

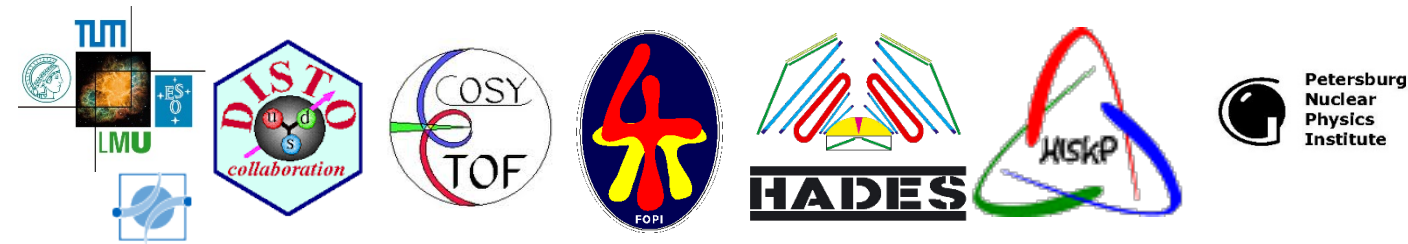
# Partial Wave Analysis of Strangeness Production at GeV Energies

Laura Fabbietti

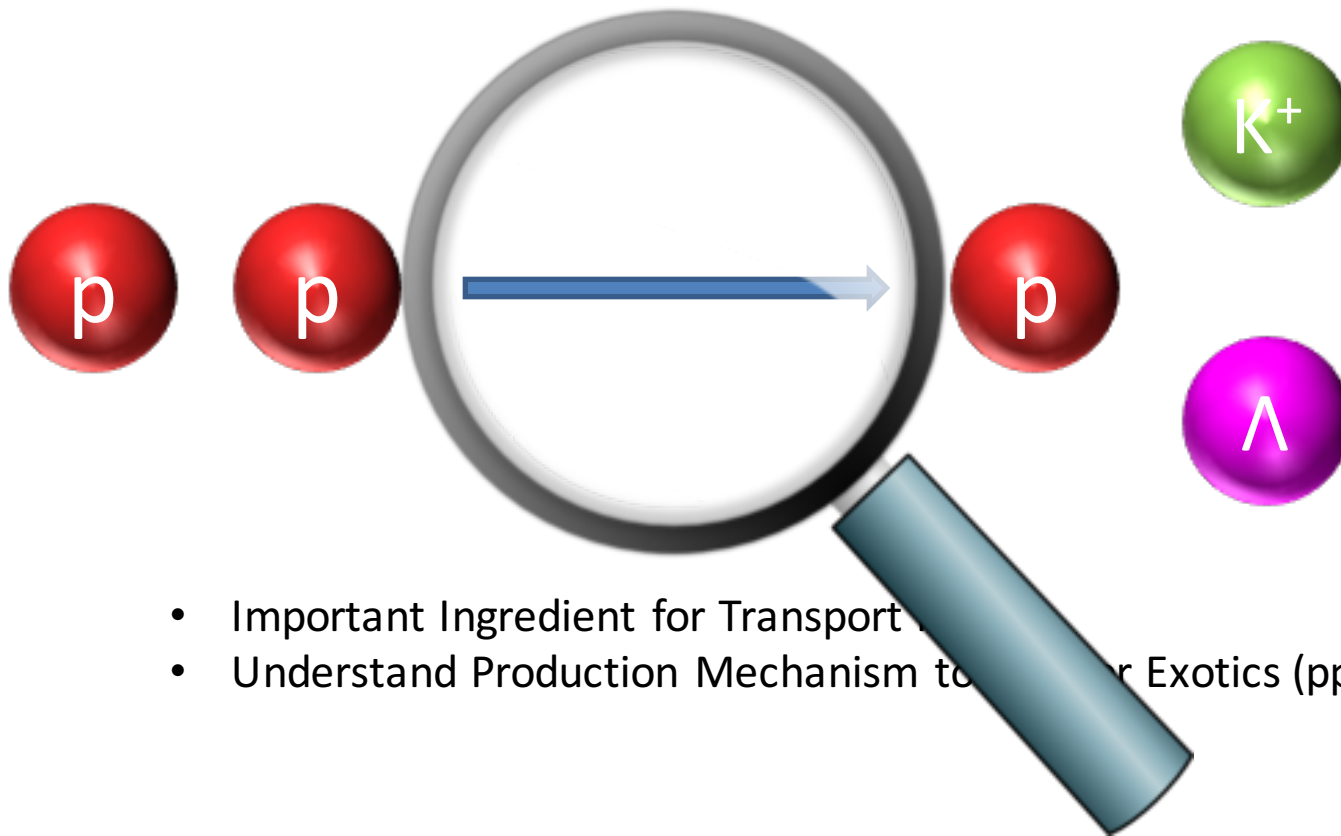
Robert Münzer, Shuna Lu,

Technische Universität München

Excellence Cluster – Origin of the Universe

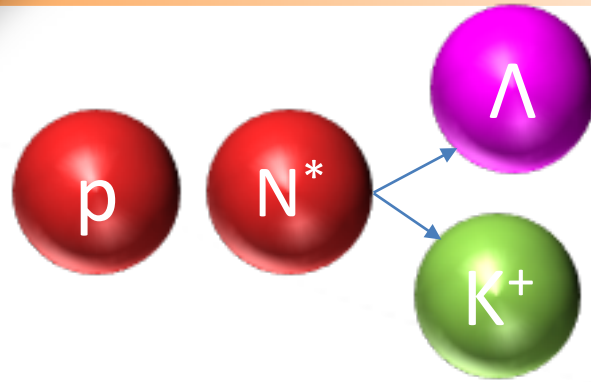


# Strangeness Production



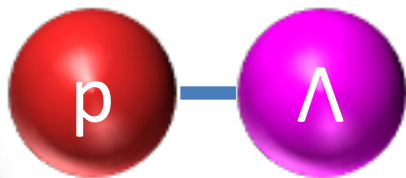
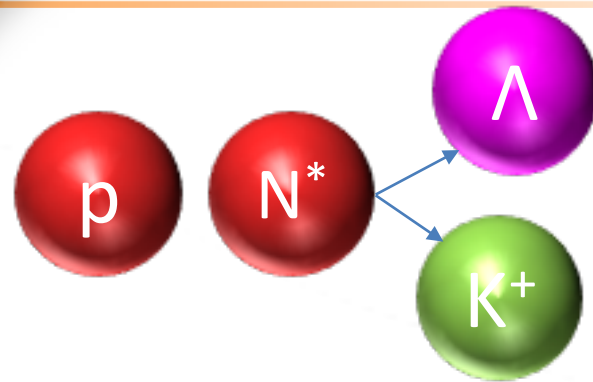
- Important Ingredient for Transport
- Understand Production Mechanism to ... Exotics (ppK<sup>-</sup>)

# Strangeness Production

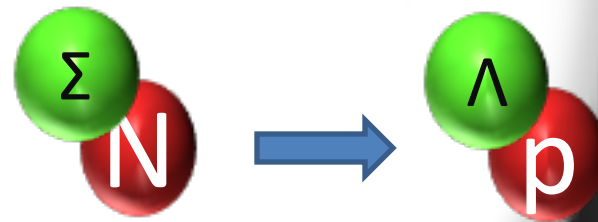


Resonance	$J^P$	Mass ( $GeV/c^2$ )	$\Gamma$ ( $MeV/c^2$ )
N*(1650)	$1/2^-$	1.655	0.150
N*(1710)	$1/2^+$	1.710	0.100
N*(1720)	$3/2^+$	1.720	0.250
N*(1875)	$3/2^-$	1.875	0.220
N*(1880)	$1/2^+$	1.870	0.235
N*(1895)	$1/2^-$	2.090	0.090
N*(1900)	$3/2^+$	1.900	0.0250

# Strangeness Production

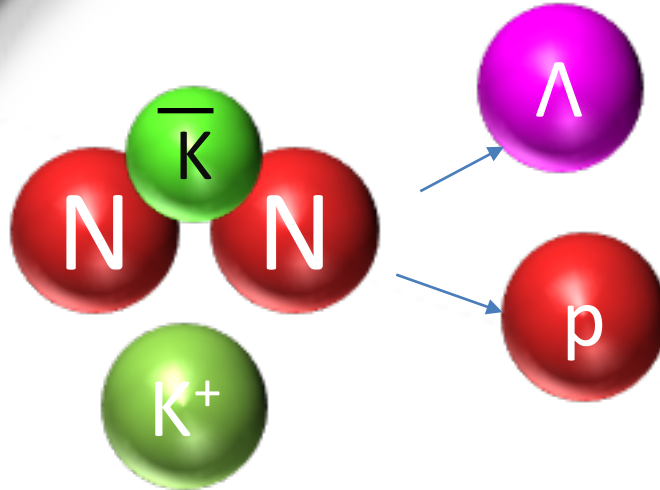


Final State Interaction  
Aka: scattering length and  
effective range



Conversion Processes

# Strangeness Production



Kaonic Bound States

# Partial Wave Analysis

## Bonn-Gatchina PWA Framework

A. Sarantsev et.al., Eur.Phys J A 25 2005

### Cross-section Decomposition

$$d\sigma = \frac{(2\pi)^4 |A|^2}{4|k|\sqrt{s}} d\phi(P, q_1, q_2, q_3), \quad P = k_1 + k_2$$

$A$  : reaction amplitude  $A \propto A \propto A_{tr}^\alpha(s)$  (Transition amplitude of wave  $\alpha$ )

$k$  : 3-momentum of the initial particle in the CM

$s - P^2 : (k_1 + k_2)^2$

$d\phi(P, q_1, q_2, q_3)$ : invariant three-particle phase space

### Parameterization of the Transition

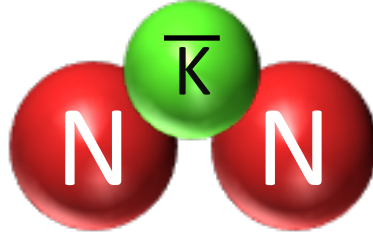
$$A_{tr}^\alpha(s) = (a_1^\alpha + a_3^\alpha \sqrt{s}) e^{a_2^\alpha}$$

$a_1^\alpha$  Constant amplitude

$a_2^\alpha$  Phase

$a_3^\alpha$  Energy dependent amp.

# Kaonic Cluster



## Theoretical Predictions

Chiral, energy dependent

	var. [DHW09, DHW08]	Fad. [BO12b, BO12a]	var. [BGL12]	Fad. [IKS10]	Fad. [RS14]
$BE$	17–23	26–35	16	9–16	32
$\Gamma_m$	40–70	50	41	34–46	49
$\Gamma_{nm}$	4–12	30			

Non-chiral, static calculations

	var. [YA02, AY02]	Fad. [SGM07, SGMR07]	Fad. [IS07, IS09]	var. [WG09]	var. [FIK <sup>+</sup> 11]
$BE$	48	50–70	60–95	40–80	40
$\Gamma_m$	61	90–110	45–80	40–85	64–86
$\Gamma_{nm}$	12			~20	~21

**Binding Energy (BE):**

10–100 MeV

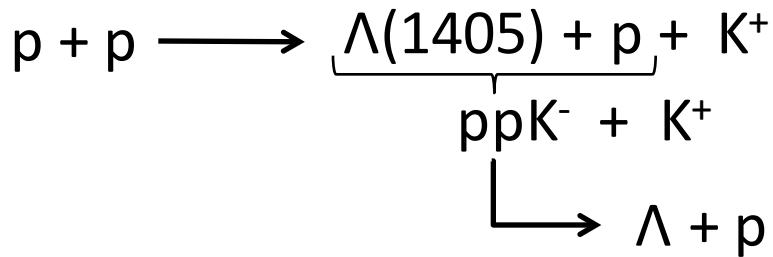
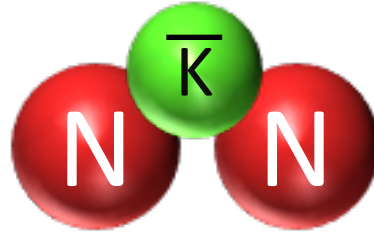
**Mesonic Decay ( $\Gamma_m$ )**

30–110 MeV

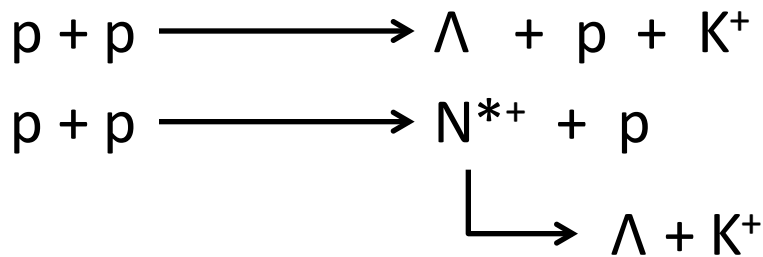
**Non-Mesonic Decay ( $\Gamma_{nm}$ )**

4–30 MeV

# Kaonic Cluster



Physical Background:



