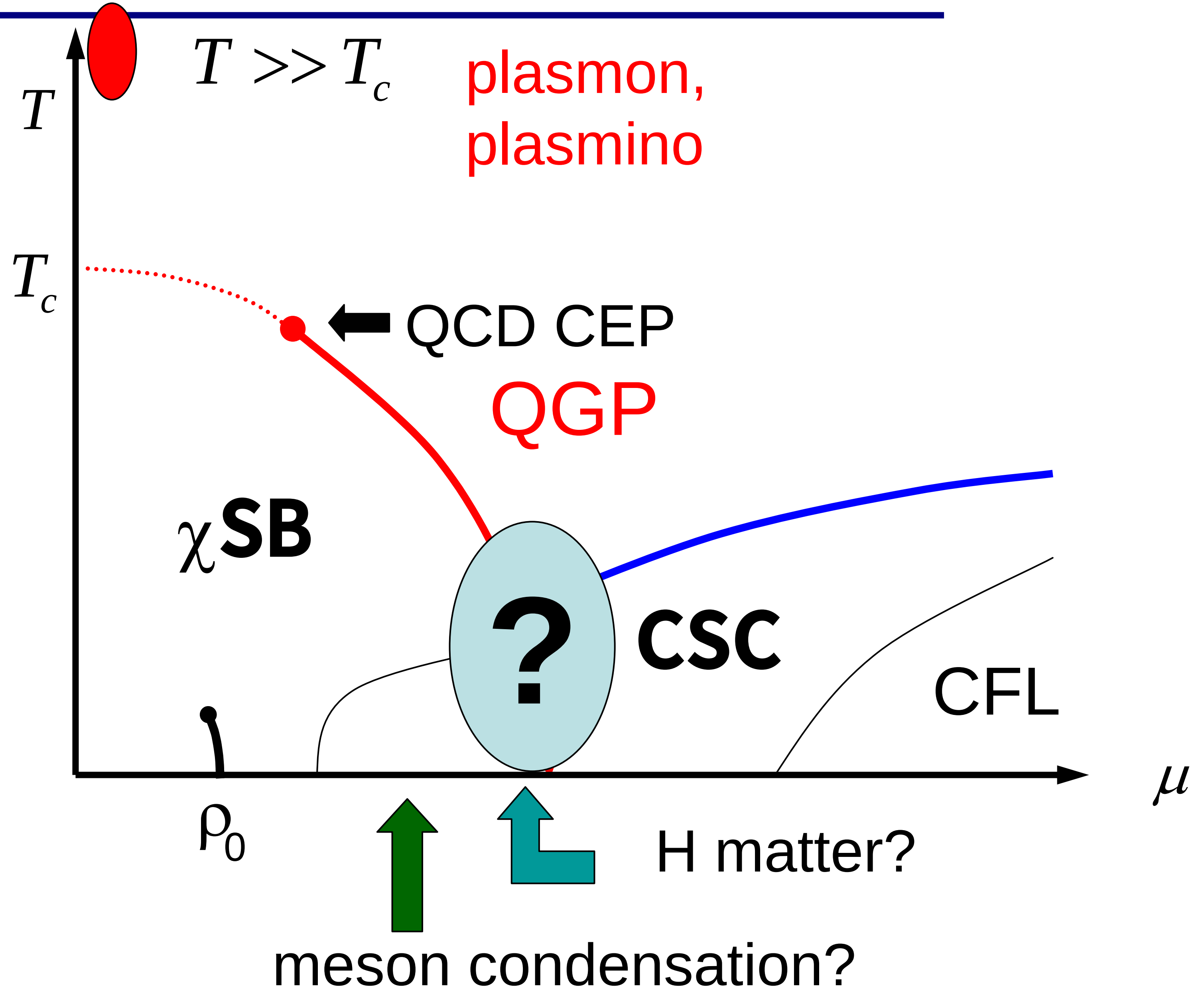
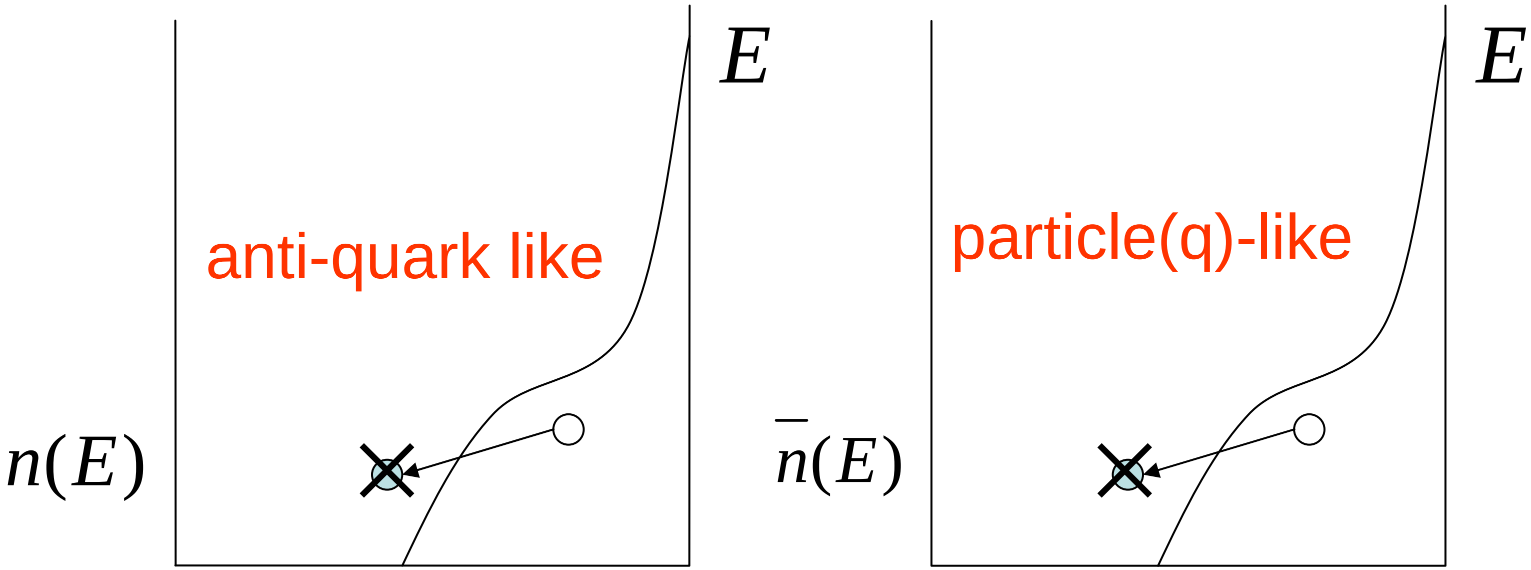
1.Introduction

QCD phase diagram and quasi-particles



quark distribution　　　　anti-q distribution



Plasmino excitation

How about when the temperature
Is lowered close to T_c ?

The wisdom of many-body theory tells us:

If a phase transition is of 2nd order or weak 1st order,

$\exists$ soft modes ; the fluctuations of the order parameter

eg. softening of 2+ phonon -→ quadrupole deformation

Gamow-Teller GR ; a soft mode of pion condensation (T.K., 1981)

Chiral Transition = a phase transition of QCD vacuum,

$\langle \bar{q}q \rangle$ being the order parameter. Lattice QCD;

There can be **hadronic excitations** (para pion and sigma)

as the soft mode of the chiral transition **in the ``QGP" phase.**

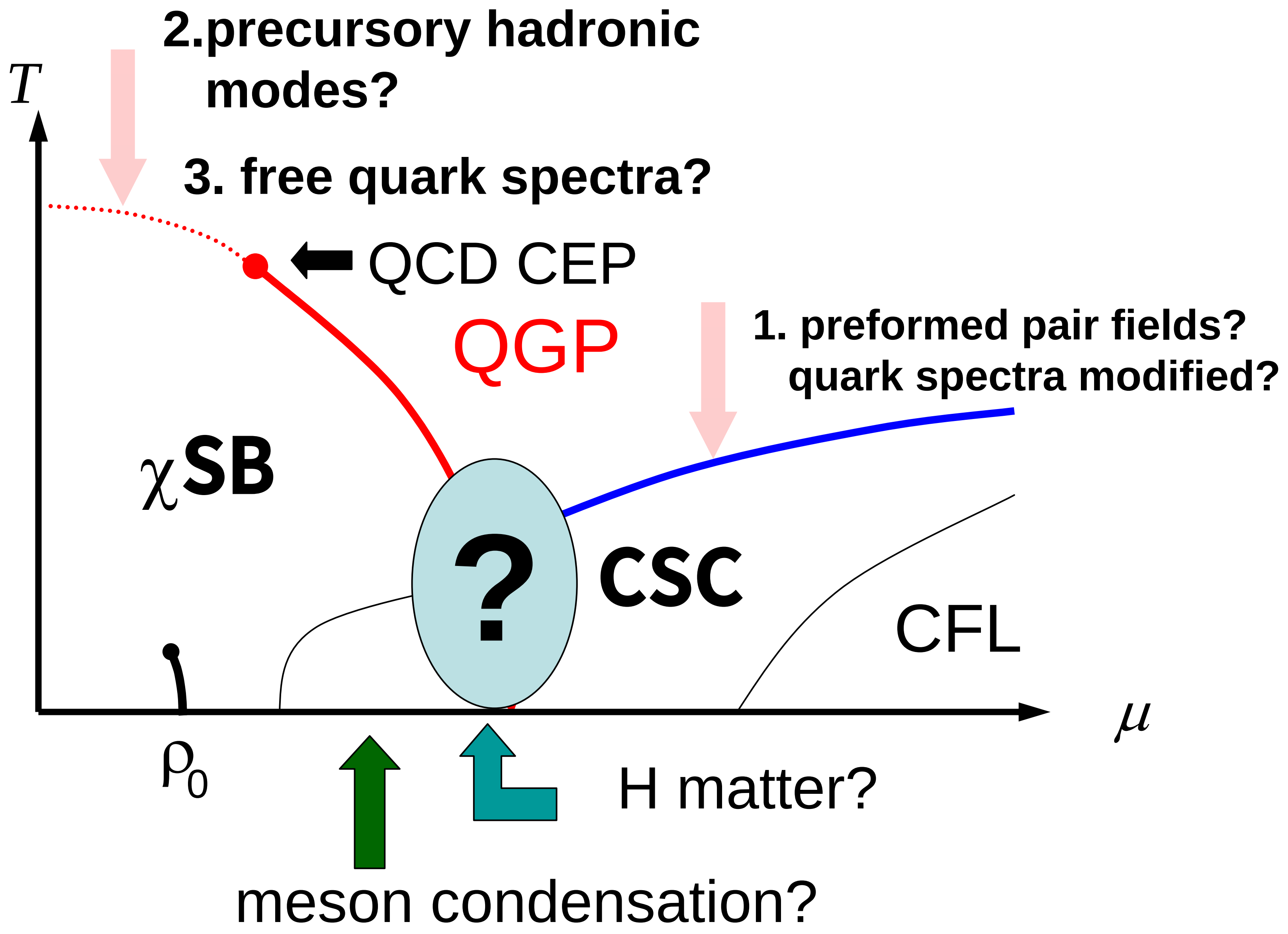
T. Hatsuda and T. K. , Phys. Rev. Lett.55('85)158; PLB71('84),1332

Prog. Theor. Phys 74 (1985), 765:

Cf. T<T_c ; the σ meson becomes the soft mode of

chiral restoration at $T \neq 0$ and/or $\rho_B \neq 0$: $m_\sigma \to 0$, $\Gamma_\sigma \to 0$

