Compositeness of near-threshold quasi-bound states

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- Introduction ~compositeness of bound state~

Part I

- Extension to the quasi-bound state
- Applications to exotic hadrons ~ $\Lambda(1405), a_0(980), f_0(980)$ ~

Part II

- Derivation without convergence of ERE
- Extended relation with the CDD pole contribution

Introduction ~exotic hadrons~

- Exotic hadrons

Hadrons which do not coincide with the predictions of the quark model.

More complicated internal structure can be expected.

- tetra quark, penta quark
- hadron molecule ⋯

It is important to reveal the internal structure of exotics.

e.g. ; Λ(1405)

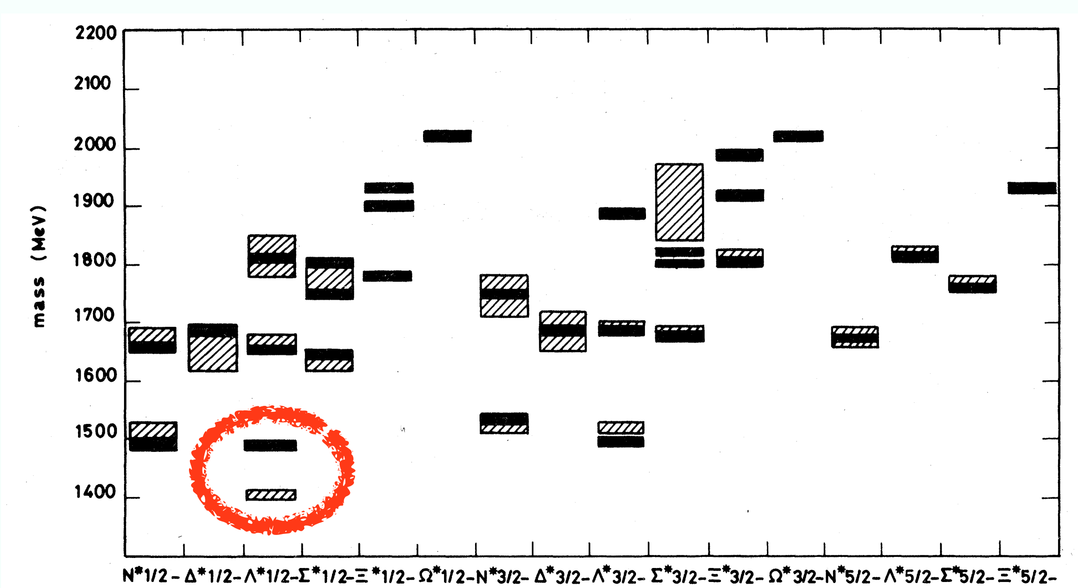
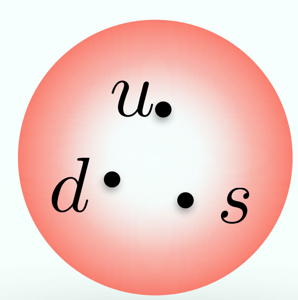


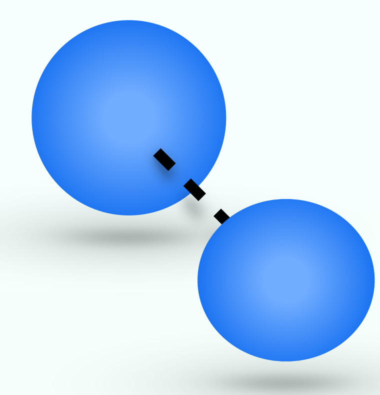
FIG. 1. Comparison of the predicted and observed spectrum of negative-parity baryons. The shaded regions corre-

N. Isgur, and G. Karl, Phys. Rev. D18, 4187 (1978)

exited Λ state(uds)



$\bar{K}N$ bound state



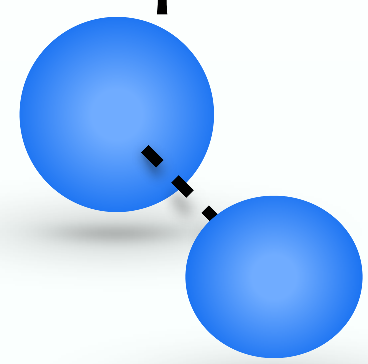
Compositeness of bound state

Previous work

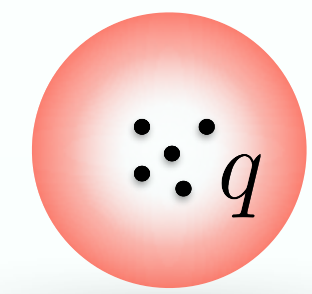
Introduced to study deuteron by Weinberg.

S. Weinberg, Phys. Rev. 137, B672 (1965)

Composite



Elementary



$X = 1$ $\longleftrightarrow$ $X = 0$

$Z = 0$ $\qquad$ $Z = 1$

Condition

- weakly bound
- stable state
- s-wave

Output

- X ;weight of composite state $(0 < X < 1)$
- Z ;wave function renormalization $(0 < Z < 1)$
- a_0 ;scattering length
- B ;binding energy

$$R = \frac{1}{\sqrt{2\mu B}}$$

(μ;reduced mass of scat. state)

$$\boxed{a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(R_{\mathrm{typ}}/R\right)\right\}}$$

typical length scale

We can extract the information of the internal structure using experimental observables.

Part I

~unstable states~

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Extension to the quasi-bound state.

System

Two channel scattering

- scattering channel $|\boldsymbol{p}\rangle$
- decay channel $|\boldsymbol{p}'\rangle$

$|\boldsymbol{p}\rangle$ can decay to $|\boldsymbol{p}'\rangle$.

Unstable quasi-bound state $|QB\rangle$ exists near $|\boldsymbol{p}\rangle$ threshold.

The interaction has a typical length scale R_{typ} .

Effective field theory

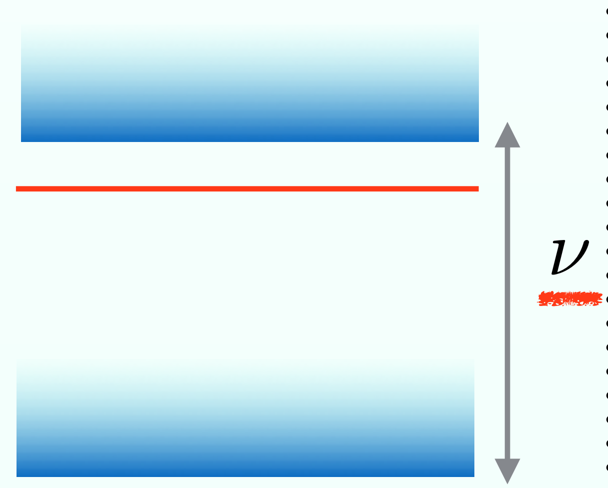
To discuss the near-threshold physics, we use following non-relativistic EFT.