## N\*+ - Resonances

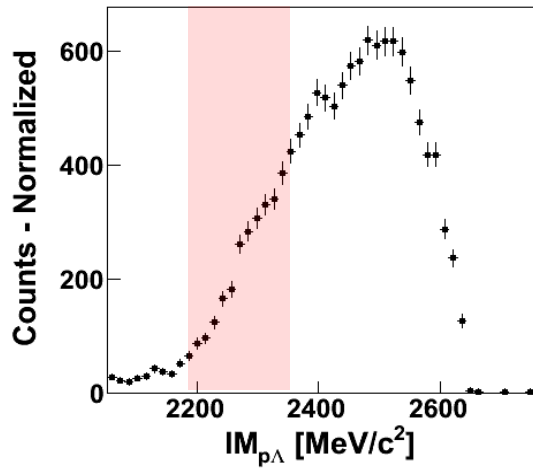
J. Beringer  
Phys.Rev. D86 (2012)

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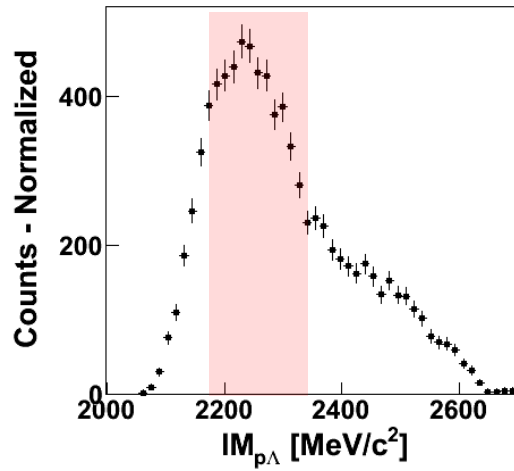


# Total Data Set

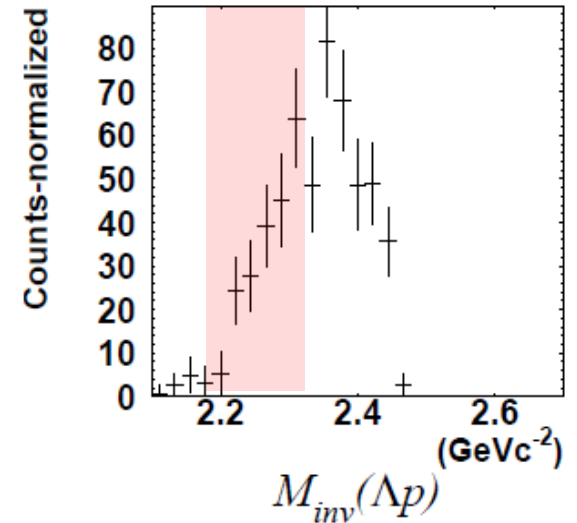
Hades Data  $E_{\text{beam}}=3.5$  GeV



Had. Wall Data  $E_{\text{beam}}=3.5$  GeV



FOPI Data  $E_{\text{beam}}=3.1$  GeV



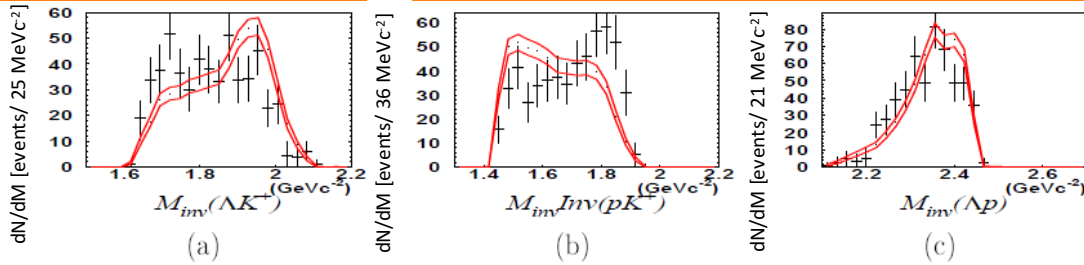
No Peak Visible  
No Signal?

R. Münzer, PhD Thesis, TUM 2014

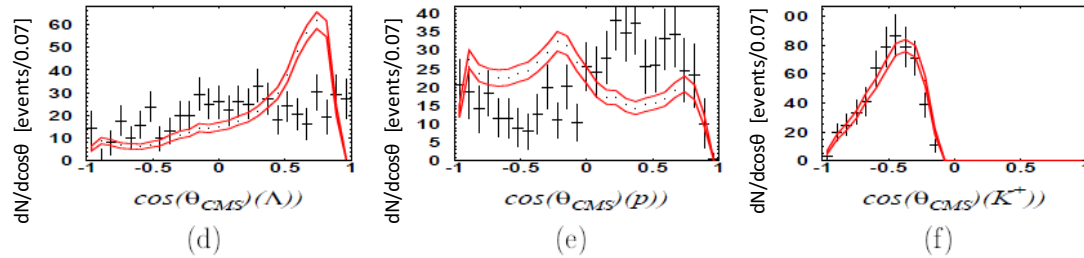
E. Epple, PhD Thesis, TUM 2014

# Phase Space Simulation

Masses



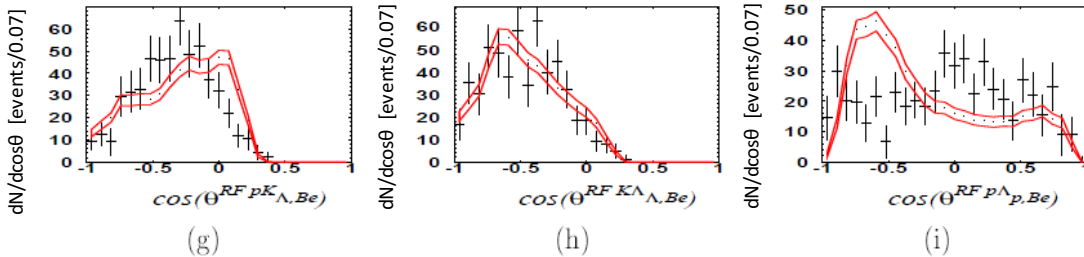
CMS Angle



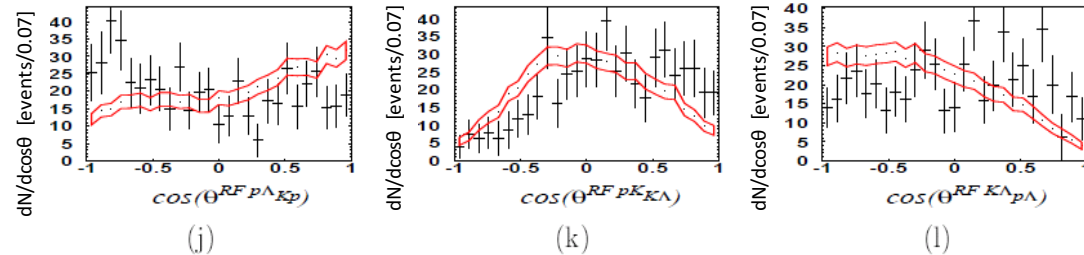
✦ Experimental Data

—  $pp \rightarrow p K^+ \Lambda$  Phase Space

G.-J.-Angle

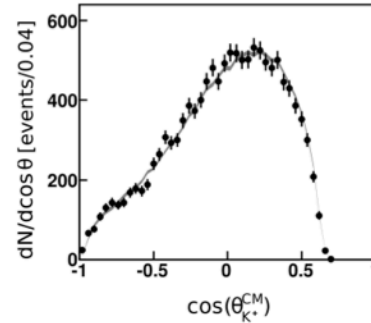
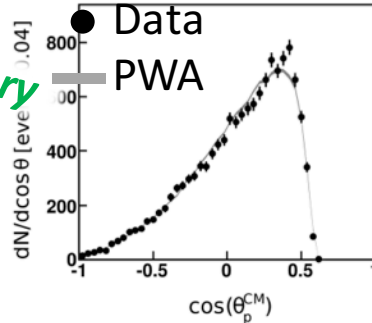
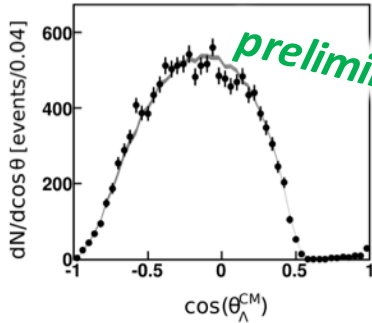


Hel. - Angle



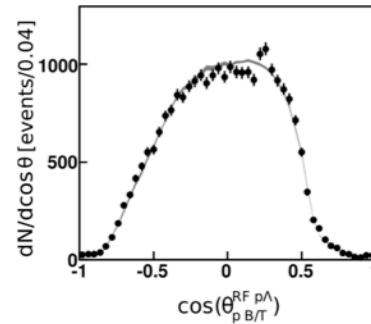
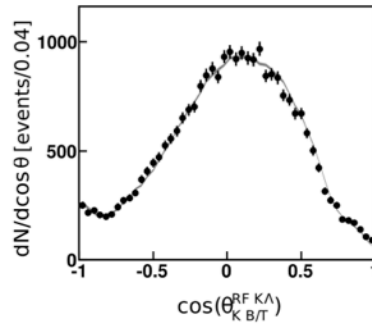
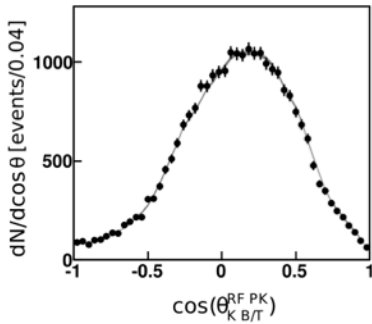
# Four Best PWA Solutions

CMS Angle



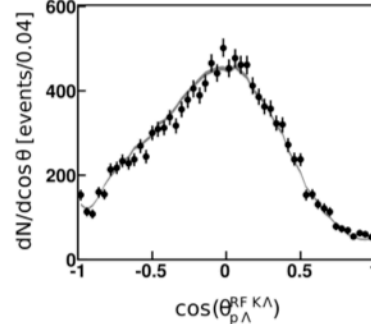
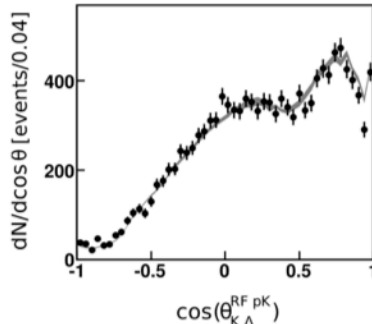
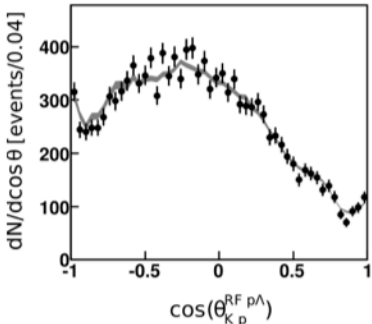
Inside HADES acceptance

G.-J.-Angle



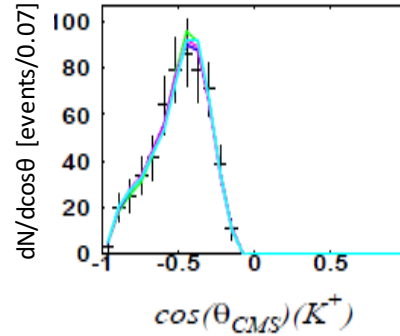
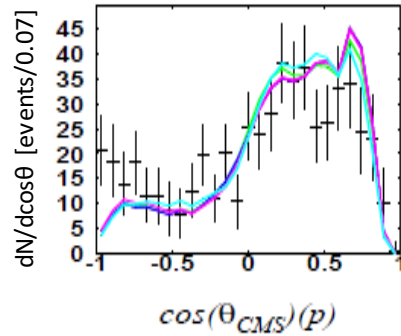
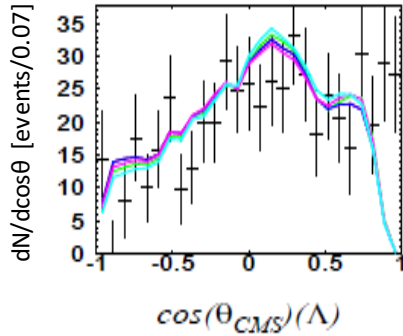
Measured Data  
PWA solutions