QCD phase diagram and quasi-particles

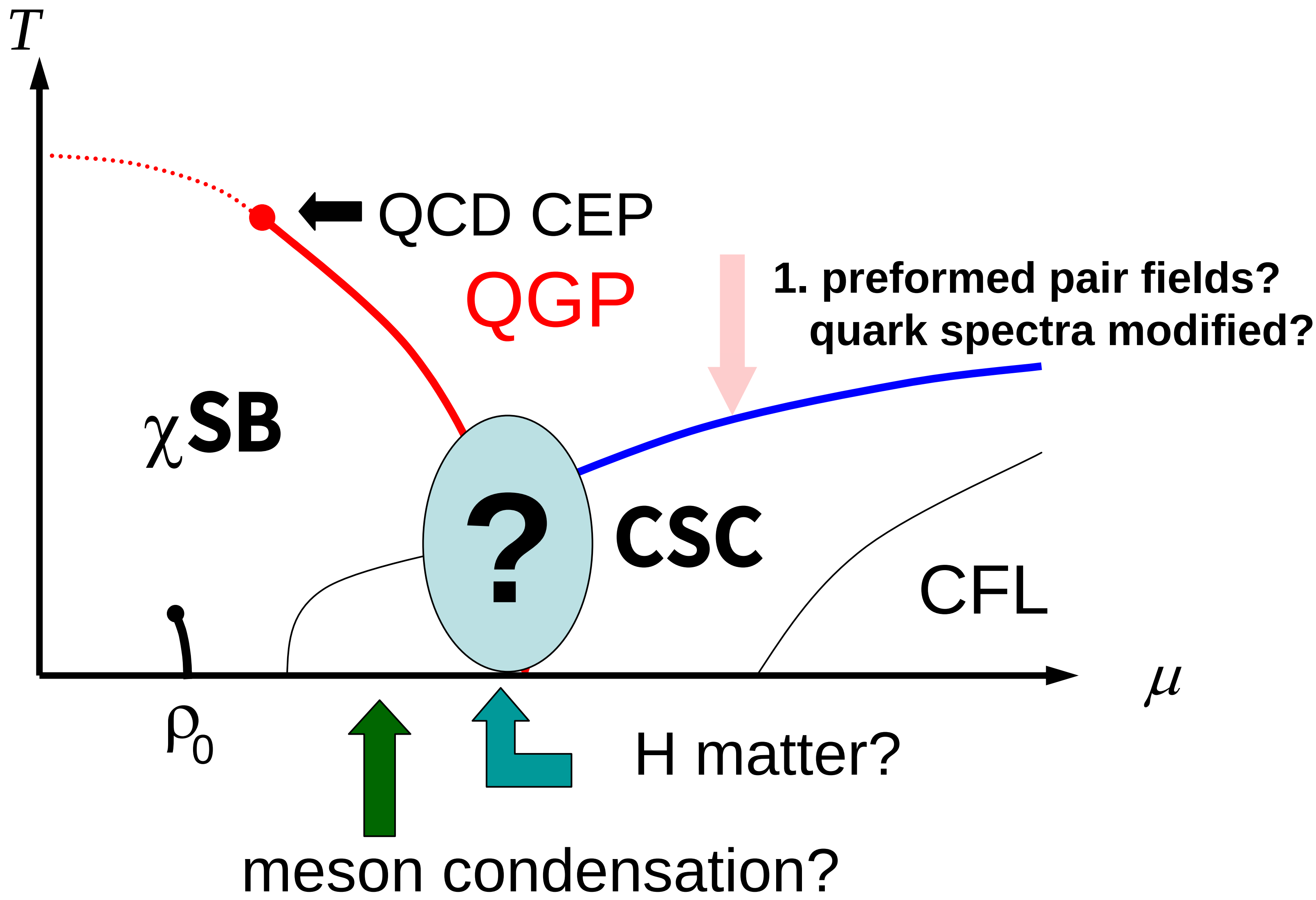


2. Precursory Phenomena of Color Superconductivity in Heated Quark Matter

Ref. M. Kitazawa, T. Koide, T. K. and Y. Nemoto,

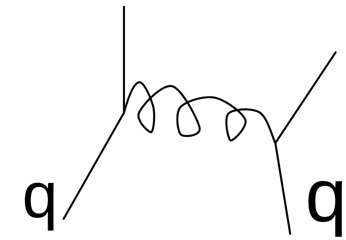
Phys. Rev. D65,091504 (2002); D70, 0965003 (2004)

QCD phase diagram



Color Superconductivity; diquark condensation

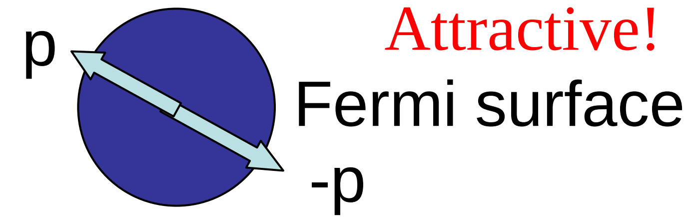
- Dense Quark Matter:
  - quark (fermion) system
  - with attractive channel in one-gluon exchange interaction.



$[3]_c \times [3]_c = [\bar{3}]_c + [6]_c$

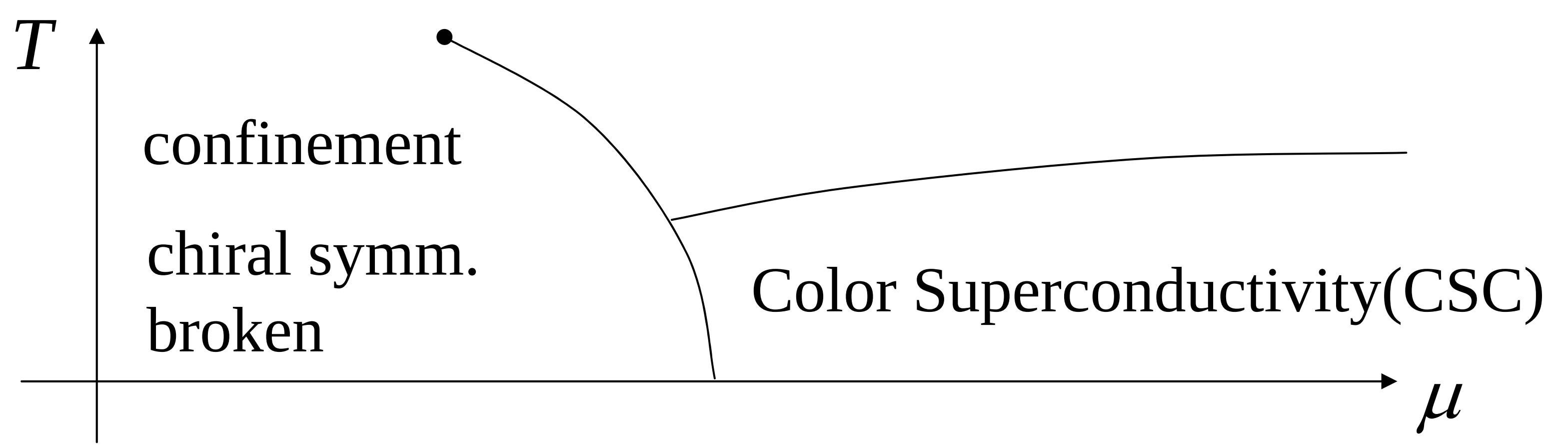
Attractive!

⇨ Cooper instability at sufficiently low T



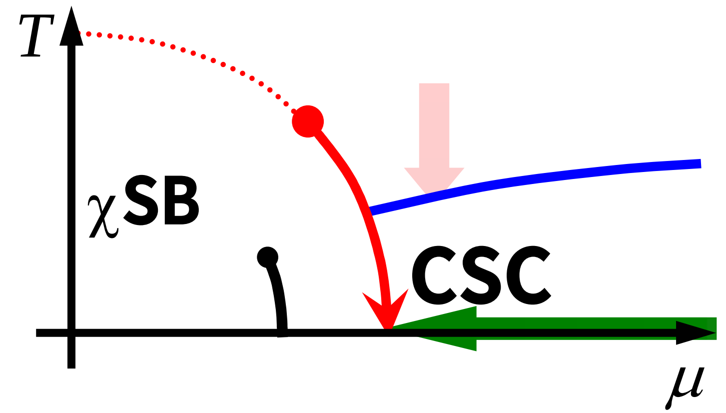
⇨ $SU(3)_c$ gauge symmetry is broken!

- Δ~100MeV at moderate density μ_q~ 400MeV



Nature of CSC in Intermediate Density (μ_q~500MeV)

- gap Δ ~ 100MeV at T=0
- order of T_c ~ 50MeV



$\Delta / E_F \sim 0.1$ ⇔ in electric SC $\Delta / E_F \sim 0.0001$

owing to Strong coupling nature of QCD

implies,

Short correlation length of Cooper pairs ⇒ Large fluctuation of the pair field is expected even at T>0 !

⇒ may be relevant to newly born neutron stars or intermediate states in heavy-ion collisions (GSI, J-PARC)

Collective Mode in CSC

Response Function of Pair Field

Linear Response

- external field: $H_{ex} = \int d\mathbf{x}\left(\Delta_{ex}^{\dagger}\bar{\psi}^{C} i\gamma_5\tau_2\lambda_2\psi + \mathrm{h.c.}\right)$

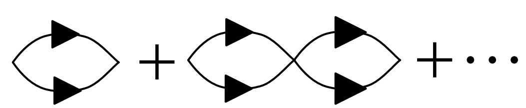
  ⇨ expectation value of induced pair field:

$$\left\langle \bar{\psi}(x) i\gamma_5\tau_2\lambda_2\psi^{C}(x)\right\rangle_{ex} = i\int_{t_0}^{t} ds \left\langle \left[H_{ex}(s), O(\mathbf{x},t)\right]\right\rangle$$

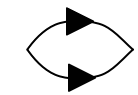
$$\begin{cases} \Delta_{ind}(x) = -2G_C\left\langle \bar{\psi}(x) i\gamma_5\tau_2\lambda_2\psi^{C}(x)\right\rangle_{ex} = \int dt' \int d\mathbf{x} D^{R}(x,x')\Delta_{ex}(x') \\ D^{R}(\mathbf{x},t) = -2G_C \underline{\left\langle \left[\bar{\psi}(x) i\gamma_5\tau_2\lambda_2\psi^{C}(x), \bar{\psi}(0) i\gamma_5\tau_2\lambda_2\psi^{C}(0)\right]\right\rangle \theta(t)} \end{cases}$$

⮡ Retarded Green function

- Fourier transformation with Matsubara formalism ⇨ $\Delta^{\dagger}(\mathbf{k},\omega_n)_{\mathrm{ind}} = \mathcal{D}(\mathbf{k},\omega_n)\Delta^{\dagger}(\mathbf{k},\omega_n)_{\mathrm{ext}}$

- RPA approx.: $\mathcal{D}(\mathbf{k},\omega_n) =$ 

$$= -\frac{G_C Q(\mathbf{k},\omega_n)}{1+G_C Q(\mathbf{k},\omega_n)}$$

with $Q(\mathbf{k},\omega_n) =$ 

After analytic continuation to real time,

$$\begin{aligned} D^R(\mathbf{k},\omega) &= -G_c Q(\mathbf{k},\omega)/(1+G_c Q(\mathbf{k},\omega)), \\ &\equiv -G_c Q(\mathbf{k},\omega)\cdot \Xi(\mathbf{k},\omega) \\ \Xi^{-1}(\mathbf{k},\omega) &\equiv 1+G_c Q(\mathbf{k},\omega). \end{aligned}$$

The spectral function;

$$\rho(\mathbf{k},\omega) = -\frac{1}{\pi}\mathrm{Im}D^R(\mathbf{k},\omega)$$

An important obeservation: at $T = T_c$;

$$\Xi^{-1}(\mathbf{k}=\mathbf{0},\omega=0)=0$$

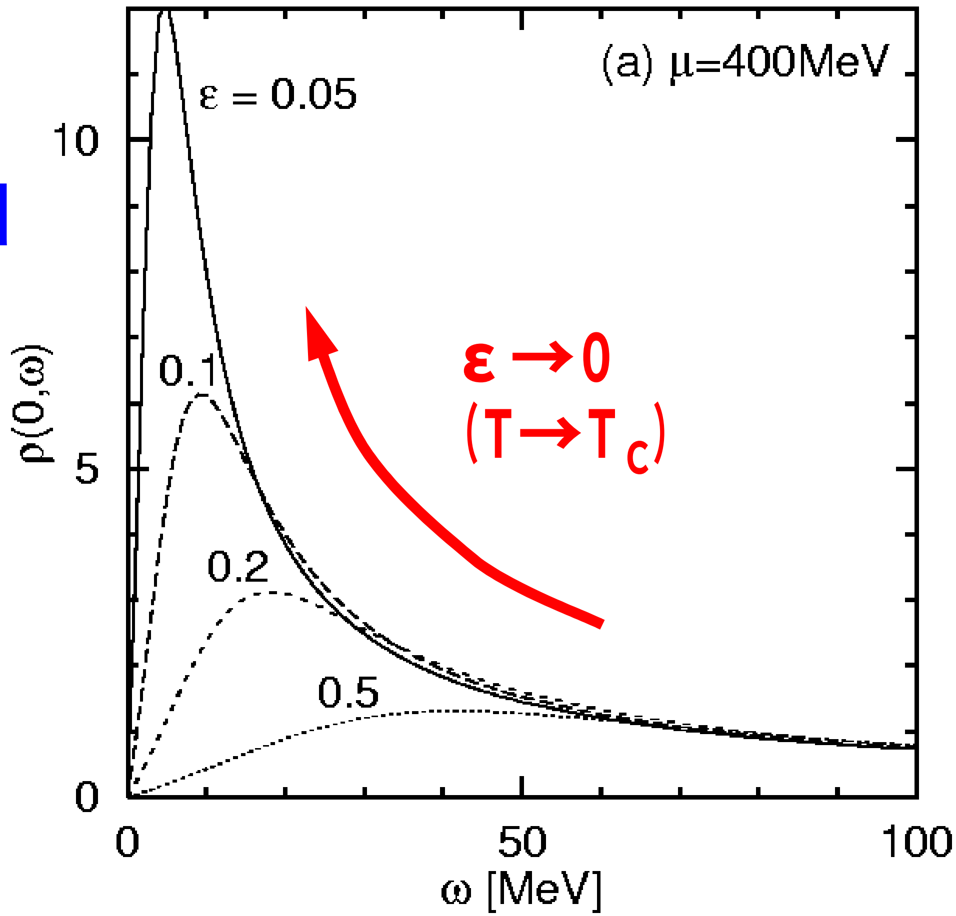
Equivalent with the gap equation (Thouless criterion)

