How do diquark fluctuations and chiral soft modes affect di-lepton production in the deconfined phase

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A conjectured QCD phase diagram

- $\chi_{SB}$
- $T$
- $T_c$
- $\mu$
- $\rho_0$
- QCD CEP
- QGP
- FAIR(GSI)
- Various phases
- CSC
- CFL
- H-dibaryon matter?
- superconducting phases
- various meson condensation?
- Ferromagnetism?
A conjectured QCD phase diagram

- QCD CEP
- QGP
- RHIC
- FAIR (GSI)
- Various phases
- CSC
- CFL
- Ferromagnetism?
- H-dibaryon matter?
- Various meson condensation?
- superconducting phases
- $\chi_{SB}$
Contents

• Part 1  The soft mode of CSC above Tc and lepton-pair production
• Part 2  The sigma mode above Tc at finite density and lepton-pair production
PART I

Precursory Phenomena of Color Superconductivity in Heated Quark Matter

Ref. M. Kitazawa, T. Koide, T. K. and Y. Nemoto
Phys. Rev. D70, 956003(2004);
Prog. Theor. Phys. 114, 205(2005),
M. Kitazawa, T.K. and Y. Nemoto,
M. Kitazawa and T. K., in preparation
Various CSC phases in $T-\mu$ plane

The phase in the highest temperature is 2SC or g2SC.

The phase in the highest temperature is 2SC or g2SC.
**Pair Fluctuations in CSC**

\[ D^R(x,t) = -2G_C \left\langle \left[ \overline{\psi}(x)i\gamma_5\tau_2\lambda_2\psi^C(x), \overline{\psi}(0)i\gamma_5\tau_2\lambda_2\psi^C(0) \right] \right\rangle \theta(t) \]

**Spectral Function of the diquark excitations**

\[ \rho(k,\omega) = -\frac{1}{\pi} \text{Im} D^R(k,\omega) \]

\[ D^R(k,\omega) = \sum \cdot \cdot \cdot \]

**Sharp peak from \( \varepsilon \sim 0.2 \)**

electric SC: \( \varepsilon \sim 0.005 \)

even in 2d-SC

**Existence of large pair fluctuations**


It may affect various observables even well above \( T_c \).
Precursory Phenomena

- quark dispersion
  - rapid increase around $p_F$
  - $\mu = 400$ MeV, $\varepsilon = 0.01$

- density of states
  - formation of the pseudogap

- specific heat
  - anomalous enhancement as $T \to T_c$

M. Kitazawa, T. Koide, T. K., Y. Nemoto, PRD 70, 056003 (’04)

QM~ High Tc SC

More accessible observables?
- dilepton-pair production, in H-I collisions.

But how?
Dilepton-pair Production Rate

\[ \frac{dR_{ee}}{d^4 q} = -\frac{\alpha}{12\pi^4 Q^2} \text{Im} \Pi^{R\mu}_\mu \frac{1}{e^{q^0/T} - 1} \]

**Photon Self-Energy**

How do the diquark fluctuations affect the photon self-energy?
Anomalous Self-energy of Photon; Aslamasov-Larkin term

Photon Self-Energy $\Pi$

$$\Pi_{AL}^{\mu\nu} = \frac{\hat{e}\gamma^\mu}{k} \cdots \frac{\hat{e}\gamma^\nu}{k}$$

$$= 3 \int d^4 q \Gamma^\mu(q, q + k) \Xi(q + k) \Gamma^\nu(q + k, q) \Xi(q)$$

- Factor 3 due to color degrees of freedom
- Pair field (T-matrix):
  $$\Xi(q) = 1 + \cdots$$

Vertex:
$$\Gamma^\mu(p_1, p_2) = \frac{\hat{e}\gamma^\mu}{p_2}$$

cf) Maki-Thompson term

Collective pairing oscillation

Sov. Phys. SS 10,875’68
Dilepton-pair Production

\[ \frac{dR_{ee}}{d^4q} = -\frac{\alpha}{12\pi^4Q^2}\text{Im}\Pi^{R\mu}_\mu \frac{1}{e^{q^0/T} - 1} \]

-per invariant mass

\[ \frac{dR_{ee}}{dM^2} = \int \frac{d^3q}{2q^0} \frac{dR_{ee}}{d^4q} \]

from AL-term

----- from free quarks

Preliminary enhancement at \( M < 150\text{MeV} \).

The peak becomes sharp as \( \varepsilon \to 0 \).

Possible experimental observable for the CSC to be seen in FAIR(GSI)?

Maybe difficult, unfortunately, because of the too-low mass enhancement.

M. Kitazawa and T.K., (2005), unpublished
Remarks:

* Effects of the Maki-Thompson term

* If $T_c$ is higher, say, 100 MeV or higher, the enhancement is more prominent.

