Understanding the nature of heavy pentaquarks and searching for them in pion-induced reactions

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Outline

- **Early study**
- **Triangle singularity (TS) mechanism**
- **TS mechanism and the heavy pentaquark “Pc”**
- **Searching for the heavy pentaquark in $\pi p \rightarrow \pi J/\psi p$**
- **Summary**

Early study in 1960s

- **Connections between kinematic singularities of the S-matrix elements and resonance-like peaks: e.g. Peierls mechanism**

R.F.Peierls, PRL6,641(1961);
R.C.Hwa, PhysRev130,2580(1963);
C.Goebel,PRL13,143(1964);
P.Landshoff&S.Treiman Phys.Rev.127,649(1962);
………

- **Some disadvantages:**

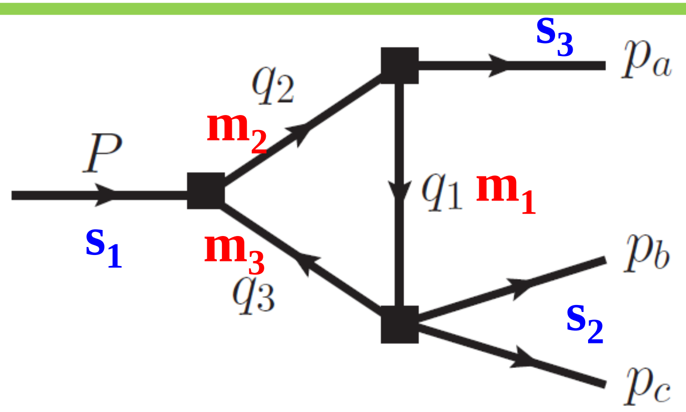
✓ **Few experiments to search for the effects;**

✓ **Low statistics;**

✓ **For the elastic scattering process, singularities of the triangle diagram will be weakened by the corresponding tree diagram without rescattering, according to the so called Schmid theorem**

C.Schmid,Phys.Rev.154,1363(1967);
I. J. R. Aitchison & C. Kacser, Phys.Rev.173,1700(1968); A.V. Anisovich, PLB345,321(1995)

Triangle Singularity Mechanism



$$P^2 = s_1, \ (p_b + p_c)^2 = s_2$$

$$p_a^2 = s_3$$

$$\Gamma_3(s_1, s_2, s_3) = \frac{-1}{16\pi^2} \int_0^1 \int_0^1 \int_0^1 da_1\, da_2\, da_3\, \frac{\delta(1 - a_1 - a_2 - a_3)}{D - i\epsilon}$$

$$D = \sum_{i,j=1}^{3} a_i a_j Y_{ij}, \ \ Y_{ij} = \frac{1}{2}\left[m_i^2 + m_j^2 - (q_i - q_j)^2\right]$$

✓ Singularity in the complex space

Necessary conditions (Landau Equation)

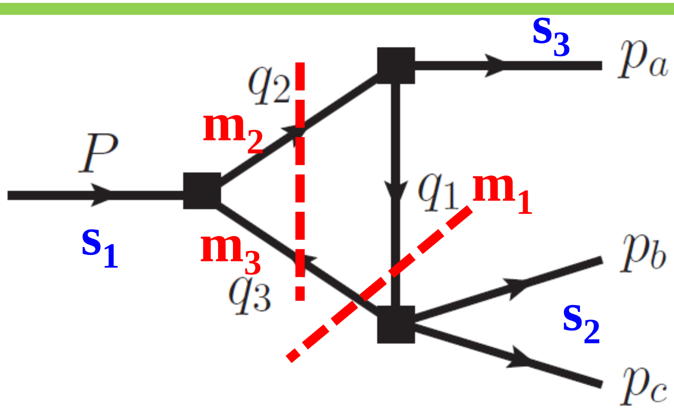
$$D = 0,$$

$$either \ \ a_j = 0 \ \ or \ \boxed{\frac{\partial D}{\partial a_j} = 0.}$$

Leading singularity

Landau, Nucl.Phys.13,181(1959)

Triangle Singularity Mechanism



$$P^2 = s_1,\ (p_b + p_c)^2 = s_2$$

$$p_a^2 = s_3$$

✓ **Singularity in the complex space**

The position of the singularity is obtained by solving

$$\det[Y_{ij}] = 0$$

Normal Threshold

s_1, s_3, $m_{1,2,3}$ fixed

$$\boxed{s_2^{\pm}} = \boxed{(m_1+m_3)^2} + \frac{1}{2m_2^2}\left[(m_1^2+m_2^2-s_3)(s_1-m_2^2-m_3^2) - 4m_2^2 m_1 m_3 \right.$$
$$\left. \pm\ \lambda^{1/2}(s_1, m_2^2, m_3^2)\lambda^{1/2}(s_3, m_1^2, m_2^2)\right], \quad \lambda(x,y,z) \equiv (x-y-z)^2 - 4yz$$

Anomalous Threshold

$$\boxed{s_1^{\pm}} = (m_2+m_3)^2 + \frac{1}{2m_1^2}\left[(m_1^2+m_2^2-s_3)(s_2-m_1^2-m_3^2) - 4m_1^2 m_2 m_3 \right.$$
$$\left. \pm\ \lambda^{1/2}(s_2, m_1^2, m_3^2)\lambda^{1/2}(s_3, m_1^2, m_2^2)\right].$$

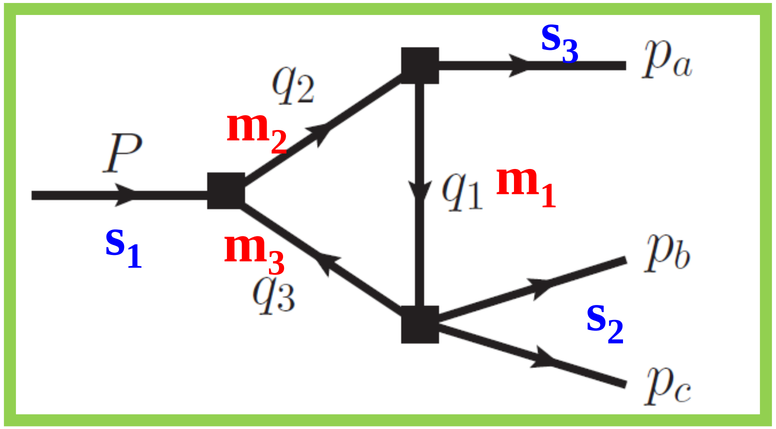
s_2, s_3, $m_{1,2,3}$ fixed

Triangle Singularity Mechanism

Single dispersion relation

$$\Gamma_3(s_1, s_2, s_3) = \frac{1}{\pi}\int_{(m_1+m_3)^2}^{\infty} \frac{ds_2'}{s_2' - s_2 - i\epsilon}\,\sigma(s_1, s_2', s_3)$$

$$\sigma(s_1, s_2, s_3) = \sigma_+ - \sigma_-$$



$$\sigma_\pm(s_1, s_2, s_3) = \frac{-1}{16\pi\lambda^{1/2}(s_1, s_2, s_3)}\log[-s_2(s_1 + s_3 - s_2 + m_1^2 + m_3^2 - 2m_2^2) - (s_1 - s_3)(m_1^2 - m_3^2) \pm \lambda^{1/2}(s_1, s_2, s_3)\lambda^{1/2}(s_2, m_1^2, m_3^2)].$$

Work in the kinematical region

$s_1 \le (m_2 + m_3)^2,\ s_3 \le (m_2 - m_1)^2 \qquad 0 < s_2 < (m_1 + m_3)^2$

By analytic continuation, it can be extended into the over threshold region

Fronsdal&Norton,J.Math.Phys.5,100(1964)

$s_1 \ge (m_2 + m_3)^2,\ (m_1 + m_3)^2 \le s_2 \le (\sqrt{s_1} - \sqrt{s_3})^2,\ 0 \le \sqrt{s_3} \le m_2 - m_1$

