ExHIC, March 28, 2016

Lambda-Lambda correlation from an integrated dynamical model

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Hiromi Hinohara
Koichi Murase

References for the model:

TH, K.Murase, P.Huovinen, Y.Nara,
Prog. Part. Nucl. Phys. **70** (2013) 108.
S.Takeuchi, K. Murase, TH, P.Huovinen, Y.Nara,
Phys. Rev. C **92** (2015) 044907.

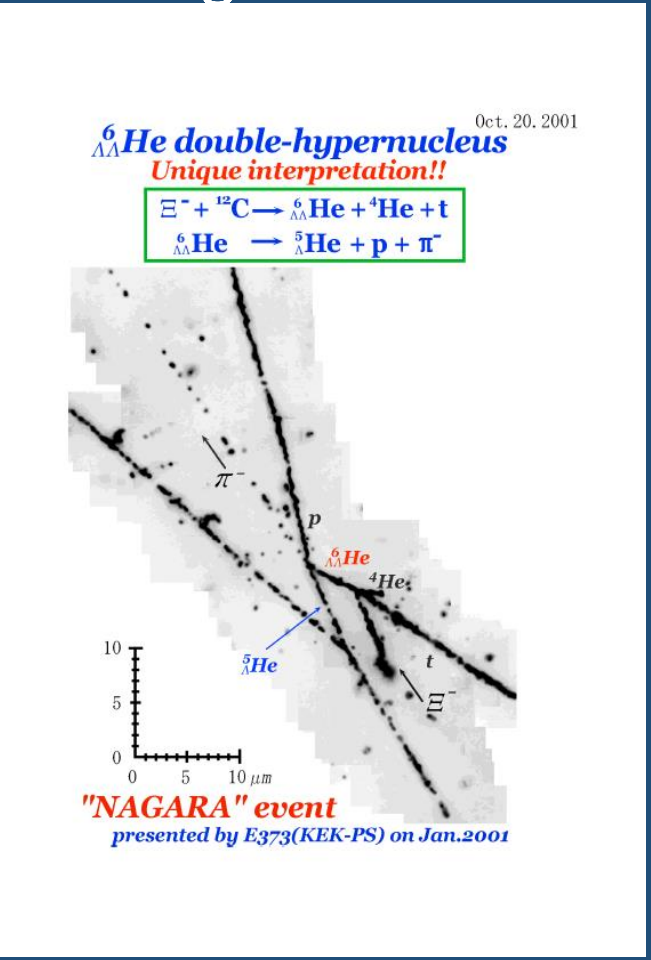
Contents

Interaction between hyperons

H dibaryon

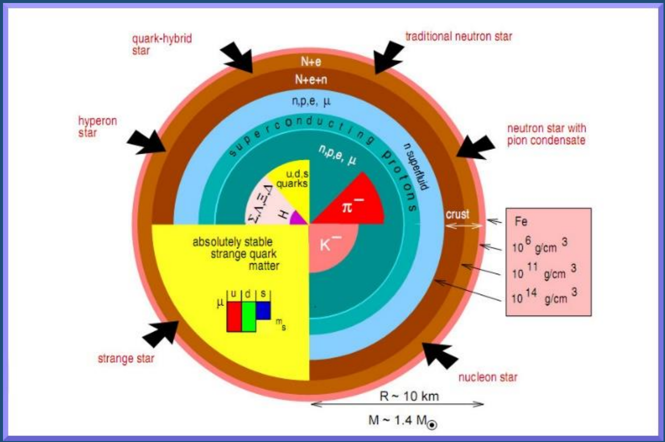
https://en.wikipedia.org/wiki/Hexaquark

Nagara event



http://www.phys.ed.gifu-u.ac.jp/Topics/NAGARA-j.htm

Neutron star core



http://nrumiano.free.fr/Estars/neutrons.html

Motivation

- Investigation from a viewpoint of high-energy nuclear collisions
  - "Lots of" hyperons produced in collisions
  → Correlation functions
- Source function
  - Parametrization for source function (talks by Morita, Shah and Ohnishi)
  - Dynamical model to estimate source function (talk by Fabbietti)
- Utilize an integrated dynamical model to analyze correlation functions of hyperons

Contents

High-energy nuclear collisions

$^{197}\mathrm{Au} + {}^{197}\mathrm{Au}$

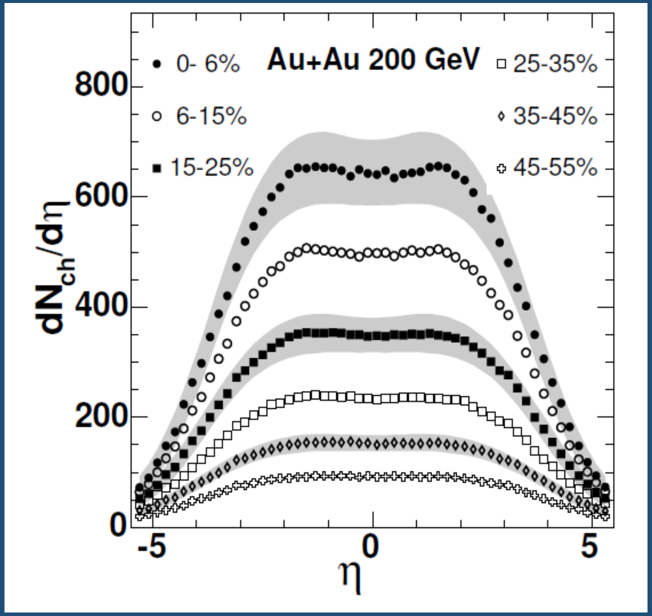
$\sqrt{s_{NN}} = 200\mathrm{GeV}$

$^{208}\mathrm{Pb} + {}^{208}\mathrm{Pb}$

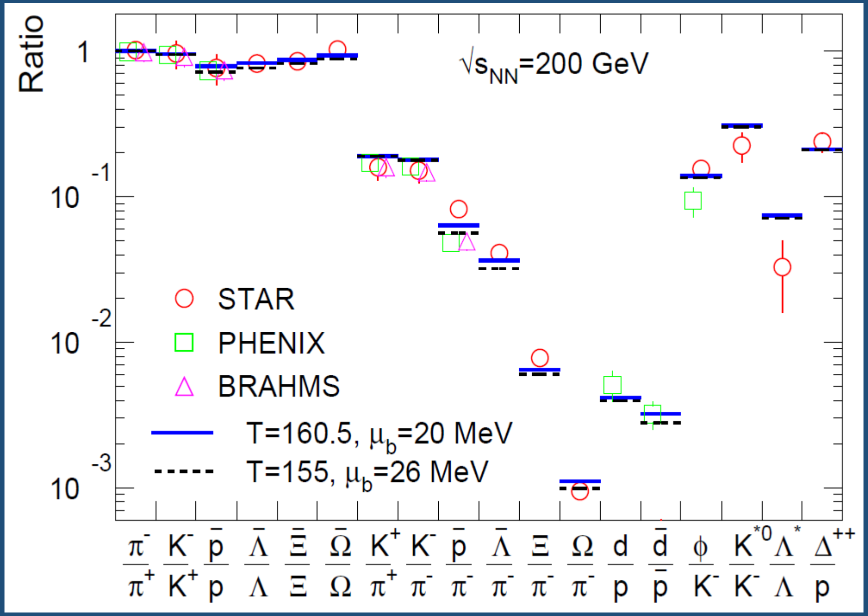
$\sqrt{s_{NN}} = 2.76(5.02)\mathrm{TeV}$

Collisional energy → Thermal energy
→ Multi-particle production

Multi-particle production



PHOBOS white paper (2005)



A.Andronic *et al*. (2005)