How about, the lepton pair emission from the color-conducting phase?
Dilepton emission rate from the CFL phase

P. Jaikumar R. Rapp and I. Zahed ('02)

Enhancement due to photon-gluon mixing in the CFL phase, and from the generalized rho meson. (not shown here)
PART II

The Case of the Chiral Transition
Chiral Transition and the sigma mode (meson)

\[ \mathcal{V}(\sigma, \pi) \]

\[ T, \rho \]

\[ \pi \]

\[ \sigma \left( \approx \langle \bar{q}q \rangle \right) \]

\[ T > T_c \quad \rho > \rho_c \]

c.f. Higgs particle in WS model

\[ \phi ; \text{Higgs field} \quad \phi = \langle \phi \rangle + \tilde{\phi} \]

\[ \sigma = \sigma_0 + \tilde{\sigma} \]

\[ \tilde{\sigma} \]

\[ m_\sigma \]

\[ \pi \]

\[ \sigma \]

\[ T < T_c \quad \rho < \rho_c \]
Fig. 2. Deconfinement and chiral symmetry restoration in 2-flavour QCD: Shown is $\langle L \rangle$ (left), which is the order parameter for deconfinement in the pure gauge limit ($m_q \to \infty$), and $\langle \bar{\psi} \psi \rangle$ (right), which is the order parameter for chiral symmetry breaking in the chiral limit ($m_q \to 0$). Also shown are the corresponding susceptibilities as a function of the coupling $\beta = 6/g^2$. 
Cf. Lattice Calculation of the generalized masses

F. Karsch, Lect. Note Phys. 583 (2002), 209. \( N_f = 2, 8^3 \times 4; \) Staggered fermion

\[ m_{\sigma}^2 = \chi_{\sigma}^{-1} \]

\[ \chi_{\sigma} = \langle (\bar{q}q)^2 \rangle \]

the softening of the \( \sigma \) with increasing \( T \)

and

a degeneracy of the \( \sigma \) and \( \pi \) at high \( T \)

What is the significance of the \( \sigma \) in hadron physics?
How about above $T_c$?
Interest in the nature of elementary modes in ‘QGP’ phase

RHIC experiments
- robust collective flow
  - good agreement with rel. hydro models
  - almost perfect liquid

(quenched) Lattice QCD
charmonium states up to 1.6-2.0 Tc
(Asakawa et al., Datta et al., Matsufuru et al. 2004)

Strongly coupled plasma rather than weakly interacting gas
The spectral function of the degenerate hadronic ``para-pion'' and the ``para-sigma'' at $T>T_c$ for the chiral transition: $T_c=164$ MeV


- response function in RPA
  
  $$D(k, \omega) = \begin{array}{c}
  \text{graph} \\
  + \\
  \text{graph} \\
  + \\
  \text{graph} \\
  + \\
  \ldots
  \end{array}$$

- spectral function
  
  $$A(k\omega) = -\frac{1}{\pi} \text{Im} D(k\omega)$$

$T \rightarrow T_c$, they become elementary modes with small width!

1. two $\gamma$ decay
2. modified quark spectrum

M. Kitazawa, Y. Nemoto and T.K. (05)
Finite $T$ and $\mu$ with finite quark mass
Phase diagram

$m_{q0} = 0$

$m_{q0} = 5.5$ MeV

Asakawa, Yazaki, (1989)
Caveats
Effects of $G_V$ on Chiral Restoration

As $G_V$ is increased,
- Chiral restoration is shifted to higher densities.
- The phase transition is weakened.

Asakawa, Yazaki ’89 /Klimt, Lutz, & Weise ’90 /T.K. ’90/ Buballa, Oertel ’96

What would happen when the CSC joins the game?
With color superconductivity transition incorporated

\( G_V / G_S = 0.35 \)

M. Kitazawa et al ('02)

(2) The first order transition between \( \chi_{SB} \) and CSC phases is weakened and eventually disappears.

(3) The region of the coexisting phase becomes broader.

Appearance of the coexisting phase becomes robust.

(4) Another end point appears from lower temperature, and hence there can exist two end points in some range of \( G_V \): \( 0.33 \sim G_V \sim 0.38 \)
Smooth variation of the quark condensate with baryon density?

S. Klimt, M. Lutz and W. Weise, PLB249 ('90)

So strong vector coupling making the chiral transition crossover at finite density!
Phase diagram

$m_{q0} = 0$

$m_{q0} = 5.5$ MeV

Asakawa, Yazaki, (1989)
What is the soft mode at CP?

Sigma meson has still a non-zero mass at CP. This is because the chiral symmetry is explicitly broken.

What is the soft mode at CP?

Phonon mode in the space-like region softens at CP.


See also, D. T. Son and M. Stephanov (2004)

It couples to hydrodynamical modes, leading to interesting dynamical critical phenomena.

Spectral function of the chiral condensate  
T-dependence ($\mu = \mu_{CP}$)
Dilepton production rate from the sigma mode at $T>T_c$

What is contributions of the sigma mode?

Enhancement around $m_\sigma \approx 2M_\phi$.