Precursory Mode in CSC

(Kitazawa, Koide, Nemoto and T.K., PRD 65, 091504(2002))

$$\rho(\mathbf{k}=\mathbf{0},\omega)$$

Spectral function of the pair field at T>0



at k=0

$$\varepsilon = \frac{T - T_C}{T_C}$$

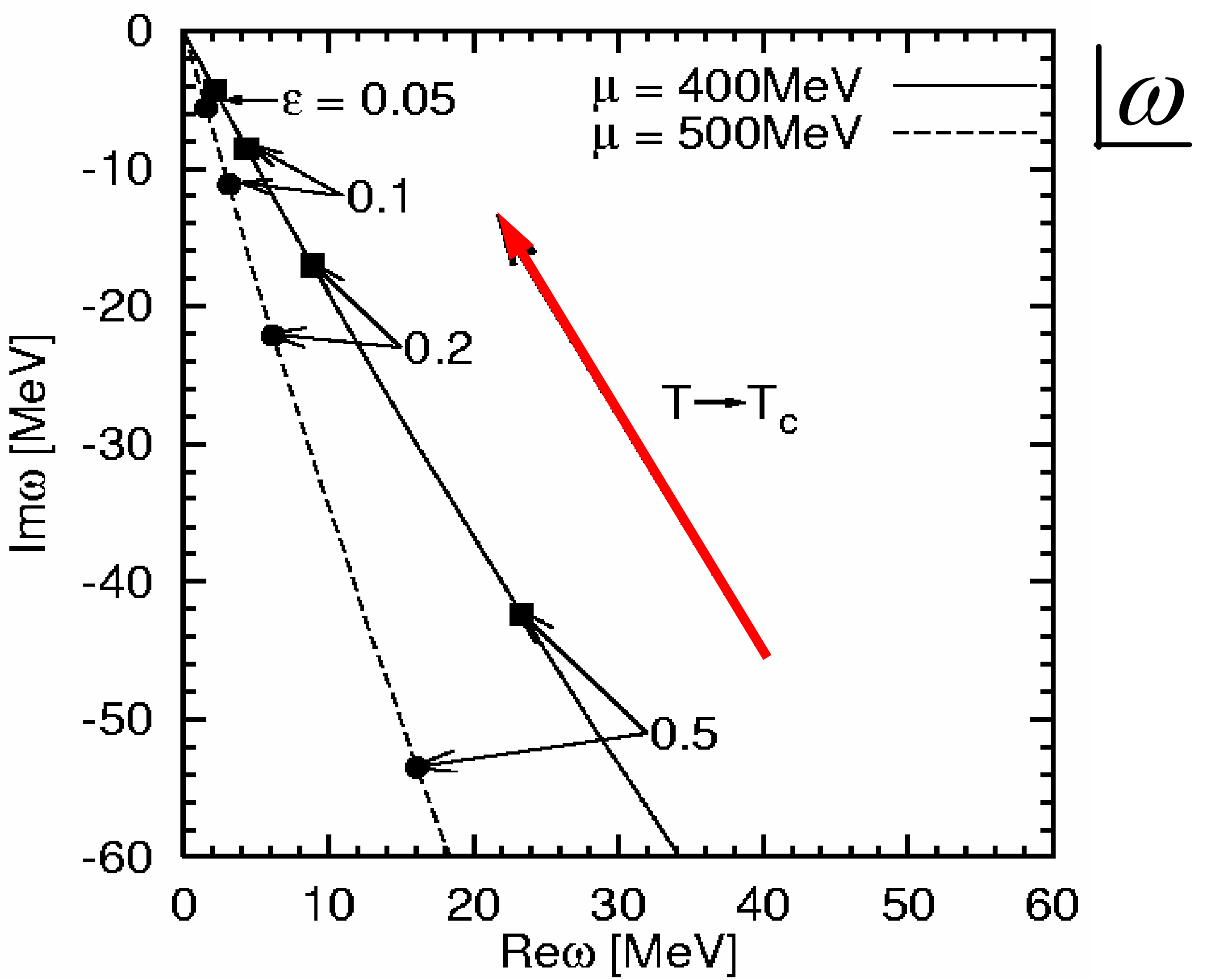
- As T is lowered toward T_C,

  The peak of ρ becomes sharp. (Soft mode) ⟹ Pole behavior

- The peak survives up to ε 0.2 ⟺ Electric SC： $\sim\varepsilon$ 0.005

The pair fluctuation as the soft mode;

--- movement of the pole of the precursory mode---



How does the soft mode affect the quark spectra?

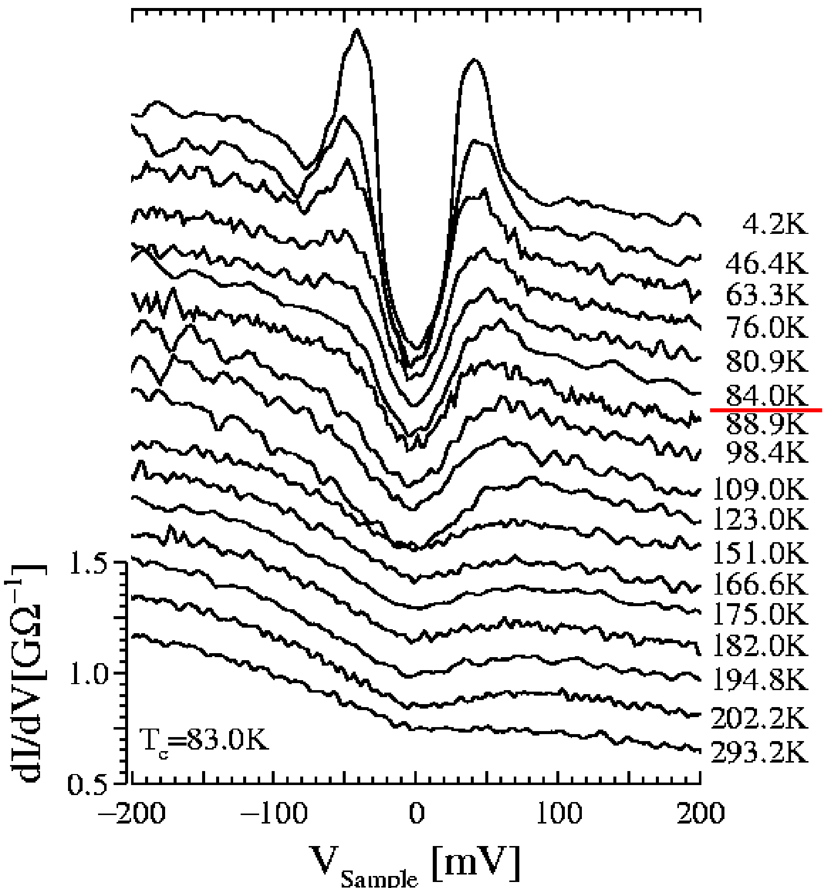
---- formation of pseudogap ----

Ref. M. Kitazawa, T. Koide, T. K. and Y. Nemoto

Phys. Rev. D70, 956003(2004);hep-ph/0309026

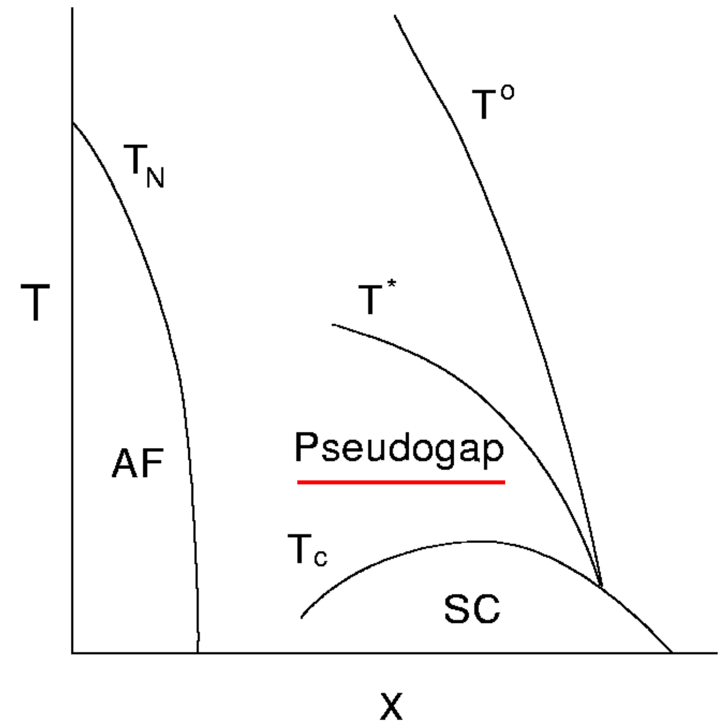
Pseudogap

:Anomalous depression of the density of state near the Fermi surface in the normal phase.



Renner et al.('96)

Conceptual phase diagram of HTSC cuprates

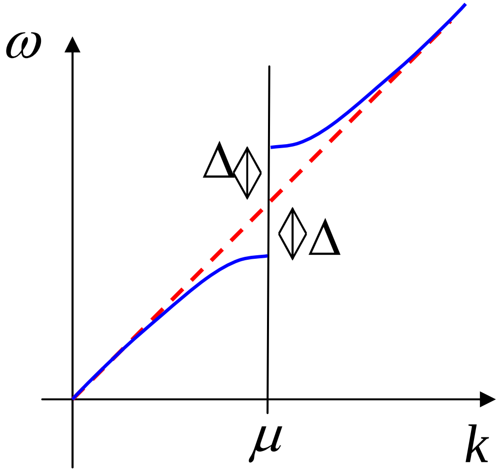


The origin of the pseudogap in HTSC is **still controversial**.

Density of State in BCS theory

- Quasi-particle energy:

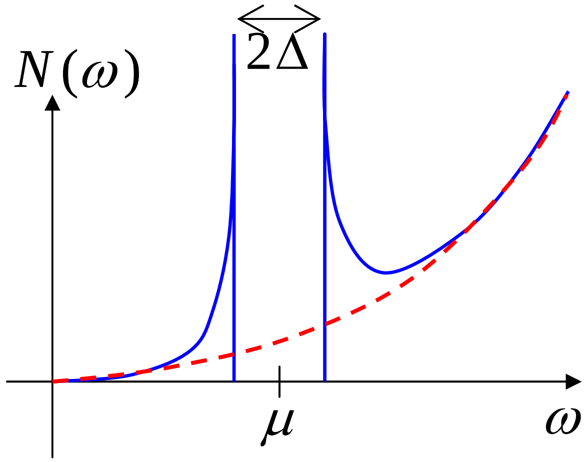
$$\omega = \mathrm{sgn}(k-\mu)\sqrt{(k-\mu)^2+\Delta^2}$$



- Density of State:

$$N(\omega) \propto k^2 \frac{dk}{d\omega}$$

$$\frac{d\varepsilon}{dk} = \frac{k-\mu}{\sqrt{(k-\mu)^2+\Delta^2}}$$



The gap on the Fermi surface becomes smaller as T is increased, and it closes at T_c.

Density of State $N(\omega)$

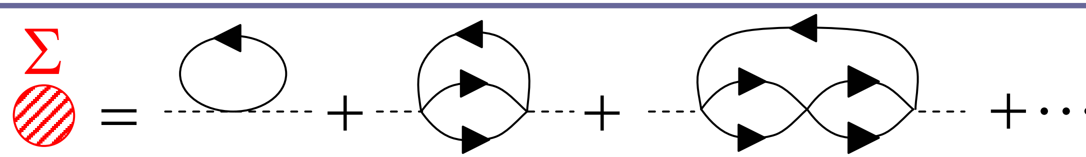
$$N = \int d^3x \left\langle \bar{\psi}\gamma^0\psi \right\rangle$$

$$N(\omega) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\rho^0(\mathbf{k},\omega) \Leftarrow \rho^0(\mathbf{k},\omega) = \frac{1}{4}\mathrm{Tr}\left[\gamma^0 \,\mathrm{Im}\, G^R(\mathbf{k},\omega)\right]$$

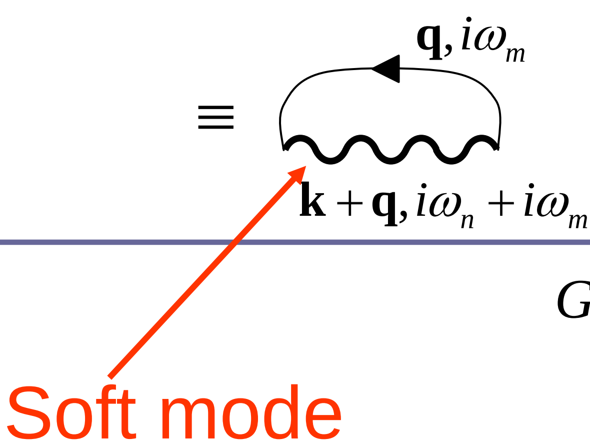
T-matrix Approximation

$$G(\mathbf{k},\omega_n) = \frac{1}{G^0(\mathbf{k},i\omega_n) - \Sigma(\mathbf{k},i\omega_n)}$$

$\Sigma(\mathbf{k},\omega_n) =$



$$\equiv$$



$$= T\sum_m \int \frac{d^3q}{(2\pi)^3}\Xi(\mathbf{k}+\mathbf{q},\omega_n+\omega_m)G^0(\mathbf{q},\omega_m)$$

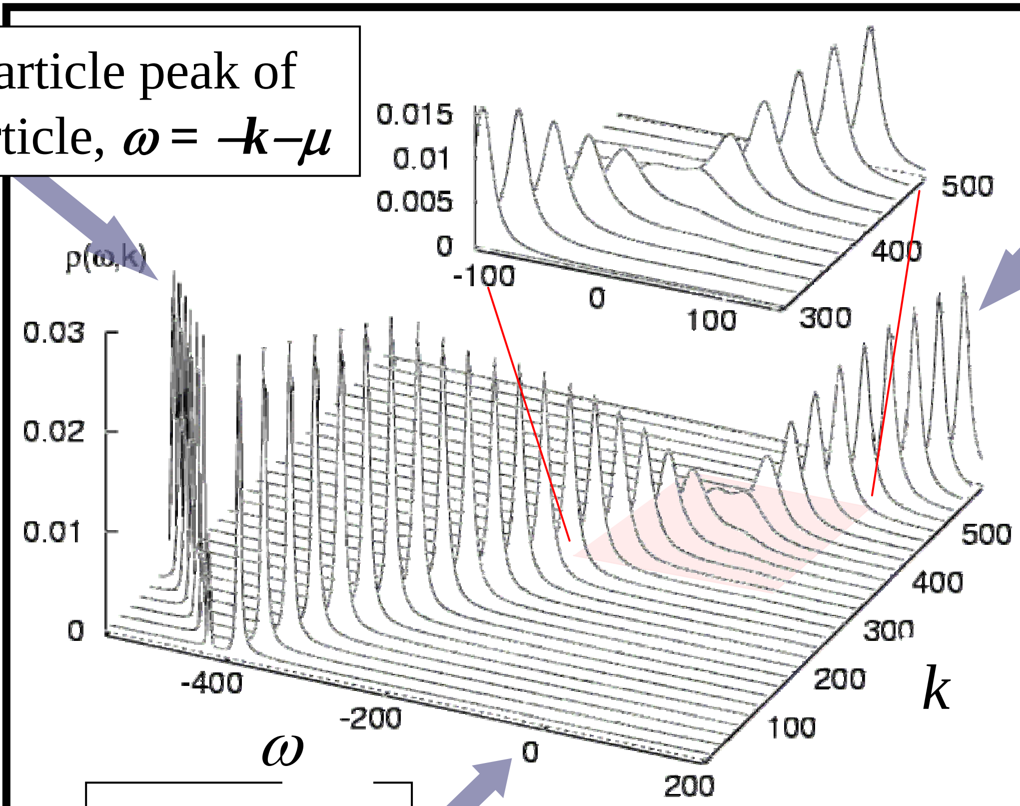
$$G^0(\mathbf{k},i\omega_n) = \left[(i\omega_n+\mu)\gamma^0 - \mathbf{k}\cdot\vec{\gamma}\right]^{-1} = \longrightarrow$$:free progagator