Free field H_{free} eigenstate

$|\boldsymbol{p}\rangle$ scattering channel

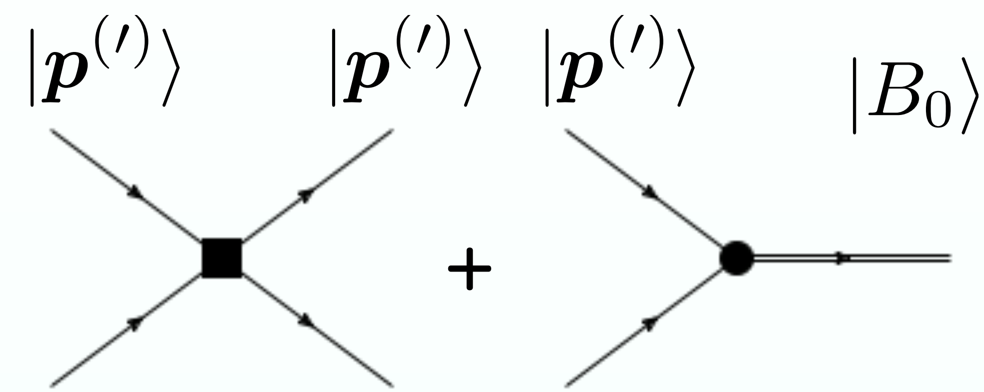
$|B_0\rangle$ discrete channel

$|\boldsymbol{p}'\rangle$ decay channel



Interaction

$\mathcal{H}_{\mathrm{int}} =$



point interaction

Eigenstate

$$H = H_{\mathrm{free}} + H_{\mathrm{int}}$$

$$H|QB\rangle = E_{QB}|QB\rangle$$

$$E_{QB} = -B - i\Gamma/2$$; complex

We consider the compositeness of $|\boldsymbol{p}\rangle$ channel ; X.

Extension to the quasi-bound state.

Definition of compositeness

Bound state

Bound state $|B\rangle$ is normalized with $\langle B|B\rangle = 1$

- $X + Z = 1$
- $0 < X, Z < 1$

$$X \equiv \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \langle B|\boldsymbol{p}\rangle\langle\boldsymbol{p}|B\rangle = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} |\langle\boldsymbol{p}|B\rangle|^2$$

The probabilistic interpretation is guaranteed for X and Z.

Quasi-bound state

To normalize unstable state, we introduce Gamow state $|\overline{QB}\rangle$.

Normalization condition becomes

$$\langle\overline{QB}|QB\rangle = \langle QB^*|QB\rangle = 1.$$

T. Berggren, Nucl. Phys. A 109 (1968)

The expectation value of the any operator becomes complex number.

- $X + Z = 1$
- $\cancel{0 < X, Z < 1}$ $X, Z \in C$

$$X \equiv \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \langle\overline{QB}|\boldsymbol{p}\rangle\langle\boldsymbol{p}|QB\rangle$$

The probabilistic interpretation is not guaranteed!

Extension to the quasi-bound state.

Y. Kamiya and T. Hyodo, arXiv:1509.00146 [hep-ph].

Phys. Rev. C. 93.035203

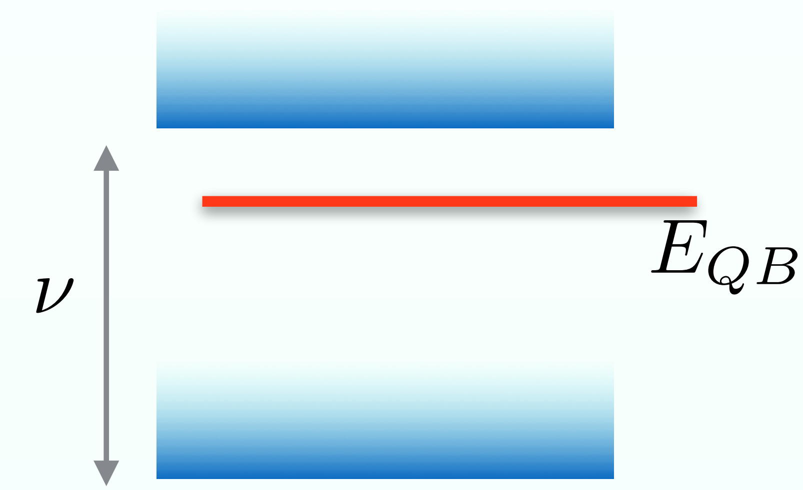
- Assuming $|E_{QB}|$ is small, we expand a_0 with respect to $1/R$.

$$a_0 = R\left[\underbrace{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\mathrm{typ}}}{R}\right|\right)}_{\text{original}} + \underbrace{\mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)}_{\text{new}}\right]$$

$$R = \frac{1}{\sqrt{-2\mu E_{QB}}}$$

$$l = \frac{1}{\sqrt{2\mu\nu}}$$

If $\underline{|R_{\mathrm{typ}}/R|}$ and $\underline{|l/R|^3}$ are sufficiently smaller than 1, we can extract X from a_0 and E_{QB} .



- Notice
  - a_0 , E_{QB}, X are all complex numbers, then above relation is established among them.
  - If the the contribution of decaying mode is neglected, the compositeness relation is same to the one for bound state.
  - The same argument is valid for the case with $\mathrm{Re}\ E_h > 0$.

Interpretation of X

Interpretation of the complex compositeness

- There is no common interpretation of the complex X.

bound state case

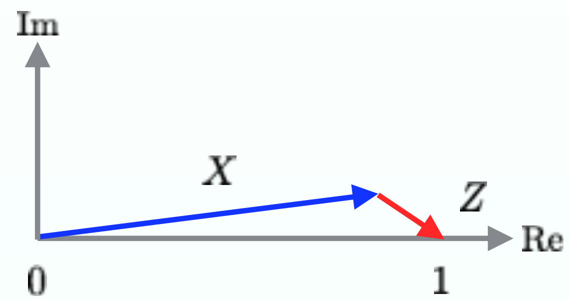
$$\begin{cases} X = 0.8 \\ Z = 0.2 \end{cases}$$

probabilistic interpretation is available

(1) close to bound state case

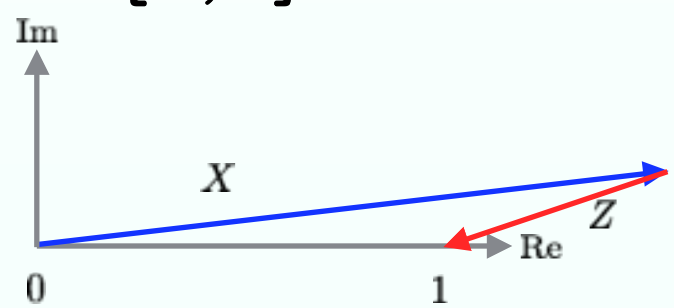
$$\begin{cases} X = 0.8 + 0.1i \\ Z = 0.2 - 0.1i \end{cases}$$

small cancellation in X+Z



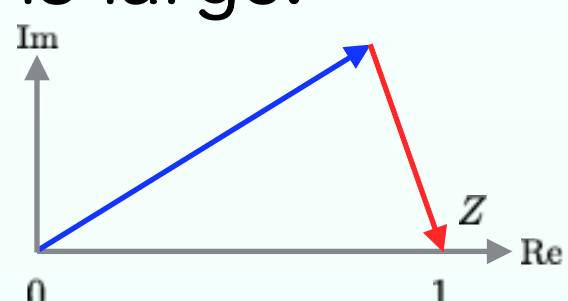
(2-a) When real part is not in [0,1]

$$\begin{cases} X = 1.9 + 0.2i \\ Z = -0.9 - 0.2i \end{cases}$$



(2-b) When imaginary part is large.

$$\begin{cases} X = 0.9 + 0.8i \\ Z = 0.1 - 0.8i \end{cases}$$



large cancellation in X+Z

When the cancelation is small,
we can interpret the complex compositeness.