Hel. - Angle



# PWA Results

CMS Angle



✦ Experimental Data

— Solution A

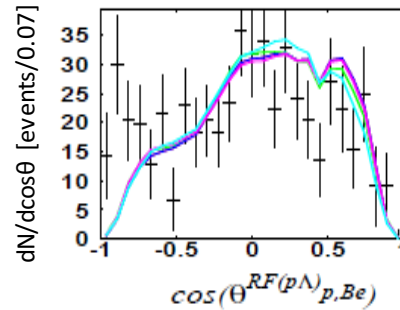
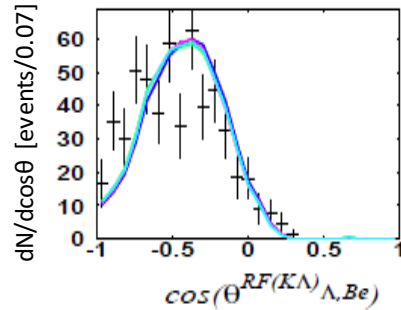
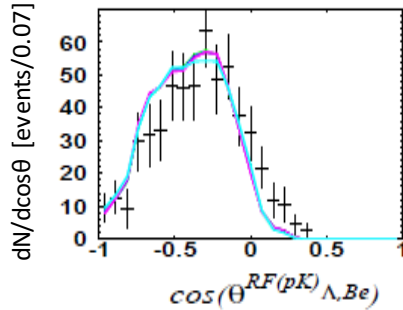
— Solution B

— Solution C

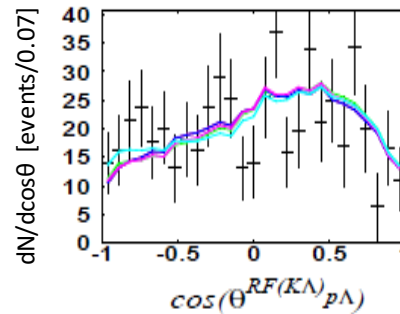
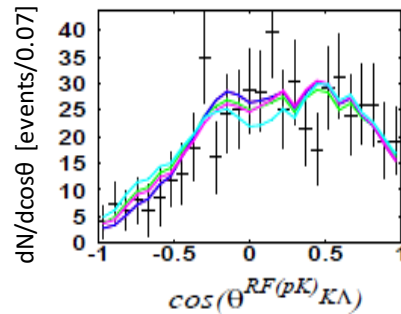
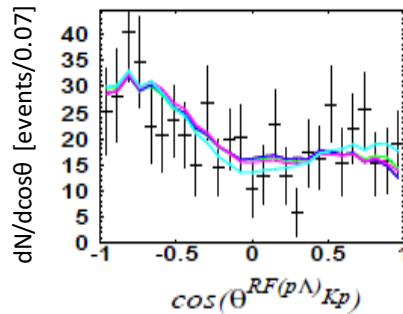
— Solution D

— Solution E

G.-J.-Angle

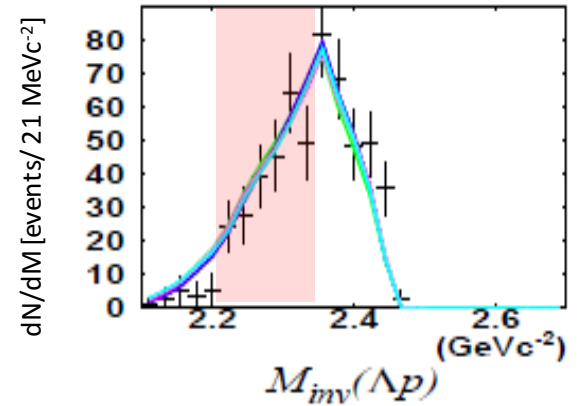
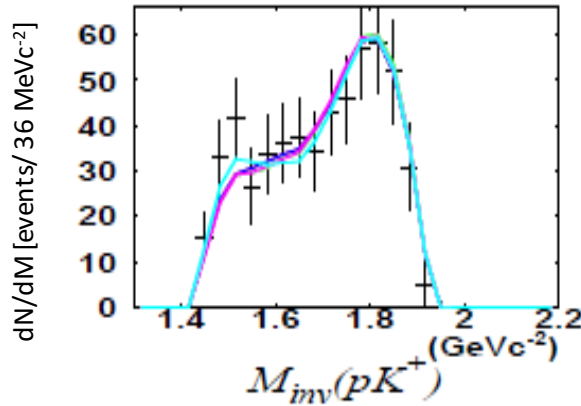
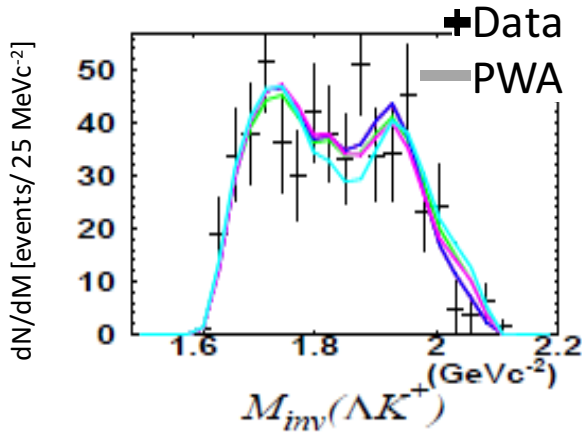


Hel. - Angle

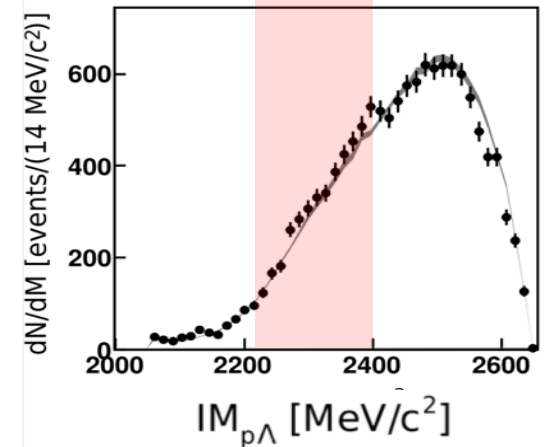
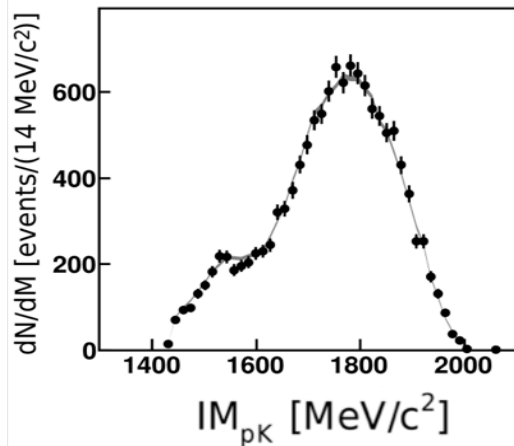
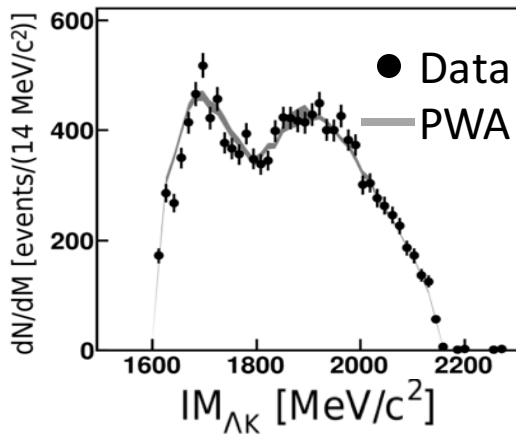


# Four Best PWA Solutions

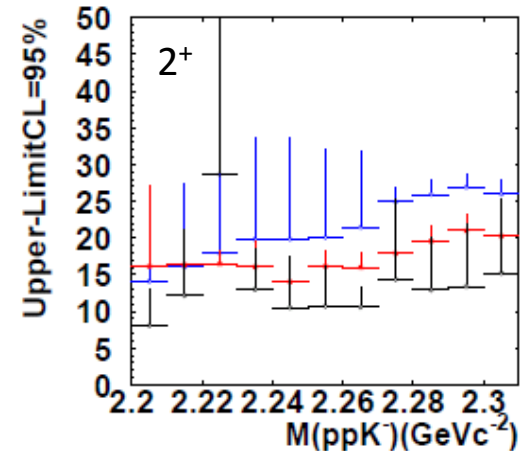
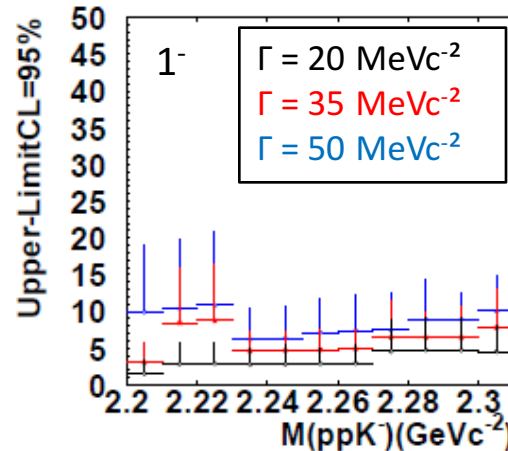
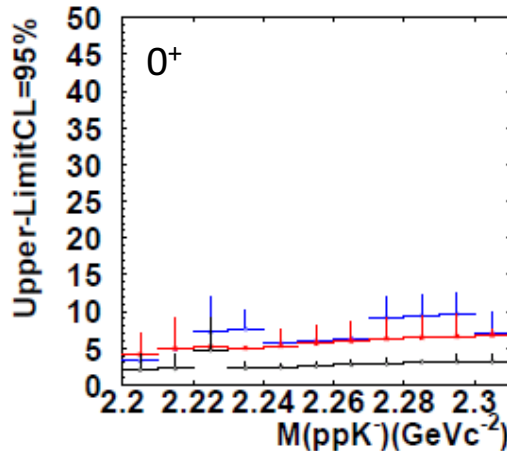
FOPI



HADES



# ppK<sup>-</sup> Upper Limit Determination



$p + p \rightarrow p + K^+ + \Lambda$   
Total Cross Section

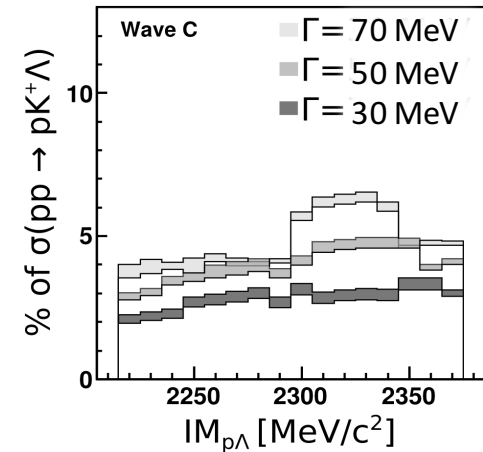
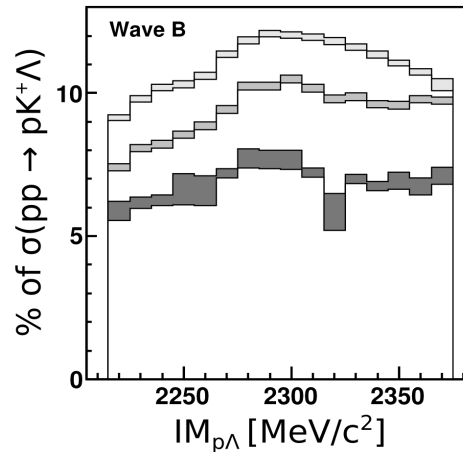
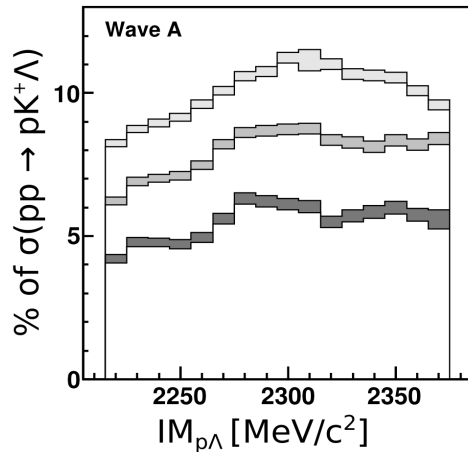
$$\sigma_{pK+\Lambda} = 41.0 \pm 12.8 \mu\text{b}$$

Interpolated from literature

Upper Limit Cross Section

$\Gamma$ (MeVc <sup>-2</sup> )	Cross Section ( $\mu\text{b}$ )
20	$7.6 \pm 1.2^{-3.5} - 22.4 \pm 3.6^{-10.7}$
35	$6.3 \pm 1.7^{-0.6} - 9.5 \pm 2.6^{-0.9}$
50	$10.2 \pm 1.8^{-4.5} - 11.6 \pm 3.4^{-0.6}$
60	$11.2 \pm 1.9^{-5.0} - 33.8 \pm 5.2^{-16.9}$
80	$11.4 \pm 2.7^{-3.8} - 35.9 \pm 5.7^{-17.4}$

# Upper Limit



Measured total cross-section:  $\sigma_{pK^+\Lambda} = 38.12 \pm 0.43_{-2.83}^{+3.55} \pm 2.67(p+p\text{-error}) - 2.9(\text{background}) \mu\text{b}$

Upper limit of  $ppK^-$  Cross Section:

$\Gamma$ (MeV $c^{-2}$ )	Cross Section ( $\mu\text{b}$ )
$0^+$	1.9 – 3.9
$1^-$	2.1 – 4.2
$2^+$	0.7 – 2.1

Production Cross Section  $\Lambda(1405)$

$$9.2 \pm 0.9 \pm 0.7 \text{ } ^{+3.3}_{-1.0} \mu\text{b}$$

HADES coll. (G. Agakishiev et al.)  
Phys. Lett. B742 (2015) 242–248.