1-Particle Spectral Function

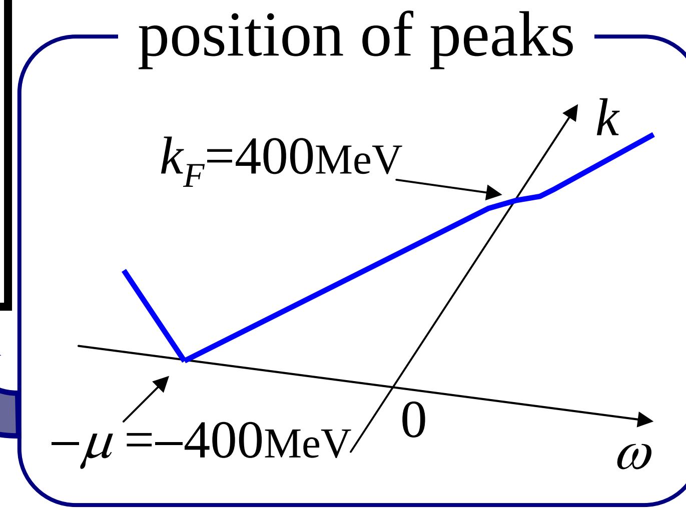
μ= 400 MeV
ε=0.01

quasi-particle peak of anti-particle, $\omega = -k-\mu$

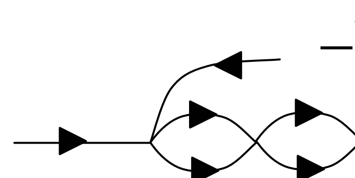
quasi-particle peak, $\omega = \omega_-(k) \sim k-\mu$



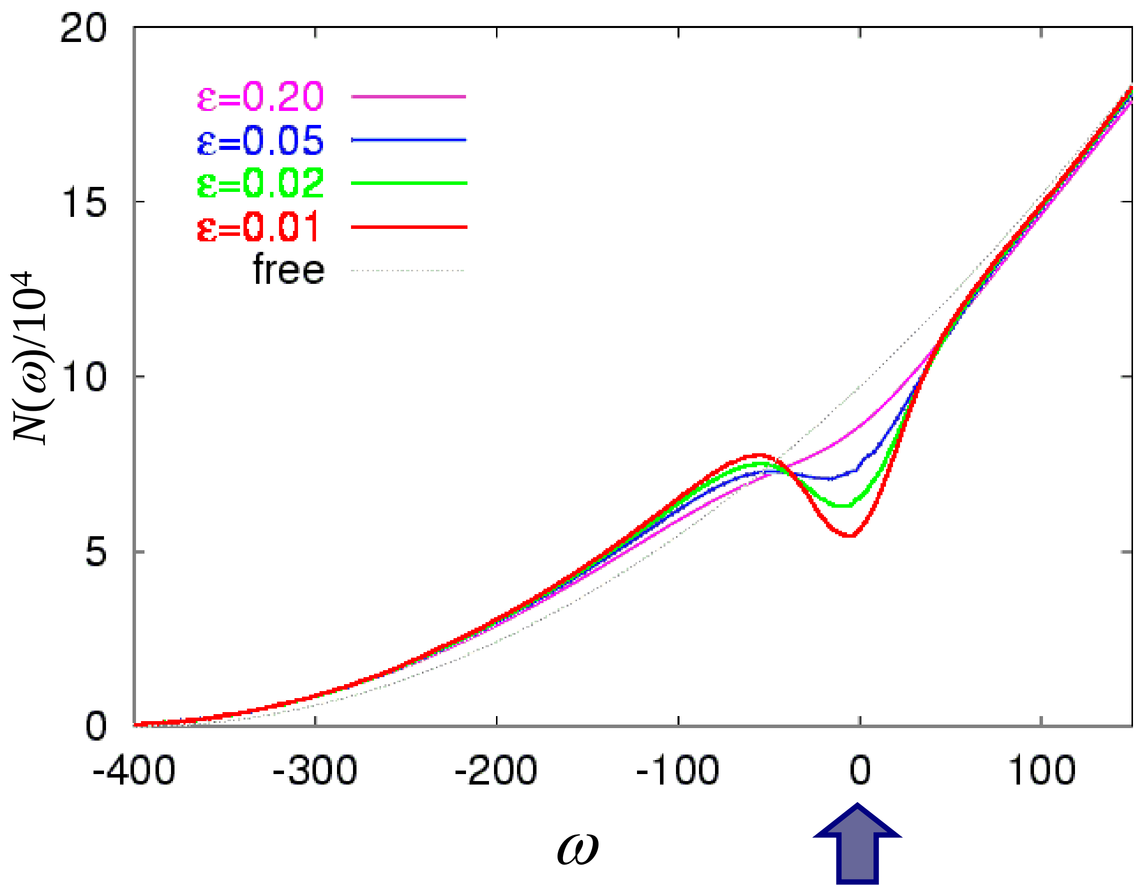
Fermi energy



- quasi-particle peaks at $\omega = \omega_-(k) \sim k-\mu$ and $\omega = -k-\mu$.
- Quasi-particle peak has a depression around the Fermi energy due to **resonant scattering**.



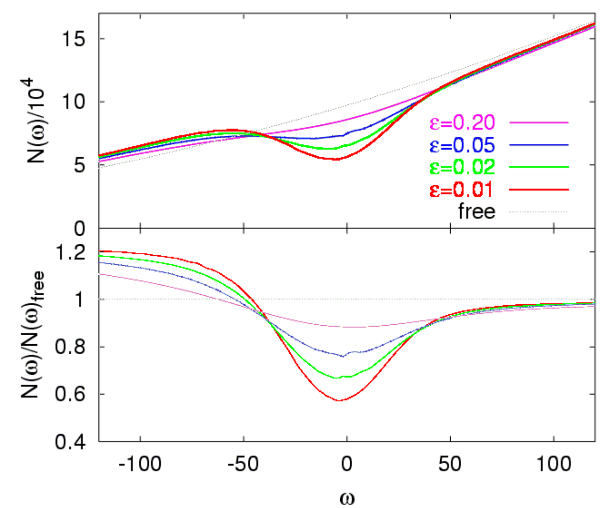
Density of state of quarks in heated quark matter



Fermi energy

$\mu = 400$ MeV

- **<u>Pseudogap structure manifests itself in $N(\omega)$.</u>**
- The pseudogap survives up to <u>$\varepsilon \sim 0.05$</u> <u>(5% above T_C).</u>



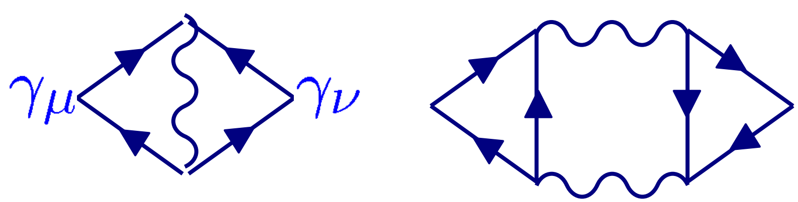
Summary of this section

- There may exist a wide T region where the precursory soft mode of CSC has a large strength.

- The soft mode induces the pseudogap, the anomalous enhancement of the specific heat C_V

Future problems:

effects of the soft mode on . **H-I coll. & proto neutron stars**

eg.1) 

anomalous lepton pair production

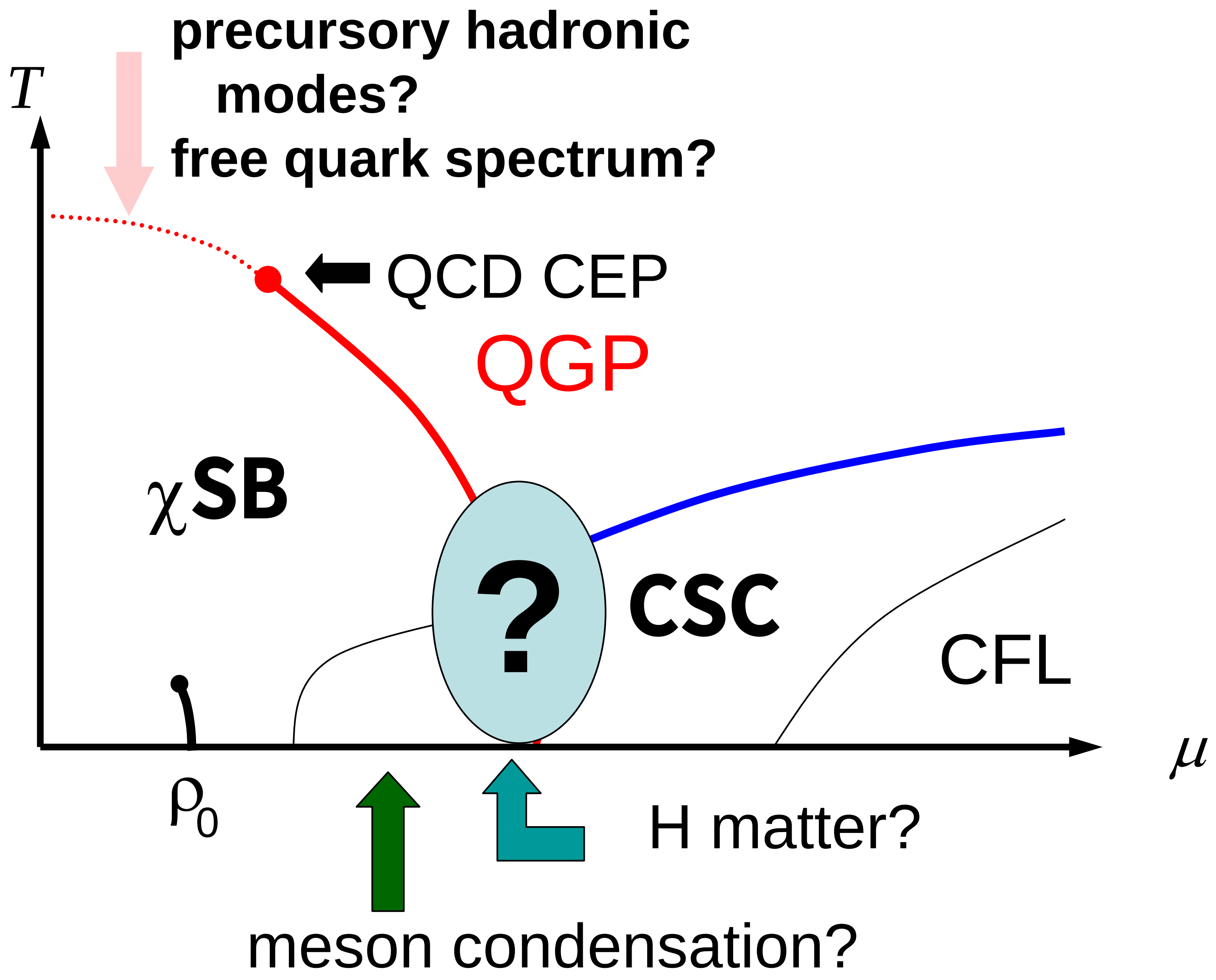
collective mode:  (ω,k)

(M.Kitazawa and T.K in progress.)