Interpretation of X

Our proposal

c.f. T. Berggren, Phys. Lett. B 33 (1979) 8

For probabilistic interpretation we define the following real quantities.

$\tilde{X}$; probability to find the scattering state in physical state

$\tilde{Z}$; probability to find the other states

U ; degree of uncertainty of the interpretation

$$\tilde{X} \equiv \frac{1-|Z|+|X|}{2}$$
$$\tilde{Z} \equiv \frac{1-|X|+|Z|}{2}$$
$$U \equiv |Z|+|X|-1$$

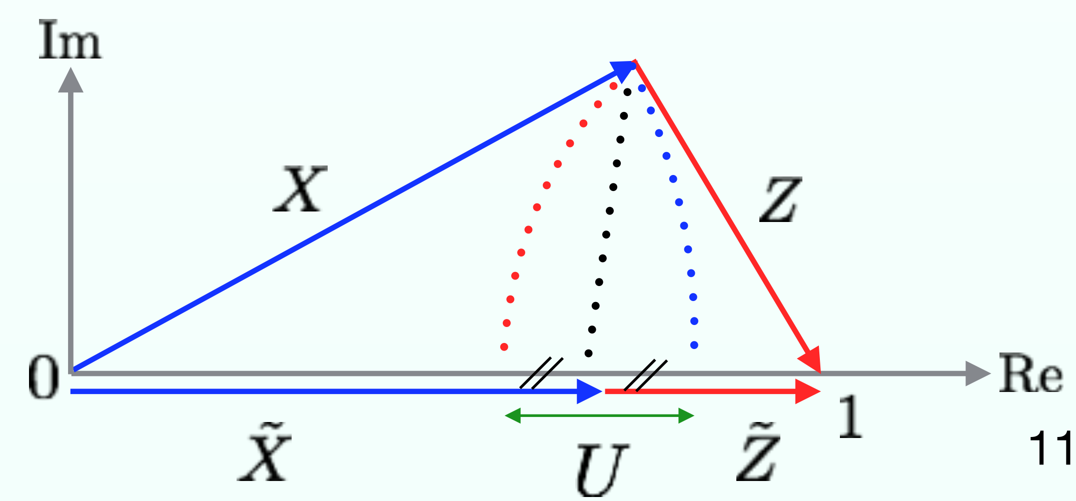
conditions :

- $\tilde{X}+\tilde{Z}=1$
- $0 \le \tilde{X}, \tilde{Z} \le 1$
- When the cancellation is 0,

  $\tilde{X}=X, \tilde{Z}=Z, U=0$.
- U becomes large

  when the cancellation becomes large.

If U is small, we interpret $\tilde{X}$ as the probability.



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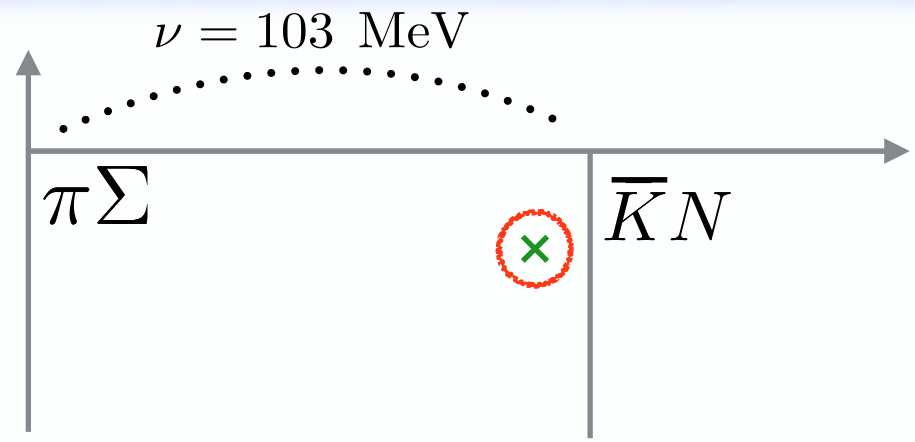
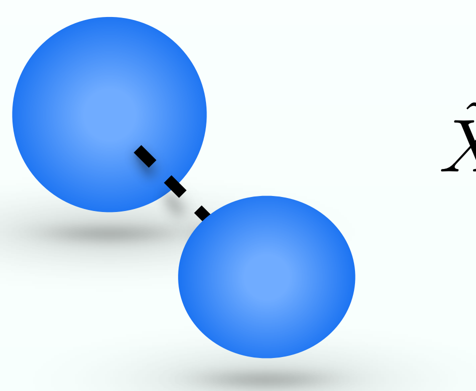
Applications to hadrons



- $\Lambda(1405)$ ($I=0$ $\bar{K}N$ scattering)

  $J^P = \frac{1}{2}^-$

$\bar{K}N$ molecule?



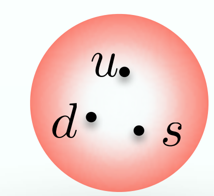
$\tilde{X} = 1$

or

other components?

$\tilde{X} = 0$

e.g.



- Λ excited states(uds)
- penta-quark state

…

R_{typ} is estimated from rho meson exchange int. $(R_{\rm typ} \sim 0.25\ \mathrm{fm})$ → $\left|\frac{R_{\rm typ}}{R}\right| \lesssim 0.17$

$R = \frac{1}{\sqrt{-2\mu E_{QB}}}$

$l = \frac{1}{\sqrt{2\mu\nu}}$

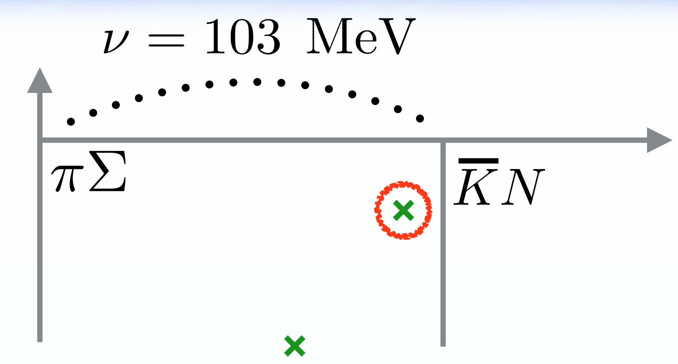
l is estimated from difference of the threshold energy → $\left|\frac{l}{R}\right|^3 \lesssim 0.14$

$$a_0 = R\left[\frac{2X}{1+X} + \underbrace{\mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)}_{\text{can be neglected}}\right] \Rightarrow X = \frac{a_0}{2R - a_0} \Rightarrow \tilde{X}, U$$

Applications to hadrons

- $\Lambda(1405)$ in $I=0$ $\bar{K}N$ scattering

  We use E_{QB} and a_0 in the following papers.