# of charged hadrons ~ 700 at midrapidty in central collisions

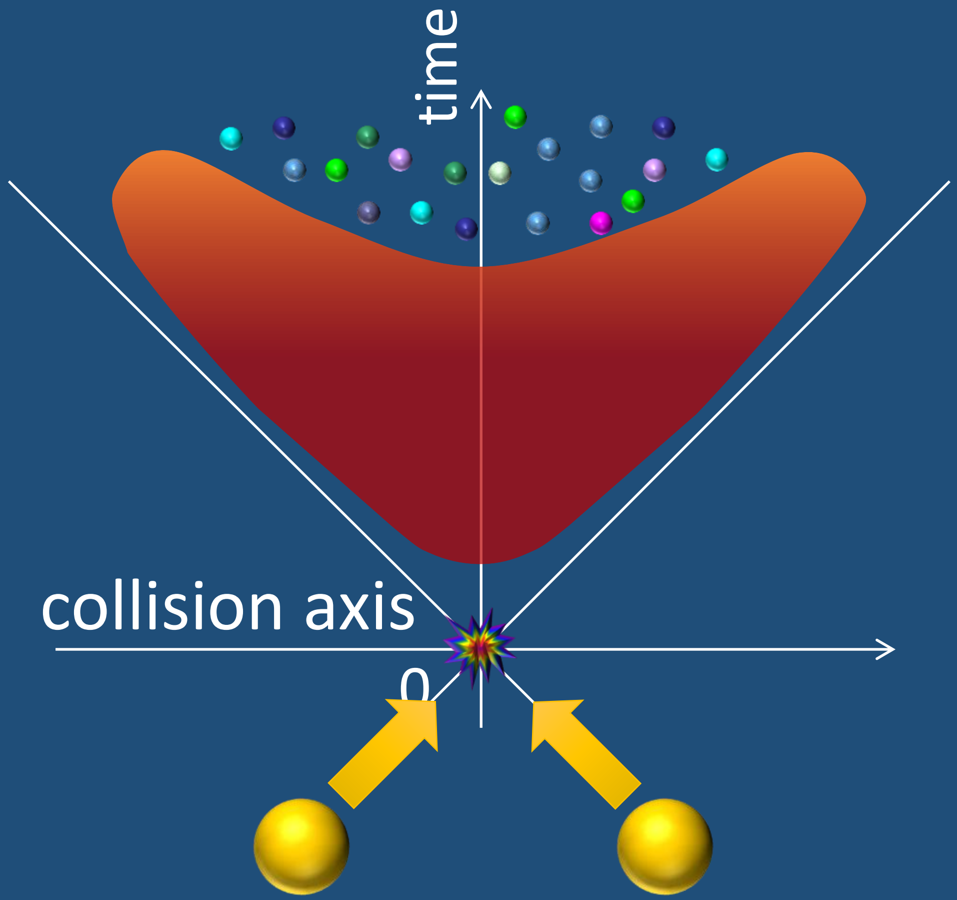
Expected # of Lambda ~1-10 per event

# of events >~10^8

High-energy nuclear experiment as hyperon factory

Contents

Integrated dynamical model



<u>Hadronic cascade model</u>

Microscopic description of Hadron gases

<u>Relativistic hydrodynamics</u>

Space-time evolution of QGP+hadronic fluids

<u>Monte-Carlo Glauber model</u>

Initial profile of matter

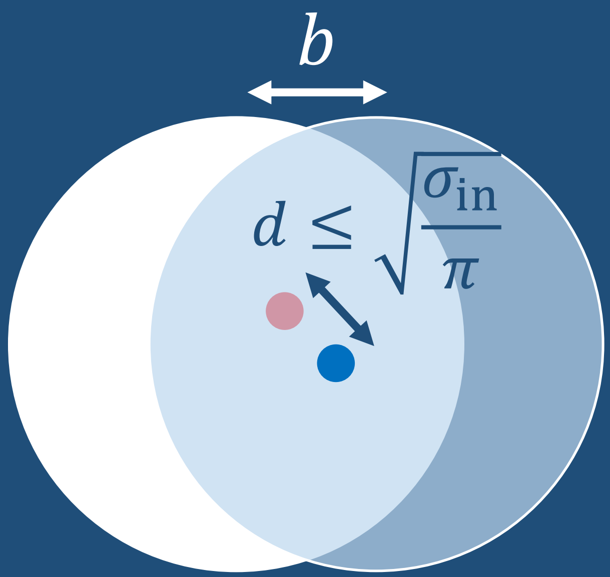
Monte-Carlo Glauber model

Collision energy $\sqrt{s_{\mathrm{NN}}}$

pp inelastic cross section σ_{in}

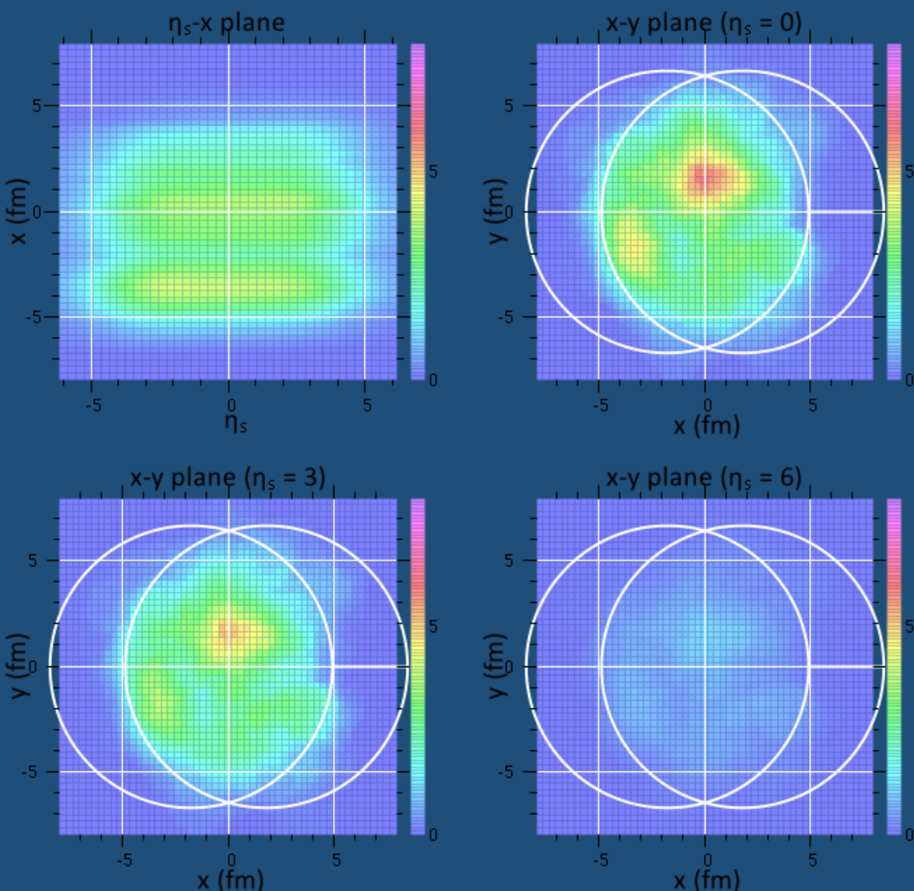
Nuclear species

Woods-Saxon distribution



$$\rho_{\mathrm{part}}(x_\perp), \rho_{\mathrm{coll}}(x_\perp)$$

MC-Glauber to hydro



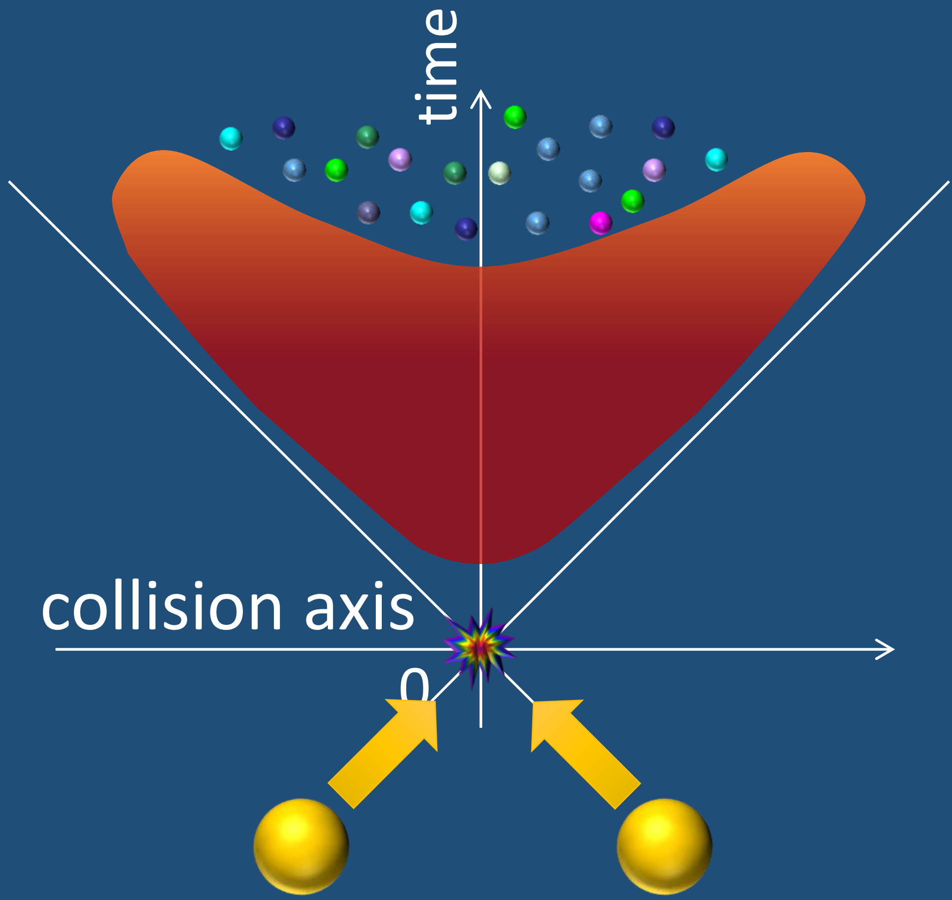
Initial time: $\tau_0 = 0.6\ \mathrm{fm}$
Soft/hard fraction: $\alpha = 0.18$
+ overall normalization
+ rapidity shape in pp

Entropy density

The other thermodynamic variables through equation of state

One example of initial condition

Integrated dynamical model



Hadronic cascade model

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Hydrodynamic equations

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (e + p)u^\mu u^\nu - pg^{\mu\nu}$$

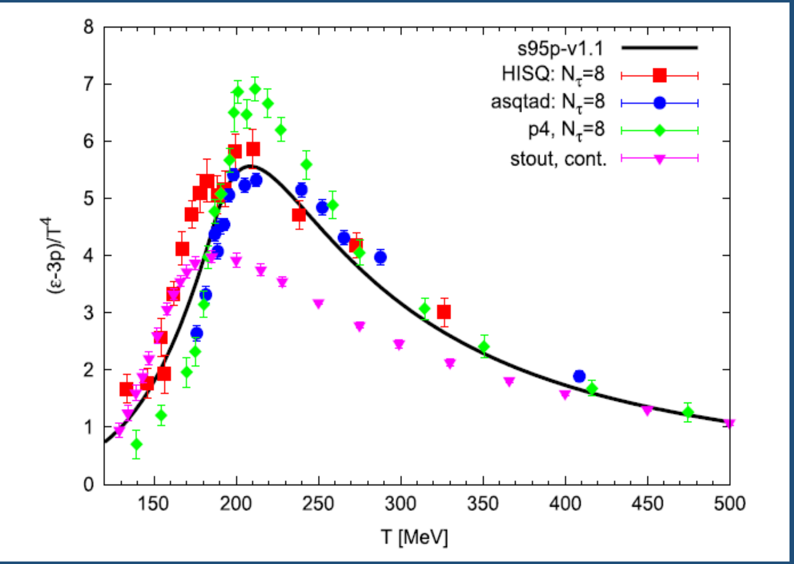
e: energy density

p: pressure

u^μ: four flow velocity

*No dissipation in this study

Equation of state



High T

→ Lattice result (hot QCD)