Spectral function of the sigma mode
Vector-scalar mixing

\[ \Pi_{\mu\nu}(q_0, \vec{q}) = e_q \gamma_\mu \begin{array}{c} \text{V-S mixing} \\ \text{V-S mixing} \end{array} \]

The above vector-scalar mixing exists if

\[ \begin{align*}
\mu &\neq 0 \\
m_q &\neq 0 \\
q &\neq 0
\end{align*} \]

Not SU(3) limit \((m_u = m_d = m_s)\)

ex: SU(2) symmetry \((m_q = m_u = m_d)\)

\[ \propto N_C (e_u + e_d) m_q \int d^3 k F(q_\mu, \vec{k}) [f(E_k - \mu) - f(E_k + \mu)] \]

cf: quark number susceptibility through the vs-mixing
Kunihiro, 1991

cf: dilepton production due to \(\sigma - \omega\) and \(\sigma - \gamma\) mixings in hadronic matter
Weldon, 1992
Wolf, Friman, Soyeur, 1998

without the notion of chiral transition, nor softening of the sigma
Di-electron Production Rate

$$\frac{d\Gamma}{dM^2} = \int \frac{d^3 q}{2q_0} \frac{d\Gamma}{d^4 q}$$

$$\frac{d\Gamma}{d^4 q} = -\alpha \text{Im} \Pi_{\mu}^{R\mu}(q_0, q) \frac{1}{12\pi^4 q^2 (\exp[q_0/T] - 1)}$$

$$\Pi_{\mu}^{R\mu}(q_0, q) = e_{q}^{\gamma\mu} e_{q}^{\gamma\mu} + e_{q}^{\gamma\mu}$$

Enhancement around $m_\sigma \approx 2M_q$ at $T > T_c$
Di-muon Production Rate along T-axis

\[
\frac{d\Gamma}{dM^2} = \int \frac{d^3q}{2q_0} \frac{d\Gamma}{d^4q} = \frac{-\alpha \text{Im} \Pi_{\mu}^{R\mu}(q_0, q)}{12\pi^4 q^2 \left(\exp\left[q_0 / T\right] - 1\right)} \left(1 + \frac{2m_\mu^2}{q^2}\right) \left(1 - \frac{4m_\mu^2}{q^2}\right)^{1/2}
\]

\[
\Pi_{\mu}^{R\mu}(q_0, \bar{q}) = e_q \gamma_\mu + e_q \gamma_\nu
\]
Di-electron Production Rate

along a pseudo-critical line

\[
\frac{d\Gamma}{dM^2} = \int \frac{d^3q}{2q_0} \frac{d\Gamma}{d^4q} = \frac{1}{12\pi^4 q^2} \frac{-\alpha \text{Im} \Pi_{\mu}^{R\mu}(q_0, q)}{\exp[q_0/T] - 1}
\]

\[
\Pi_{\mu}^{R\mu}(q_0, q) = e_q \gamma_{\mu} e_q \gamma_{\mu} + e_q \gamma_{\nu} e_q \gamma_{\nu}
\]

Graph showing di-electron production rate along a pseudo-critical line, with and without fluctuations.
Di-muon Production Rate along a pseudo-critical line

\[
\frac{d\Gamma}{dM^2} = \int \frac{d^3 q}{2q_0} \frac{d\Gamma}{d^4 q} = \frac{\alpha \text{Im} \Pi_{\mu}^{R\mu}(q_0, q)}{12\pi^4 q^2 (\exp[q_0/T] - 1)} \left( 1 + \frac{2m_{\mu}^2}{q^2} \right) \left( 1 - \frac{4m_{\mu}^2}{q^2} \right)^{1/2}
\]

\[
\Pi_{\mu}^{R\mu}(q_0, \bar{q}) = e_q^\gamma e_{q_\nu} + e_q^\gamma e_{q_\nu}
\]

Graph showing di-muon production rate as a function of mass (M) with different temperatures (Tc) and with/without fluctuations.
What is the origin of an enhancement around 0.3 GeV?
Remarks on the sigma mode in the hadronic phase at finite $T$ and density.
Lattice Calculations in full QCD of the sigma mass

The SCALAR collaboration, PRD70, 034504(2004)
The poles of the S matrix in the complex mass plane for the sigma meson channel:


\[ \Im M \] = \Gamma / 2

See also, I. Caprini, G. Colangelo and H. Leutwyler, PRL(2006); H. Leutwyler, hep-ph/0608218; \( M_{\sigma} = 441 - i 272 \) MeV

Softening?
A conjectured QCD phase diagram

see T. Hatsuda and T.K., nucl-th/0112027; contribution to Chiral 01.
Fig. 6. Spectral function for the $\sigma$ at $\rho_B / \rho_0 = 2$.

see also, G. Wolf et al (1998);
O. Teodorescu et al (2001)
Summary and concluding remarks

• The notion of the soft modes of QCD phase transitions was emphasized; they may be hadronic excitations above Tc.

• The soft modes of color-superconductivity above Tc may cause an enhancement of the electron-pair production in very-low mass region.

• Off the chiral limit, the lepton-pair production due to the specific mode around the QCD critical point is enhanced around \(2 \text{ MeV} \sim 300-400\text{MeV}\), which might account (at least partly) for the excess of lepton pairs seen in CERES experiment.