(1) Y. Ikeda, T. Hyodo and W. Weise, Nucl. Phys. A 881 98 (2012)

(2) M. Mai and U. G. Meissner, Nucl. Phys. A 900, 51 (2013)

(3) Z. H. Guo and J. A. Oller, Phys. Rev. C 87, 035202 (2013)

(4)M. Mai and U.-G. Meißner, Eur. Phys. J. A 51, 30 (2015).

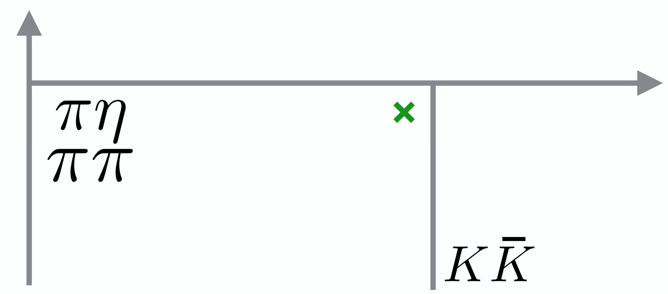
Ref.	E_{QB} (MeV)	a_0 (fm)	X	$\tilde{X}$	U
(1)	-10-i26	1.39-i0.85	1.3+i0.1	1.0	0.5
(2)	-4-i8	1.81-i0.92	0.6+i0.1	0.6	0.0
(3)	-13-i20	1.30-i0.85	0.9-i0.2	0.9	0.1
(4)-1	2-i10	1.21-i1.47	0.6+i0.0	0.6	0.0
(4)-2	- 3-i12	1.52-i1.85	1.0+i0.5	0.8	0.6

- U is small enough. —> $\tilde{X}$ can be considered as the probability.
- $\tilde{X}$ is close to 1.

$\boxed{\Lambda(1405)\ :\ \bar{K}N \text{ composite dominance}}$

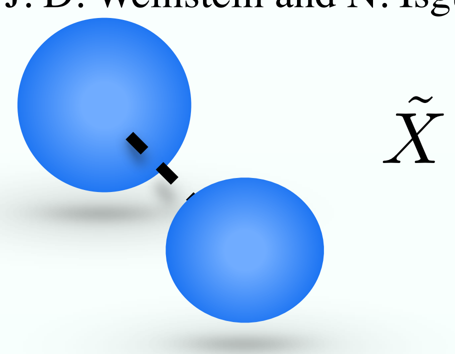
Applications to hadrons

- $a_0(980)$、$f_0(980)$ ($K\bar{K}$ scattering)
  $(I=1)$ $(I=0)$
  $J^{PC}=0^{++}$



$K\bar{K}$ molecule ?

J. D. Weinstein and N. Isgur, PRD 41 (1990)



$\tilde{X}=1$

or

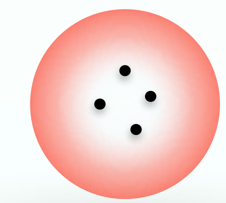
other components?

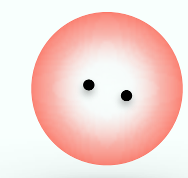
e.g.

$\tilde{X}=0$

- tetra quark state
  R. L. Jaffe, PRD 15 (1977)
- $q\bar{q}$ meson state

…





$$\left|\frac{R_{\mathrm{typ}}}{R}\right| \lesssim 0.17 \qquad \left|\frac{l}{R}\right|^3 \lesssim 0.04$$

$$a_0 = R\left[\frac{2X}{1+X} + \underbrace{\mathcal{O}\left(\left|\frac{R_{\mathrm{typ}}}{R}\right|\right)} + \underbrace{\mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)}\right] \Rightarrow X = \frac{a_0}{2R-a_0} \Rightarrow \tilde{X}, U$$

can be neglected

Applications to hadrons

- $a_0(980)$ in $K\bar{K}$ scattering

  We determine E_{QB} and a_0 from Flatte parameters which are obtained experimental analysis.

c. f. : V. Baru et al. Phys. Lett. B 586, 53 (2004)

T. Sekihara and S. Kumano, Phys. Rev. D 92, 034010 (2015)

(1) G. S. Adams et al. [CLEO Collaboration], Phys. Rev. D 84, 112009 (2011)

(2) F. Ambrosino et al. [KLOE Collaboration], Phys. Lett. B 681, 5 (2009)

(3) D. V. Bugg, Phys. Rev. D 78, 074023 (2008)

(4) S. Teige et al. [E852 Collaboration], Phys. Rev. D 59, 012001 (1999)

Set	E_{QB} (MeV)	a_0 (fm)	X	$\tilde{X}$	U
(1)	31-i70	-0.03-i0.53	0.2-i0.2	0.3	0.1
(2)	3-i25	0.17-i0.77	0.2-i0.2	0.2	0.1
(3)	9-i36	0.05–i0.63	0.2-i0.2	0.2	0.1
(4)	15-i29	-0.13-i0.52	0.1-i0.4	0.1	0.1

- U is small enough. —> $\tilde{X}$ can be considered as the probability.
- $\tilde{X}$ is close to 0.

$\boxed{a_0(980) : \text{small } K\bar{K} \text{ fraction}}$

Applications to hadrons

- $f_0(980)$ in $K\overline{K}$ scattering

  We determine E_{QB} and a_0 from Flatte parameters which are obtained experimental analysis.

c. f. T. Sekihara and S. Kumano, Phys. Rev. D 92, no. 3, 034010 (2015)

(1) T. Aaltonen et al. [CDF Collaboration], Phys. Rev. D 84, 052012 (2011)

(2) F. Ambrosino et al. [KLOE Collaboration], Phys. Lett. B 634, 148 (2006)

(3) A. Garmash et al. [Belle Collaboration], Phys. Rev. Lett. 96, 251803 (2006)

(4) M. Ablikim et al. [BES Collaboration], Phys. Lett. B 607, 243 (2005)

(5) J. M. Link et al. [FOCUS Collaboration], Phys. Lett. B 610, 225 (2005)

(6) M. N. Achasov et al., Phys. Lett. B 485, 349 (2000)

Ref.	E_{QB} (MeV)	a_0 (fm)	X	$\tilde{X}$	U
(1)	19-i30	0.02-i0.95	0.3-0.3	0.4	0.2
(2)	-6 -i10	0.84-i0.85	0.3-i0.1	0.3	0.0
(3)	-8 -i28	0.64-i0.83	0.4-i0.2	0.4	0.1
(4)	10-i18	0.51-i1.58	0.7-i0.3	0.6	0.1
(5)	-10-i29	0.49-i0.67	0.3-i0.1	0.3	0.0
(6)	10-i7	0.52-i2.41	0.9-i0.2	0.9	0.1

- U is small enough. —> $\tilde{X}$ can be considered as the probability.
- Values of $\tilde{X}$ are not consistent.