Low T

→ Hadron resonance gas

See also, P.Huovinen and P.Petreczky, NPA837, 26 (2010)

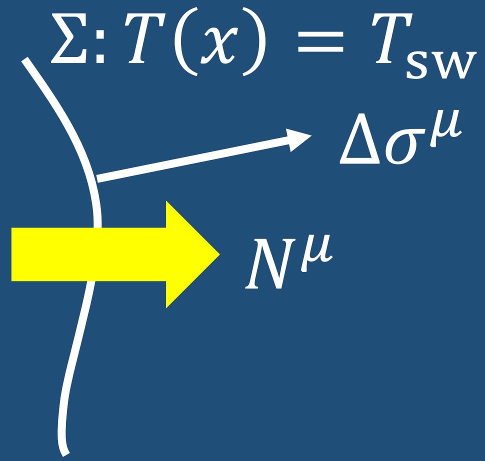
Hydro to cascade

One-particle distribution from a fluid element

F.Cooper, G.Frye (1974)

$$E\frac{d^3\Delta N}{d^3k} = \frac{g}{(2\pi)^3}\frac{k\cdot\Delta\sigma}{\exp[k\cdot u/T_{\mathrm{sw}}]\pm 1}$$

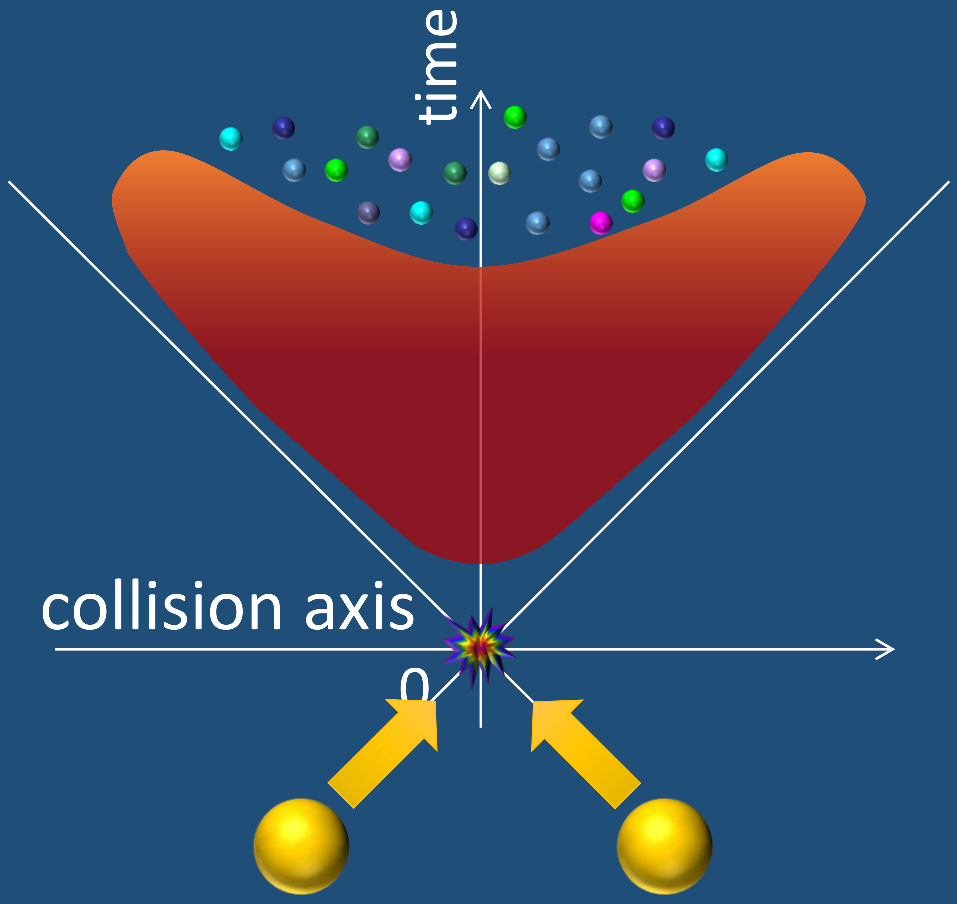
$$T_{\mathrm{sw}} = 155\ \mathrm{MeV}$$



Thermal dist. in switching hypersurface Σ boosted by fluid velocity u^μ

Output from hydro as initial condition for cascade

Integrated dynamical model



Hadronic cascade model
Microscopic description of Hadron gases

Relativistic hydrodynamics
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Initial profile of matter

Hadronic cascade model (JAM)

Incoherent convolution of hadron-hadron scattering and resonance decays

Some remarks on strangeness sector

- $\bar{K}N$ and KN incoming channel

$$\bar{K}N \to \pi Y, \bar{K}N \to Y^*, KN \to K^*N$$

*Inverse process through detailed balance

- Other processes → Additive quark model

$$\sigma_{\mathrm{tot}} = \sigma_{NN}\frac{n_1}{3}\frac{n_2}{3}\left(1 - 0.4\frac{n_{s1}}{n_1}\right)\left(1 - 0.4\frac{n_{s2}}{n_2}\right)$$

See also, Y.Nara *et al.*, PRC61, 024901 (2000),
TH and Y.Nara, PTEP 01A203 (2012).

Emission rate

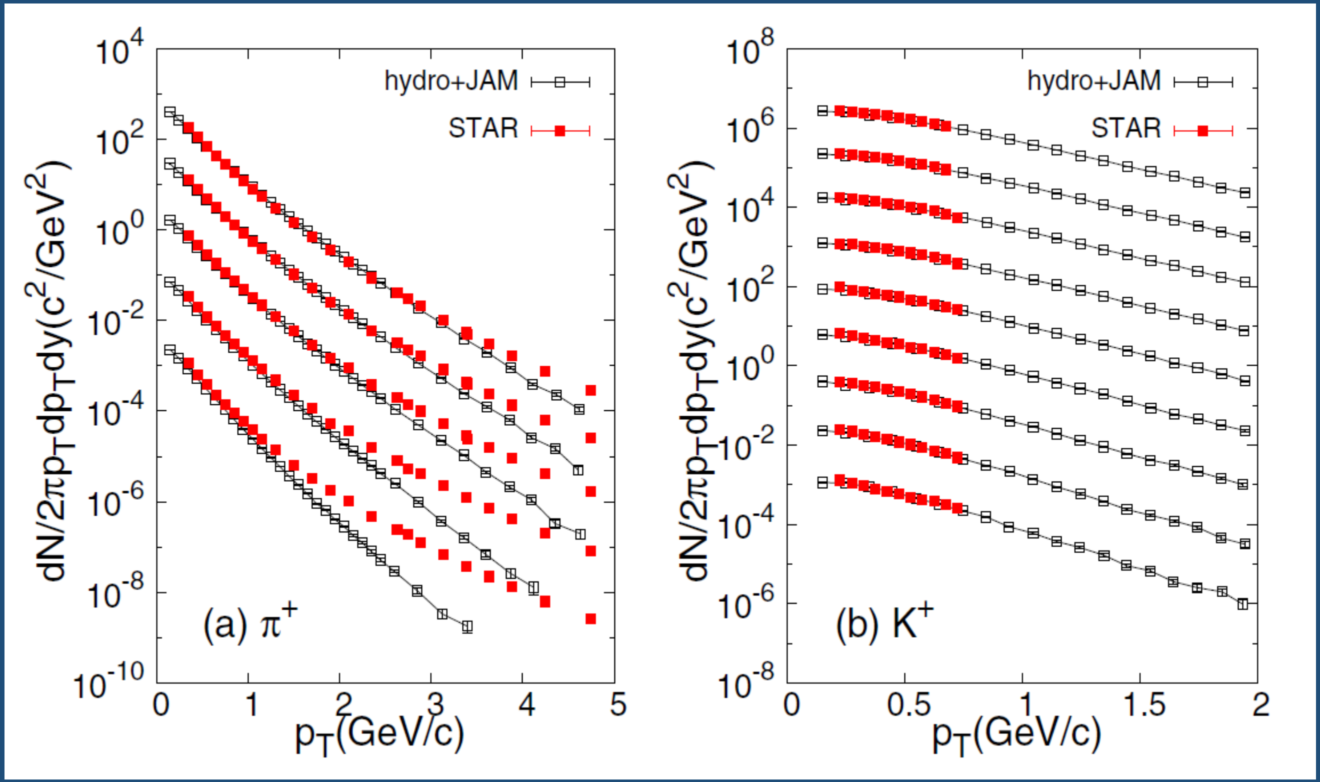
Particle emission rate with momentum $\boldsymbol{k}$ at the last interaction point x