Brach points of the log function $s_2^{\pm}$

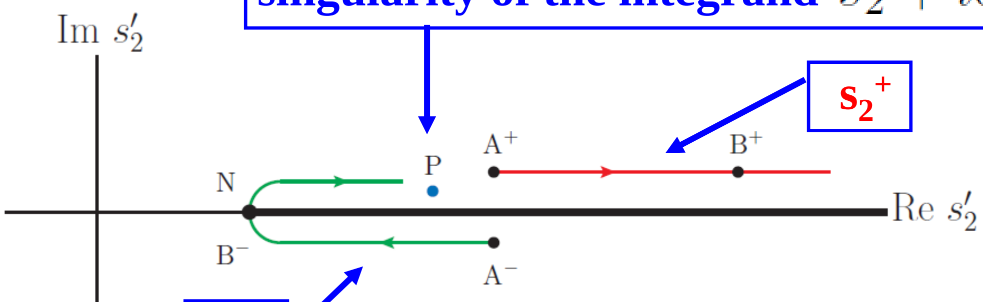
Triangle Singularity Mechanism

Locations of $s_2^{\pm}$ in the s_2'-plane can be determined by

$$s_2^{\pm}(s_1 + i\epsilon) = s_2(s_1) + i\epsilon \frac{\partial s_2^{\pm}}{\partial s_1}$$

singularity of the integrand $s_2 + i\epsilon$

s_2^+



s_2^-

Trajectory of the branch points s_2^-, s_2^+ with s_1 increases from normal threshold to infinity in the complex s_2'-plane

$A^{\pm}$ s_{1N} $s_1 = (m_2 + m_3)^2, \;\; B^{\pm}: \;\; s_1 = (m_2 + m_3)^2 + \frac{m_3}{m_1}[(m_2 - m_1)^2 - s_3]$ $=s_{1C}$

$A^-: \;\; s_2^- = (m_1 + m_3)^2 + \frac{m_3}{m_2}[(m_2 - m_1)^2 - s_3] - i\epsilon$ $=s_{2C}$

$B^-: \;\; s_2^- = (m_1 + m_3)^2$ $=s_{2N}$

s_2^- and P will pinch the integral contour

This pinch singularity is the triangle singularity (TS)

Triangle Singularity Mechanism

Locations of $s_2^{\pm}$ in the s$_2$'-plane can be determined by

$$s_2^{\pm}(s_1 + i\epsilon) = s_2(s_1) + i\epsilon \frac{\partial s_2^{\pm}}{\partial s_1}$$

singularity of the integrand $s_2 + i\epsilon$

Trajectory of the branch

"The kinematic conditions for the existence of singularities on the physical boundary are equivalent to the condition that the relevant Feynman diagram be interpretable as a picture of an energy and momentum-conserving process occurring in space-time, with all internal particles real, on the mass shell and moving forward in time."

Coleman&Norton, Nuovo Cimento 38,5018 (1965)

Fronsdal&Norton,J.Math.Phys.5, 100(1964)

………

TS Kinematic Region

Normal threshold and critical point

$$s_{1N} = (m_2+m_3)^2,\ s_{1C} = (m_2+m_3)^2 + \frac{m_3}{m_1}[(m_2-m_1)^2 - s_3],$$

$$s_{2N} = (m_1+m_3)^2,\ s_{2C} = (m_1+m_3)^2 + \frac{m_3}{m_2}[(m_2-m_1)^2 - s_3],$$

$$\Delta_{s_1} = \sqrt{s_1^-} - \sqrt{s_{1N}},$$

$$\Delta_{s_2} = \sqrt{s_2^-} - \sqrt{s_{2N}}.$$

Discrepancy between anomalous and normal threshold

When $s_2 = s_{2N}$

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2+m_3)}[(m_2-m_1)^2 - s_3],$$

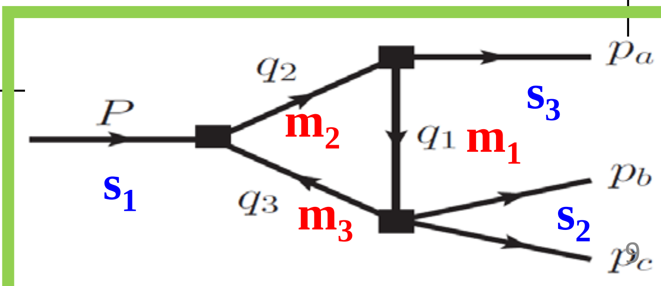
$$\Delta_{s_2}^{\max} = \sqrt{s_{2C}} - \sqrt{s_{2N}} \approx \boxed{\frac{m_3}{2m_2(m_1+m_3)}}\underline{[(m_2-m_1)^2 - s_3]}.$$

When $s_1 = s_{1N}$

Largest discrepancy

Enlarge

How to amplify the discrepancy between normal and anomalous threshold?



Liu, Oka, Zhao, PLB753,297 (2016)
arXiv:1507.01674

TS Kinematic Region

Normal threshold and critical point

$$s_{1N} = (m_2+m_3)^2,\ s_{1C} = (m_2+m_3)^2 + \frac{m_3}{m_1}[(m_2-m_1)^2 - s_3],$$
$$s_{2N} = (m_1+m_3)^2,\ s_{2C} = (m_1+m_3)^2 + \frac{m_3}{m_2}[(m_2-m_1)^2 - s_3],$$

$$\Delta_{s_1} = \sqrt{s_1^-} - \sqrt{s_{1N}},$$
$$\Delta_{s_2} = \sqrt{s_2^-} - \sqrt{s_{2N}}.$$

Discrepancy between anomalous and normal threshold

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2+m_3)}[(m_2-m_1)^2 - s_3],$$
$$\Delta_{s_2}^{\max} = \sqrt{s_{2C}} - \sqrt{s_{2N}} \approx \frac{m_3}{2m_2(m_1+m_3)}[(m_2-m_1)^2 - s_3].$$

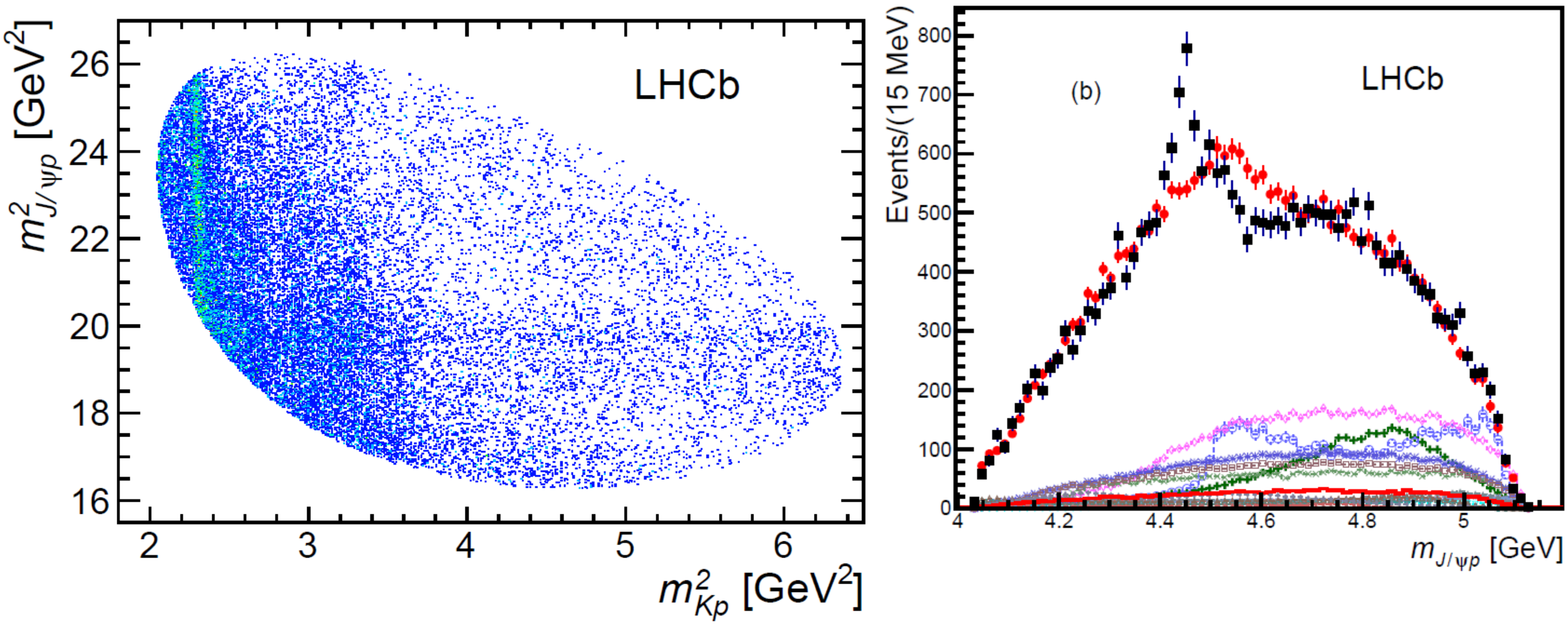
Largest discrepancy

How to amplify the discrepancy between normal and anomalous threshold?