More precise analysis is needed.

Part II

~CDD pole contribution~

CDD pole and weak-binding relation

CDD(Castillejo Dalitz Dyson) pole(E_c) and internal structure

$$\boxed{\text{CDD pole : } f(E_c) = 0}$$

L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. 101, 453 (1956).

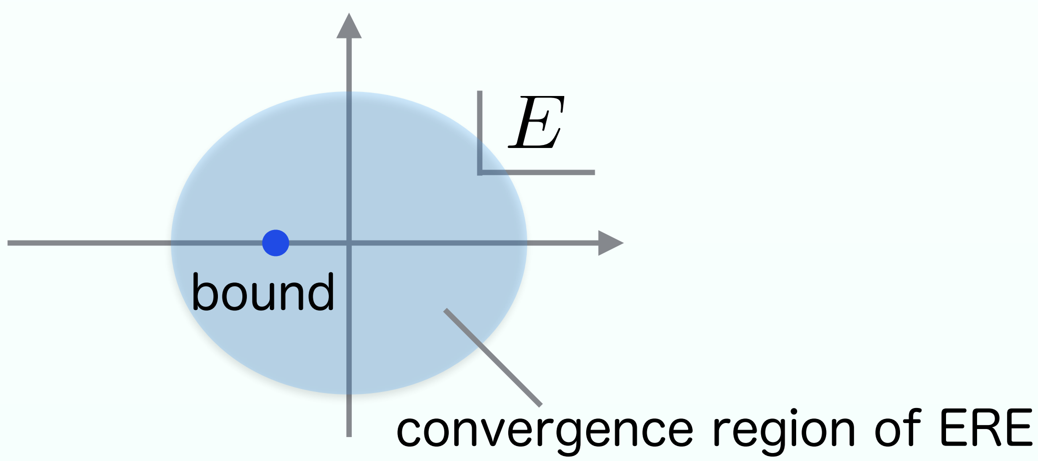
G. F. Chew and S. C. Frautschi, Phys. Rev. 124, 264 (1961).

・represents the contribution from outside of the model

V. Baru et al, Eur. Phys. J. A44, 93 (2010), 1001.0369.
T. Hyodo, Phys. Rev. Lett. 111 (2013) 132002.
Z.-H. Guo and J. A. Oller, Phys. Rev. D93, 054014 (2016), 1601.00862.

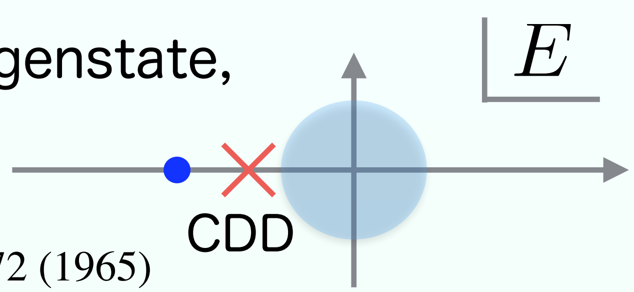
Condition of the weak-binding relation

In the derivation of the relation
we assume that effective range expansion (ERE)
work well at the pole of eigenstate.



$$f(E) = \left[-\frac{1}{a_0} + \frac{r_e}{2}p^2 - ip\right]^{-1} \quad \text{(s-wave)}$$

When CDD pole lies near the threshold and ERE fails to describe the eigenstate,
weak-binding relation is not available.



S. Weinberg, Phys. Rev. 137, B672 (1965)

If CDD pole lies near the threshold, we cannot use the previous weak-binding relation to study internal structure.

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Derivation without convergence of ERE

- Compositeness

T. Sekihara, T. Hyodo, and D. Jido, PTEP 2015, 063D04 (2015), 1411.2308.
Y. Kamiya, T. Hyodo in preparation.

$$X \equiv \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} |\langle \boldsymbol{p}|B\rangle|^2$$

$$\boxed{X = -g^2 \underline{G'(E_B)}}$$

$$H|B\rangle = E_B|B\rangle \quad (E_B < 0)$$

$|B\rangle$: bound state

- The leading term of the $G'(E_B)$ is cutoff independent.

T. Sekihara, T. Hyodo, and D. Jido, PTEP 2015, 063D04 (2015), 1411.2308.

$$G'(E_B) = \frac{\mu}{4\pi E_B R}\left\{1 + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

$$R \equiv 1/\sqrt{-2\mu E_B}$$

$R_{\rm typ}$: typical length scale of int. $(\sim 1/\Lambda)$

$$G(E) = \frac{1}{2\pi^2}\int_0^\Lambda p^2 dp \frac{1}{E - p^2/(2\mu) + i0^+}$$

Derivation without convergence of ERE

- Compositeness

T. Sekihara, T. Hyodo, and D. Jido, PTEP 2015, 063D04 (2015), 1411.2308.
Y. Kamiya, T. Hyodo in preparation.

$$X \equiv \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} |\langle \boldsymbol{p}|B\rangle|^2$$

$$H|B\rangle = E_B|B\rangle \quad (E_B < 0)$$

$|B\rangle$: bound state

$$\boxed{X = -g^2 G'(E_B)}$$

- g^2 is cutoff independent.

T. Hyodo, NSTAR proceedings, arXiv:1511.00870.

With the slight change of the cutoff $\Lambda \to \Lambda + \delta\Lambda$ keeping $T(E)$ invariant, represent of g^2 does not change.

approximate g^2 with ERE

$$f(E) = [\underline{p\cot\delta} - ip]^{-1}$$

$$p\cot\delta = -\frac{1}{a_0} + \frac{r_e}{2}p^2 + \mathcal{O}(R_{\text{eff}}^3 p^4)$$

$$g^2 = -\lim_{E\to E_B} \frac{2\pi}{\mu}(E - E_B)f(E)$$

R_{eff} : range scale characterizing ERE

$$g^2 = \frac{2\pi}{\mu^2}\frac{1}{R - r_e + R\mathcal{O}((R_{\text{eff}}/R)^3)}$$

Derivation independent of effectiveness of ERE

Compositeness

$$G(E) = \frac{1}{2\pi^2}\int_0^\Lambda p^2 dp \frac{1}{E - p^2/(2\mu) + i0^+}$$

$$\boxed{X = -g^2 G'(E_B)}$$

$$g^2 = -\lim_{E\to E_B}\frac{2\pi}{\mu}(E - E_B) f(E)$$

$$\Rightarrow \quad X = \frac{1}{1 - \frac{r_e}{R} + \mathcal{O}\left(\left(\frac{R_{\mathrm{eff}}}{R}\right)^3\right)}\left[1 + \mathcal{O}\left(\frac{R_{\mathrm{typ}}}{R}\right)\right]$$

- Two independent expansions are used to derive the relation.

(1) R_{typ}/R : ratio of typical length scale of int. R_{typ} and R.

(2) R_{eff}/R : ratio of length scale characterizing ERE R_{eff} and R.

- When the both expansions converge well, compositeness can be estimated from experimental observables (re, R).
- If R_{eff} satisfies $R_{\mathrm{eff}} \lesssim R_{\mathrm{typ}}$, above relation reduces to the Weinberg's relation.