$$S(x, \boldsymbol{k}) = E\frac{d^7N}{d^3k d^4x}$$

One-particle momentum distribution

$$W(k) = E\frac{d^3N}{d^3k} = \int d^4x S(x, \boldsymbol{k})$$

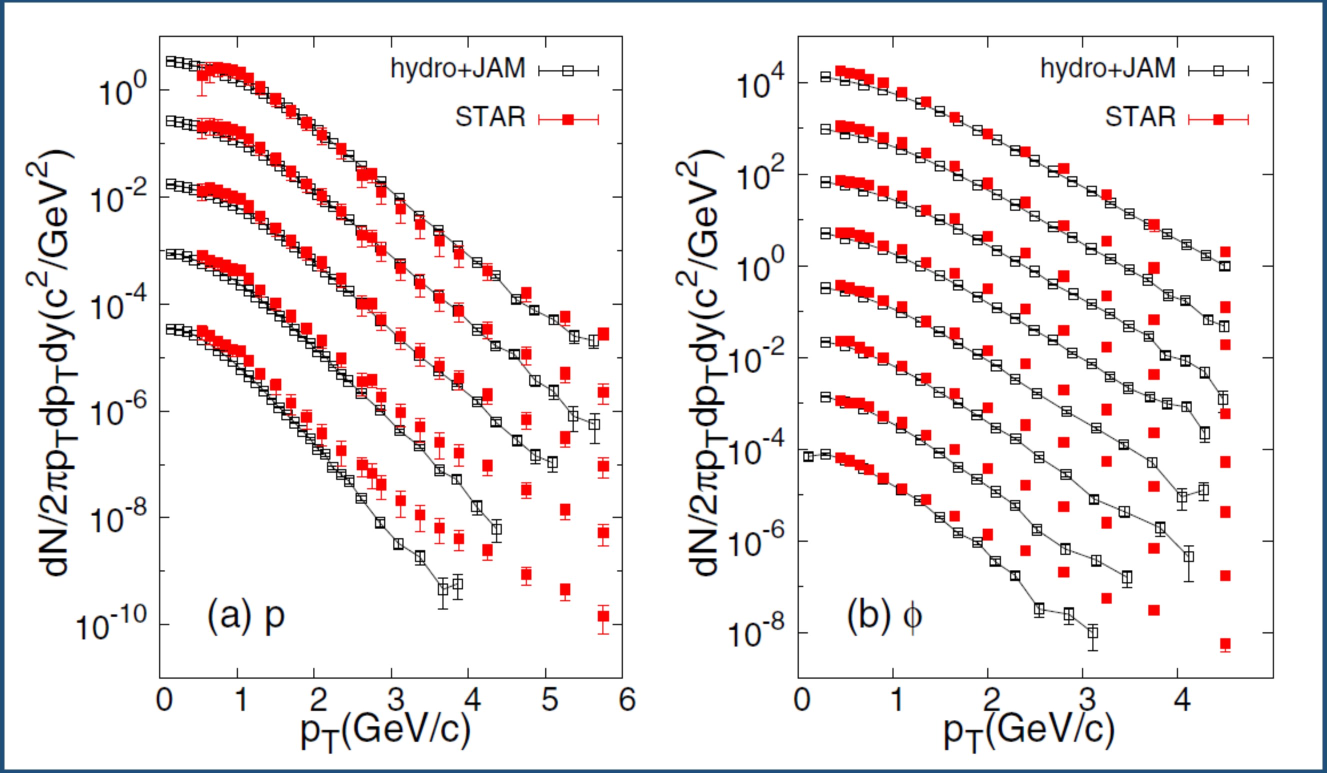
Transverse momentum spectra of pion and kaon



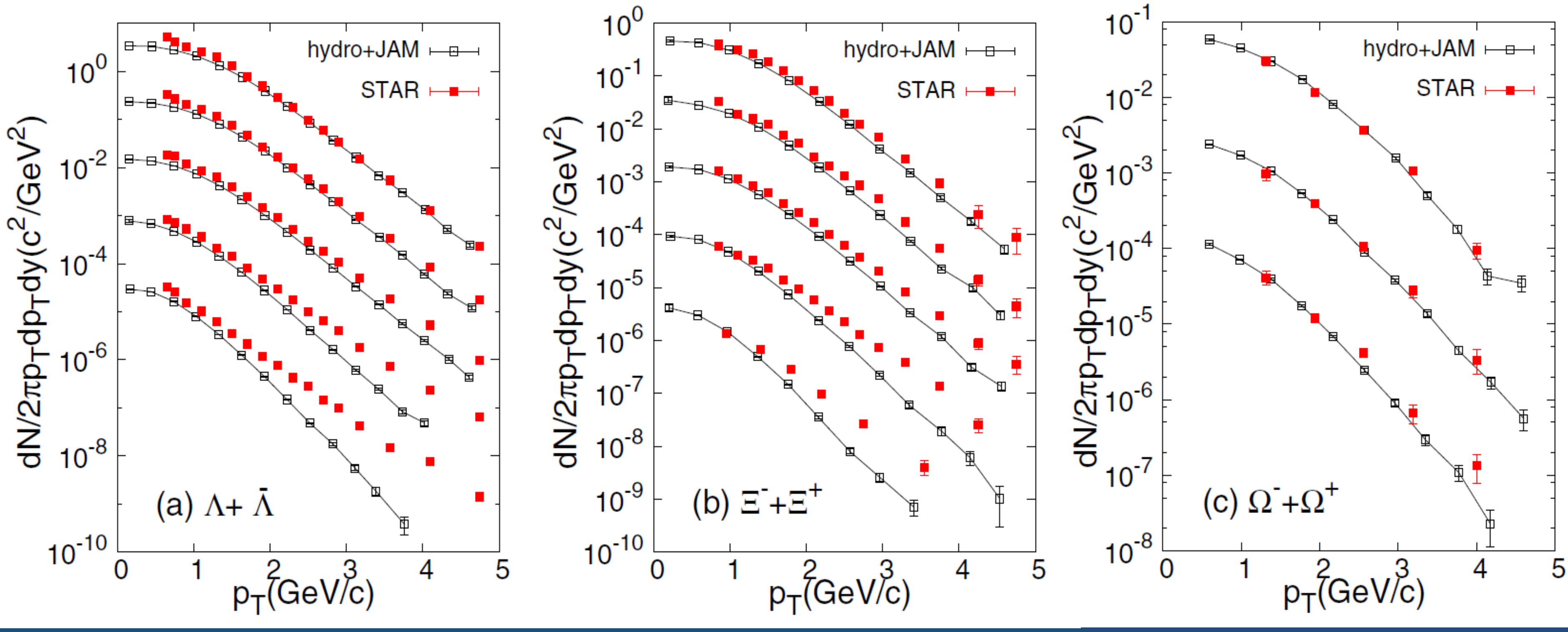
From top to bottom: Central to peripheral