# Multi-PWA



# Data Sets

Experiment	$E_B$ [GeV]	$pK^+\Lambda$ Statistics	Status
COSY-TOF	1.96	~160k	In Preparation (not used in the analysis)
DISTO	2.15	121 k	Available
COSY-TOF	2.16	43 k	Available
COSY-TOF	2.16	~90k	In Preparation (not used in the analysis)
DISTO	2.5	304 k	Available
DISTO	2.85	424 k	Available
FOPI	3.1	0.9 k	Single PWA
HADES	3.5	21 k	Single PWA

# COSY-TOF Spectrometer

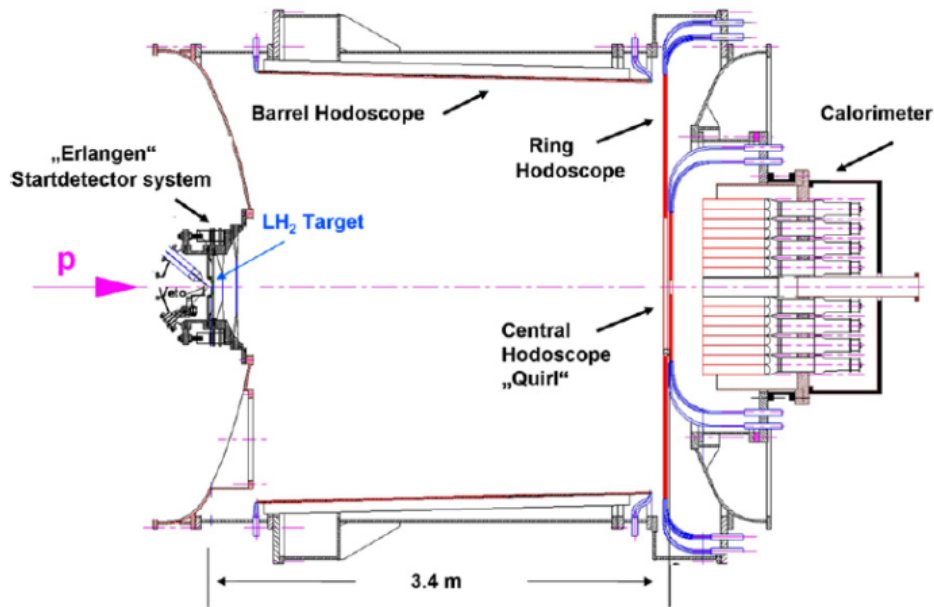
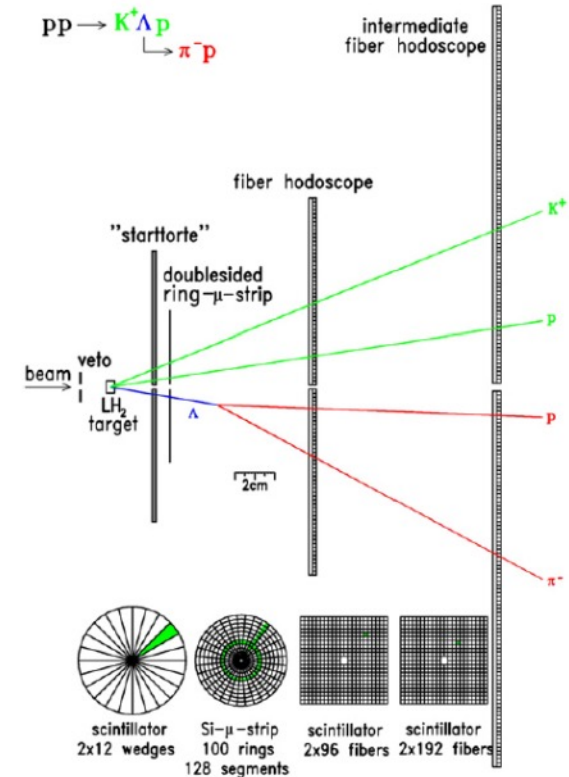


Fig. 1. Schematic view of the COSY-TOF detector.



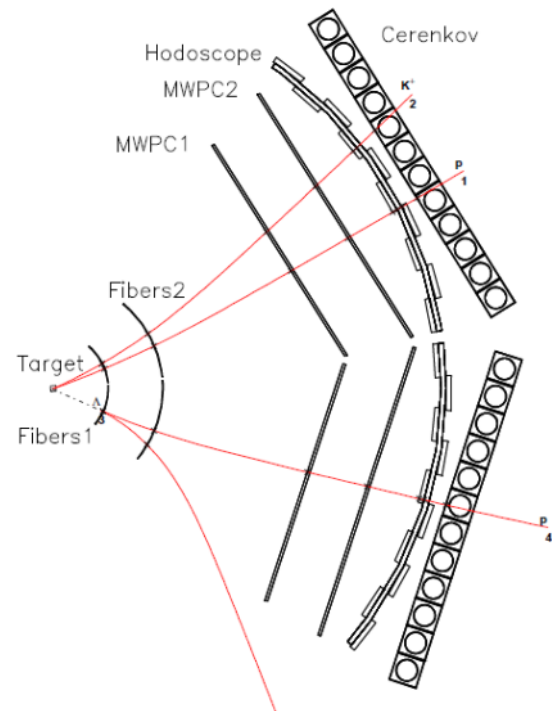
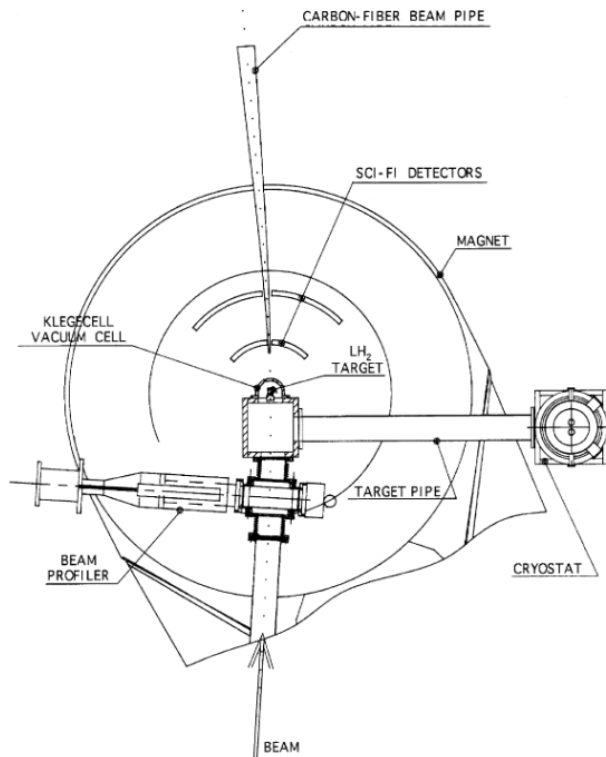
Acceptance:  $1^\circ$ - $60^\circ$  (polar),  $2\pi$  (azimuthal)

Sec. Vertex:  $\sigma_{x,y} < 1\text{mm}$ ,  $\sigma_z < 3\text{mm}$

$$\sigma_{TOF} = 300\text{ps}$$

$$\sigma_{MM(pK)} = 16\text{ MeV}/c^2$$

# DISTO Spectrometer



Acceptance: 23°-43° (polar), 2 $\pi$  (azimuthal)

$$\sigma_p = 5\%$$

$$\sigma_{MM(pK)} = 30 \text{ MeV}/c^2$$

# Combined Analysis

## 1. Solution for HADES+FOPI+DISTO25

- Energy Range wide enough for energy dependence
- High energy for higher N\*-Resonances
- Three datapoint to pin down set
- Easier to make manual changes in parameter set

## 2. Include Stepwise further data sample

1. Cosy216 / DISTO21 / DISTO28
  - Inclusion do not require further manual modification of data parameter set.

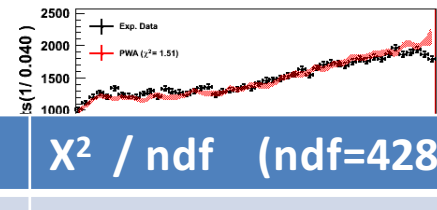
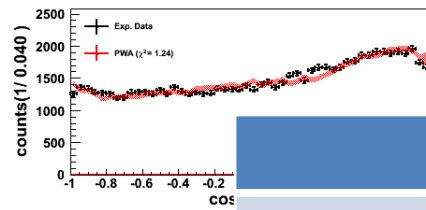
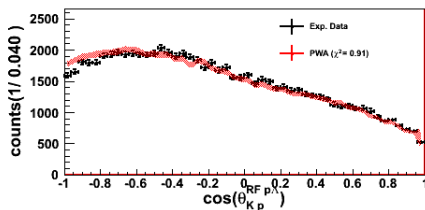
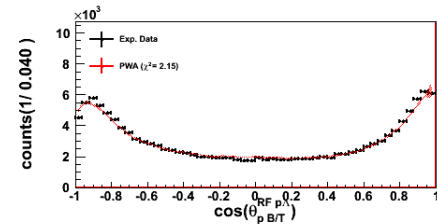
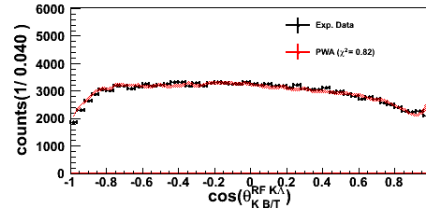
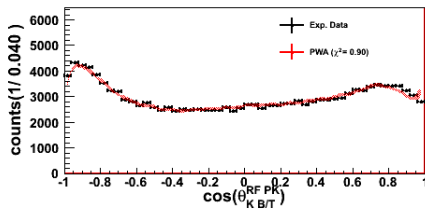
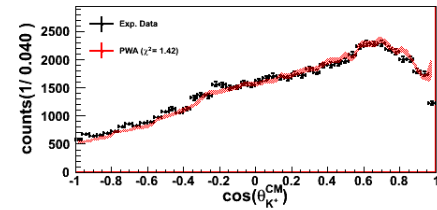
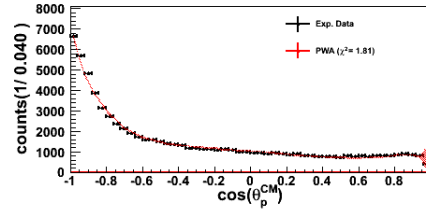
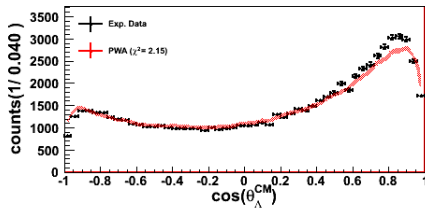
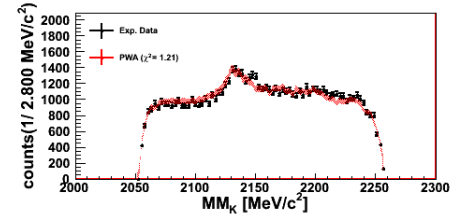
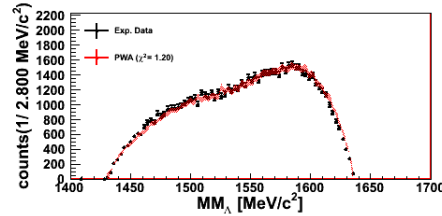
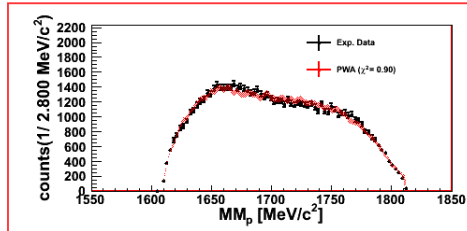
# Parameter Scan

pKΛ non resonant up to F wave \*

Include different N\* Resonances

Solution	A	B	C	D	E
Loglike	-67142	-67018	-66878	-66504	-66405
$\frac{\chi^2}{ndf}$ ( $ndf = 4547$ )	9,50	9,98	9,98	10,01	10,34
N*(1650)	+	+	+	+	+
N*(1710)	+	+	+	+	+
N*(1720)	+	+	+	+	-
N*(1875)	+	+	-	-	+
N*(1880)	+	+	+	+	+
N*(1895)	+	+	+	+	+
N*(1900)	-	+	+	-	+
$\Sigma N$ ( $0^+$ )	+	+	+	+	+