Derivation independent of effectiveness of ERE

Compositeness

$$G(E) = \frac{1}{2\pi^2}\int_0^{\Lambda} p^2 dp \frac{1}{E - p^2/(2\mu) + i0^+}$$

$$g^2 = -\lim_{E\to E_B} \frac{2\pi}{\mu}(E - E_B) f(E)$$

$$\boxed{X = -g^2 G'(E_B)}$$

$$X = \frac{1}{1 - \frac{r_e}{R} + \underline{\mathcal{O}\left(\left(\frac{R_{\rm eff}}{R}\right)^3\right)}} \left[1 + \underline{\mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)}\right]$$

- Two independent expansions are used to derive the relation.

(1) $R_{\rm typ}/R$: ratio of typical length scale of int. $R_{\rm typ}$ and R .

(2) $R_{\rm eff}/R$: ratio of length scale characterizing ERE $R_{\rm eff}$ and R .

- When the both expansions converge well,
  compositeness can be estimated from experimental observables (re, R).

- If $R_{\rm eff}$ satisfies $R_{\rm eff} \lesssim R_{\rm typ}$, above relation reduces to the Weinberg's relation.

If ERE does not describe the bound state $\left((R_{\rm eff}/R)^3 \gtrsim 1\right)$,
the approximation of the coupling constant should be improved.

Contents

- Introduction ~compositeness of bound state~

Part I

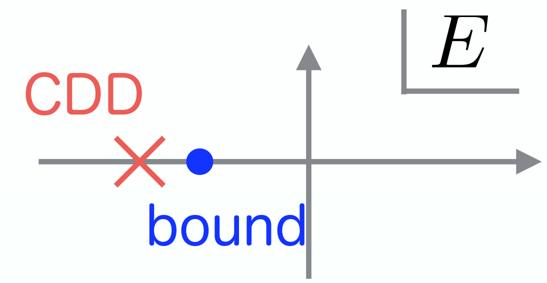
- Extension to the quasi-bound state
- Applications to exotic hadrons ~ $\Lambda(1405), a_0(980), f_0(980)$ ~

Part II

- Derivation without convergence of ERE
- Extended relation with the CDD pole contribution

Extended relation with the CDD pole contribution

- To take account of the contribution of CDD pole



$$\boxed{X = -\underline{g^2} G'(E_B)}$$

$$f(E) = [\underline{p \cot \delta} - ip]^{-1}$$

$$\frac{b_0 + b_1 p^2}{1 + c_1 p^2} + \mathcal{O}(R_{\text{Padè}}^5 p^6)$$ Pade approximation

Y. Kamiya, T. Hyodo in preparation.

$$\boxed{X = \left[1 - \frac{4R(a_0 - R)^2}{a_0^2 r_e} + \mathcal{O}\left(\left(\frac{R_{\text{Padè}}}{R}\right)^5\right)\right]^{-1} \left(1 + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right)}$$

Even when the ERE does not describe the bound state,
we can estimate the compositeness using experimental observables.

Extended relation with the CDD pole contribution

Verification with model

We compare the effectiveness of the estimation using the previous and extended weak-binding relation.