Liu, Oka, Zhao, PLB753,297 (2016) arXiv:1507.01674

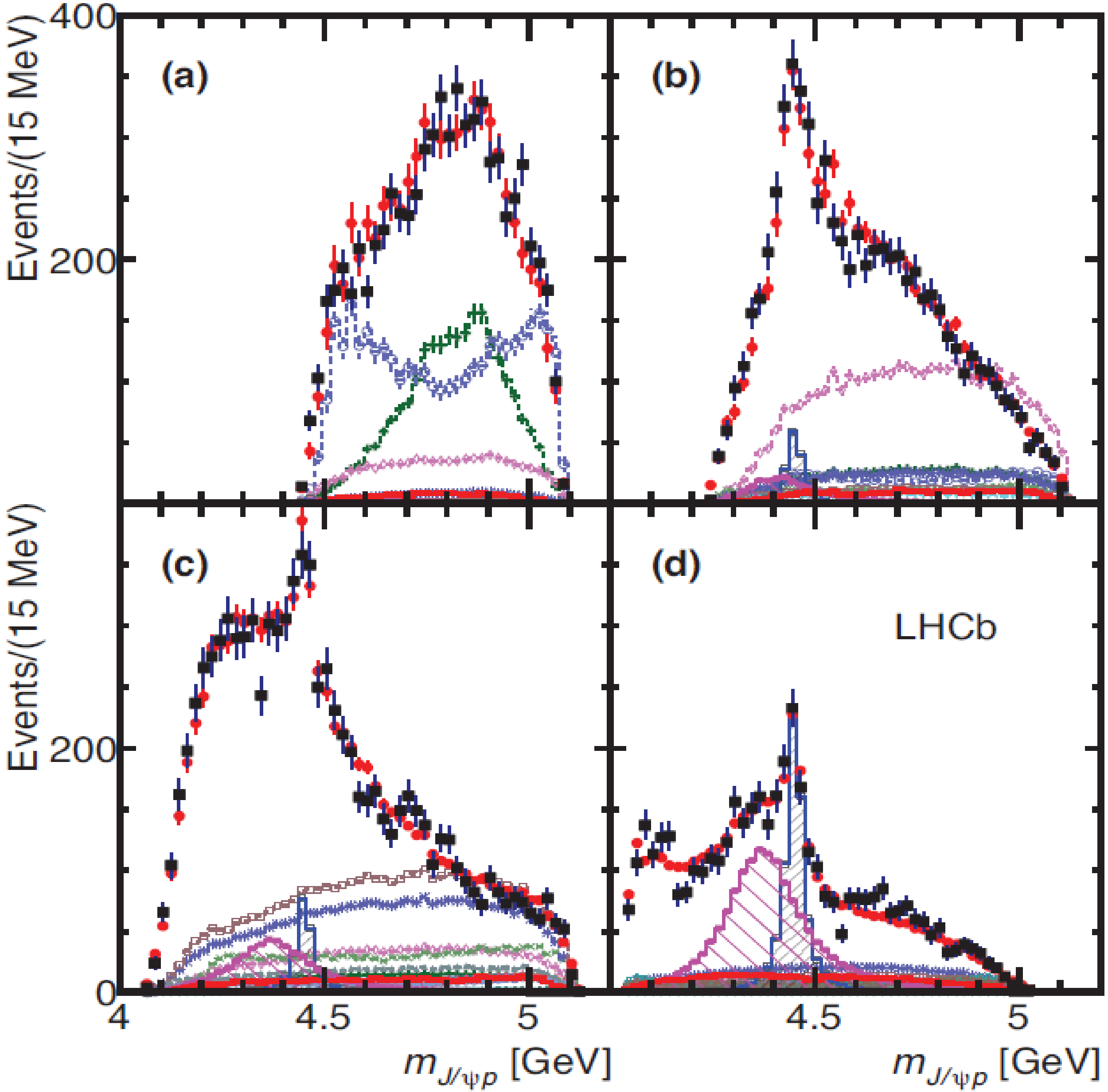
If the discrepancy is larger, maybe it could be used to distinguish the kinematic singularities from genuine particles.