Multiplied by 10^x for each result

Transverse momentum spectra of proton and phi

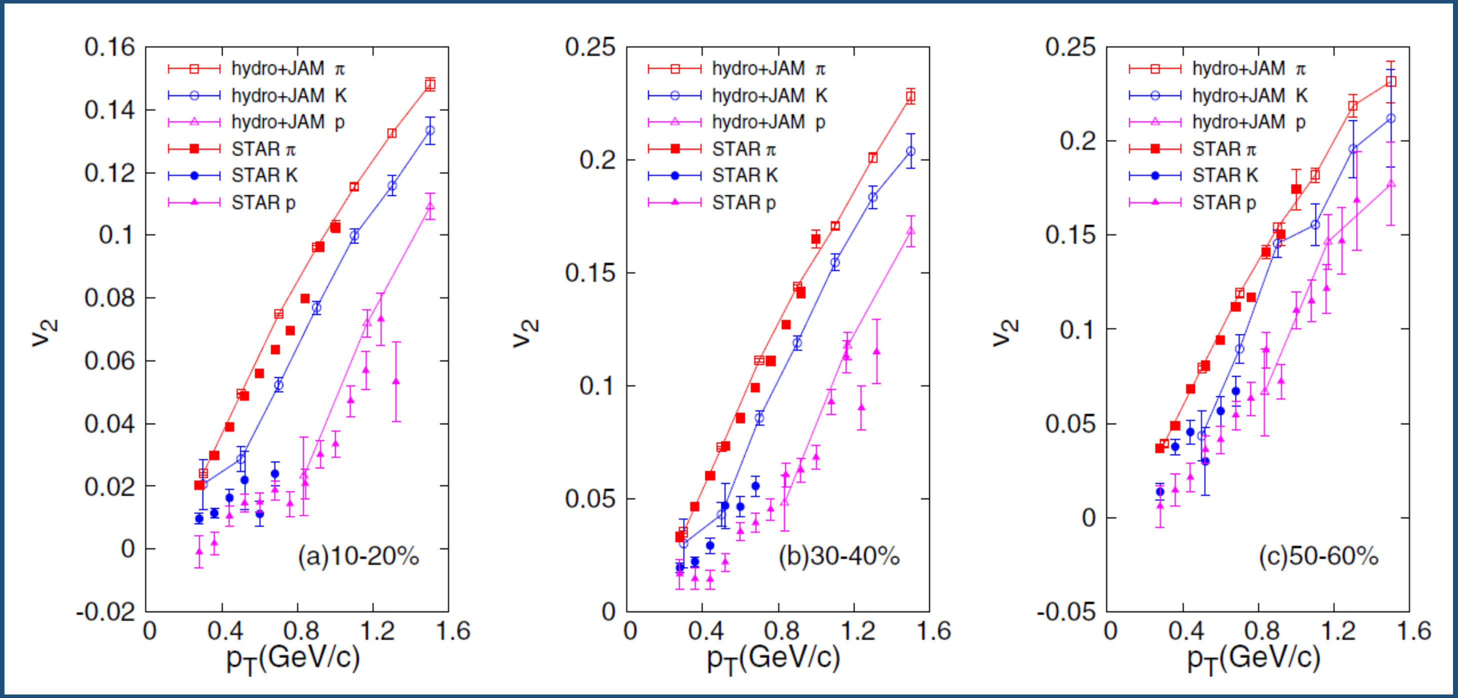


Transverse momentum spectra of hyperon

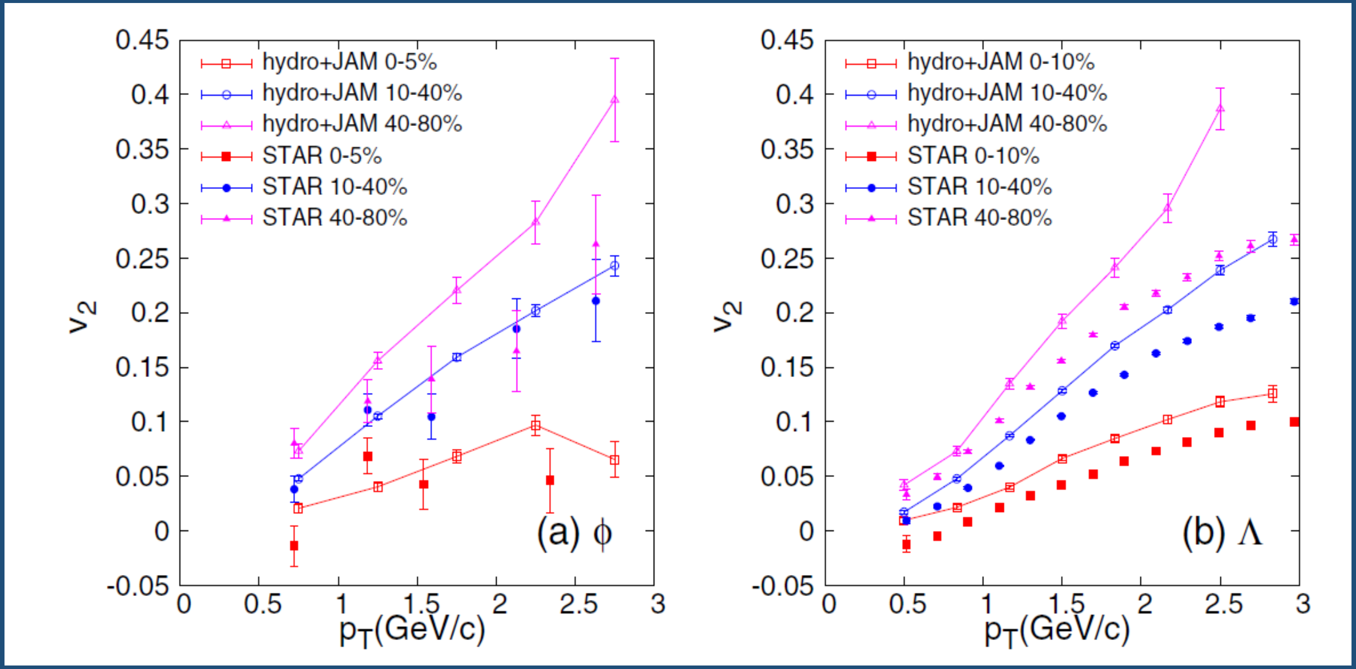


$\Sigma^0 \to \Lambda + \gamma$ included

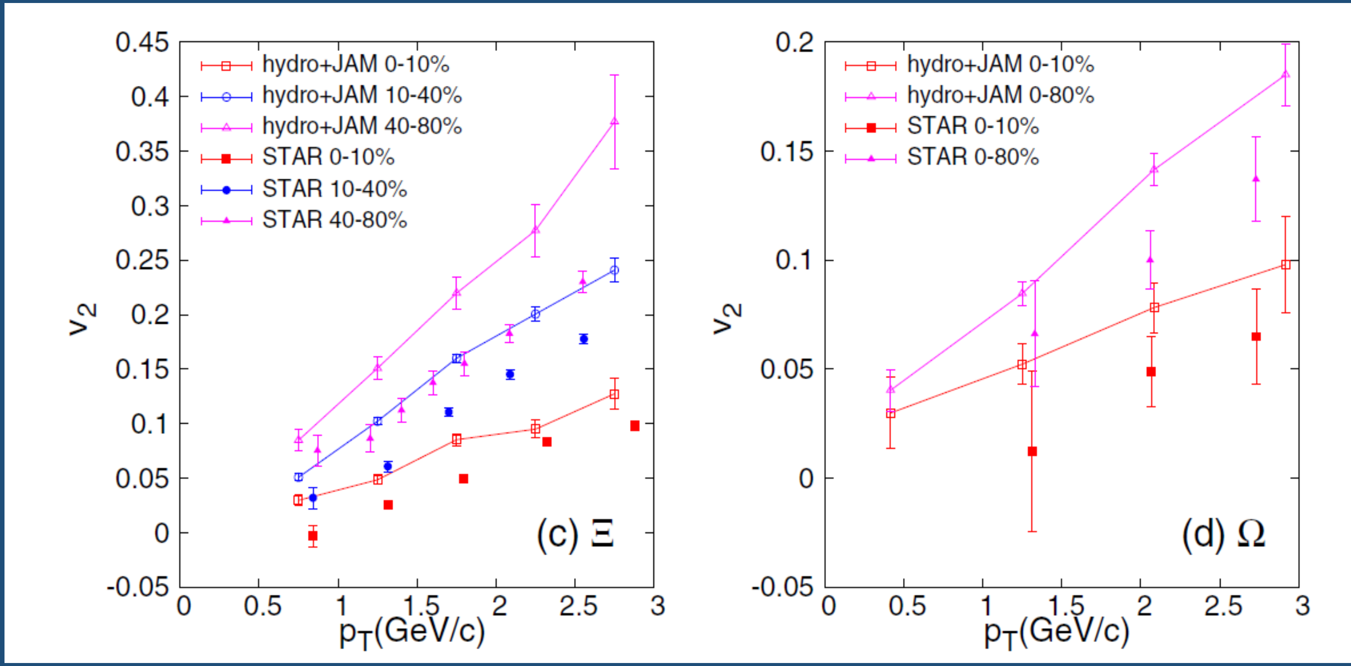
Elliptic flow parameter of pion, kaon and proton



Elliptic flow parameter of phi and Lambda



Elliptic flow parameter of Xi and Omega



Short summary so far

Reasonable reproduction of transverse momentum spectra and elliptic flow parameters including strange particles by using an integrated dynamical model

Contents

Full use of emission rate

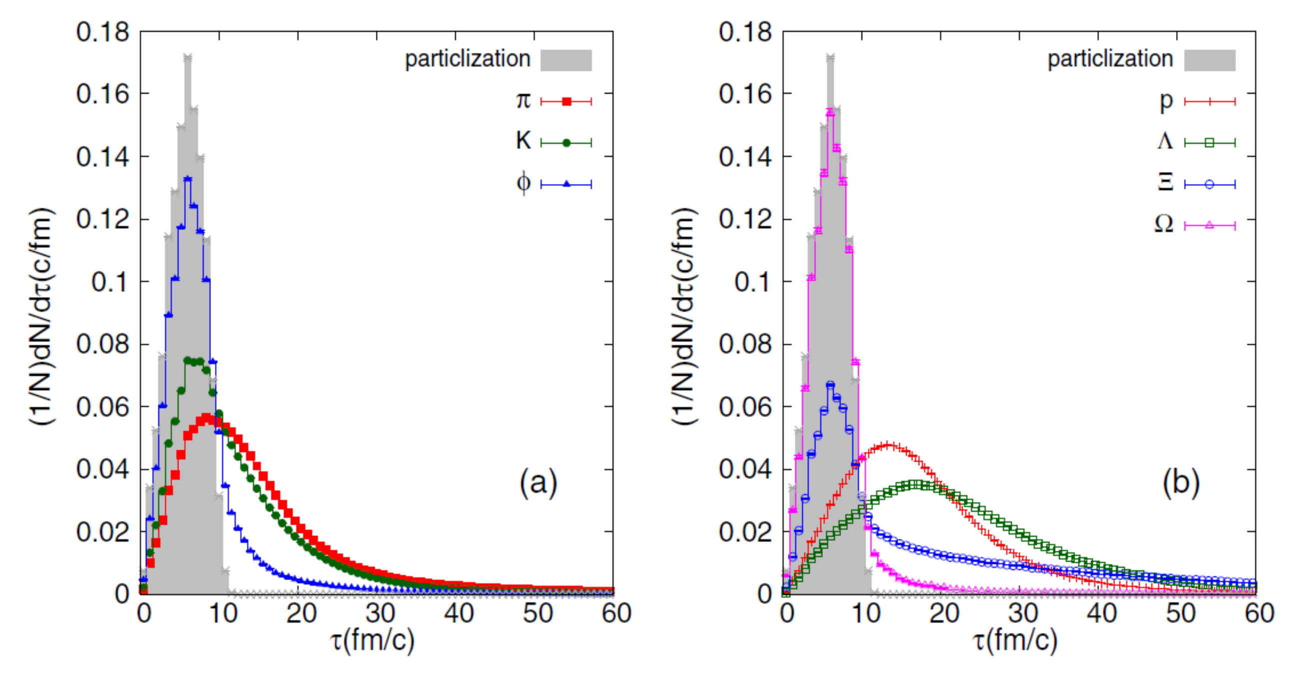
$$S(x, \boldsymbol{k}) = E \frac{d^7 N}{d^3 k \, d^4 x}$$

Utilize information about space-time distribution

- Femtoscopy
  - HBT radii
  - Source imaging
  - Information about interaction between two particles
- …