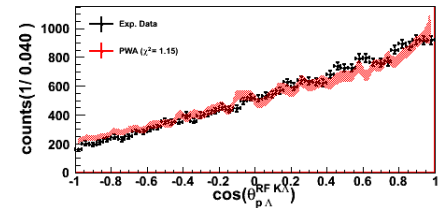
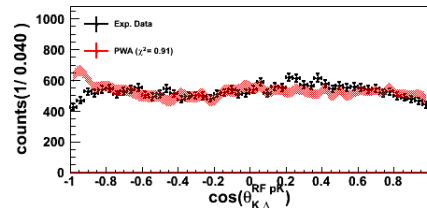
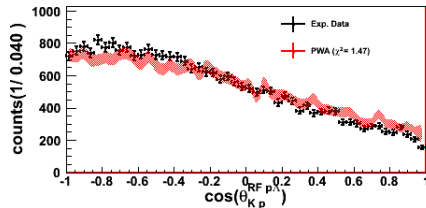
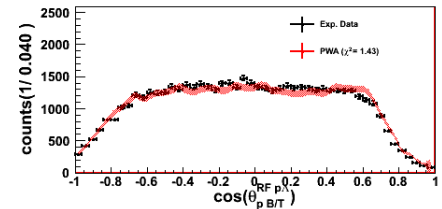
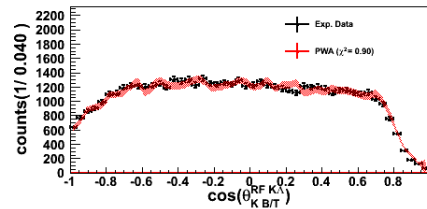
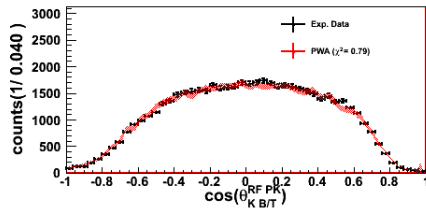
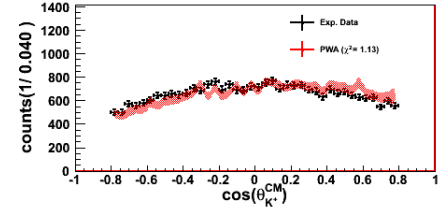
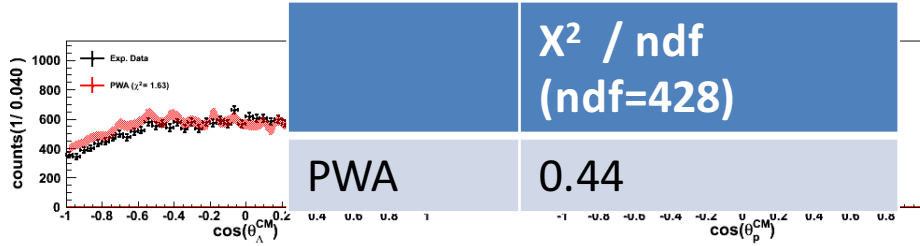
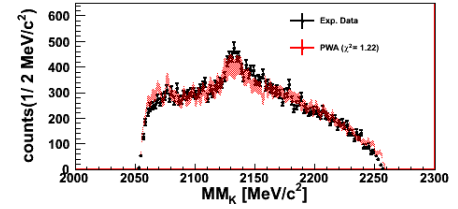
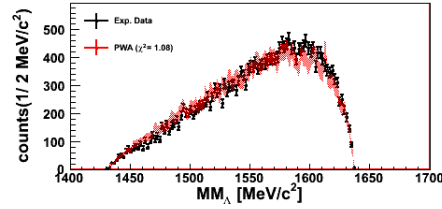
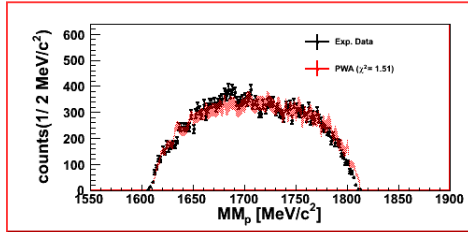
# DISTO@2.14 GeV



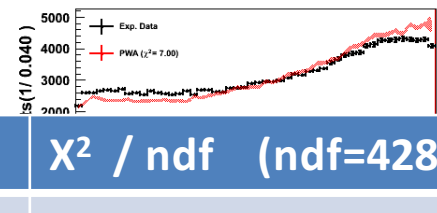
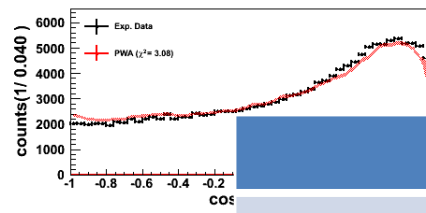
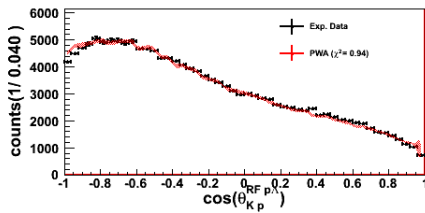
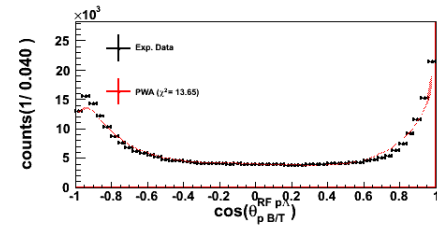
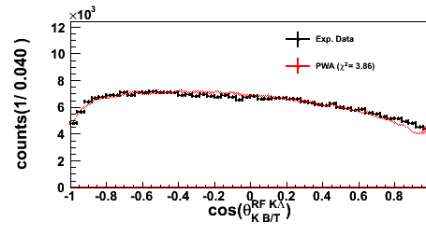
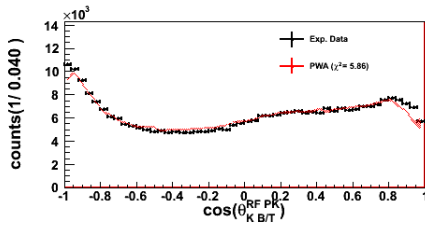
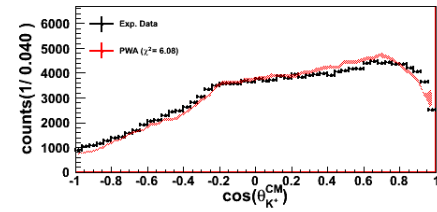
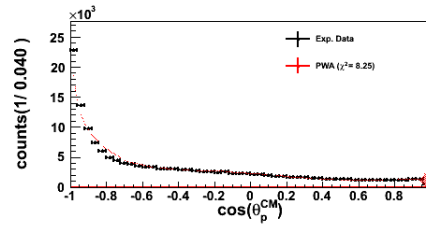
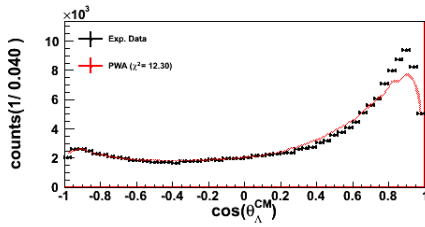
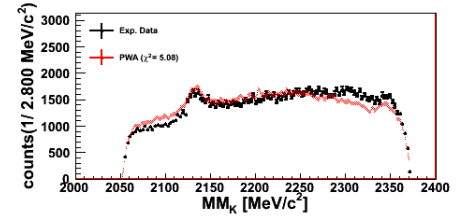
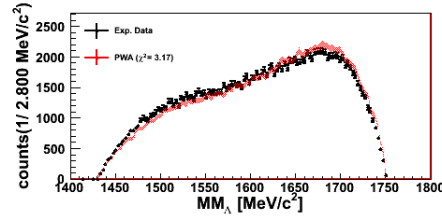
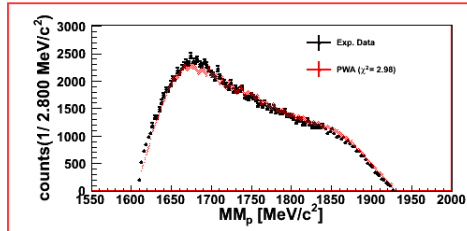
**X<sup>2</sup> / ndf (ndf=428)**  
**PWA 1.52**



# COSY-TOF@2.16 GeV



# DISTO@2.5 GeV



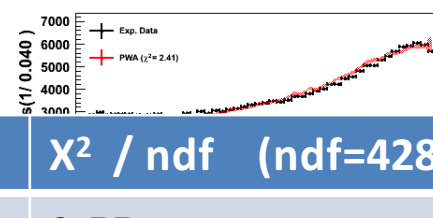
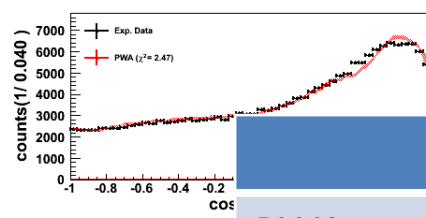
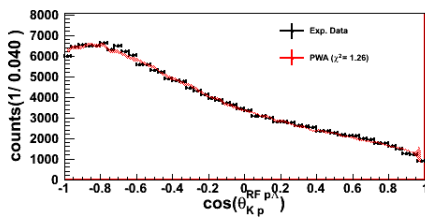
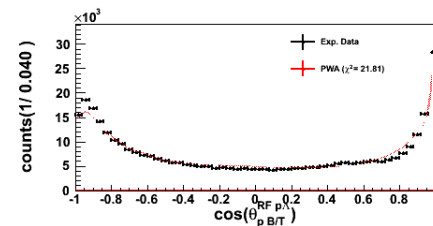
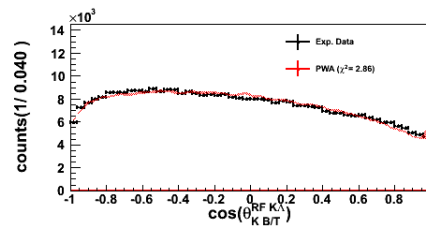
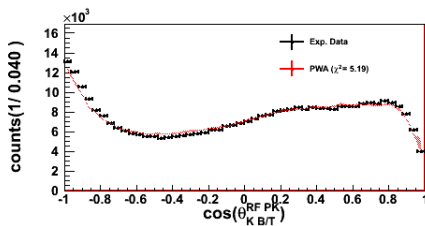
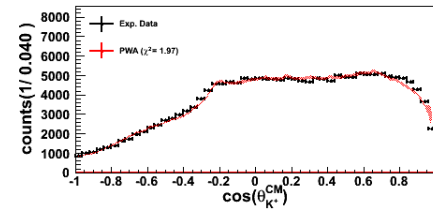
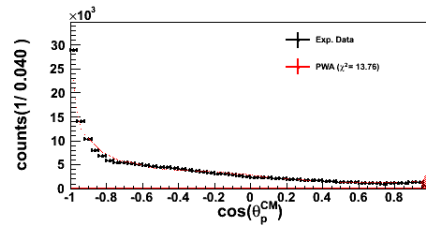
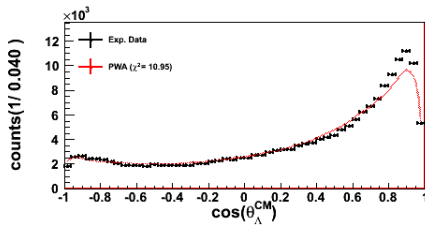
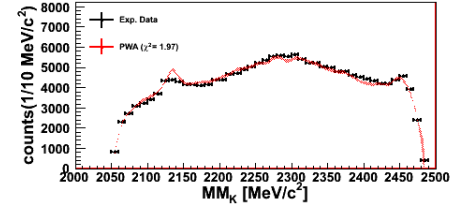
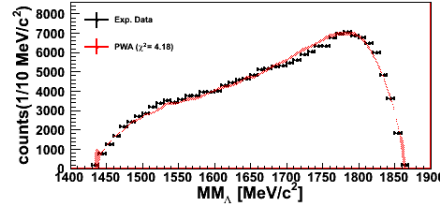
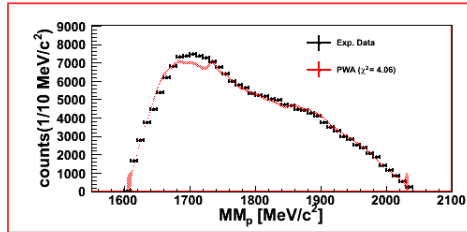
$\chi^2 / \text{ndf}$  (ndf=428)  
PWA 2.56