Heavy pentaquark "Pc" observed in LHCb



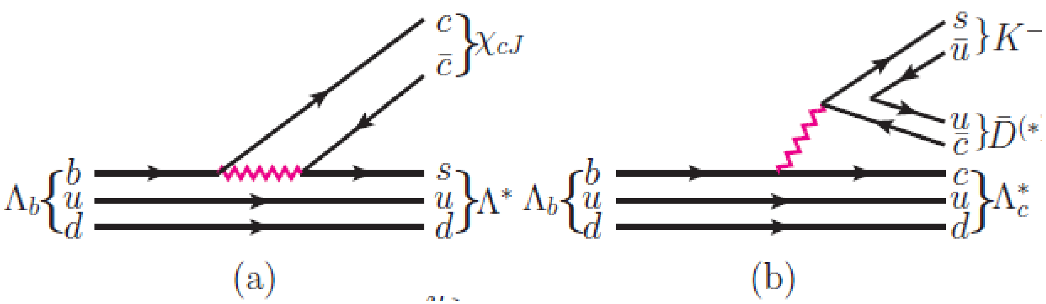
LHCb, arXiv: 1507.03414

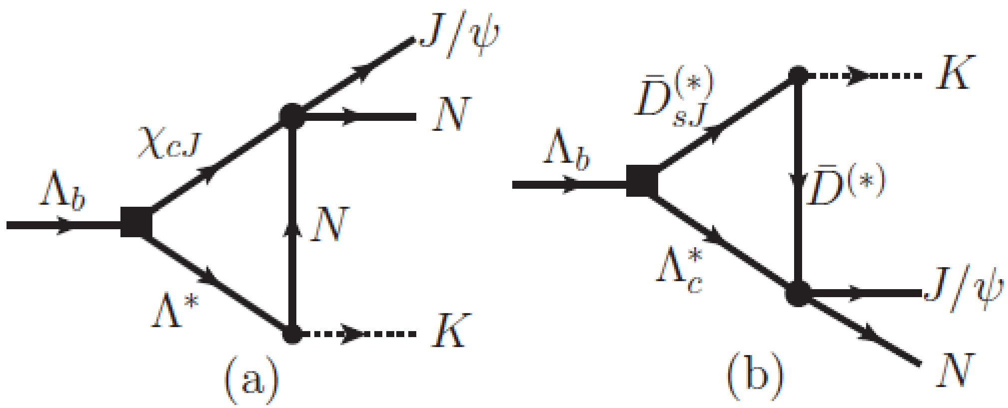
Heavy pentaquark "Pc" observed in LHCb

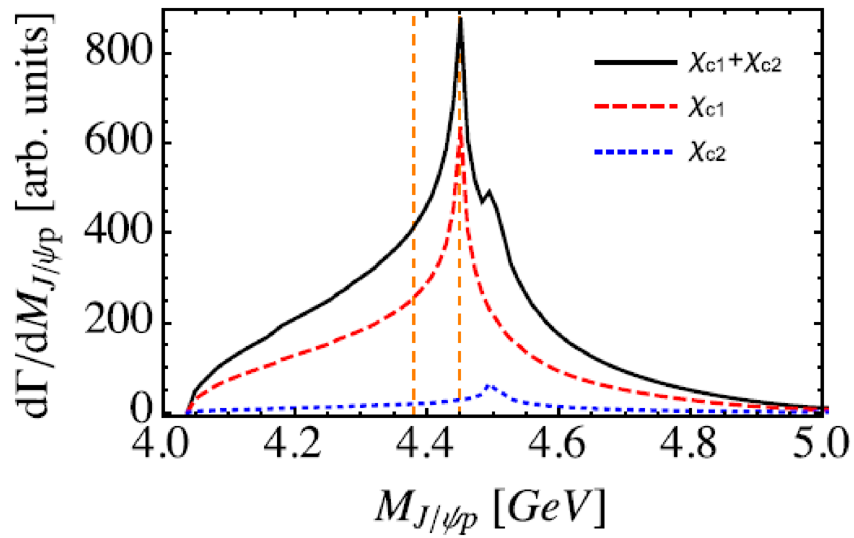


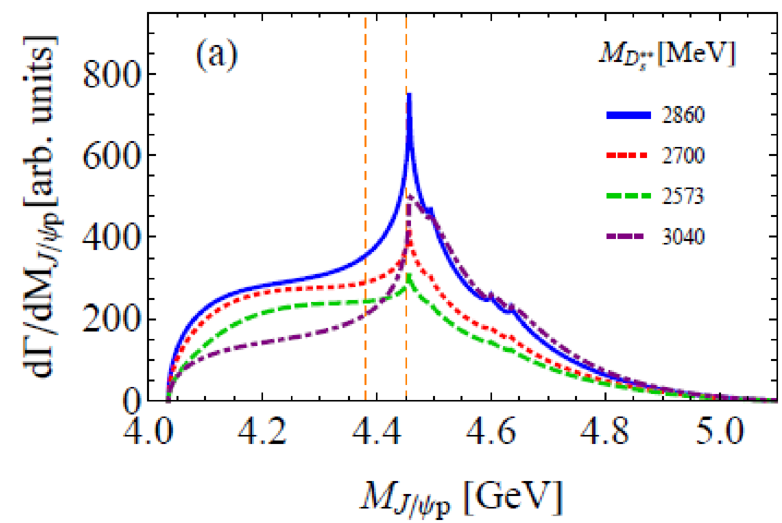
Various intervals of Kp invariant mass for the fit with two Pcs

TS mechanism and the heavy pentaquark "Pc"









Thresholds [GeV]	$\chi_{c0}(1P)\ 0^+$	$\chi_{c1}(1P)\ 1^+$	$\chi_{c2}(1P)\ 2^+$
$p\ 1/2^+$	4.353	$\boxed{4.449}$	4.494

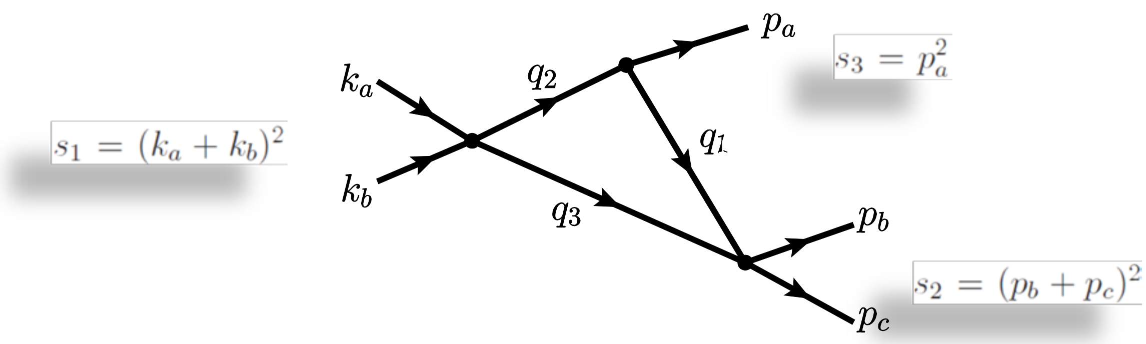
Thresholds [GeV]	$\Lambda_c(2286)\ 1/2^+$	$\Lambda_c(2595)\ 1/2^-$	$\Lambda_c(2625)\ 3/2^-$
$\bar{D}(1865)\ 0^-$	4.151	$\boxed{4.457}$	4.493
$\bar{D}^*(2007)\ 1^-$	4.293	4.599	4.635

Liu,Wang,Zhao, arXiv:1507.05359

Guo et al, arXiv:1507.04950

Searching for the heavy pentaquark in $\pi p \to \pi J/\psi p$

TS mechanism is a highly process-dependent mechanism.
Different kinds of processes are required to check it.

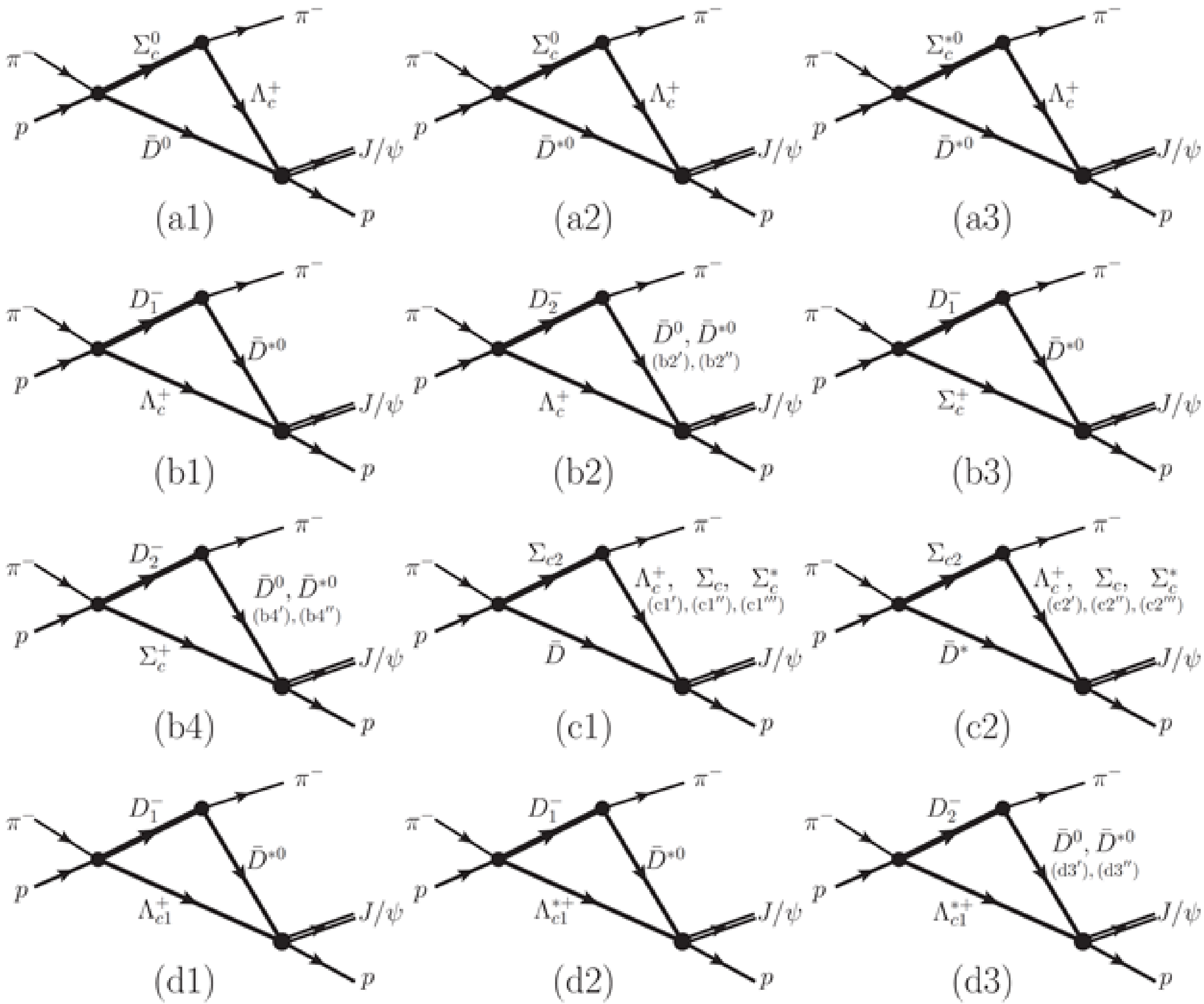


$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2+m_3)}[(m_2-m_1)^2 - s_3],$$
$$\Delta_{s_2}^{\max} = \sqrt{s_{2C}} - \sqrt{s_{2N}} \approx \frac{m_3}{2m_2(m_1+m_3)}[(m_2-m_1)^2 - s_3].$$

Requirements:

- **The quantity $[(m_2 - m_1)^2 - s_3]$ should be larger (phase space should be larger);**
- **Particle q_2 should not be too broad;**
- **Balance between the phase space and width.**