Distribution of the last interaction time

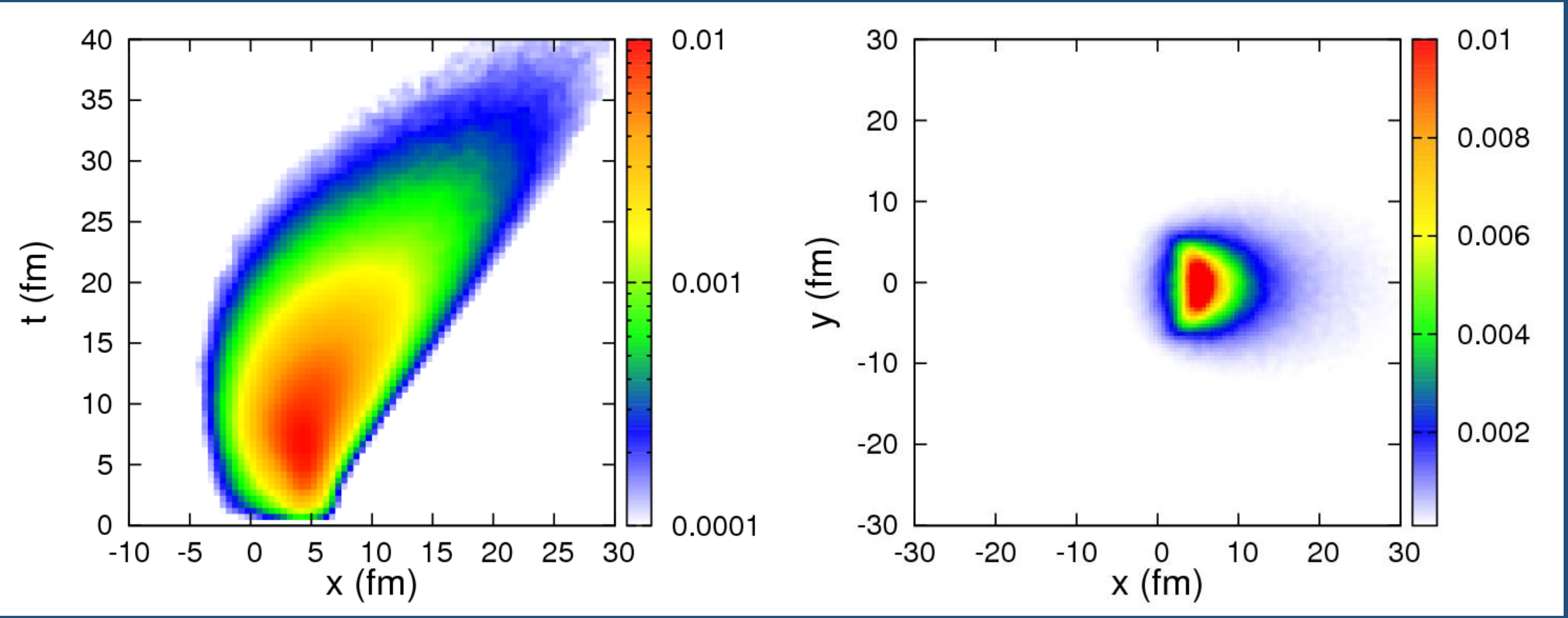
Au+Au 200 GeV, min. bias, $|y| < 1$



Early decouple of multi-strange hadrons
→ Sensitive to interaction with other particles

Distribution of particle emission point

Pion case



$S(t, x, 0.3 < k_x < 0.9\text{GeV})$ $S(x, y, 0.3 < k_x < 0.9\text{GeV})$

Non-trivial shape of distribution
→ No longer simple Gaussian

Koonin-Pratt equation

$$C_2(\boldsymbol{q}) = \frac{W_2(\boldsymbol{k}_1, \boldsymbol{k}_2)}{W_1(\boldsymbol{k}_1)W_2(\boldsymbol{k}_2)}$$

$$= 1 + \frac{\int K(\boldsymbol{q}, \boldsymbol{r}) D_{\boldsymbol{P}}(\boldsymbol{r}, \boldsymbol{q}) d^3 r}{\int D_{\boldsymbol{P}}(\boldsymbol{r}, \boldsymbol{q}) d^3 r}$$

$K(\boldsymbol{Q}, \boldsymbol{r})$: Kernel

→Information about two-particle wave functions

$D_{\boldsymbol{P}}(\boldsymbol{r})$: Un-normalized source function

→ Information about relative distance of emission points

S.E.Koonin, Phys. Lett. 70B, 43 (1977),
S.Pratt et al., Phys. Rev. C 42, 2646 (1990)

Kernel for spin ½ particles

$$K(\boldsymbol{q},\boldsymbol{r}) = |\Psi_{12}|^2 - 1$$
$$= \frac{1}{4}|\Psi_s|^2 + \frac{3}{4}|\Psi_t|^2 - 1$$

Ψ_{12}: Two-particle wave function in scattering state in Pair Co-Moving System (PCMS)

Ψ_s: Spin-singlet (symmetric in coordinate space)

Ψ_t: Spin-triplet (anti-symmetric in coordinate space)

Schrödinger eq.

Schrödinger eq. in the radial coordinate with s-wave approximation $(E > 0)$

$$\frac{d^2u(r)}{dr^2} = \frac{2\mu}{\hbar^2}[V(r) - E]u(r)\,,\ \mu = \frac{m_\Lambda}{2}$$

"Initial value problem"

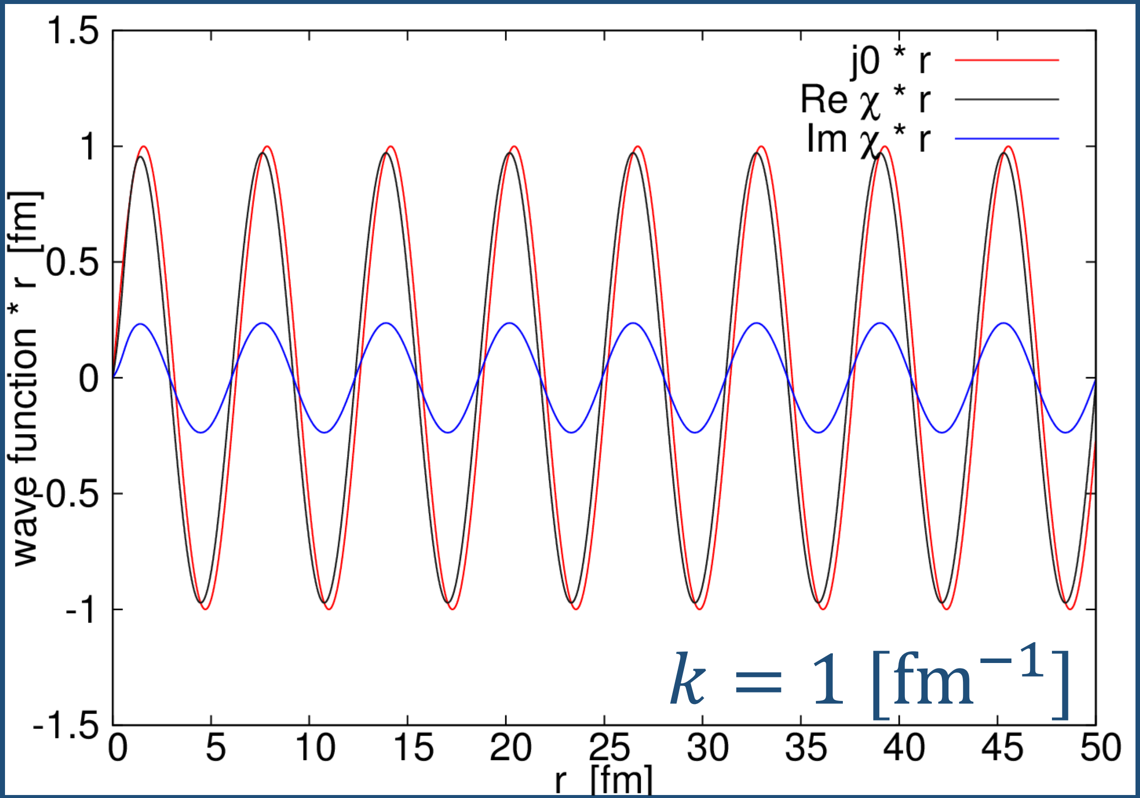
$u(0) = 0$: Regular at $r = 0$

$\frac{du}{dr}(r = 0) = 1$: Later determined by normalization

Normalized solution (asymptotically parametrized by phase shift)

$$u(r) = r \times \chi(r) \to \frac{1}{k} e^{i\delta} \sin(kr + \delta)$$

Wave function in scattering state



$$V_{\Lambda\Lambda}(r) = V_1 \exp(-r^2/{\mu_1}^2) + V_2 \exp(-r^2/{\mu_2}^2)$$

"fss2" potential model

$V_1 = -103.9$ [MeV]
$V_2 = 658.2$ [MeV]
$\mu_1 = 0.92$ [fm]
$\mu_2 = 0.41$ [fm]

Y.Fujiwara *et al*. (2007)

Solving Schrödinger eq. numerically
→ Wave function for kernels

Wave function in kernel

s-wave approximation:

$$\Psi_\mathrm{s} = \sqrt{2}\left[\cos\left(\frac{\boldsymbol{q}\cdot\boldsymbol{r}}{2}\right) + \underline{\chi(r)} - j_0\left(\frac{qr}{2}\right)\right]$$

s-wave scattering state

$$\Psi_\mathrm{t} = i\sqrt{2}\underbrace{\left[\sin\left(\frac{\boldsymbol{q}\cdot\boldsymbol{r}}{2}\right)\right]}_{\text{plane wave}}$$

*No scattering wave in triplet state due to parity $(-1)^l$ in s-wave approximation

