Coalescence Model for Resonances

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ExHIC workshop,
YITP, March 30, 2016

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1. Introduction

coalescence model :

simple picture of composite particle production in heavy ion collisions

commonly used tool for hadronization and formation of molecular states

production probability of a composite particle γ (bound state of a and b)

$$P_\gamma = \iint \frac{d\mathbf{k}d\mathbf{k}'}{(2\pi)^6} \varphi_\gamma(\mathbf{k})^* \rho(\mathbf{k},\mathbf{k}') \varphi_\gamma(\mathbf{k}'),$$

$\mathbf{k},\mathbf{k}'$: momenta of a, b relative motion

$\varphi_\gamma(\mathbf{k})$: bound state wave function in $\mathbf{k}$ space

$\rho(\mathbf{k},\mathbf{k}')$: density matrix for particle source (collision complex)

How to calculate P_γ when γ is a resonance in a, b scattering system ?

Use the resonance wave function with $\varphi_\gamma(\mathbf{k})^* \to \varphi_\gamma(-\mathbf{k})$?

The resulting P_γ would be complex and its physical meaning becomes unclear.

Experimentally, the resonance is observed as a peak in the invariant mass spectrum of the a, b scattering system.

We formulate here a coalescence model for resonance production in the way it is observed experimentally.

2. Model of S-wave resonance

Lee-type model of S-wave resonance

a particle c coupled to two particles, a and b, generating a resonance

c in c.m. system : $|c>$

a, b with relative momentum $\mathbf{k}$: $|\mathbf{k}>$

hamiltonian of the coupled system

$$H = H_0 + V,$$

$$< c|H_0|c >= E_0,$$

$$< \mathbf{k}|H_0|\mathbf{k}' >= (2\pi)^3\delta(\mathbf{k} - \mathbf{k}')E_k,$$

$$< \mathbf{k}|V|c >= gv(k),$$

g, $v(k)$: real

a, b scattering state with asymptotic momentum $\mathbf{p}$: $\varphi_{\mathbf{p}}^{\pm}$

$$< \mathbf{k}|\varphi_{\mathbf{p}}^{\pm} >= (2\pi)^3\delta(\mathbf{k}-\mathbf{p}) + \frac{gv(k)}{E_p^{\pm}-E_k} < c|\varphi_{\mathbf{p}}^{\pm} >,$$

$$< c|\varphi_{\mathbf{p}}^{\pm} >= \frac{1}{E_p^{\pm}-E_0}\int \frac{dk}{(2\pi)^3} gv(k) < \mathbf{k}|\varphi_{\mathbf{p}}^{\pm} >,$$

$$E_p^{\pm} = E_p \pm i\eta$$

solution of the coupled equation

$$< c|\varphi_{\mathbf{p}}^{\pm} >= \frac{gv(p)}{E_p^{\pm}-E_0-\Sigma(E_p^{\pm})},$$

$$< \mathbf{k}|\varphi_{\mathbf{p}}^{\pm} >= (2\pi)^3\delta(\mathbf{k}-\mathbf{p}) + \frac{T(\mathbf{k},\mathbf{p};E_p^{\pm})}{E_p^{\pm}-E_k},$$

full off-shell T-matrix for complex E

$$T(\mathbf{k},\mathbf{k}';E) = \frac{g^2 v(k) v(k')}{E - E_0 - \Sigma(E)},$$

$$\Sigma(E) = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{(g v(k))^2}{E - E_k}.$$

resonance state : φ_r

$$(\mathcal{E}_r - E_k) < \mathbf{k}|\varphi_r >= gv(k) < c|\varphi_r >$$
$$(\mathcal{E}_r - E_0) < c|\varphi_r >= g \int \frac{d\mathbf{k}}{(2\pi)^3} v(k) < \mathbf{k}|\varphi_r >$$

eigenvalue equation

$$\mathcal{E}_r - E_0 = \Sigma(\mathcal{E}_r).$$

$\mathcal{E}_r$: pole of T-matrix in the complex E plane

normalization condition determines $< c|\varphi_r >$

$$< c|\varphi_r >^2 + \int \frac{d\mathbf{k}}{(2\pi)^3} < -\mathbf{k}|\varphi_r >< \mathbf{k}|\varphi_r >= 1,$$
$$\Longrightarrow \qquad < c|\varphi_r >^2 = (1 - \partial_E \Sigma(E)|_{E=\mathcal{E}_r})^{-1}.$$