πp→πJ/ψp via the open-charm loops



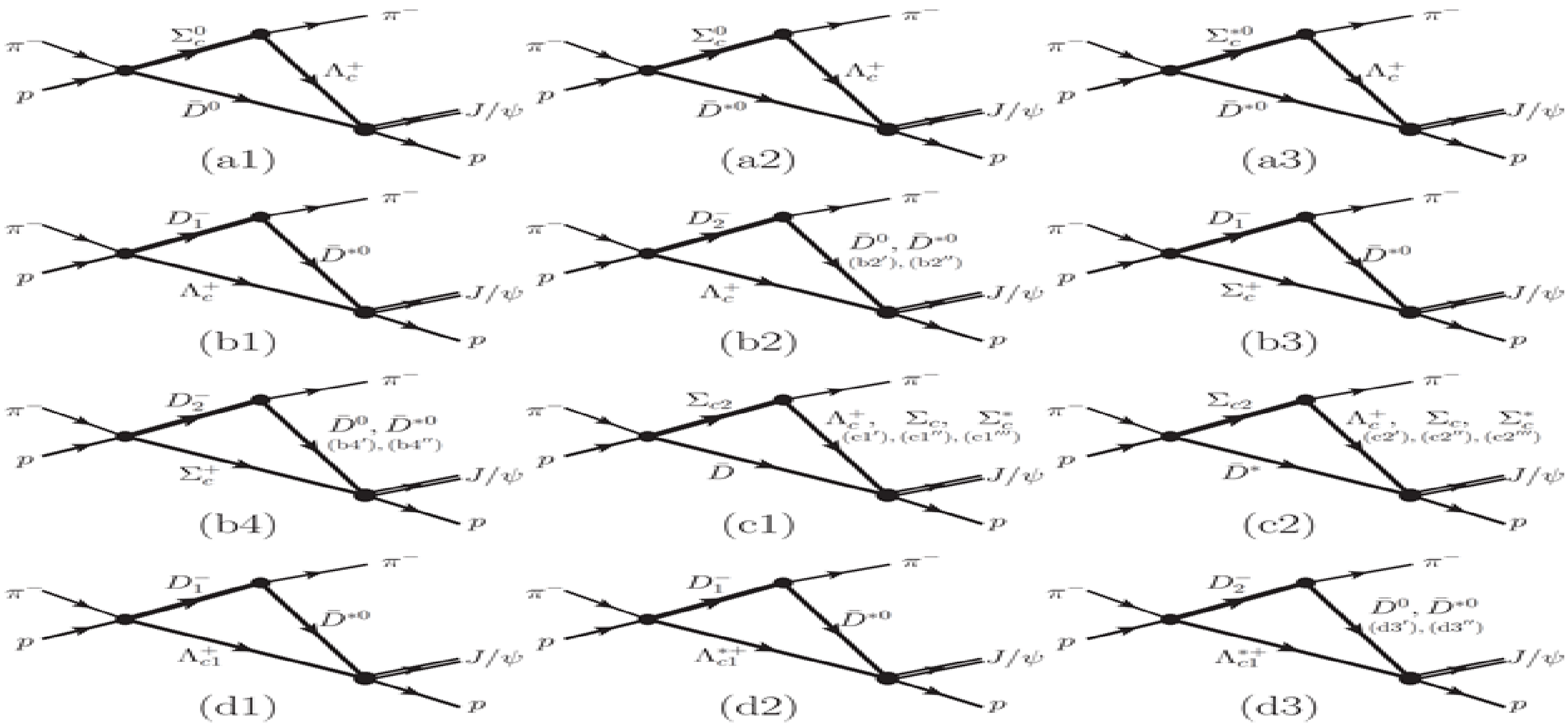
Liu, Oka, arXiv: 1602.07069

$\pi p \to \pi J/\psi p$ via the open-charm loops

Particle in PDG	Notation	J^P	Mass [MeV]	Width [MeV]
Λ_c^+	Λ_c^+	$\frac{1}{2}^+$	2286.46 ± 0.14	
$\Lambda_c(2595)^+$	Λ_{c1}^+	$\frac{1}{2}^-$	2592.25 ± 0.28	2.6 ± 0.6
$\Lambda_c(2625)^+$	Λ_{c1}^{*+}	$\frac{3}{2}^-$	2628.11 ± 0.19	<0.97
$\Sigma_c(2455)^{++}$	Σ_c^{++}	$\frac{1}{2}^+$	2453.98 ± 0.16	2.26 ± 0.25
$\Sigma_c(2455)^+$	Σ_c^+	$\frac{1}{2}^+$	2452.9 ± 0.4	<4.6
$\Sigma_c(2455)^0$	Σ_c^0	$\frac{1}{2}^+$	2453.74 ± 0.16	2.16 ± 0.26
$\Sigma_c(2520)^{++}$	Σ_c^{*++}	$\frac{3}{2}^+$	2517.9 ± 0.6	14.9 ± 1.5
$\Sigma_c(2520)^+$	Σ_c^{*+}	$\frac{3}{2}^+$	2517.5 ± 2.3	<17
$\Sigma_c(2520)^0$	Σ_c^{*0}	$\frac{3}{2}^+$	2518.8 ± 0.6	14.5 ± 1.5
$\Sigma_c(2800)^{++}$	Σ_{c2}^{++}	$\frac{3}{2}^-?$	2801^{+4}_{-6}	75^{+22}_{-17}
$\Sigma_c(2800)^+$	Σ_{c2}^+	$\frac{3}{2}^-?$	2792^{+14}_{-5}	62^{+60}_{-40}
$\Sigma_c(2800)^0$	Σ_{c2}^0	$\frac{3}{2}^-?$	2806^{+5}_{-7}	72^{+22}_{-15}
D^0	D^0	0^-	1864.84 ± 0.07	
$D^\pm$	$D^\pm$	0^-	1869.61 ± 0.10	
$D^*(2007)^0$	D^{*0}	1^-	2006.96 ± 0.10	<2.1
$D^*(2010)^\pm$	$D^{*\pm}$	1^-	2010.26 ± 0.07	0.0834 ± 0.0018
$D_1(2420)^0$	D_1^0	1^+	2421.4 ± 0.6	27.4 ± 2.5
$D_2(2460)^0$	D_2^0	2^+	2462.6 ± 0.6	49.0 ± 1.3
$D_2(2460)^\pm$	$D_2^\pm$	2^+	2464.3 ± 1.6	37 ± 6

Notations of the charmed hadrons in relevant with our discussion

πp→πJ/ψp via the open-charm loops



I) (a1)-(a3): Both $\bar{D}$ and Y_c are S-wave states,

II) (b1)-(b4): $\bar{D}$ is a P-wave state and Y_c is an S-wave state,

III) (c1)-(c2): $\bar{D}$ is an S-wave state and Y_c is a P-wave state,

IV) (d1)-(d3): Both $\bar{D}$ and Y_c are P-wave states.

$\pi p \to \pi J/\psi p$ via the open-charm loops

TS Kinematic Region

Diagram #	$\sqrt{s_{1N}}$	$\sqrt{s_{1C}}$	$\Delta_{s_1}^{\max}$	$\sqrt{s_{2N}}$	$\sqrt{s_{2C}}$	$\Delta_{s_2}^{\max}$
(a1)	4317.7	4318.5	0.773	4151.4	4152.1	0.75
(a2)	4459.9	4460.7	0.806	4293.5	4294.3	0.78
(a3)	4524.5	4527.7	3.28	4293.5	4296.7	3.14
(b1)	4708.0	4726.3	18.4	4293.5	4310.2	16.7
(b2′)	4750.9	4794.5	43.7	4151.4	4189.2	37.8
(b2″)	4750.9	4773.5	22.7	4293.5	4314.0	20.4
(b3)	4874.3	4893.4	19.1	4459.9	4477.1	17.3
(b4′)	4917.2	4962.4	45.2	4317.7	4356.7	39.0
(b4″)	4917.2	4940.7	23.5	4459.9	4481.0	21.1
(c1′)	4665.8	4687.2	21.4	4151.4	4171.0	19.6
(c1″)	4665.8	4674.1	8.28	4317.7	4325.6	7.83
(c1‴)	4665.8	4670.7	4.83	4382.3	4387.0	4.62
(c2′)	4808.0	4830.3	22.3	4293.5	4313.9	20.4
(c2″)	4808.0	4816.6	8.65	4459.9	4468.0	8.16
(c2‴)	4808.0	4813.0	5.05	4524.5	4529.3	4.82
(d1)	5013.7	5033.2	19.6	4599.2	4616.9	17.7
(d2)	5049.5	5069.2	19.7	4635.1	4652.9	17.8
(d3′)	5092.4	5139.2	46.8	4493.0	4533.1	40.2
(d3″)	5092.4	5116.7	24.3	4635.1	4656.8	21.8