Source function

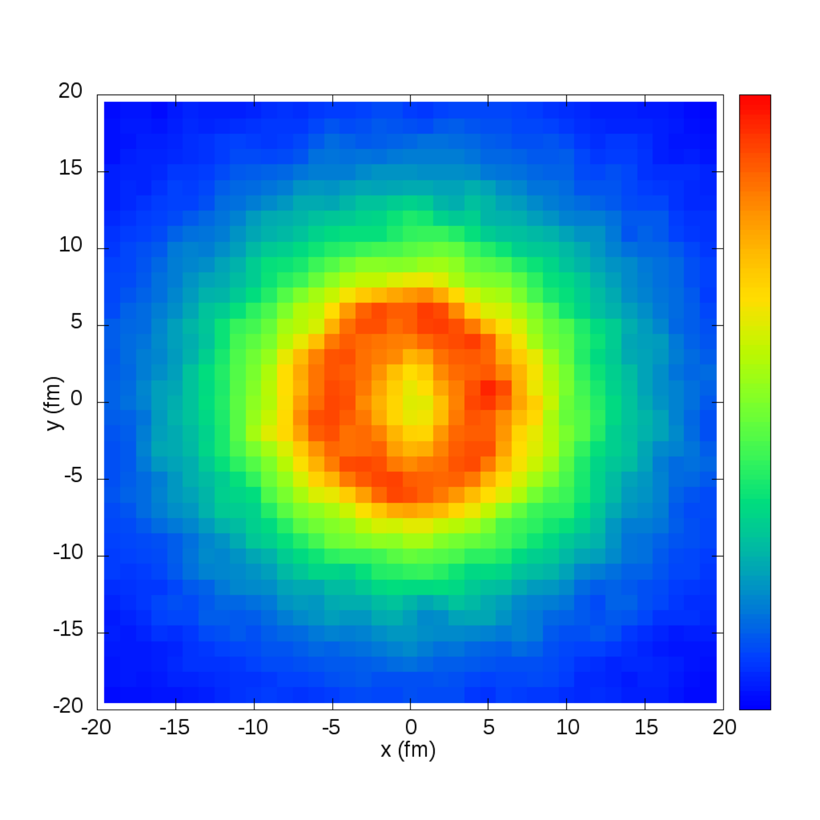
$$D_{\boldsymbol{P}}(\boldsymbol{r},\boldsymbol{q}) = \int d^4x_1 d^4x_2\, S_1 S_2 \delta^3(\boldsymbol{r}-\boldsymbol{x}_1-\boldsymbol{x}_2)$$

$$= \int dt \int d^4R\, S(x_1,\boldsymbol{k}_1) S(x_2,\boldsymbol{k}_2)$$

$S(x,\boldsymbol{k})$: One-particle emission rate
$t = t_1 - t_2$: Relative emission time
$\boldsymbol{r} = \boldsymbol{x}_1 - \boldsymbol{x}_2$: Relative distance of emission points in PCMS
$R = (x_1 + x_2)/2$: Four-dimensional center-of-mass coordinates
$\boldsymbol{P} = (\boldsymbol{k}_1 + \boldsymbol{k}_2)/2$: Average momentum (vanishing in PCMS)

Emission rate of Lambda

Lambda case



$S(x, y)$

Au+Au, $\sqrt{s_{\mathrm{NN}}} = 200$ GeV
0-80% centrality
|rapidity|<0.5

- No contribution from long-lived resonances
- No longer Gaussian as is the case for pions
- A dip at origin
- Typical length ~ 5 fm

Correlation function in model calculations

Two Λs from the same event

$(x_1, \boldsymbol{k}_1)$ $(x_2, \boldsymbol{k}_2)$

Variable transformation

$(t', \boldsymbol{r}', \boldsymbol{q}')$ $(R', \boldsymbol{P}')$

Lorentz transformation to PCMS

$(\boldsymbol{r}, \boldsymbol{q})$

$$C_2(\boldsymbol{q}) = 1 + \frac{\sum_{\mathrm{event}} \sum_{\mathrm{combination}} K(\boldsymbol{q}, \boldsymbol{r})}{\sum_{\mathrm{event}} \sum_{\mathrm{combination}} 1\,(\boldsymbol{q}, \boldsymbol{r})}$$

Feed-down contribution

At least one in the Λ-pair from the following decay channels $(\mathrm{B.R.}\sim 100\%)$

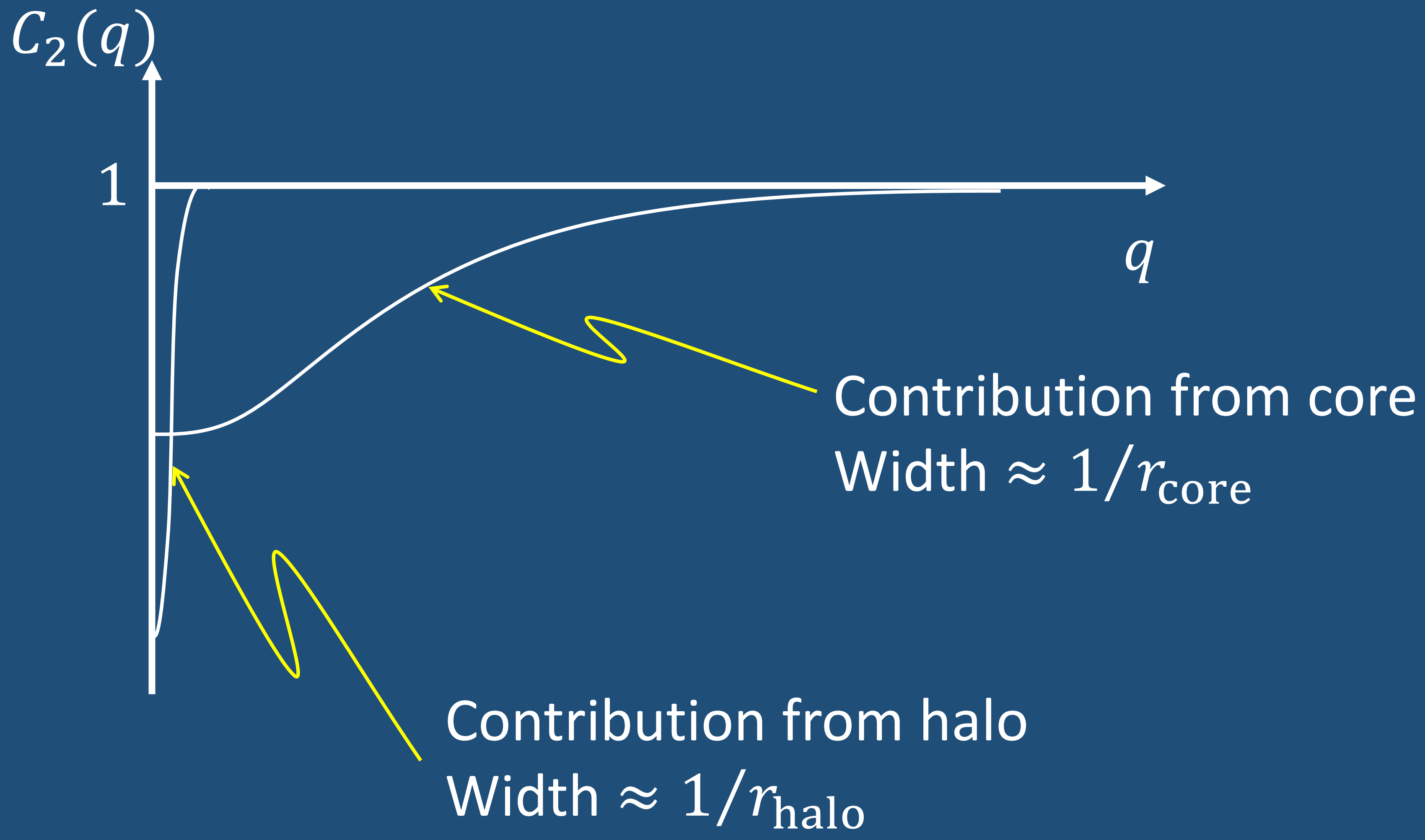
$$\Sigma^0 \to \Lambda\gamma,\ \Xi^- \to \Lambda\pi^- \text{ or } \Xi^0 \to \Lambda\pi^0,$$

assume

$$K(\boldsymbol{q},\boldsymbol{r}) \to 0 \text{ as } |\boldsymbol{r}| \gg 1/|\boldsymbol{q}|$$

# of events: 300K min. bias
→ Analysis of 0-80% centrality → 240K events

Effect of feed-down contribution



Discussion on feed-down contribution

$$C_2(\boldsymbol{q}) = 1 + \frac{\sum_{\Lambda\Lambda} K(\boldsymbol{q},\boldsymbol{r}) + \sum_{\mathrm{FD}} 0}{N_{\Lambda\Lambda}(\boldsymbol{q}) + N_{\mathrm{FD}}(\boldsymbol{q})}$$

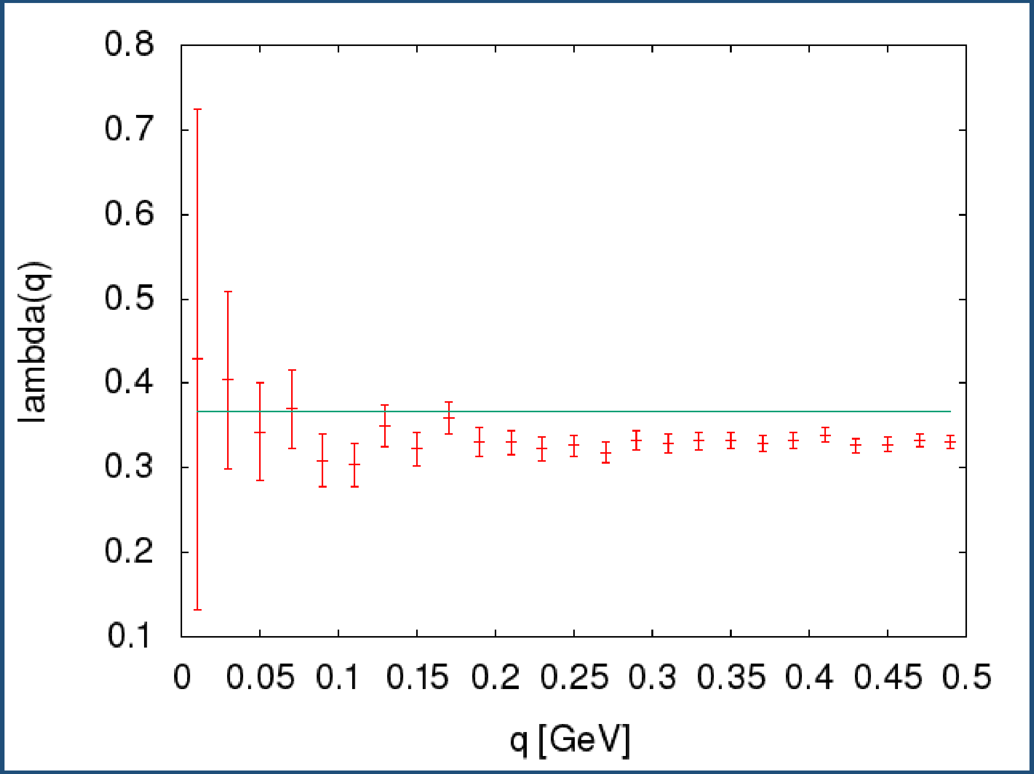
$$= 1 + \underline{\lambda(\boldsymbol{q})} \boxed{\frac{\sum_{\Lambda\Lambda} K(\boldsymbol{q},\boldsymbol{r})}{N_{\Lambda\Lambda}(\boldsymbol{q})}}$$