3. Coalescence model for scattering states

collision complex : ensemble of free particles, a, b, and c, described by density matrix $\hat{\rho}$

$$< \mathbf{k}|\hat{\rho}|\mathbf{k}' >= \rho(\mathbf{k},\mathbf{k}'), \quad < c|\hat{\rho}|c >= \rho_c,$$

probability of finding two-particle system with relative momentum $\mathbf{p}$ in coalescence model

$$P(\mathbf{p}) =< \varphi^-_{\mathbf{p}}|\hat{\rho}|\varphi^-_{\mathbf{p}} >= P_{ab}(\mathbf{p}) + P_c(\mathbf{p}),$$

$$P_{ab}(\mathbf{p}) = \iint \frac{d\mathbf{k}d\mathbf{k}'}{(2\pi)^6}\rho(\mathbf{k},\mathbf{k}') < \mathbf{k}|\varphi^-_{\mathbf{p}} >^* < \mathbf{k}'|\varphi^-_{\mathbf{p}} >$$

$$P_c(\mathbf{p}) = \rho_c| < c|\varphi^-_{\mathbf{p}} > |^2 = \frac{\rho_c(gv(p))^2}{|E^-_p - E_0 - \Sigma(E^-_p)|^2}.$$

P_{ab} : further decomposed into free term $P_{ab}^{(0)}$, interaction term $P_{ab}^{(1)}$, and interference term $P_{ab}^{(2)}$

$$P_{ab}(\mathbf{p}) = P_{ab}^{(0)}(\mathbf{p}) + P_{ab}^{(1)}(\mathbf{p}) + P_{ab}^{(2)}(\mathbf{p}),$$

$$P_{ab}^{(0)}(\mathbf{p}) = \rho(\mathbf{p},\mathbf{p}),$$

$$P_{ab}^{(1)}(\mathbf{p}) = \iint \frac{d\mathbf{k}d\mathbf{k}'}{(2\pi)^6}\rho(\mathbf{k},\mathbf{k}')\frac{T^*(\mathbf{k},\mathbf{p};E_p^-)T(\mathbf{k}',\mathbf{p};E_p^-)}{(E_p^+ - E_{k'})(E_p^- - E_k)},$$

$$P_{ab}^{(2)}(\mathbf{p}) = 2\mathrm{Re}\left(\int \frac{d\mathbf{k}}{(2\pi)^3}\rho(\mathbf{p},\mathbf{k})\frac{T(\mathbf{k},\mathbf{p};E_p^-)}{E_p^- - E_k}\right).$$

completeness of $\varphi_{\mathbf{p}}^-$

$$\int \frac{d\mathbf{p}}{(2\pi)^3}(P_{ab}^{(1)}(\mathbf{p}) + P_{ab}^{(2)}(\mathbf{p})) = 0, \quad \int \frac{d\mathbf{p}}{(2\pi)^3}P_c(\mathbf{p}) = \rho_c.$$