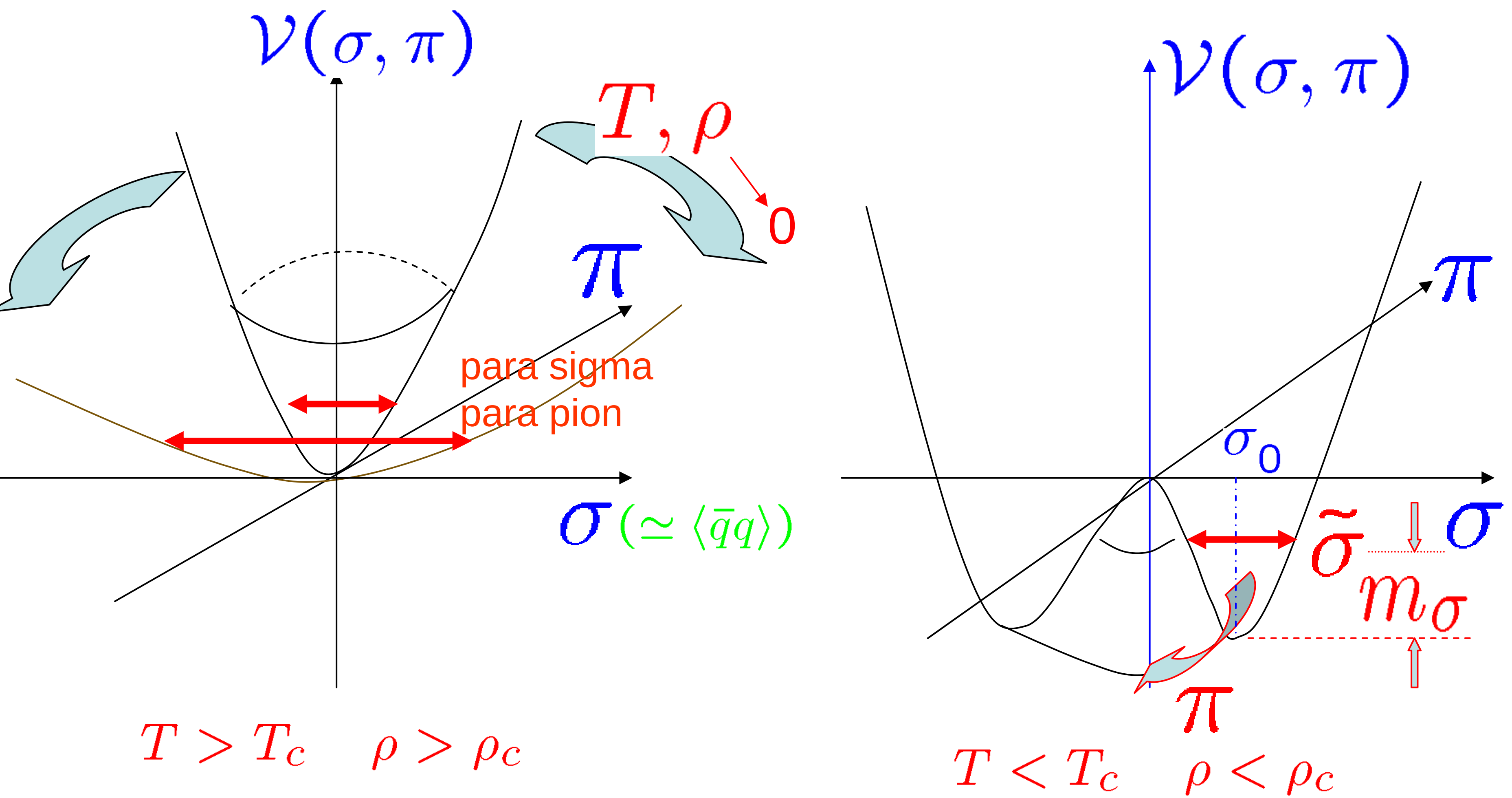
eg.2) C_V ; cooling of newly born stars

3. Precursory Hadronic Mode
and
Single Quark Spectrum
above
Chiral Phase Transition

QCD phase diagram and quasi-particles



Chiral Transition and the collective modes



$$\sigma = \sigma_0 + \tilde{\sigma}$$

c.f. Higgs particle in WSH model

ϕ ; Higgs field ➡ $\phi = \langle \phi \rangle + \tilde{\phi}$

Higgs particle

Hadronic Modes in the 'QGP' Phase

The `para-sigma' and `para-pion'

T. Hatsuda and T. K.,(1985)

The driving force leading to the phase transition should be strong enough to form the collective modes even at $T > T_c$

T. Hatsuda and T. K. , Phys. Rev. Lett.55('85)158; PLB71('84),1332 ; Prog. Theor. Phys 74 (1985), 765.

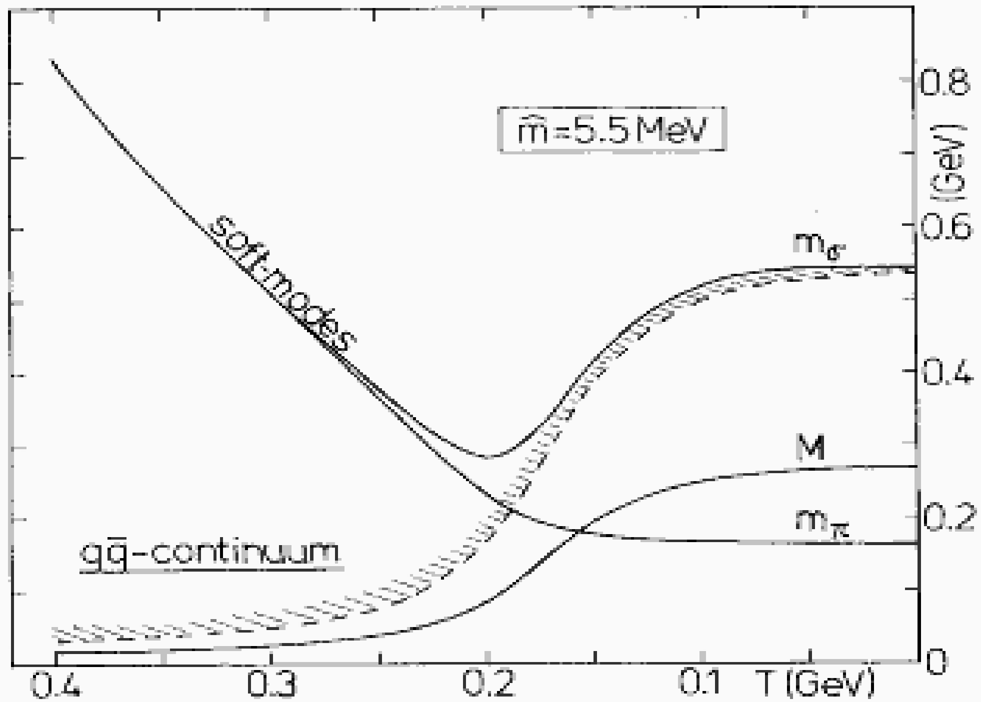


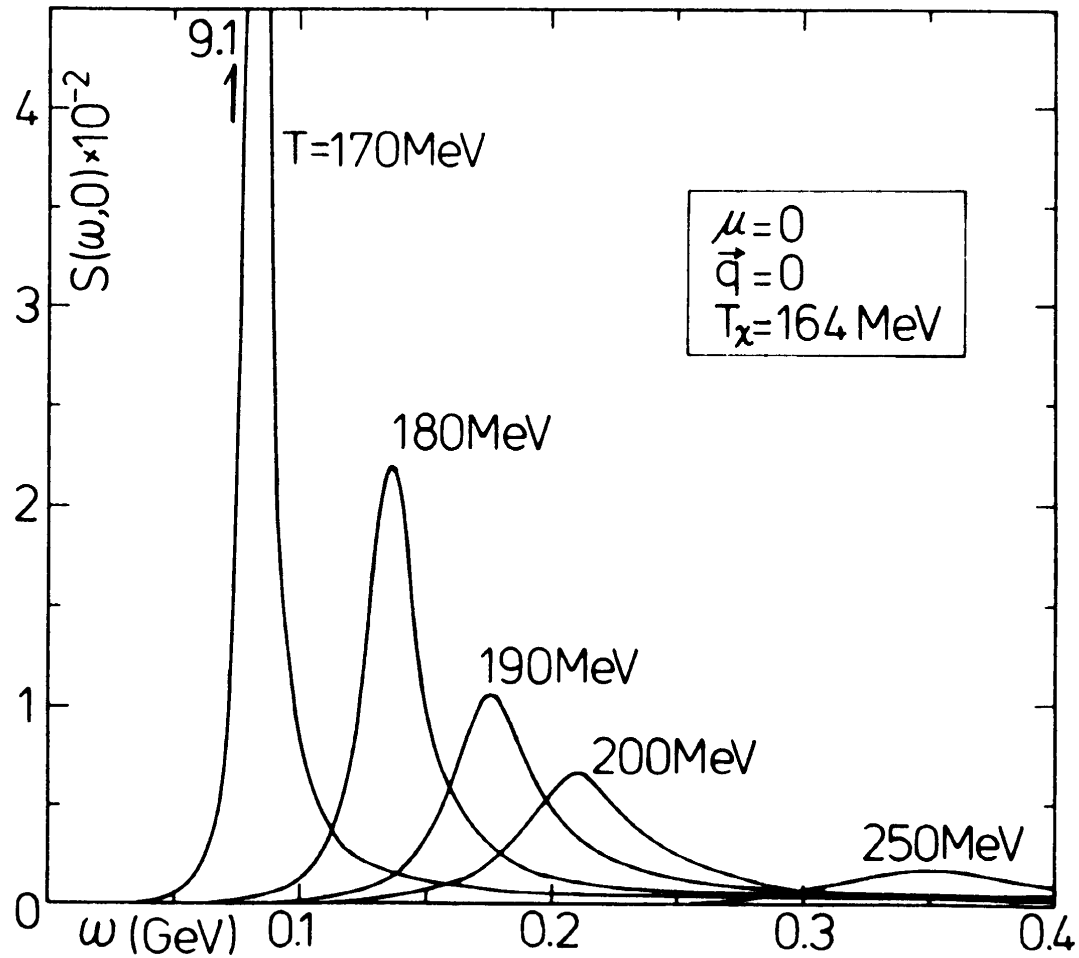
FIG. 3. Dynamical quark mass $M = M_D(T,\mu) + \hat{m}$, and the masses of σ mode (m_σ) and π mode (m_π). The dashed line denotes the $2M$ threshold from which the $q\bar{q}$ continuum starts.

Large ← T

The spectral function of the degenerate ``para-pion" and the ``para-sigma" at T>Tc for the chiral transition: Tc=164 MeV

T. Hatsuda and T.K. (1985)

The idea is rediscovered and revived :
E. Shuryak and I. Zahed, hep-ph/307267,
G.E. Brown,C.H. Lee, M. Rho and E. Shuryak, hep-ph/0312175;
M.Gyulassy and L.McLerran, Nucl-th/0405013



How does the soft mode affect a single quark spectrum near Tc?

Y. Nemoto (RIKENBNL , Nagoya U.)
M. Kitazawa (Kyoto)
T. K. (YITP)

(in preapration)

Method

- **low-energy effective theory of QCD**

  4-Fermi type interaction (Nambu-Jona-Lasinio with 2-flavor)

$$L = \bar{q} i\gamma \cdot q + G_S[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2]$$

τ：SU(2) Pauli matrices

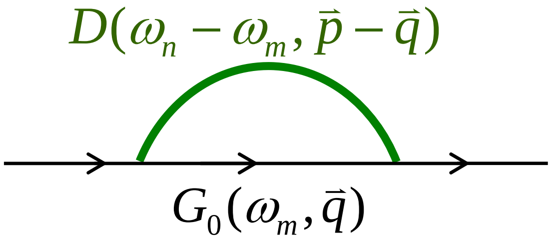
$$G_S = 5.5 \cdot 10^{-6}\mathrm{GeV}^{-2}, \Lambda = 631\mathrm{MeV}$$

$m_u = m_d = 0$ chiral limit finite m_u, m_d : future work

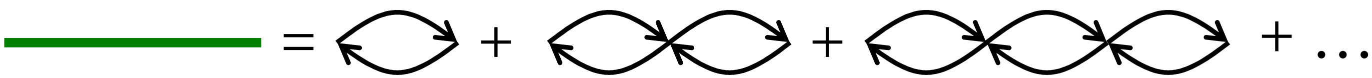
- **Chiral phase transition takes place at Tc=193.5 MeV(2nd order).**

- **Self-energy of a quark (above Tc)**

$$\Sigma(\omega_n, \vec{p}) = \mathrm{T}\sum_m \int \frac{d^3q}{(2\pi)^3} D(\omega_n - \omega_m, \vec{p} - \vec{q}) G_0(\omega_m, \vec{q})$$



$D(\omega_n, \vec{p}) =$



scalar and pseudoscalar parts

$$\Sigma^R(\omega, p) = \Sigma(\omega_n, p)|_{i\omega_n = \omega + i\varepsilon}$$: imaginary time → real time

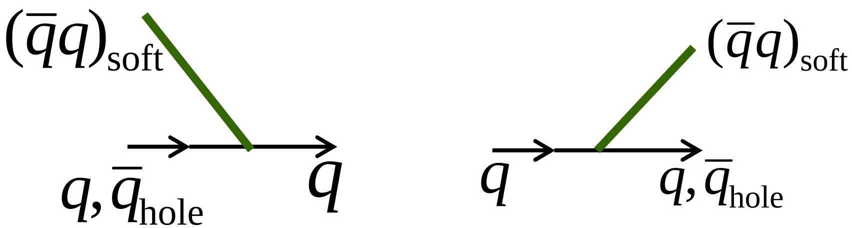
Self-Energy and Spectral Func.