# DISTO@2.85 GeV

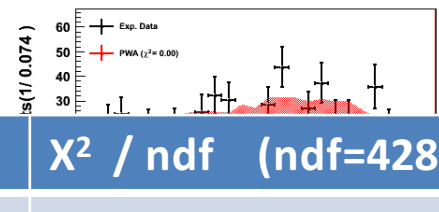
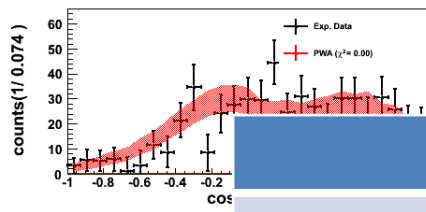
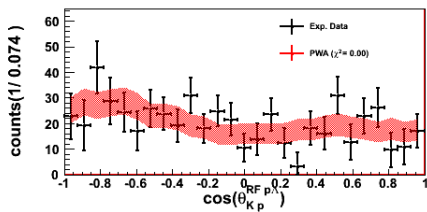
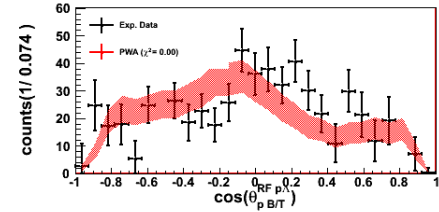
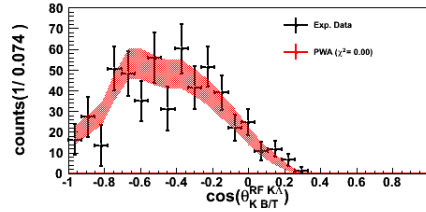
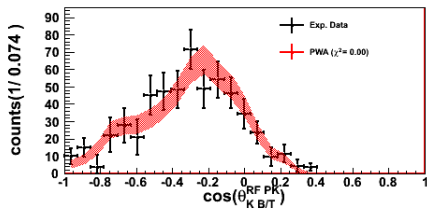
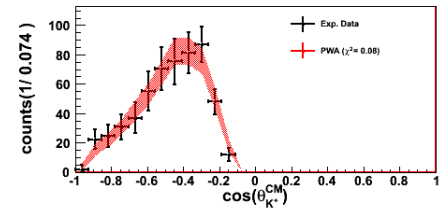
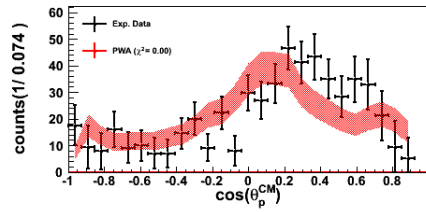
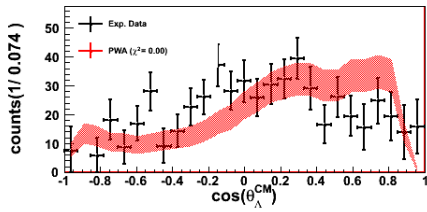
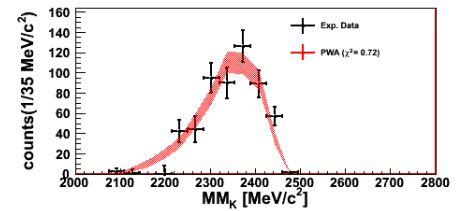
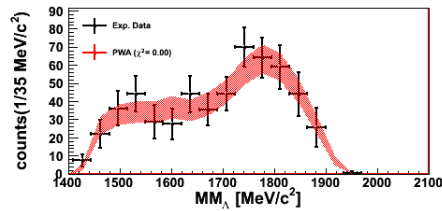
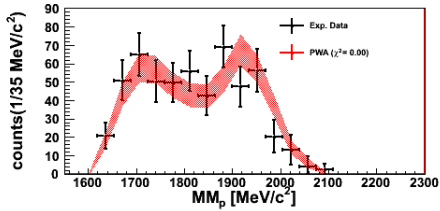
No Kaonic Cluster included



$\chi^2 / \text{ndf}$  (ndf=428)  
PWA 3.55



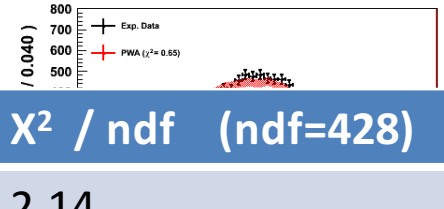
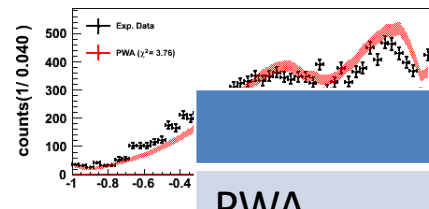
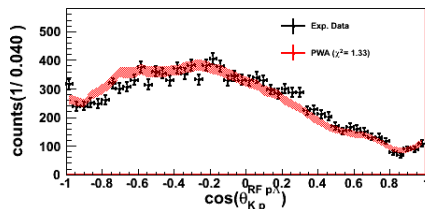
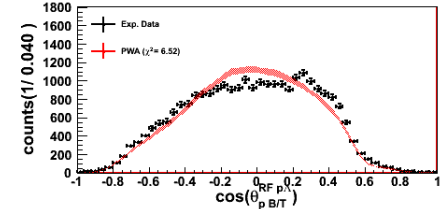
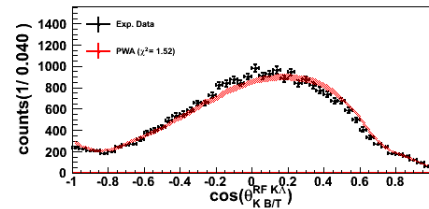
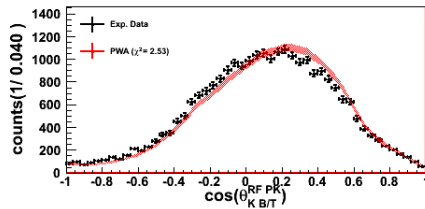
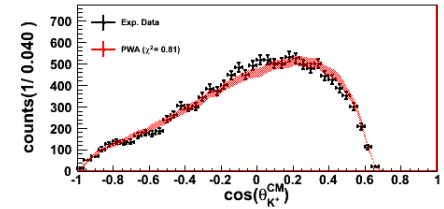
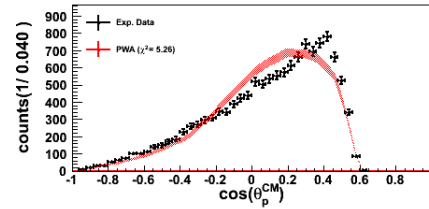
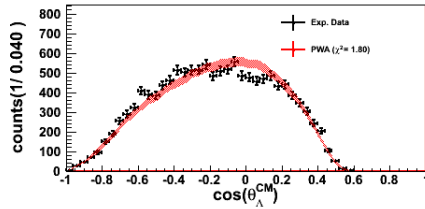
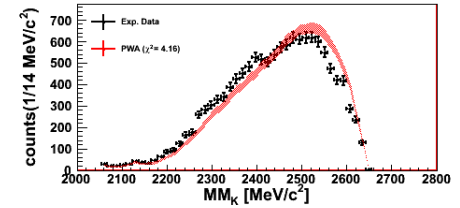
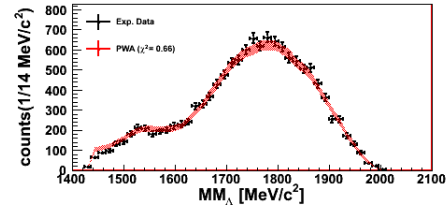
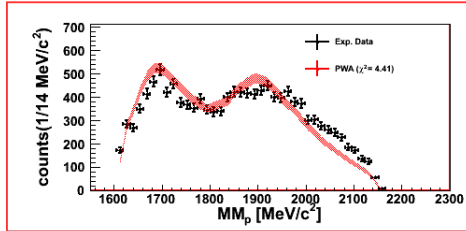
# FOPI



**$\chi^2 / \text{ndf}$  (ndf=428)**  
**PWA 0.91**



# HADES



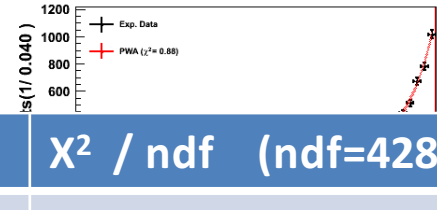
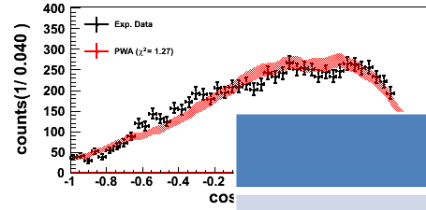
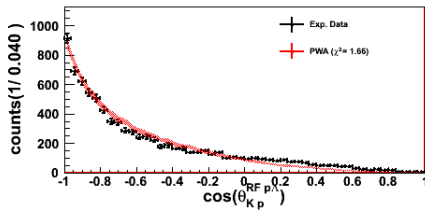
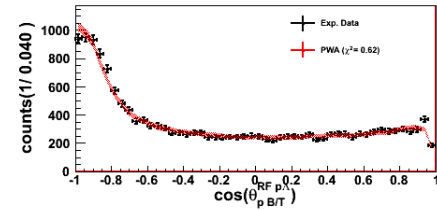
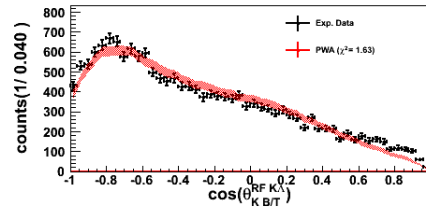
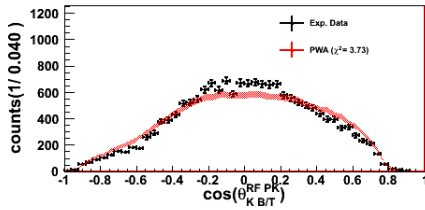
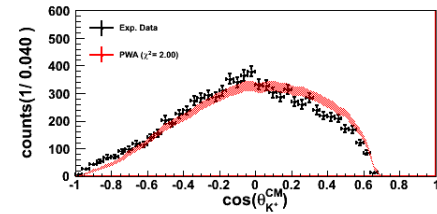
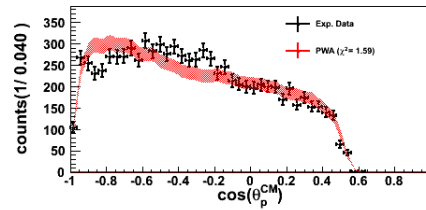
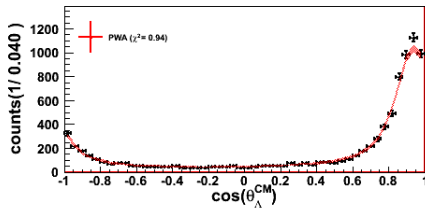
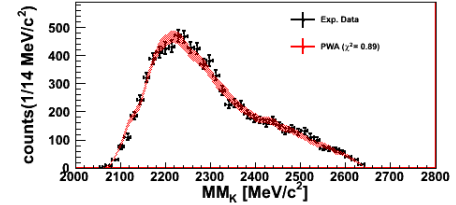
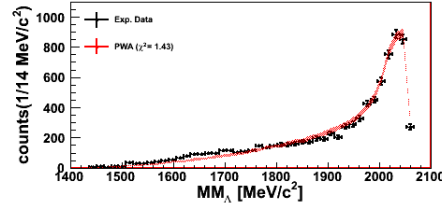
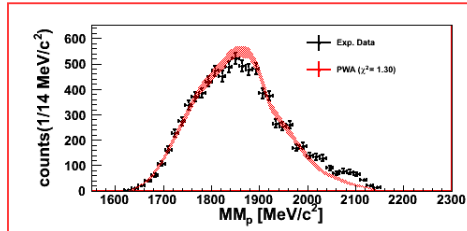
$\chi^2 / \text{ndf}$  (ndf=428)