Formalism

The model is built in the framework of heavy hadron chiral perturbation theory (HHChPT)

Charmed Meson Superfiled:

$$H_{1a} = \frac{1+\not{v}}{2}[D^{*}_{a\mu}\gamma^{\mu} - D_a\gamma_5],$$

$$H_{2a} = [\bar{D}^{*}_{a\mu}\gamma^{\mu} + \bar{D}_a\gamma_5]\frac{1-\not{v}}{2},$$

$$T^{\mu}_{1a} = \frac{1+\not{v}}{2}\left\{ D^{\mu\nu}_{2a}\gamma_\nu - \sqrt{\frac{3}{2}}D_{1a\nu} \times\gamma_5\left[g^{\mu\nu} - \frac{1}{3}\gamma^{\nu}(\gamma^{\mu} - v^{\mu})\right]\right\},$$

$$T^{\mu}_{2a} = \left\{ \bar{D}^{\mu\nu}_{2a}\gamma_\nu + \sqrt{\frac{3}{2}}\bar{D}_{1a\nu} \times\gamma_5\left[g^{\mu\nu} - \frac{1}{3}\gamma^{\nu}(\gamma^{\mu} - v^{\mu})\right]\right\}\frac{1-\not{v}}{2},$$

$$\bar{H}_{1a,2a} = \gamma^0 H^{\dagger}_{1a,2a}\gamma^0,\ \bar{T}_{1a,2a} = \gamma^0 T^{\dagger}_{1a,2a}\gamma^0,$$

Formalism

Charmed Baryon Superfiled:

$$\mathcal{T}_i = \frac{1}{2}\epsilon_{ijk}\frac{1+\not{v}}{2}(B_{\bar{3}})_{jk},$$

$$\mathcal{R}_\mu = \frac{1}{\sqrt{3}}(\gamma_\mu + v_\mu)\gamma_5\frac{1+\not{v}}{2}\Lambda_{c1}^{+} + \frac{1+\not{v}}{2}\Lambda_{c1\mu}^{*+},$$

$$\mathcal{S}_\mu^{ij} = \frac{1+\not{v}}{2}B_{6\mu}^{*ij} + \frac{1}{\sqrt{3}}(\gamma_\mu + v_\mu)\gamma_5\frac{1+\not{v}}{2}B_6^{ij},$$

$$\mathcal{X}_{\mu\nu}^{ij} = \frac{1}{\sqrt{10}}[(\gamma_\mu + v_\mu)\gamma_5 g_\nu^\alpha + (\gamma_\nu + v_\nu)\gamma_5 g_\mu^\alpha]X_\alpha^{ij},$$

$$(B_{\bar{3}})_{ij} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}_{ij} \qquad (B_6)_{ij} = \begin{pmatrix} \Sigma_c^{++} & \frac{1}{\sqrt{2}}\Sigma_c^+ & \frac{1}{\sqrt{2}}\Xi_c^{\prime +} \\ \frac{1}{\sqrt{2}}\Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}}\Xi_c^{\prime 0} \\ \frac{1}{\sqrt{2}}\Xi_c^{\prime +} & \frac{1}{\sqrt{2}}\Xi_c^{\prime 0} & \Omega_c^0 \end{pmatrix}_{ij}$$

$$(X)_{ij} = \begin{pmatrix} \Sigma_{c2}^{++} & \frac{1}{\sqrt{2}}\Sigma_{c2}^+ & \frac{1}{\sqrt{2}}\Xi_{c2}^{\prime +} \\ \frac{1}{\sqrt{2}}\Sigma_{c2}^+ & \Sigma_{c2}^0 & \frac{1}{\sqrt{2}}\Xi_{c2}^{\prime 0} \\ \frac{1}{\sqrt{2}}\Xi_{c2}^{\prime +} & \frac{1}{\sqrt{2}}\Xi_{c2}^{\prime 0} & \Omega_{c2}^0 \end{pmatrix}_{ij}$$

Formalism

Effective Lagrangian, combing the chiral symmetry and the heavy quark spin symmetry

$$\begin{aligned}\mathcal{L}_{\text{meson}} &= i\frac{h'}{\Lambda_\chi}\mathrm{Tr}\left[\bar{H}_{2a}T^{\mu}_{2b}\gamma^{\nu}\gamma_5(D_\mu\mathcal{A}_\nu + D_\nu\mathcal{A}_\mu)_{ba}\right] + \text{ h.c.},\\
\mathcal{L}_{\text{baryon}} &= -\sqrt{3}g_2\mathrm{Tr}\left[\bar{B}_{\bar{3}}\mathcal{A}^{\mu}\mathcal{S}_\mu + \bar{\mathcal{S}}_\mu\mathcal{A}^{\mu}B_{\bar{3}}\right]\\
&+ ih_{10}\epsilon_{ijk}\bar{\mathcal{T}}_i(D_\mu\mathcal{A}_\nu + D_\nu\mathcal{A}_\mu)_{jl}\mathcal{X}^{\mu\nu}_{kl}\\
&+ h_{11}\epsilon_{\mu\nu\sigma\lambda}v^{\lambda}\mathrm{Tr}\left[\bar{\mathcal{S}}^{\mu}(D^{\nu}\mathcal{A}_\alpha + D_\alpha\mathcal{A}^{\nu})\mathcal{X}^{\alpha\sigma}\right],\end{aligned}$$

$$\mathcal{A}_\mu = \frac{i}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right)$$

Cho, PRD50,3295 (1994); PLB285,145(1992); Prijol&Yan, PRD56,5483(1997); Cheng&Chua, PRD75,014006

$$\xi = e^{i\mathcal{M}/f_\pi}\ ,\ \mathcal{M} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

$$D_\mu = \partial_\mu + \mathcal{V}_\mu \qquad \mathcal{V}_\mu = \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right)$$

Formalism

Coupling constants can be determined by the corresponding decay widths

$$h' = 0.43,$$
$$g_2 = 0.565,$$
$$|h_{10}| = 0.85.$$

By means of the quark model

$$|h_{11}| = \sqrt{2}|h_{10}|$$

Prijol&Yan, PRD56,5483(1997)

D-wave decay modes (advantage for the TS mechanism)

$$\bar{D}_1 \to \bar{D}^*\pi,\ \bar{D}_2 \to \bar{D}^{(*)}\pi \quad \Sigma_{c2} \to \Lambda_c\pi/\Sigma_c^{(*)}\pi$$

Formalism

Effective Lagrangian for $\bar{D}Y_c \to J/\psi p$

$$\mathcal{L}_{\mathrm{ct}} = g_{\Lambda_c}\bar{N}H_2\bar{J}\mathcal{T}_3 + g_{\Sigma_c}\bar{N}\gamma_\mu\gamma_5 H_2\bar{J}\mathcal{S}^\mu + ig_{\Lambda_{c1}}\partial_\mu\bar{N}H_2\bar{J}\mathcal{R}^\mu,$$

$$J = \frac{1+\not{v}}{2}[\psi(nS)^\mu\gamma_\mu - \eta_c(nS)\gamma_5]\frac{1-\not{v}}{2}$$
$$\bar{J} = \gamma^0 J\gamma^0.$$

For these short range interactions, from the dimensional analysis, it is expected that the couplings g_{Λ_c} (g_{Σ_c}) and $g_{\Lambda_{c1}}$ are of the order of magnitude of m_D^{-2} and m_D^{-3} respectively, where m_D is the mass of the D meson. Notice that in HHChPT, the heavy filed H_2 (J) in Eq. (26) will contain a factor $\sqrt{M_{H_2}}$ $(\sqrt{M_J})$ for normalization. In addition, we will estimate the scattering amplitudes in the static limit , which means the four velocity is set to $v = (1,0,0,0)$.