exact compositeness in this model

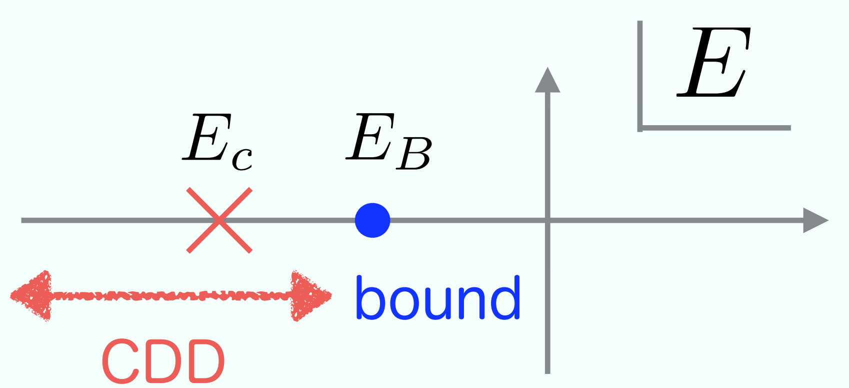
$$X_{\text{exact}} = -g^2 G'(E_B)$$

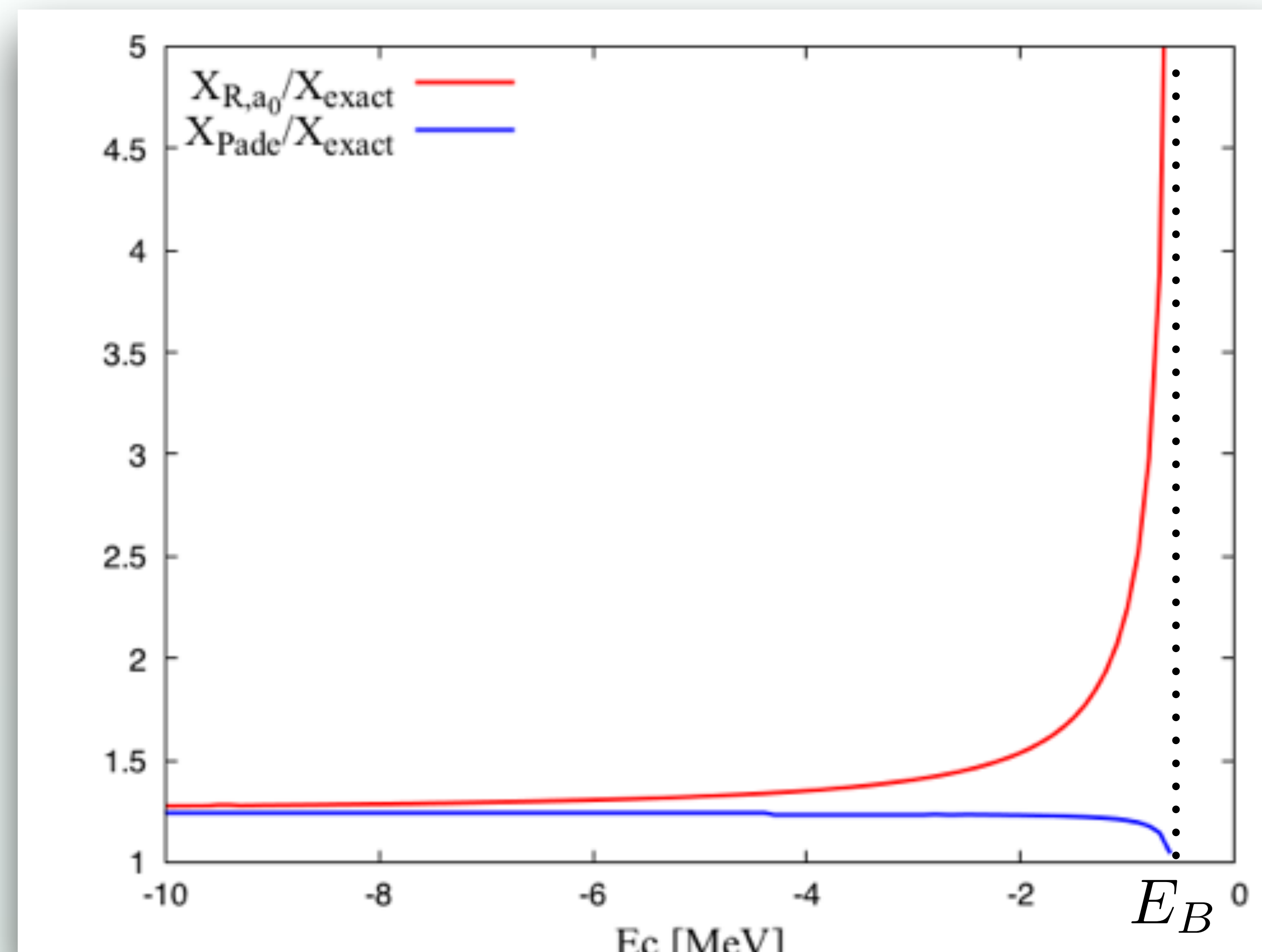
previous relation

$$X_{R,a_0} = \frac{a_0}{2R - a_0}$$

extended relation

$$X_{\text{Padè}} = \left[1 - \frac{4R(a_0 - R)^2}{a_0^2 r_e}\right]^{-1}$$





The estimation of the compositeness is improved, when the CDD pole lies near the threshold.

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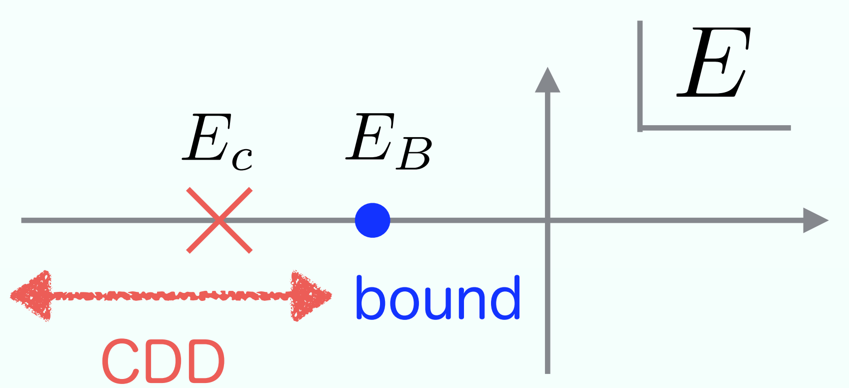

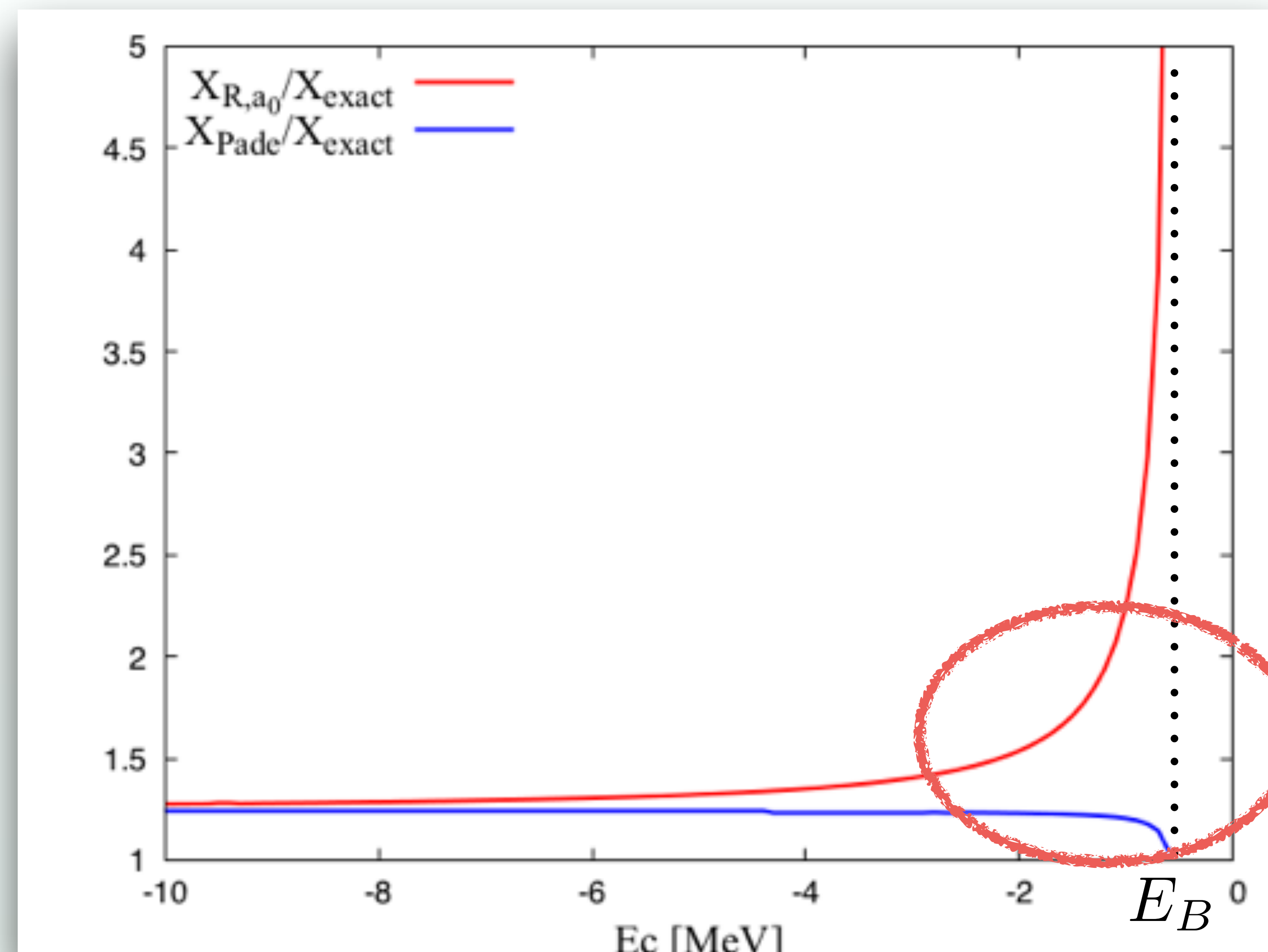

The estimation of the compositeness is improved, when the CDD pole lies near the threshold.

Conclusions ~Part I~

Conclusions

Y. Kamiya and T. Hyodo, arXiv:1509.00146 [hep-ph].

Phys. Rev. C. 93.035203

- We extend the weak-binding relation to quasi-bound states.

$$\boxed{a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(|R_{\mathrm{typ}}/R|\right) + \mathcal{O}\left(|l/R|^3\right)\right\}}$$

⇒ If the absolute value of the eigenenergy is small enough, the compositeness is model-independently determined only from observables.