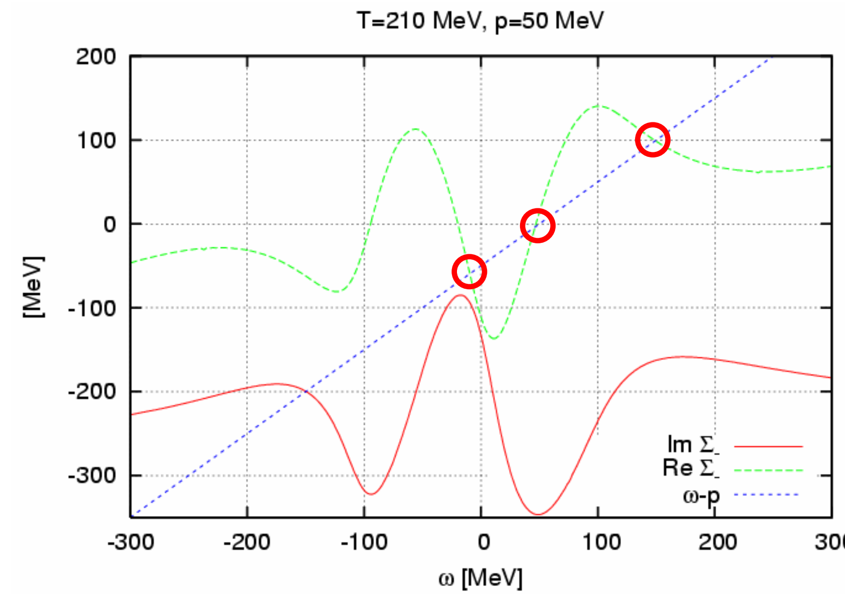
self-energy

dispersion relation for a quark

$$\omega - |\vec{p}| - \mathrm{Re}\,\Sigma_{-}(\omega, p) = 0$$

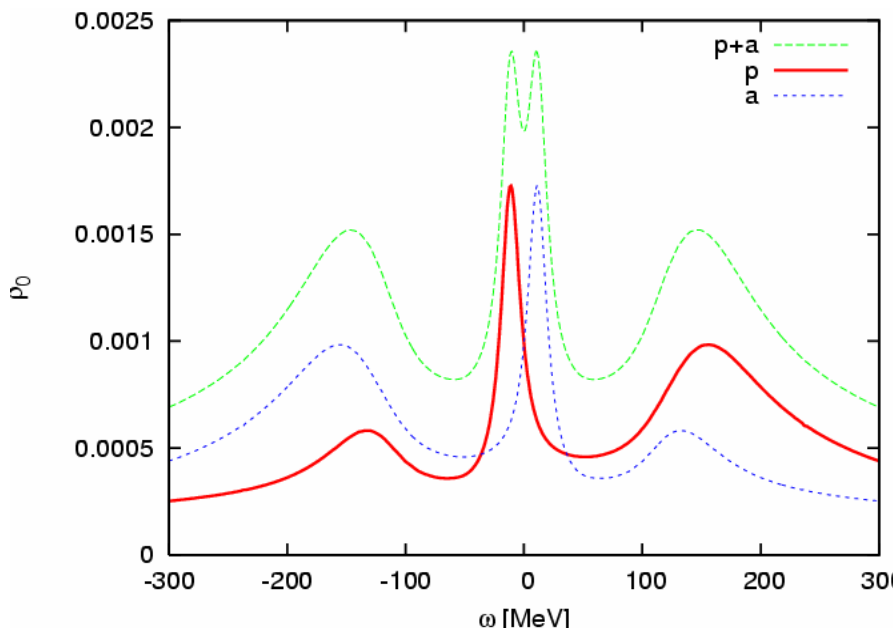
two resonant scatterings





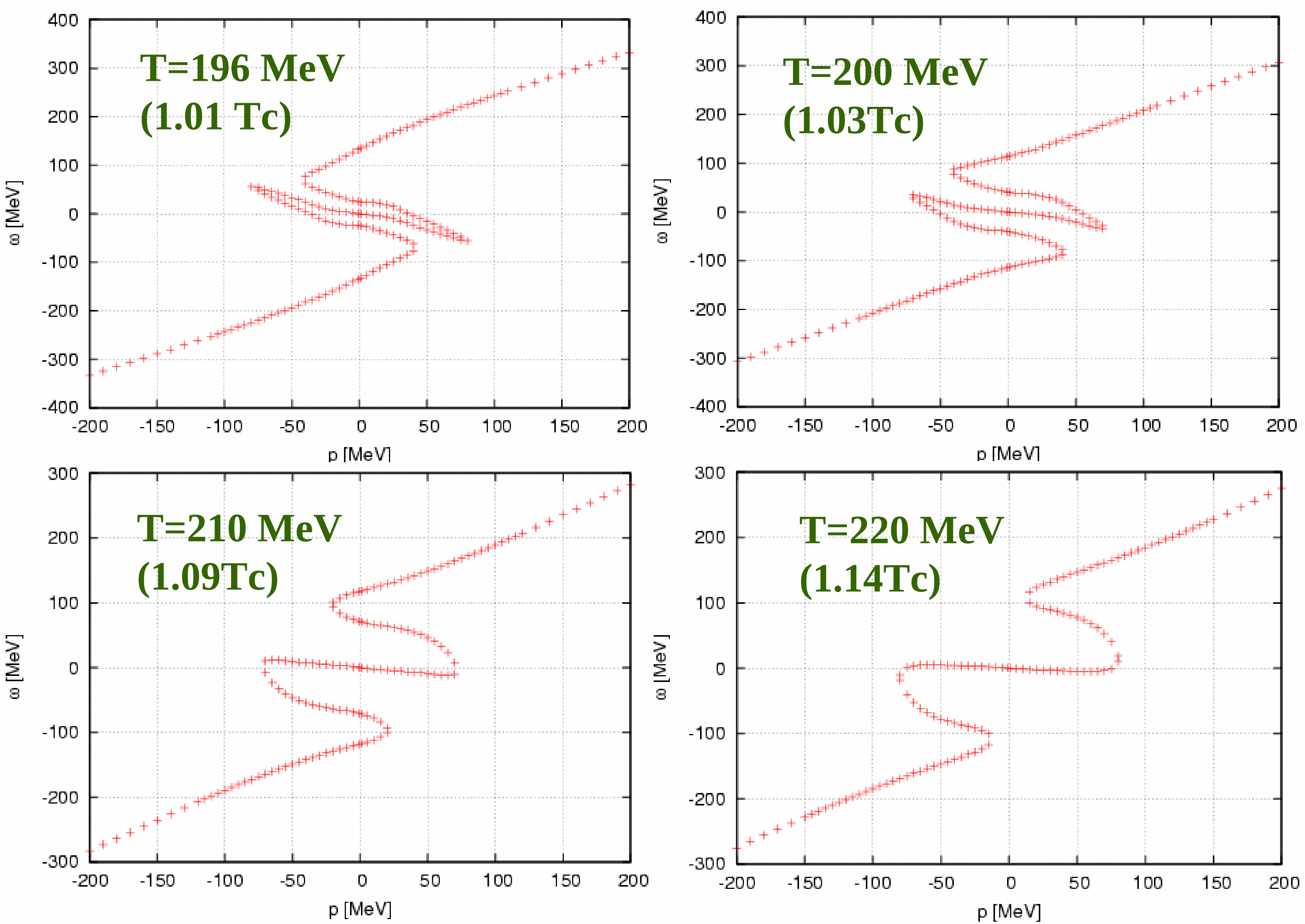
spectral function

three peaks: one is at ω>0 and the others are at ω<0.

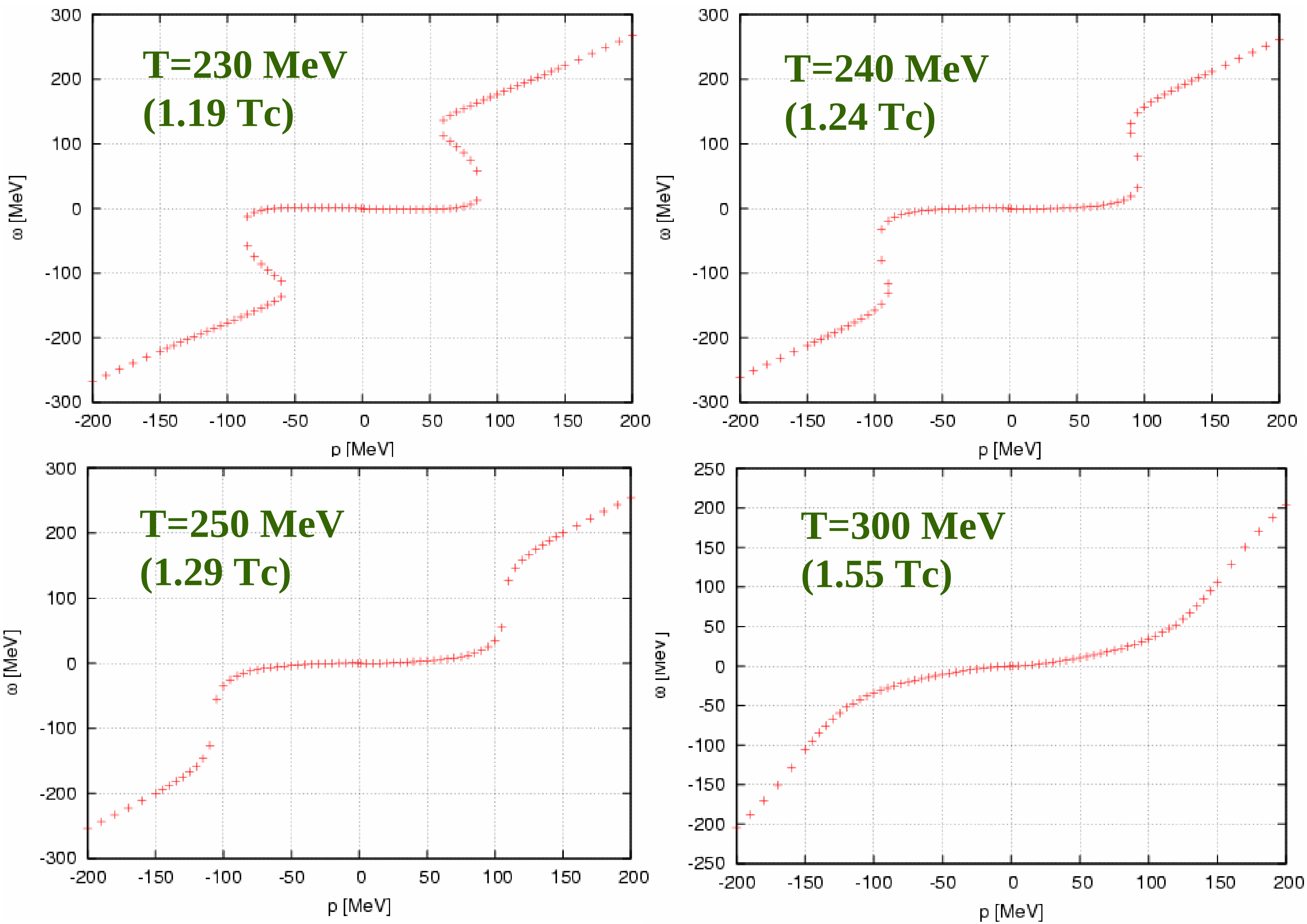


Antiquark sector is similar.