4. Numerical examples

analytically solvable choice

kinetic energy and form factor

$$E_k = \frac{k^2}{2m}, \quad v(k) = \frac{1}{k^2 + \mu^2}.$$

self-energy

$$2m\Sigma(E) = -\frac{\lambda}{(\mu - ip_E)^2}, \quad \lambda = \frac{m^2 g^2}{2\pi\mu}, \quad p_E^2 = 2mE$$

$$p_{E_p^\pm} = \pm\sqrt{2mE_p} = \pm p, \quad E_p, p \geq 0$$

full off-shell T-matrix

$$2mT(k,k';E) = \frac{8\pi\lambda\mu}{(k^2+\mu^2)(k'^2+\mu^2)}\left(p_E^2 - p_0^2 + \frac{\lambda}{(\mu - ip_E)^2}\right)^{-1}.$$

$$p_0^2 = 2mE_0$$

poles : $p_E^2 - p_0^2 + \frac{\lambda}{(\mu - ip_E)^2} = 0$

resonance pole : p_r, $p_r^2 = 2m\mathcal{E}_r$:

solution with $\mathrm{Re}(p_r) > 0, \quad \mathrm{Im}(p_r) < 0$

resonance wave function φ_r

$$< c|\varphi_r >^2 = \left(1 + \frac{i\lambda}{p_r(\mu - ip_r)^3}\right)^{-1}.$$

Choice of parameters

Mass and Interaction Parameters,

simplified model of $\Lambda(1405)$

(a, b) : (π, Σ)

c : $\bar{K}N$ bound state without coupling to $\pi\Sigma$

set A : simulating $\Lambda(1405)$

set B : broader resonance

set C : narrower one

Mass and Interaction Parameters

set	$m(\mathrm{GeV})$	$\mu(\mathrm{GeV})$	$\lambda(\mathrm{GeV}^4)$	$E_0(\mathrm{GeV})$
A	0.125	0.5	3.0×10^{-3}	0.12
B	0.125	0.5	6.25×10^{-3}	0.15
C	0.125	0.5	6.25×10^{-4}	0.10

Resonance Properties

set	$\mathcal{E}_r(\mathrm{GeV})$	$p_r(\mathrm{GeV})$	$< c\vert\varphi_r >^2$
A	$0.080 - i0.026$	$0.143 - i0.023$	$1.157 - i0.116$
B	$0.047 - i0.065$	$0.126 - i0.064$	$1.758 - i0.383$
C	$0.092 - i0.0052$	$0.152 - i0.0043$	$1.024 - i0.019$

Hyodo-Kamiya proposal for compositeness

$$Z = < c|\varphi_r >^2, \quad X = 1 - Z,$$

$$\tilde{X} = (1 + |X| - |Z|)/2, \quad \tilde{Z} = (1 + |Z| - |X|)/2,$$

$$U = (|X| + |Z| - 1)/2$$

set	$\|X\|$	$\|Z\|$	$\tilde{X}$	$\tilde{Z}$	U
A	0.195	1.163	0.016	0.984	0.179
B	0.849	1.799	0.025	0.975	0.824
C	0.031	1.024	0.004	0.996	0.028

Density Matrix for Collision Complex (Source)

density matrix used in ExHIC papers

$$\rho(\mathbf{k},\mathbf{k}') = N(2\pi)^4\delta(\mathbf{k}-\mathbf{k}')\delta(k_z)\exp(-\beta\frac{k_T^2}{2m})$$

$$\int \frac{d\mathbf{k}}{(2\pi)^3}\rho(\mathbf{k},\mathbf{k}) = n_{ab}, \quad \Longrightarrow \quad N = \frac{2\pi\beta n_{ab}}{Vm},$$

$$(2\pi)^3\delta(\mathbf{0}) \longrightarrow V$$

this gives divergent $P_{ab}^{(1)}$ and $P_{ab}^{(2)}$

$\delta(\mathbf{k}-\mathbf{k}')$ implies infinitely extended source

approximation justified for bound state but not for scattering state modified density matrix "ExHIC"