- We propose an interpretation of complex X.

$$\boxed{\tilde{X} \equiv \frac{1-|Z|+|X|}{2}, \qquad U \equiv |X|+|Z|-1}$$

⇒ If the uncertainty U is small, we interpret $\tilde{X}$ as the probability.

- We apply the method to exotic hadrons and discuss the internal structures.

⇒ $\Lambda(1405)$: $\bar{K}N$ composite dominance

$a_0(980)$: not $K\bar{K}$ dominance

Conclusions ~Part II~

Conclusions

Y. Kamiya, T. Hyodo in preparation.

- Convergence of ERE is assumed in the previous derivation.
- We derive the weak-binding relation without assuming the convergence of ERE.
- With the Pade approximation, we take into account of the contribution of the near-threshold CDD pole and derive the extended weak-binding relation.

$$\Rightarrow \quad \boxed{X = \left[1 - \frac{4R(a_0 - R)^2}{a_0^2 r_e} + \mathcal{O}\left(\left(\frac{R_{\text{Padè}}}{R}\right)^5\right)\right]^{-1} \left(1 + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right)}$$

- With model calculation, it is confirmed that the compositeness is accurately estimated even if the CDD pole lies near the threshold.

Back up slides

Interpretation of X

- X is a complex number.

(1) close to bound state case

$$\begin{cases} X = 0.8 - 0.1i \\ Z = 0.2 + 0.1i \end{cases}$$

The probabilistic interpretation is seemed to be possible.

small cancellation in X+Z

(2-a) When real part is not in [0,1]

$$\begin{cases} X = \underline{1.9} - 0.2i \\ Z = \underline{-0.9} + 0.2i \end{cases}$$

(2-b) When imaginary part is large.

$$\begin{cases} X = 0.9 \underline{- 0.8i} \\ Z = 0.1 \underline{+ 0.8i} \end{cases}$$

It is difficult to interpret X as a probability.

large cancellation in X+Z

Is there any good prescription to interpret the complex value?

Interpretation of X

- Examples of $\tilde{X},\ \tilde{Z}$

(1)

$$\begin{cases} X = 0.8 - 0.1i \\ Z = 0.2 + 0.1i \end{cases} \longrightarrow \begin{cases} \tilde{X} = 0.8 \\ \tilde{Z} = 0.2 \\ U = 0.0 \end{cases}$$

(2-a)

$$\begin{cases} X = \underline{1.9} - 0.2i \\ Z = \underline{-0.9} + 0.2i \end{cases} \longrightarrow \begin{cases} \tilde{X} = 1.0 \\ \tilde{Z} = 0.0 \\ U = 1.8 \end{cases}$$

(2-b)

$$\begin{cases} X = 0.9 \underline{- 0.8i} \\ Z = 0.1 \underline{+ 0.8i} \end{cases} \longrightarrow \begin{cases} \tilde{X} = 0.7 \\ \tilde{Z} = 0.3 \\ U = 1.0 \end{cases}$$

Flatte parametrization

- To get E_h and a_0 from Flatte parameters

$$T = \frac{1}{M^2 - s - i\left(g_1 \rho_{\alpha\pi} + g_2 \rho_{K\bar{K}}\right)}$$

for a0(980) : α denote η

f0(980) : α denote π

$$\rho_{\alpha\beta} = 2p_{\alpha\beta}/\sqrt{s}$$

g1 and g2 were determined fitting the scattering amplitude.

$\boxed{E_h}$

Find pole position of the T matrices

$\boxed{a_0}$

Normalize Kbar-K amplitude f(s) so as to f(s) satisfies

$$f(s)^{-1} \to -a_0 - ik + \mathcal{O}(k^2)$$

(k is a momentum of K or Kbar)

$$a_0 = -f(0)$$

Power counting(1)

Neglecting collection terms,

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(|R_{\text{typ}}/R|\right) + \mathcal{O}\left(|l/R|^3\right)\right\}$$

the compositeness relation is rewritten

by scattering length a_0 and effective range r_e.

$$X(a_0, r_e) = \left(1 - \frac{2r_e}{a_0}\right)^{-1/2}$$

$$X \sim 0 \longrightarrow |r_e/a_0| \sim \infty$$
$$X \sim 0.5 \longrightarrow |r_e/a_0| \sim 1.5$$
$$X \sim 1 \longrightarrow |r_e/a_0| \sim 0$$

- $\Lambda(1405)$

Ref.	E_{QB} (MeV)	a_0 (fm)	X	$\tilde{X}$	$\left\lvert\frac{r_e}{a_0}\right\rvert$
(1)	-10-i26	1.39-i0.85	1.3+i0.1	1.0	0.2
(2)	-4-i8	1.81-i0.92	0.6+i0.1	0.6	0.7
(3)	-13-i20	13.0-i0.85	0.9-i0.2	0.9	0.2
(4)-1	2-i10	1.21-i1.47	0.6+i0.0	0.6	0.7
(4)-2	- 3-i12	1.52-i1.85	1.0+i0.5	0.8	0.4

Power counting(2)

$$X(a_0, r_e) = \left(1 - \frac{2r_e}{a_0}\right)^{-1/2} \Rightarrow \begin{array}{l} X \sim 0 \longrightarrow |r_e/a_0| \sim \infty \\ X \sim 0.5 \longrightarrow |r_e/a_0| \sim 1.5 \\ X \sim 1 \longrightarrow |r_e/a_0| \sim 0 \end{array}$$

$a_0(980)$

Set	E_{QB} (MeV)	a_0 (fm)	X	$\tilde{X}$	$\left\vert\frac{r_e}{a_0}\right\vert$
(1)	31-i70	-0.03-i0.53	0.2-i0.2	0.3	4.8
(2)	3-i25	0.17-i0.77	0.2-i0.2	0.2	6.5
(3)	9-i36	0.05–i0.63	0.2-i0.2	0.2	7.2
(4)	14- i 5	-0.13-i2.19	0.8-i0.4	0.7	0.5
(5)	15-i29	-0.13-i0.52	0.1-i0.4	0.1	13

Power counting(3)

$$X(a_0, r_e) = \left(1 - \frac{2r_e}{a_0}\right)^{-1/2} \quad \Rightarrow \quad \begin{array}{lll} X \sim 0 & \longrightarrow & |r_e/a_0| \sim \infty \\ X \sim 0.5 & \longrightarrow & |r_e/a_0| \sim 1.5 \\ X \sim 1 & \longrightarrow & |r_e/a_0| \sim 0 \end{array}$$

$f_0(980)$

Set	E_{QB} (MeV)	a_0 (fm)	X	$\tilde{X}$	$\left\lvert\frac{r_e}{a_0}\right\rvert$
(1)	19-i30	0.02-i0.95	0.3-0.3	0.4	2.6
(2)	-6 -i10	0.84-i0.85	0.3-i0.1	0.3	5.4
(3)	-8 -i28	0.64-i0.83	0.4-i0.2	0.4	2.1
(4)	10-i18	0.51-i1.58	0.7-i0.3	0.6	0.7
(5)	-10-i29	0.49-i0.67	0.3-i0.1	0.3	4.0
(6)	10-i7	0.52-i2.41	0.9-i0.2	0.9	0.2