Feed-down correction

$$\lambda(\boldsymbol{q}) = \frac{N_{\Lambda\Lambda}}{N_{\Lambda\Lambda} + N_{\mathrm{FD}}}$$

Correlation of Lambda solely from core region

Feed-down correction factor



$$\lambda_{\mathrm{MFO}} = \left(1 - \frac{N^{\mathrm{p}}}{N_{\mathrm{tot}}}\right)^2$$

In our simulations,

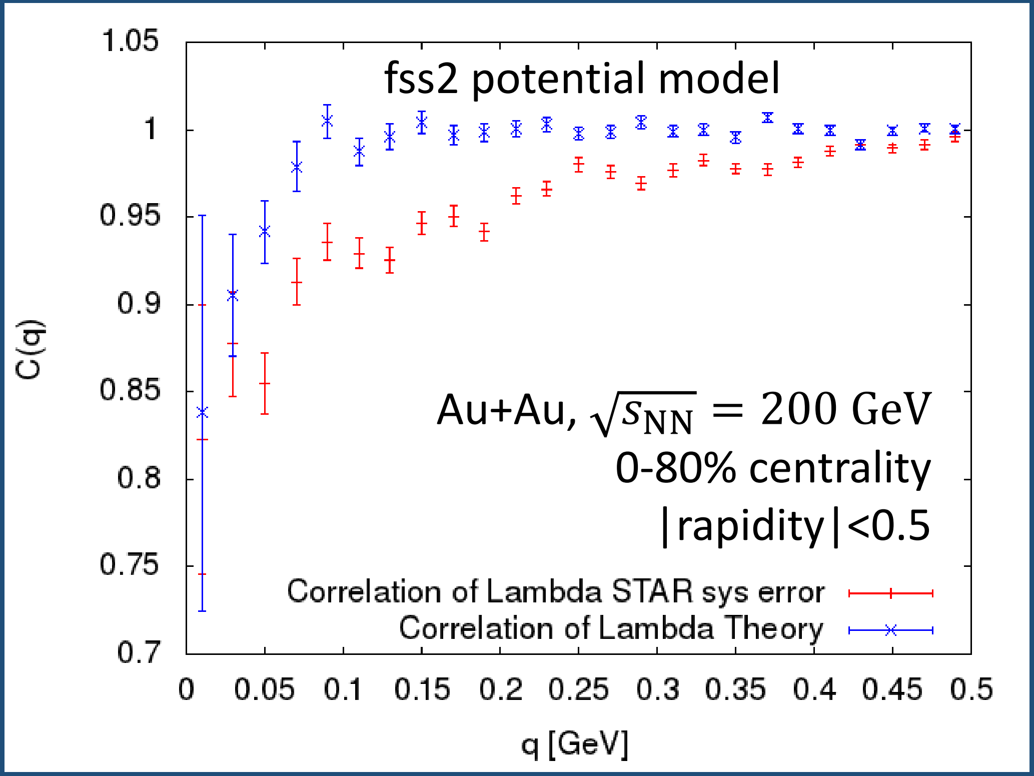
$$\lambda_{\mathrm{MFO}} \sim 0.3667$$

$$\sim (0.6056)^2$$

$$\lambda(q) \approx \lambda_{\mathrm{MFO}}$$

*MFO: K.Morita, T.Furumoto, A.Ohnishi, PRC91, 024916 (2015).

Lambda-Lambda correlation function



- Small correlation at $q > \sim 0.1$ GeV
- Need to check other potential model
- May need to gain more statistics

*# of events:
~10^5 events (this study)
~10^8 events (STAR)

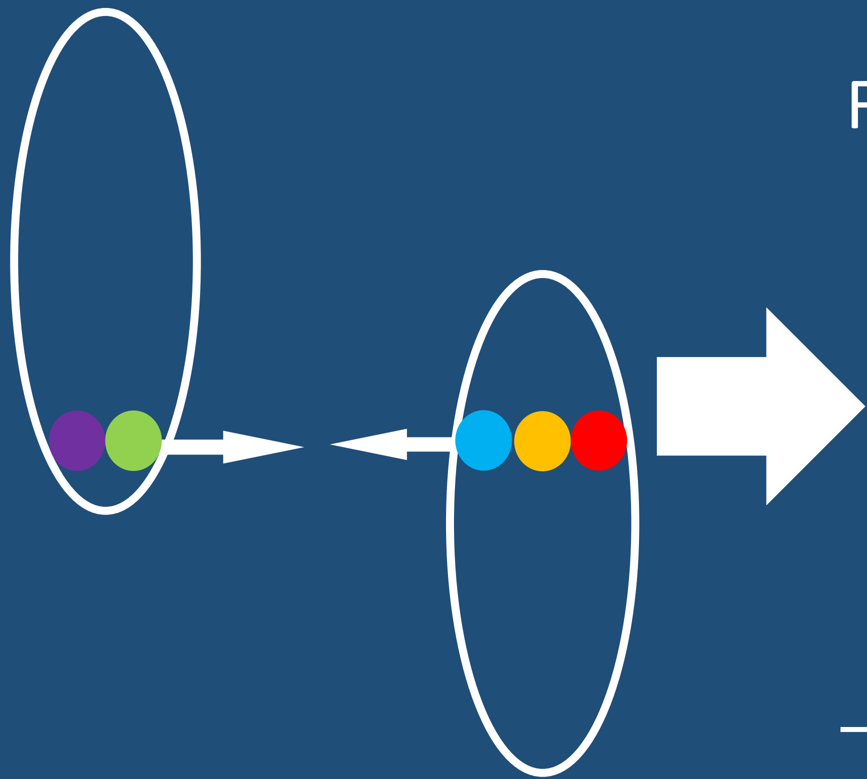
STAR, PRL114, 022301 (2015).

Contents

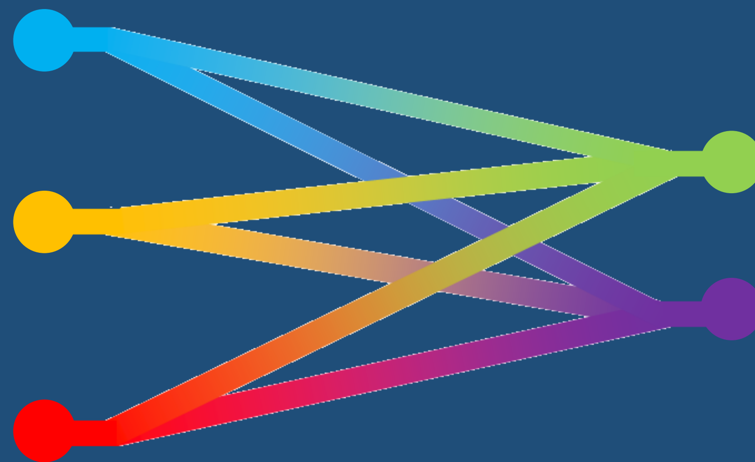
Summary and outlook

- First attempt to calculate Lambda-Lambda correlation function in high energy nuclear collisions within an integrated dynamical model
    - Non-trivial shape of emission rate of Lambda
    - Large contribution from long-lived resonances
    - Deviation from the STAR data within a certain model potential (“fss2”)
- Constraint of Lambda-Lambda potential from data (any other combination?)

Modified BGK model



Formation of hadron strings



$-Y_{\mathrm{beam}} \leftarrow$ rapidity $\rightarrow Y_{\mathrm{beam}}$

Number of participants
From Glauber model

Parametrization of
entropy density dist.

Some details of initial conditions in longitudinal direction

$$s_0(\tau_0, \eta_s, x_\perp) = \frac{dS}{\tau_0 d\eta_s d^2 x_\perp}$$

$$= \frac{C}{\tau_0} f^{pp}(\eta_s) \left[\frac{1-\alpha}{2} \left(\frac{Y_b - \eta_s}{Y_b} \rho_A(x_\perp) + \frac{Y_b + \eta_s}{Y_b} \rho_B(x_\perp) \right) + \alpha \rho_{\mathrm{coll}}(x_\perp) \right]$$

$$f^{pp}(\eta_s) = \exp\left[-\theta(|\eta_s| - \Delta\eta) \frac{(|\eta_s| - \Delta\eta)^2}{\sigma_\eta^2} \right]$$

RHIC energy:

$C = 15.0, \alpha = 0.18, \tau_0 = 0.6 \text{ fm}, \Delta\eta = 1.3, \sigma_\eta = 2.1$