Formalism

Effective Lagrangian for $\pi^- p \to \bar{D} Y_c$, with the fewest derivatives

$$\mathcal{L}_{\rm a1} = \frac{g_{a1}}{2m_D} \mathbb{D}\, \tau \cdot \pi\ \tau \cdot \bar{\mathbf{\Sigma}}_{\mathbf{c}}\, N,$$

$$\mathcal{L}_{\rm a2} = \frac{g_{a2}}{2m_D} \mathbb{D}^*_\mu\, \tau \cdot \pi\ \tau \cdot \bar{\mathbf{\Sigma}}_{\mathbf{c}} \gamma_5 \gamma^\mu N,$$

$$\mathcal{L}_{\rm a3} = \frac{g_{a3}}{2m_D} \mathbb{D}^*_\mu\, \tau \cdot \pi\ \tau \cdot \bar{\mathbf{\Sigma}}^{*\mu}_{\mathbf{c}}\, N,$$

$$\mathcal{L}_{\rm b1} = \frac{i g_{b1}}{\sqrt{2} m_D^2} \mathbb{D}^\mu_1\, \tau \cdot \partial_\mu \pi\ \bar{\Lambda}_c\, N,$$

$$\mathcal{L}_{\rm b2} = \frac{i g_{b2}}{\sqrt{2} m_D^2} \mathbb{D}^{\mu\nu}_2\, \tau \cdot \partial_\mu \pi\ \bar{\Lambda}_c \gamma_5 \gamma_\nu N,$$

$$\mathcal{L}_{\rm b3} = \frac{i g_{b3}}{2 m_D^2} \mathbb{D}^\mu_1\, \tau \cdot \partial_\mu \pi\ \tau \cdot \bar{\mathbf{\Sigma}}_{\mathbf{c}}\, N,$$

$$\mathcal{L}_{\rm b4} = \frac{i g_{b4}}{2 m_D^2} \mathbb{D}^{\mu\nu}_2\, \tau \cdot \partial_\mu \pi\ \tau \cdot \bar{\mathbf{\Sigma}}_{\mathbf{c}} \gamma_5 \gamma_\nu N,$$

$$\mathcal{L}_{\rm c1} = \frac{i g_{c1}}{2 m_D^2} \mathbb{D}\, \tau \cdot \partial_\mu \pi\ \tau \cdot \bar{\mathbf{\Sigma}}^\mu_{\mathbf{c2}}\, N,$$

$$\mathcal{L}_{\rm c2} = \frac{i g_{c2}}{2 m_D^2} \mathbb{D}^*_\mu\, \tau \cdot \partial_\nu \pi\ \tau \cdot \bar{\mathbf{\Sigma}}^\mu_{\mathbf{c2}} \gamma_5 \gamma^\nu N,$$

$$\mathcal{L}_{\rm d1} = \frac{g_{d1}}{\sqrt{2} m_D} \mathbb{D}^\mu_1\, \tau \cdot \pi\ \bar{\Lambda}_{c1} \gamma_5 \gamma_\mu N,$$

$$\mathcal{L}_{\rm d2} = \frac{g_{d2}}{\sqrt{2} m_D} \mathbb{D}^\mu_1\, \tau \cdot \pi\ \bar{\Lambda}^*_{c1\mu}\, N,$$

$$\mathcal{L}_{\rm d3} = \frac{g_{d3}}{\sqrt{2} m_D} \mathbb{D}^{\mu\nu}_2\, \tau \cdot \pi\ \bar{\Lambda}^*_{c1\mu} \gamma_5 \gamma_\nu N,$$

$\mathbb{D}$ = $(D^0,\ D^+)$

Open-charm Production

Experimental and theoretical constraints on the open-charm productions in πN collisions:

$\sigma(\pi^- p \rightarrow D^{*-}\Lambda_c^+)$ and $\sigma(\pi^- p \rightarrow D^{*-}\Sigma_c^+)$

upper limit at 13 GeV is about 7 nb BNL, PRL55, 154(1985)

Effective Lagrangian method and Regge approach: at the order of 1 nb S.H. Kim et al, PRD92, 094001 (2015); arXiv: 1405.3445

Generalized parton picture: at the order of nb

S. Kofler et al, PRD91, 054027 (2015)

Our assumptions:

$\sigma(\pi^- p \rightarrow \bar{D} Y_c)$ **is 1 nb at 20 GeV pion-beam energy**

Open-charm Production

Coupling constants according to the assumpions

$\sigma(\pi^- p \to \bar{D} Y_c)$ **is 1 nb at 20 GeV pion-beam energy**

g_{a1}	g_{a2}	g_{a3}	g_{b1}	g_{b2}	g_{b3}
0.035	0.020	0.025	0.024	0.018	0.023
g_{b4}	g_{c1}	g_{c2}	g_{d1}	g_{d2}	g_{d3}
0.017	0.027	0.009	0.024	0.026	0.029

Width Effect of the Intermediate State

Adopt the Breit-Wigner type propagator in the triangle loop integral

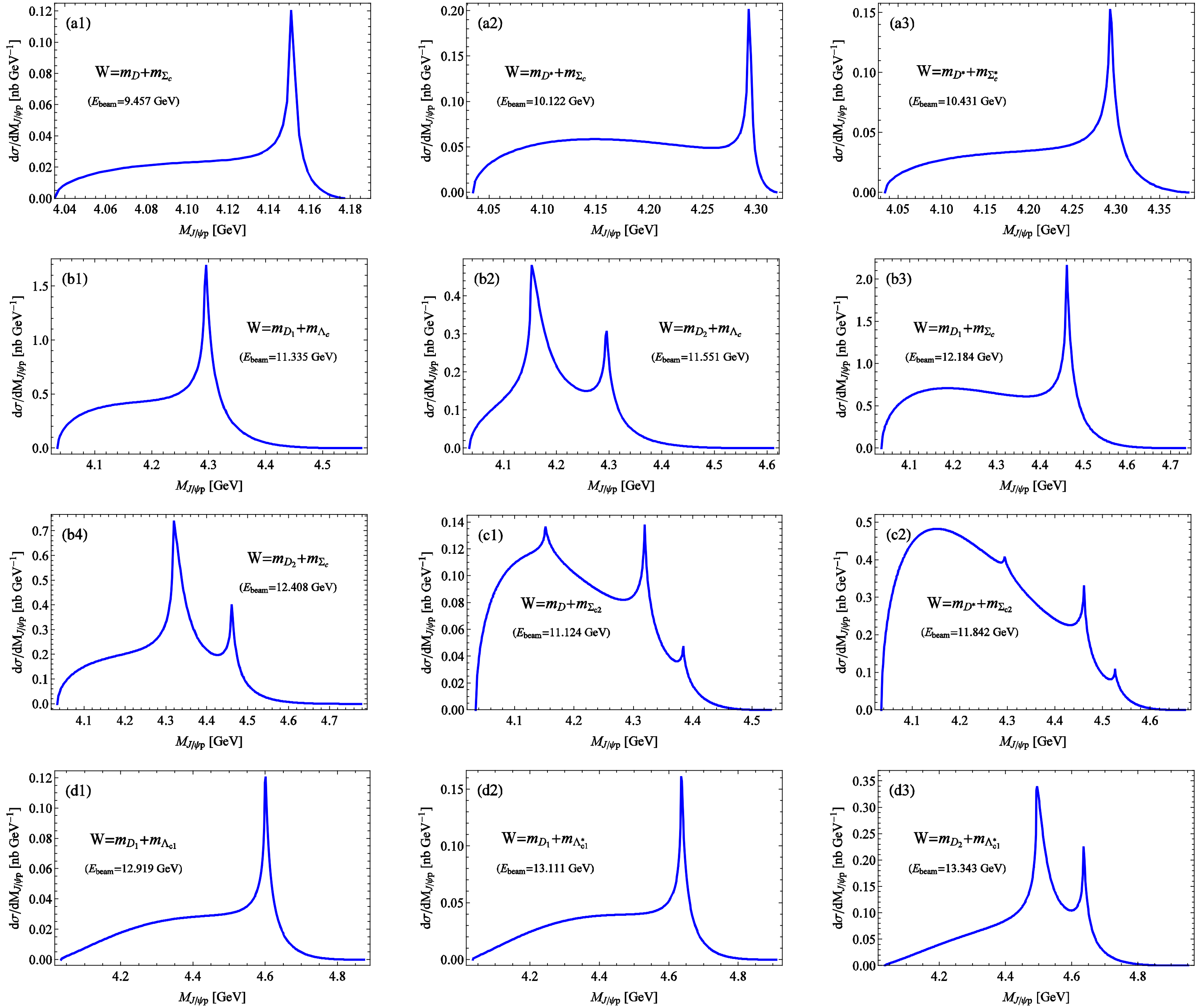
$$G_{\mathcal{D}}^{(0)} = \frac{i}{q_{\mathcal{D}}^2 - m_{\mathcal{D}}^2 + i\, m_{\mathcal{D}}\Gamma_{\mathcal{D}}},$$

$$G_{Y_c}^{(\frac{1}{2})} = \frac{i\, m_{Y_c}(1 + \not{v})}{q_{Y_c}^2 - m_{Y_c}^2 + i\, m_{Y_c}\Gamma_{Y_c}},$$