PWA

2.14



# HADES - WALL

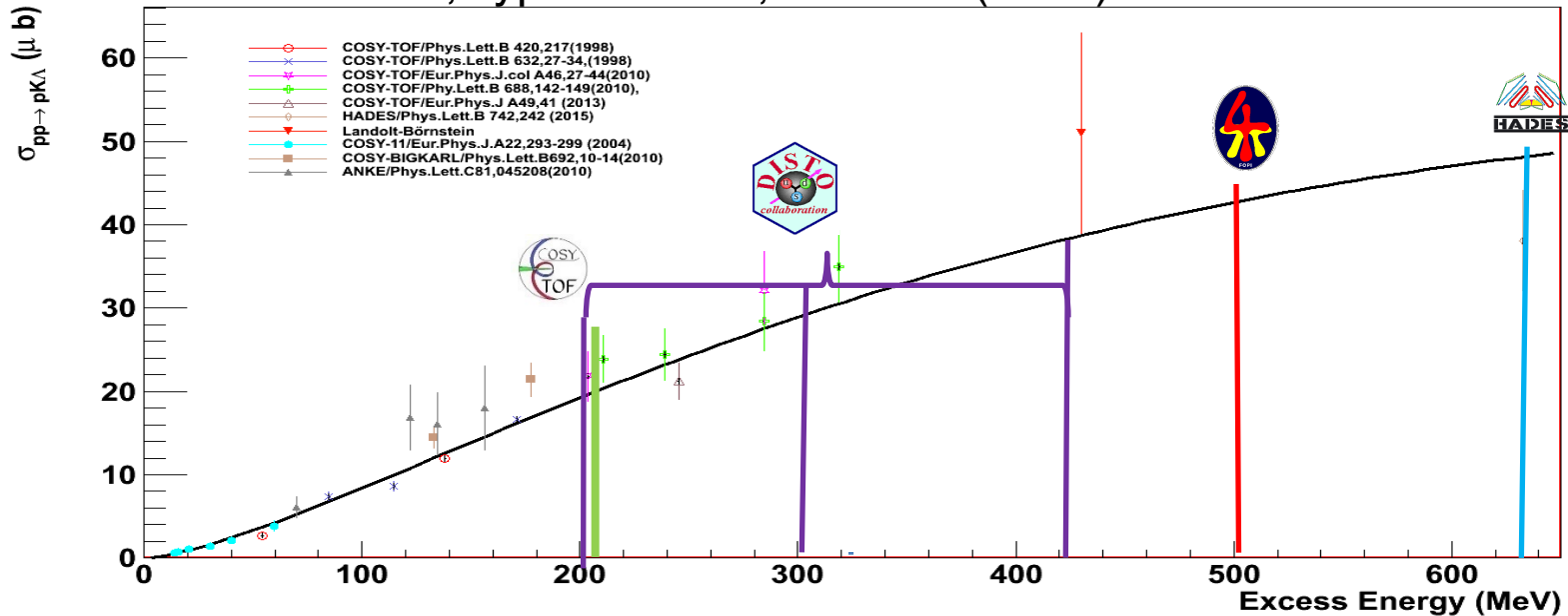


	$\chi^2 / \text{ndf}$ (ndf=428)
PWA	1.86



# Total Cross Section

R.Muenzer et al., Hyperfine 233, 159-166 (2016)



Value:

$$\sigma_{pK\Lambda} = C_1 \left( 1 - \frac{s_0}{(\sqrt{s_0} + \epsilon)^2} \right)^{C_2} \left( \frac{s_0}{(\sqrt{s_0} + \epsilon)^2} \right)^{C_3}$$

$$C_1 = 4.03 \pm 0.57 \cdot 10^2$$

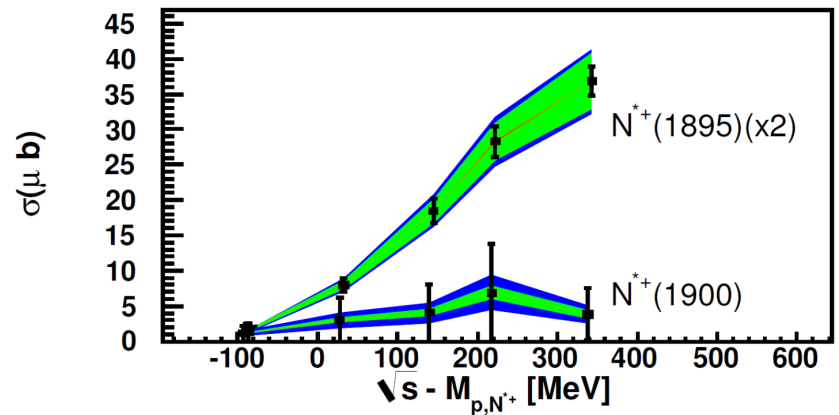
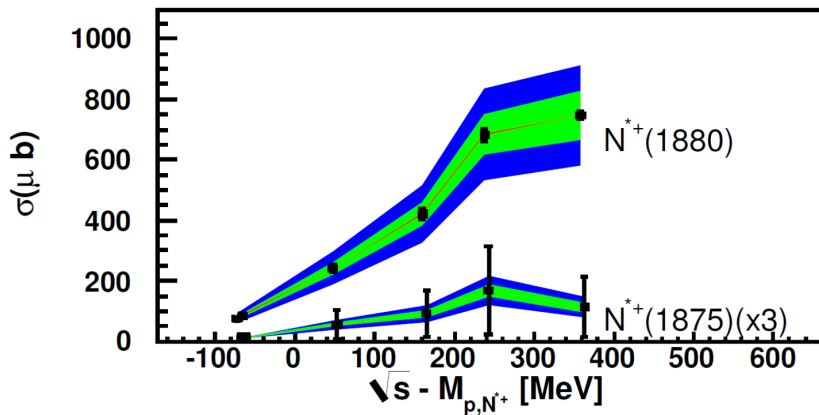
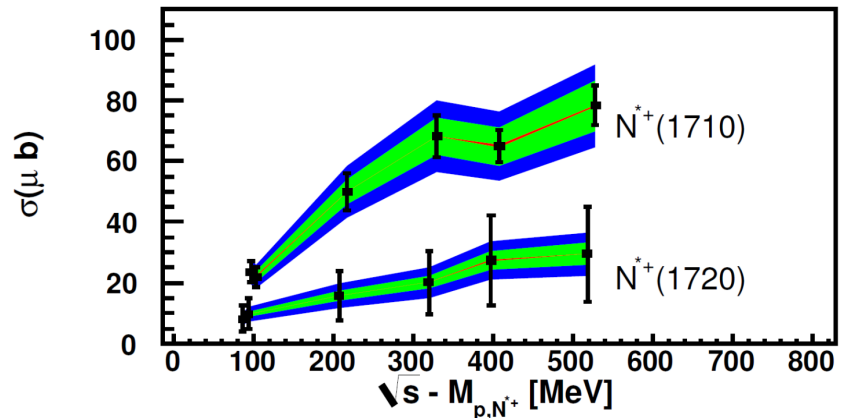
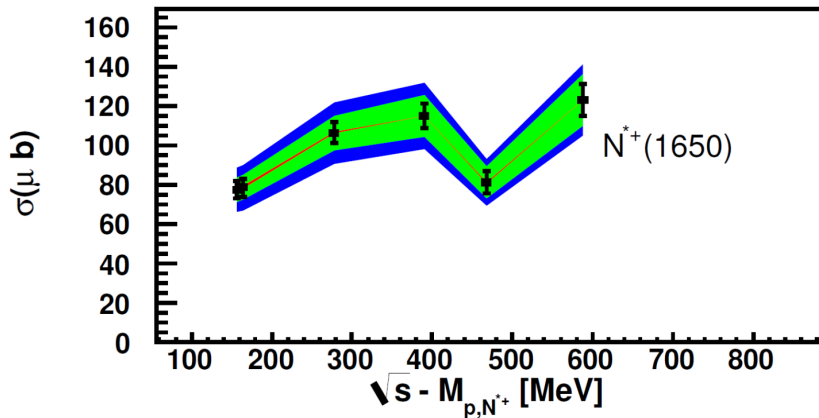
$$C_2 = 1.49 \pm 0.04$$

$$C_3 = 1.43 \pm 0.39$$

# Branching Ratio

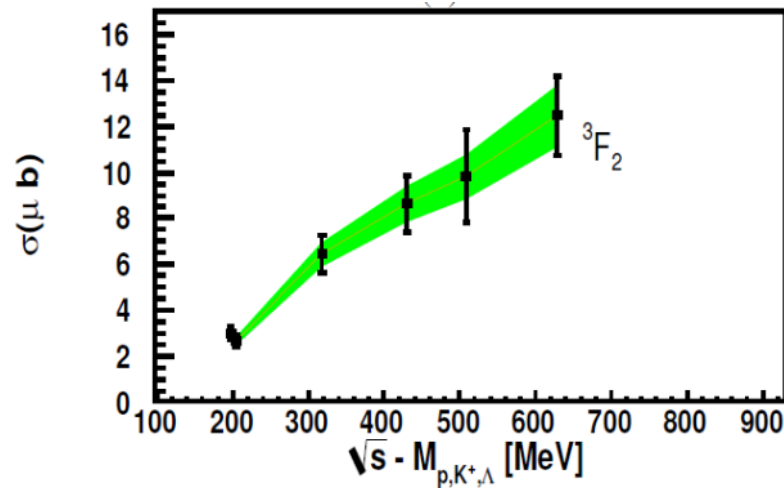
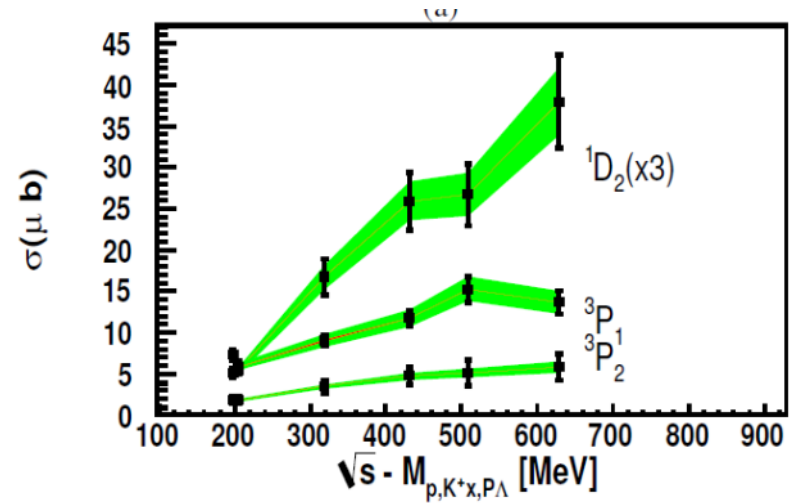
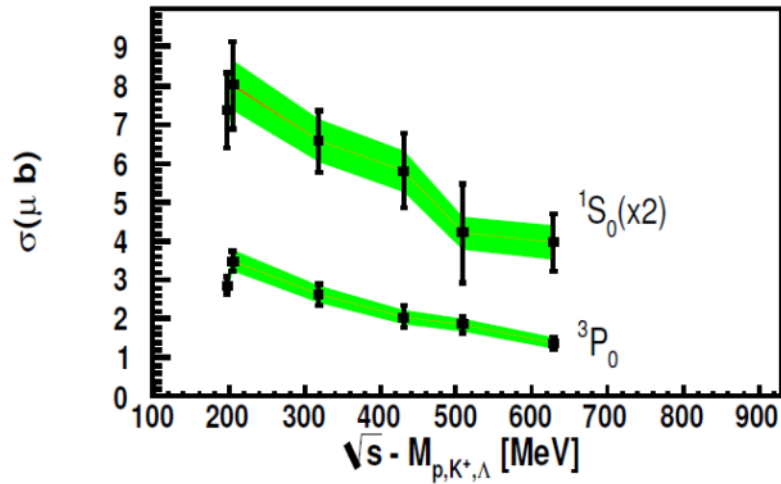
	Mass [GeV/c <sup>2</sup> ]	Width [GeV/c <sup>2</sup> ]	$\Gamma_{\Lambda K}/\Gamma_{All}$ %
N(1650)S <sub>11</sub>	1.655	0.150	3-11
N(1710)P <sub>11</sub>	1.710	0.200	5-25
N(1720)D <sub>13</sub>	1.720	0.250	1-15
N(1875)D <sub>13</sub>	1.875	0.220	4±2
N(1880)P <sub>11</sub>	1.870	0.235	2±1
N(1895)S <sub>11</sub>	1.895	0.090	18±5
N(1900)P <sub>13</sub>	1.900	0.250	0-10

# Cross Section



Non Resonant-Resonant: 20-80

# Initial State





# Final State Interaction in PWA

$$A_{2b}^{\beta} = \frac{\sqrt{s_i}}{1 + \frac{1}{2} r^{\beta} q^2 a_{p\Lambda}^{\beta} + i q a_{p\Lambda}^{\beta} q^{2L} / F(q, r^{\beta}, L)}$$

$a_{p\Lambda}^{\beta}$  Scattering Length

$r^{\beta}$  Effective Range of System

$$\alpha_s = -1.43 \pm 0.36 \pm 0.09 \text{ fm} \quad \alpha_t = -1.88 \pm 0.38 \pm 0.10 \text{ fm}$$

$$r_s = 1.31 \pm 0.24 \pm 0.16 \text{ fm} \quad r_t = 1.04 \pm 0.78 \pm 0.15 \text{ fm}$$

Source	$^1S_0 a_{\Lambda-p}$ [fm]	$^1S_0 r_{\Lambda-p}$ [fm]	$^3S_1 a_{\Lambda-p}$ [fm]	$^3S_1 r_{\Lambda-p}$ [fm]	$\langle a_{\Lambda-p} \rangle$ [fm]
This work	$-1.43 \pm 0.36 \pm 0.09$	$1.31 \pm 0.24 \pm 0.16$	$1.88 \pm 0.38 \pm 0.10$	$1.04 \pm 0.78 \pm 0.15$	
NLO <sup>2</sup> [15]	-2.91	2.78	-1.54	2.72	-1.88 <sup>3</sup>
LO <sup>2</sup> [15]	-1.91	1.40	-1.23	2.13	-1.4 <sup>3</sup>
[16]	$-1.8_{-4.2}^{+2.3}$	-	$-1.6_{-0.8}^{+1.1}$	-	-
[17]	-	-	-	-	$-1.25 \pm 0.08 \pm 0.03$
[18]	-	-	$-1.31_{-0.49}^{+0.32} \pm 0.3 \pm 0.16$	-	$-1.233 \pm 0.014 \pm 0.3 \pm 0.12$

[15] Haidenbauer et al. Nuclear Physics A, 915, 24-58 (2013)

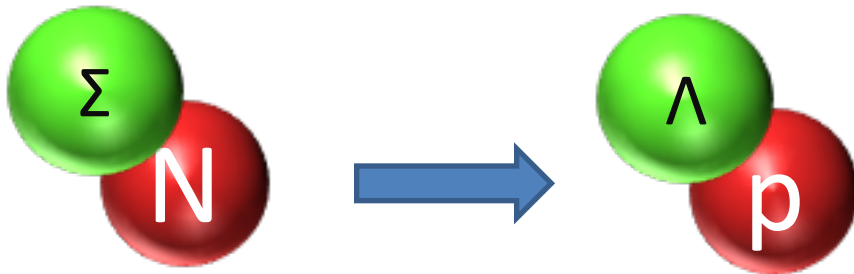
[16] G. Alexander et al., Phys. Rev. 173, 1452 (1968).