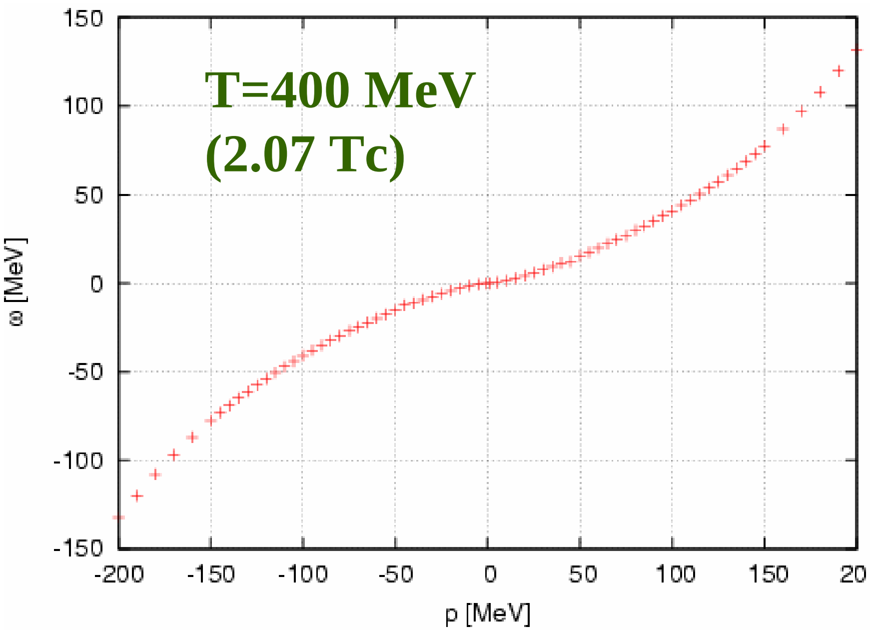
Dispersion Relations of Quarks

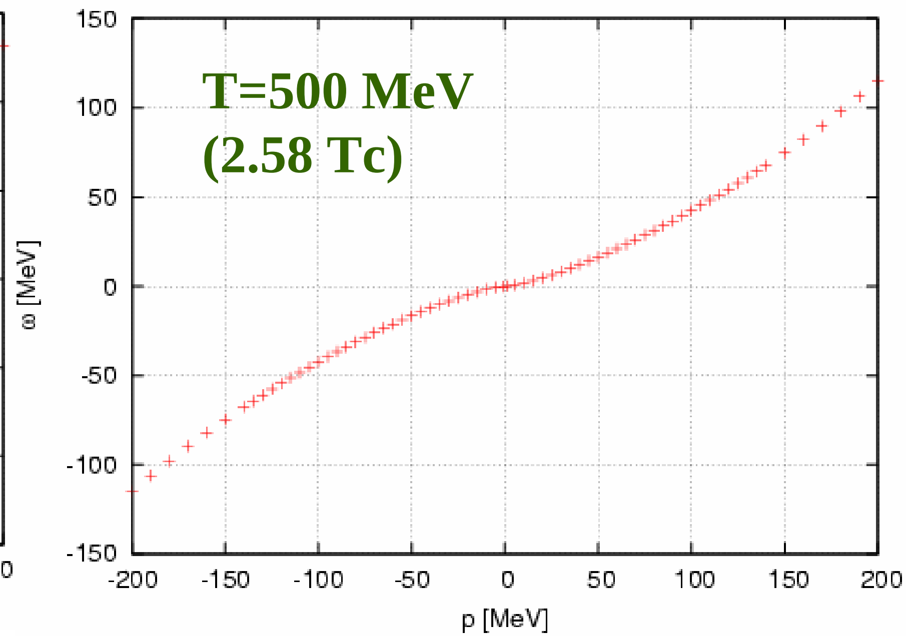


Dispersion Relations

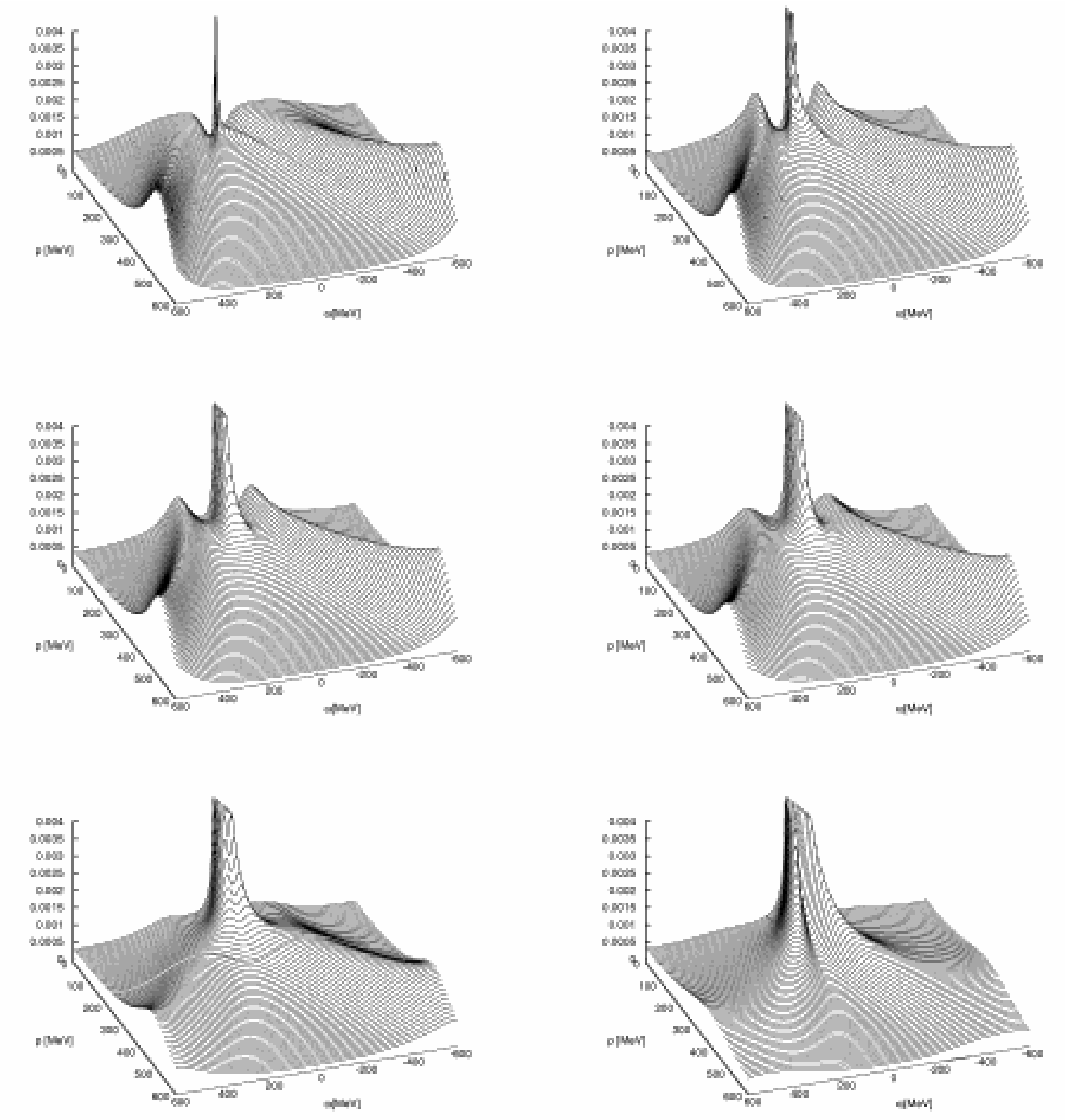


Dispersion Relations

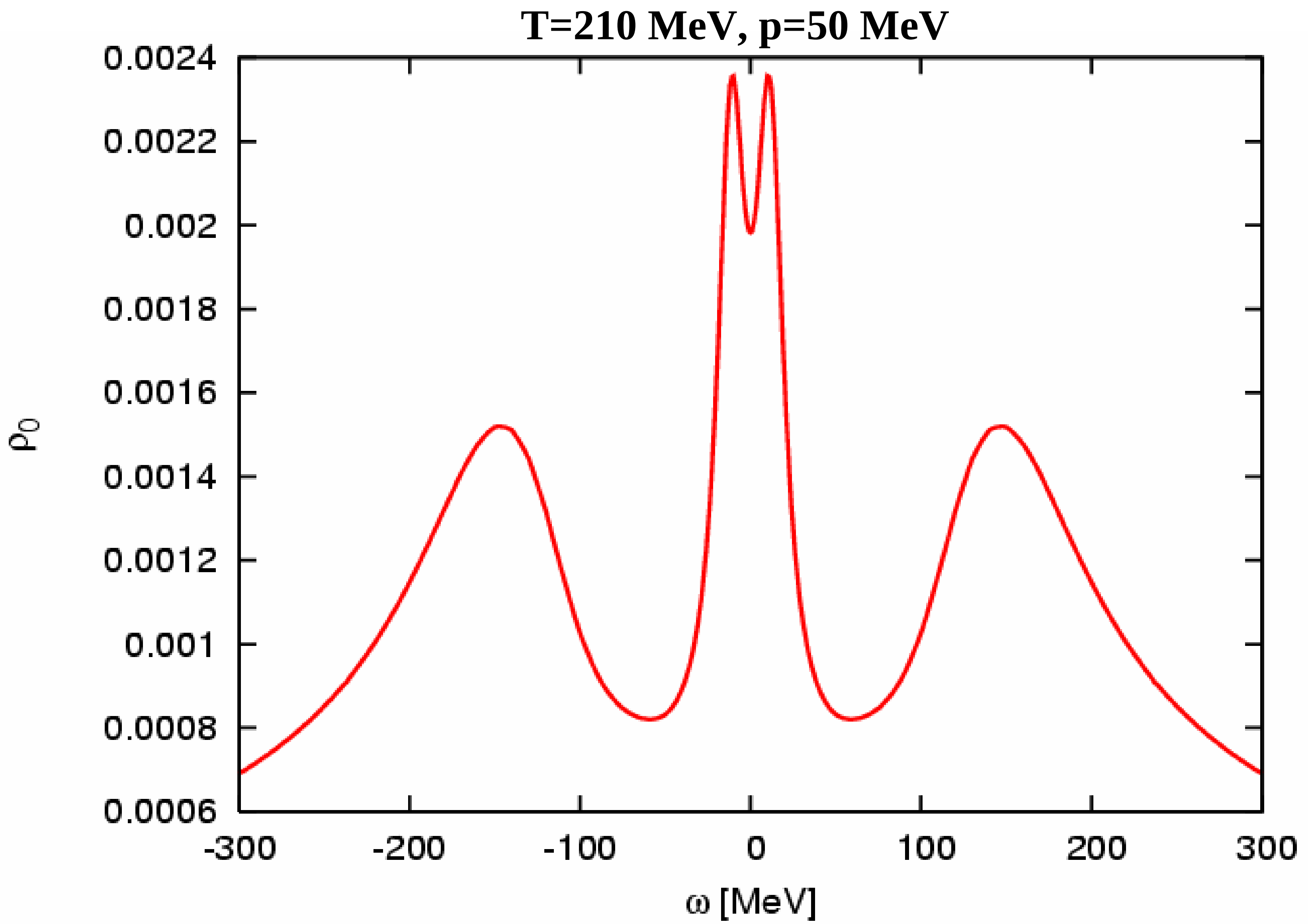




Spectral function of the quarks

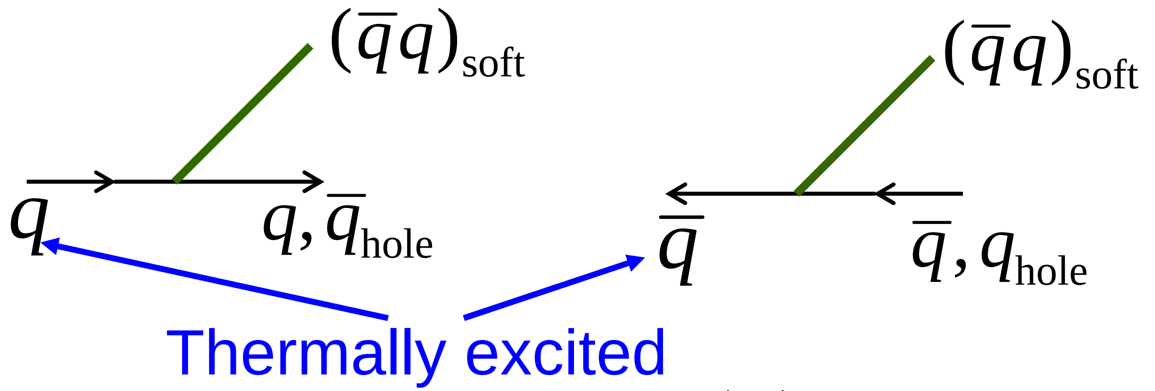


Spectral Function of Quarks

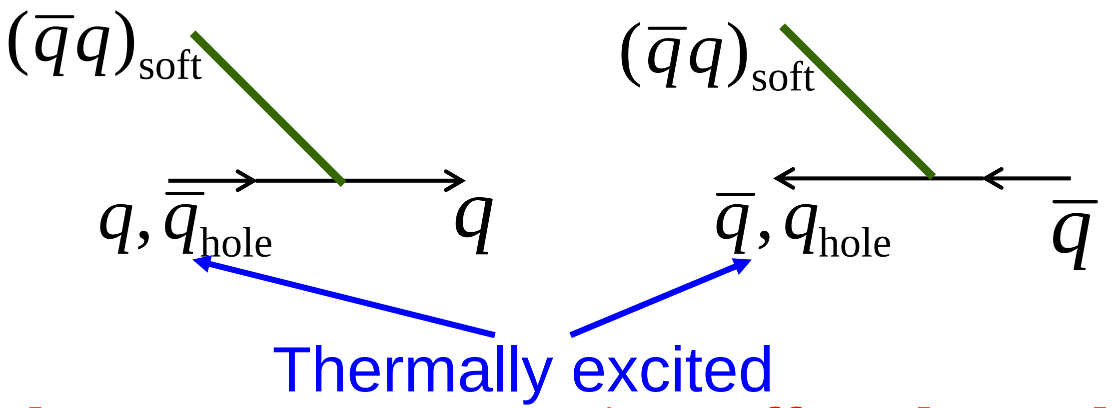


Resonant Scatterings

- **For the ω>0 soft mode,** $(\omega_q = \omega + \omega_{q'})$

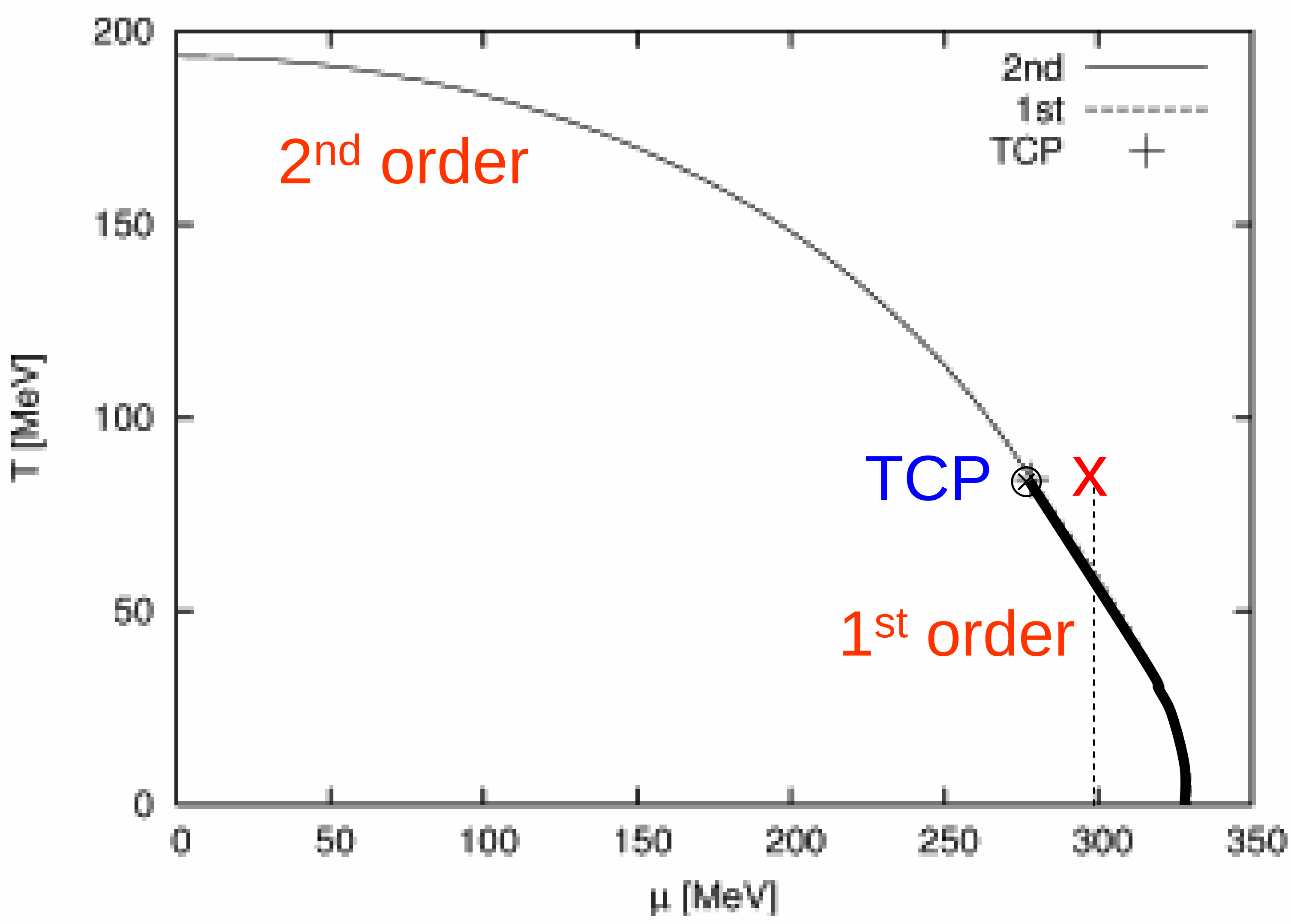


- **For the ω<0 soft mode,** $(\omega_q + |\omega| = \omega_{q'})$

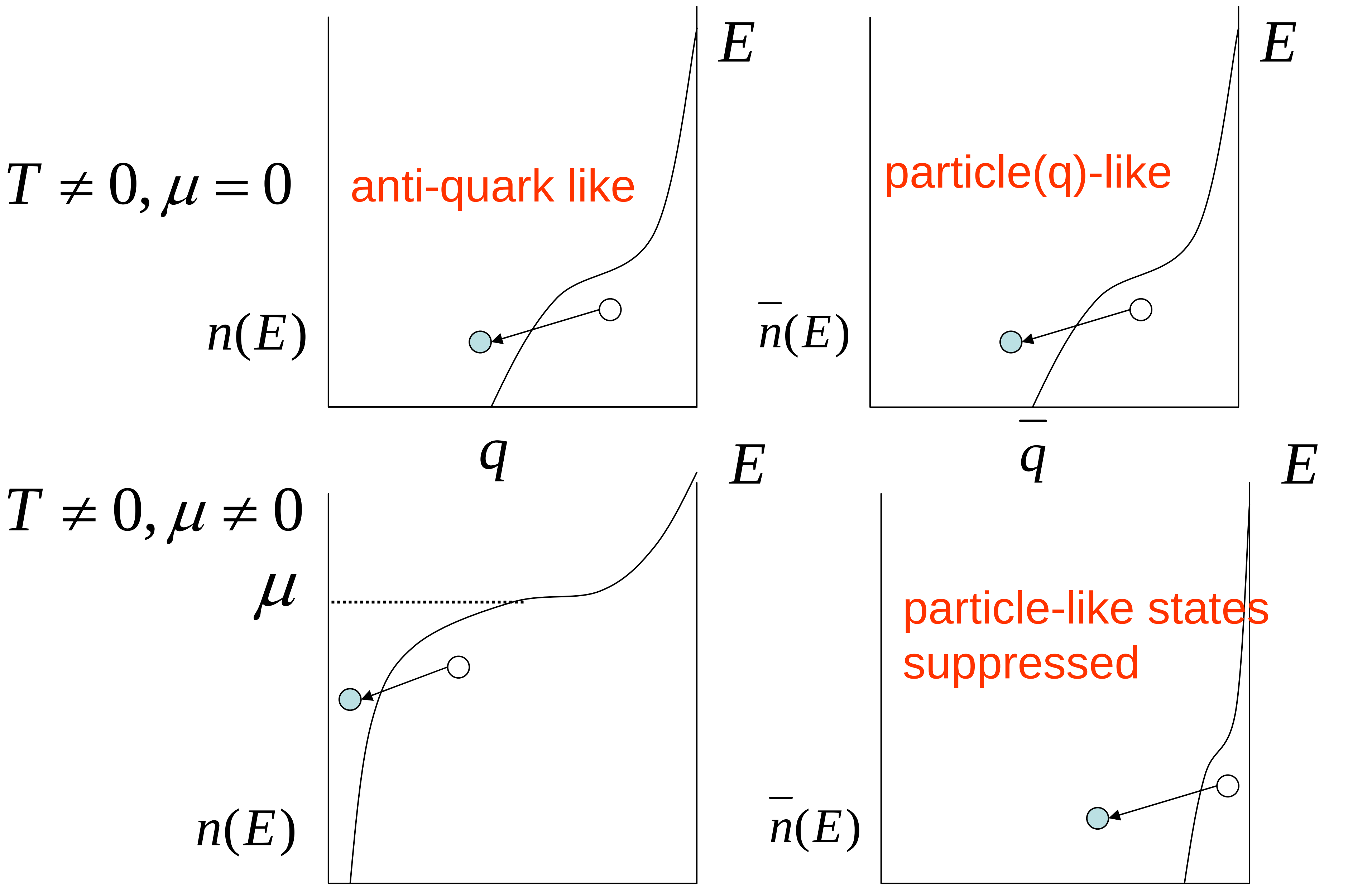


These resonant scatterings affect the peaks of the spectral functions in a non-trivial way.

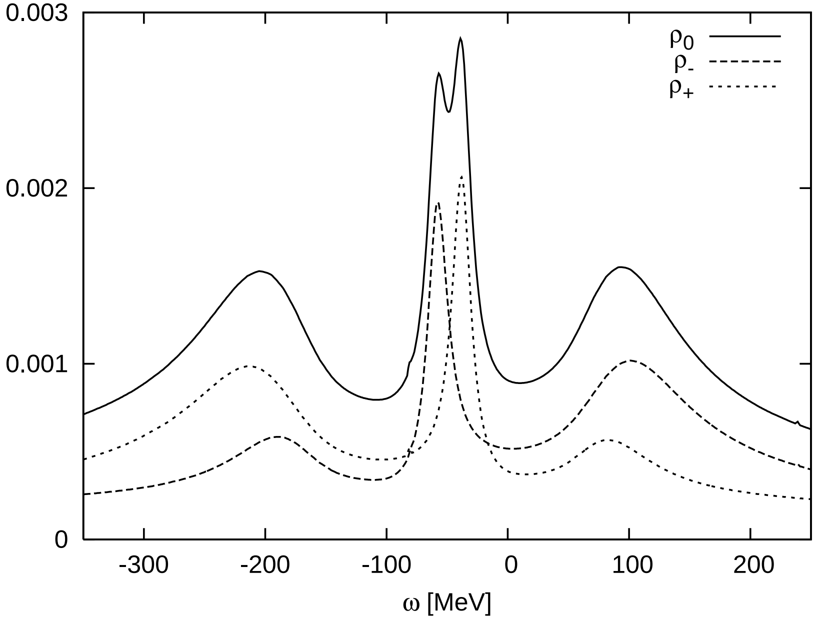
Phase diagram calculated in NJL model



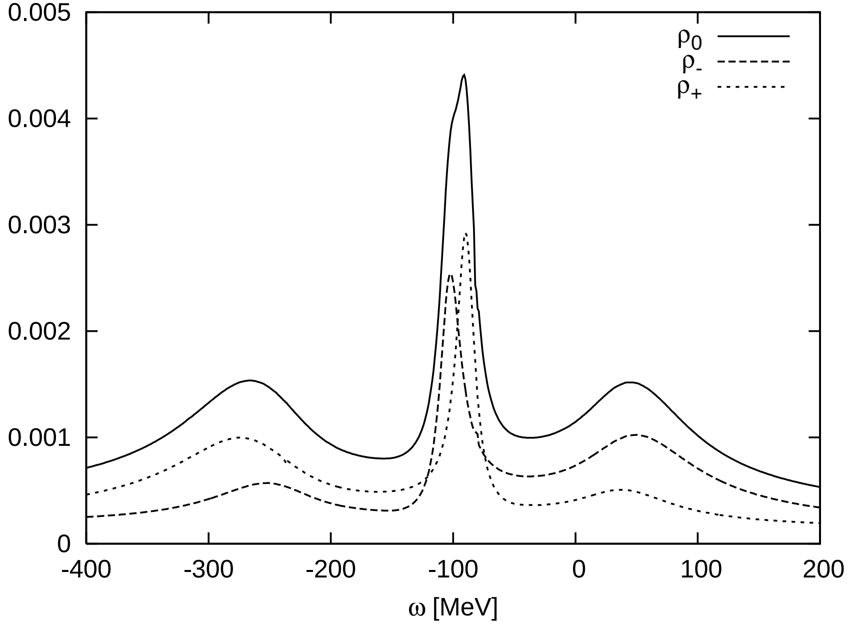
Finite μ dependence; asymmetry between q and $\bar{q}$



$(T,\mu)=(210,50)$



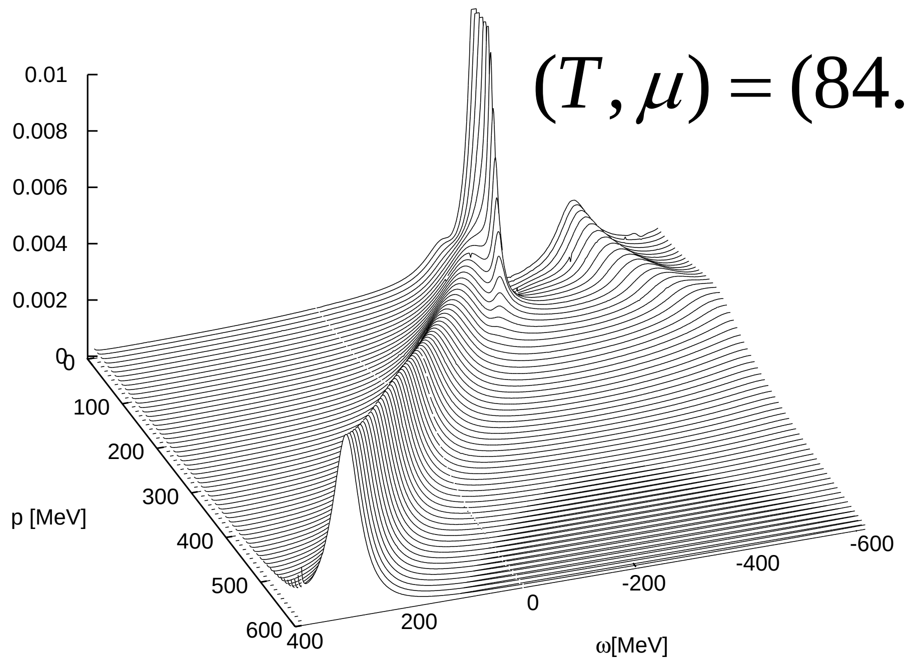