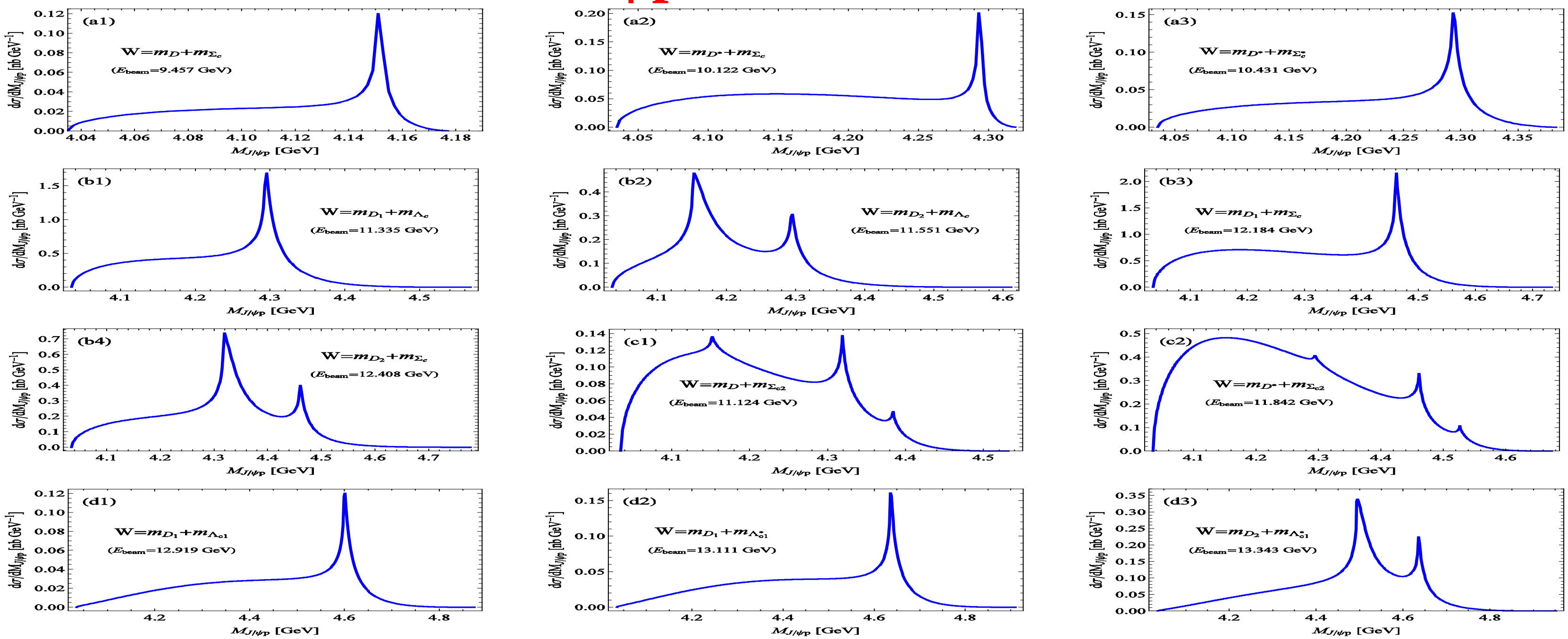
Rarita-Schwinger wave functions for higher spin particles

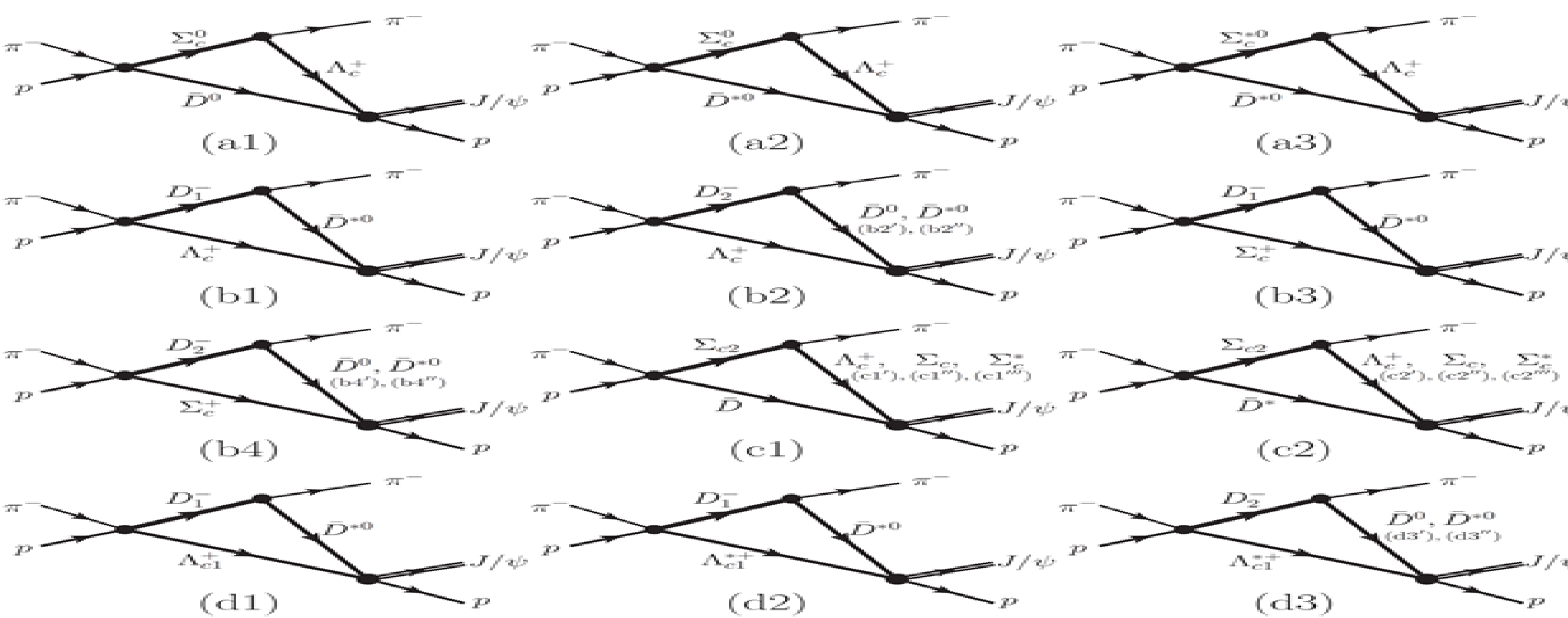
BW type propagator will remove the TS from the physical boundary by a small distance, if the corresponding decay width is smaller. **I.J.R. Aitchison & C. Kacser, PR133, B1239 (1964)**

Numerical Results: J/ψp Invariant Mass Distribution



Numerical Results: J/ψp Invariant Mass Distribution





Numerical Results: Total Cross Section

Diagram #	Cross Section [nb]
(a1)	0.003
(a2)	0.015
(a3)	0.011
(b1)	0.153
(b2)	0.056
(b3)	0.330
(b4)	0.113
(c1)	0.032
(c2)	0.166
(d1)	0.016
(d2)	0.024
(d3)	0.056

Summary

- **Kinematic singularities (TS) of the rescattering amplitude will behave themselves as peaks in the invariant mass distribution, which may imply that non-resonance interpretations for some resonance-like structures is possible.**
- **Being different from the genuine resonances, the TS mechanism is a highly process-dependent mechanism, and very sensitive to the kinematic configurations.**
- **The forthcoming J-PARC pion-induced experiment may offer us a good opportunity to check different kinematic or dynamic mechanisms and clarify the ambiguities, with its high luminosity.**