[17] M. Roeder et al., Eur. Phys. J. A 49, 157 (2013)

[18] Hauenstein 2014

# The $\Sigma N$ Cusp Effect

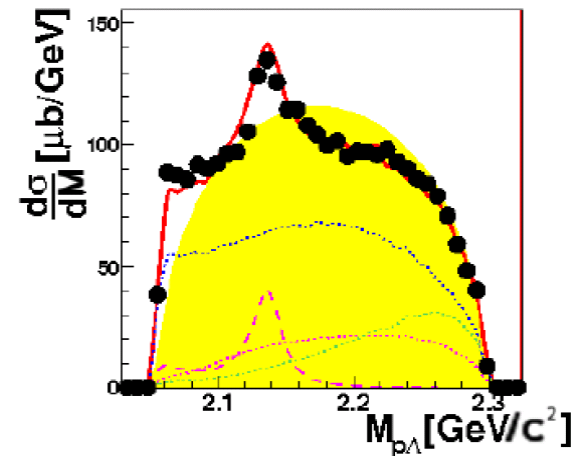
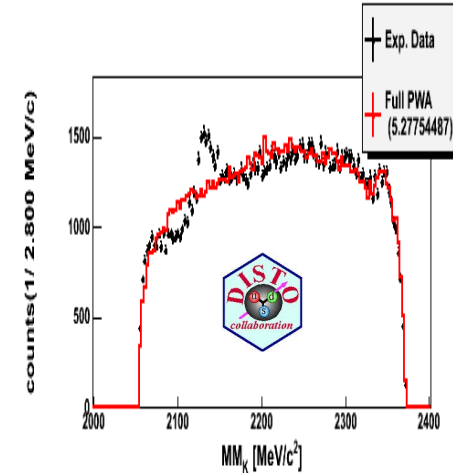
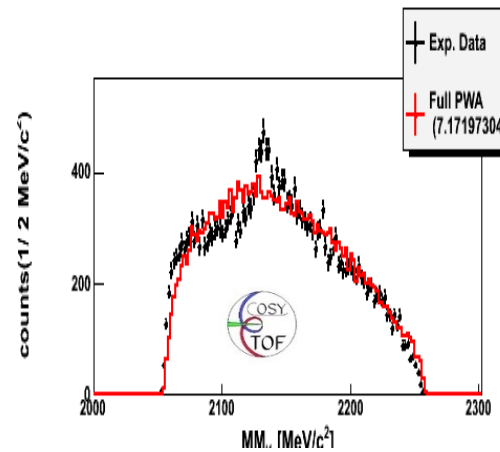
Coupled Channel:



At Threshold :  $2130 \text{ MeVc}^{-2}$

Quantum Number of Cusp:  $0^+ / 1^+ (L=0,2)$

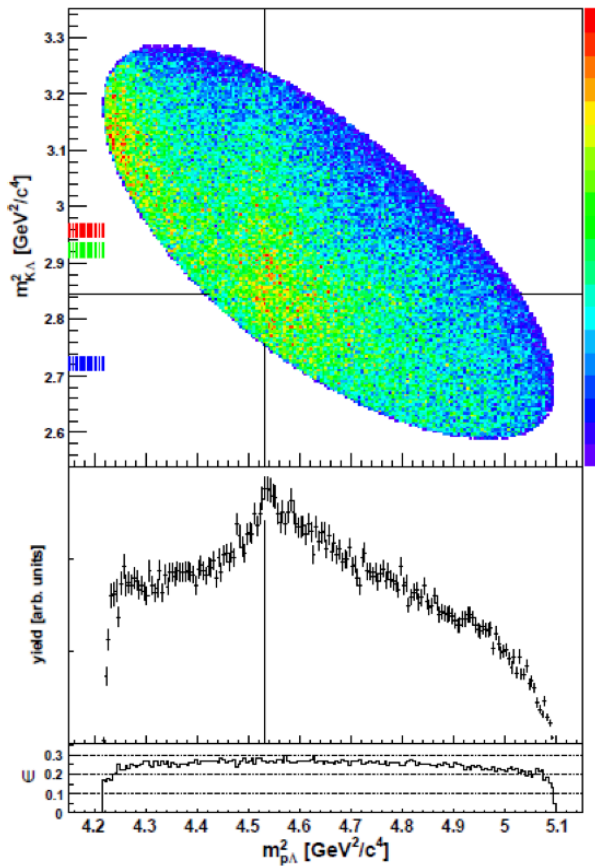
Spectral Function:  
Breit Wigner  
Flatté



S.Abd El-Samad, Eur.Phys.J A49(2013)

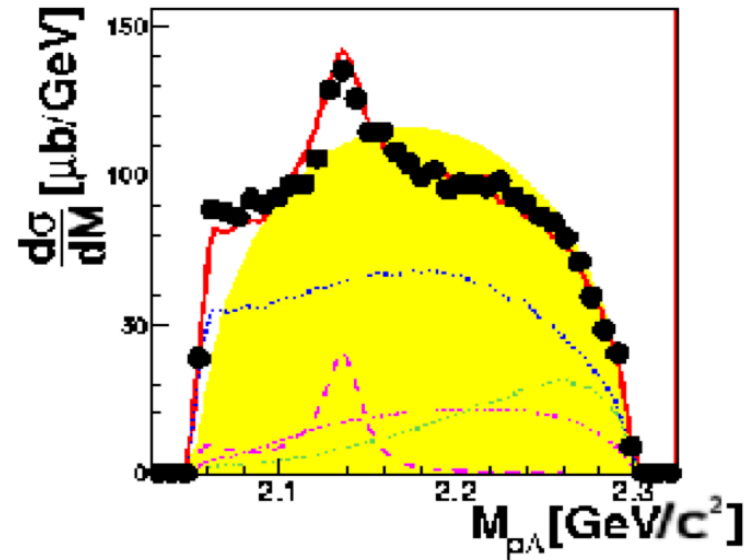
# Previous Observations

● Experimental Observation:



● Cosy-TOF Analysis:

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Solid line: Full MC simulation

Shaded areas: phase-space distributions

# Cusp Spectral Function

The Breit-Wigner:

$$\frac{d\sigma_{p\Lambda}}{dm_{p\Lambda}} \approx \frac{1}{|m_R^2 - m_{p\Lambda}^2 - i m_{p\Lambda} \Gamma|^2}$$

Mass  $M_{cusp} = 2.13\text{GeV}$ , With  $\Gamma = 0.02\text{GeV}$

$g_{p\Sigma} \ll g_{p\Lambda}$  Symmetric  
 $g_{p\Sigma} \gg g_{p\Lambda}$  Antisymmetric

Above the  
threshold

Below the  
threshold

The Flatté parametrization:

$$\frac{d\sigma_{p\Lambda}}{dm_{p\Lambda}} \approx \frac{\Gamma_{p\Lambda}}{|m_R^2 - m_{p\Lambda}^2 - i m_{p\Lambda} (\Gamma_{p\Lambda} + \Gamma_{p\Sigma})|^2}$$

$$\Gamma_{p\Lambda} = g_{p\Lambda} * q_{p\Lambda} \quad \Gamma_{p\Sigma} = g_{p\Sigma} * q_{p\Sigma}$$

$$q_{p\Sigma} = \frac{\sqrt{(m_{p\Sigma}^2 - (m_\Sigma + m_p)^2) * (m_{p\Sigma}^2 - (m_p - m_\Sigma)^2)}}{2 m_{p\Sigma}}$$

$$q_{p\Sigma} = i * \frac{\sqrt{((m_\Sigma + m_p)^2 - m_{p\Sigma}^2) * (m_{p\Sigma}^2 - (m_p - m_\Sigma)^2)}}{2 m_{p\Sigma}}$$

# Data Set

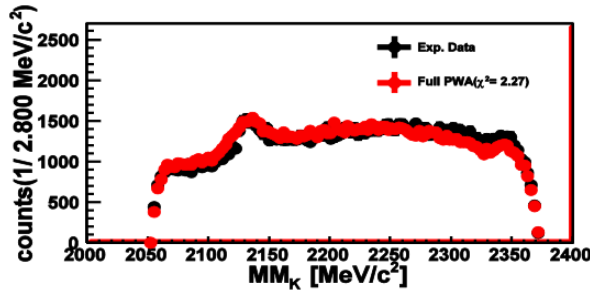
- Combined data analysis

Experiment	$E_{\text{beam}}$ (GeV)	Sqrt(S)(GeV)	Statistics	Polar.
DISTO[12][13]	2.14	2.75	76982	Y
DISTO[12][13]	2.5	2.85	80000	Y
DISTO[11][12][13]	2.85	2.98	182597	Y
COSY-TOF[14]	2.16	2.75	43662	Y

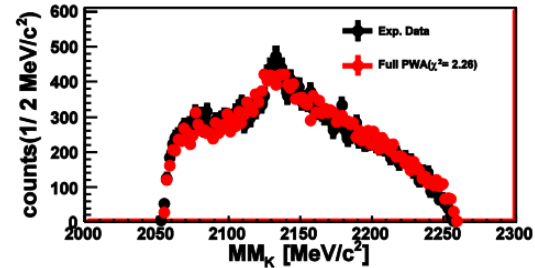
- The data samples are fitted parallel in BG-Framework.
- The parameters are optimized to explained all data samples
- The parallel fitting assure, that different detector acceptance and efficiency for different spectrometers and beam energies are not mixed.

# Results

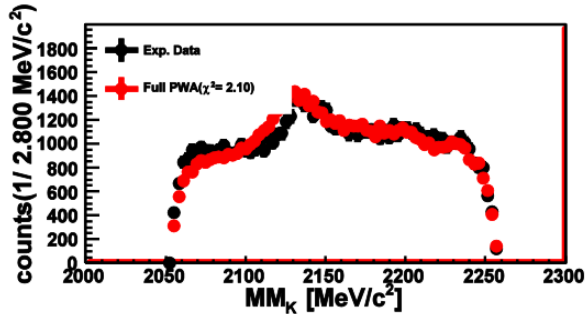
- DISTO 2.5GEV



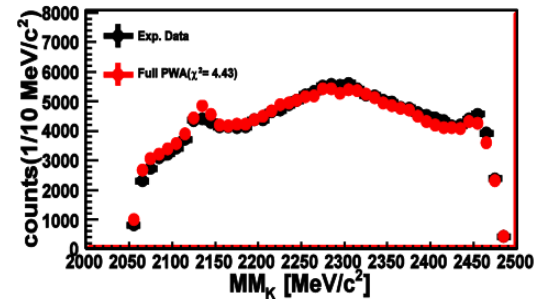
- COSY-TOF 2.16GEV



- DISTO 2.14GEV



- DISTO 2.85GEV



- Final coupling constants:  $g_{N\Sigma} = 1.55 \pm 0.08 \times 10^{-2}$ ,  $g_{p\Lambda} = 0.30 \pm 0.03 \times 10^{-2}$
- Threshold mass value from the fit:  $m_R = 2.13 \pm 0.006 \text{ GeV}/c^2$

# Summary and Outlook

- Combined Analysis for COSY & DISTO & HADES & FOPI completed
- Systematical Analysis performed
- Excitation Function for  $N^*$  and  $p\text{KL}$  extracted
- Scattering Length  $p\text{-}\Lambda$  separate for Singlet and Triplet
- Cusp Wave included too ( not discussed here)
- Common upper limit for Kaonic Bound states to come soon