$(T,\mu)=(210,100)$



just around the
Tri-critical point;

$(T_c,\mu_c)=(84.2,278.6)$

$(T,\mu)=(84.16,300)$



Summary of this section

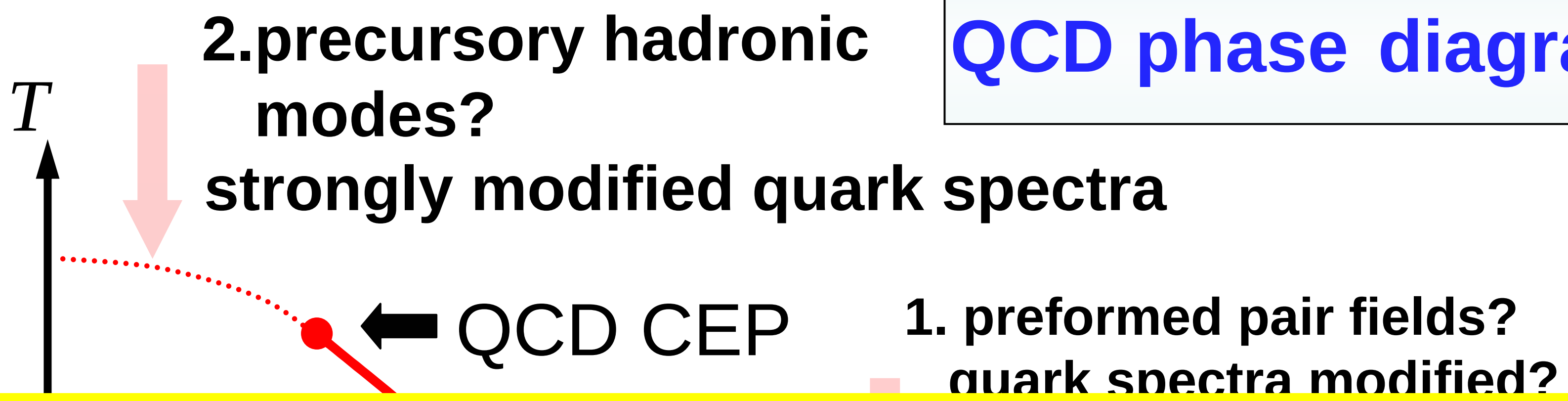
- **Near (above) Tc, the quark spectrum at long-frequency and long wave-length is modified drastically by the soft mode for the chiral condensate, $\langle \bar{q} q \rangle$.**
- **The many-peak structure of the spectral function can be understood in terms of two resonant scatterings at small ω and p of a quark and an antiquark.**
- **CSC : The Fermi surface is significant.**
  Chiral: Antiquarks are significant. (antiquark holes)

Future

- finite quark mass effects. (2^{nd} order → crossover)
- finite density (tricritical point, critical end-point)
- phenomenological applications

Summary of the Talk

QCD phase diagram



`QGP' itself seems surprisingly rich in physics!

Condensed matter physics of strongly coupled Quark-Gluon systems will constitute a new field of fundamental physics.