(transversely finite (cylinder))

$$\rho(\mathbf{k},\mathbf{k}') = \tilde{N}(2\pi)^2\delta(k_z - k'_z)\delta(k_z)\exp(-\alpha(\mathbf{k_T} - \mathbf{k}'_T)^2) \\ \times \exp(-\beta\frac{(\mathbf{k}_T + \mathbf{k}'_T)^2}{8m}),$$

$$2\pi\delta(0) \longrightarrow L, \quad \tilde{N} = \frac{2\pi\beta n_{ab}}{Lm}, \quad 4\pi\alpha L = V$$

spherically symmetric density matrix

$$\rho(\mathbf{k},\mathbf{k}') = \tilde{N}\exp(-\alpha(\mathbf{k} - \mathbf{k}')^2 - \frac{\beta}{8m}(\mathbf{k} + \mathbf{k}')^2)$$

$$\tilde{N} = (\frac{2\pi\beta}{m})^{3/2}n_{ab}$$

P's for "ExHIC" density matrix

$$P^{(1)}_{ab}(\mathbf{p}) = \frac{8\pi\lambda\mu}{(p^2+\mu^2)^2|p^2-p_0^2+\frac{\lambda}{(\mu+ip)^2}|^2} I(E_p)$$

$$I(E) = \frac{2\lambda\mu\tilde{N}}{\pi}$$

$$\times \int_0^\infty \int_0^\infty \frac{dk dk' k k' \exp(-\alpha_+(k^2+k'^2)) I_0(2\alpha_- k k')}{(k^2+\mu^2)(k'^2+\mu^2)(2mE-k^2+i\eta)(2mE-k'^2-i\eta)}$$

$$P^{(2)}_{ab}(\mathbf{p}) = \tilde{N} 2\pi\delta(p_z) \mathrm{Re}\left(\frac{8\lambda\mu}{(p^2+\mu^2)(p^2-p_0^2+\frac{\lambda}{(\mu+ip)^2})} \times \int_0^\infty \frac{dk k \exp(\alpha_+(p^2+k^2)) I_0(2\alpha_- pk)}{(k^2+\mu^2)(p^2-k^2-i\eta)} \right)$$

$$\alpha_\pm = \alpha \pm \frac{\beta}{8m}$$

$$P_c(\mathbf{p}) = \frac{8\pi\mu\lambda\rho_c}{(p^2+\mu^2)^2|p^2-p_0^2+\frac{\lambda}{(\mu+ip)^2}|^2}$$

$P_{ab}^{(1)}$, P_c : isotropic particle emissiom

$P_{ab}^{(2)}$: transverse particle emission

P's for spherically symmetric density matrix

$$P_{ab}^{(1)}(\mathbf{p}) = \frac{2(\lambda\mu)^2 \tilde{N}}{\pi^2 m^2 \alpha_- (p^2+\mu^2)^2 |p^2 - p_0^2 + \frac{\lambda}{(\mu+ip)^2}|^2} \times \iint_0^\infty \frac{dk dk' k k' \exp(-\alpha_+(k^2+{k'}^2)) \sinh(2\alpha_- k k')}{(k^2+\mu^2)({k'}^2+\mu^2)(E_p^+ - E_k)(E_p^- - E_k)},$$

$$P_{ab}^{(2)}(\mathbf{p}) = 2\mathrm{Re}\left(\frac{\lambda\mu\tilde{N}}{\pi m \alpha_- p (p^2+\mu^2)(p^2 - p_0^2 + \frac{\lambda}{(\mu+ip)^2})} \times \int_0^\infty \frac{dk k \exp(\alpha_+(p^2+k^2)) \sinh(2\alpha_- p k)}{(k^2+\mu^2)(E_p^- - E_k)} \right),$$

$P_{ab}^{(1)}$, $P_{ab}^{(2)}$, P_c : isotropic particle emission density matrix parameters :

follow ExHIC papers except finite transverse size

$1/\beta = T_F(125\mathrm{MeV})$, $V = 4\pi\alpha L = V_F(10,860\mathrm{fm}^3)$

$n_{\pi\Sigma}$: statistical model with $T_H(175\mathrm{MeV})$, $V_H(1,908\mathrm{fm}^3)$

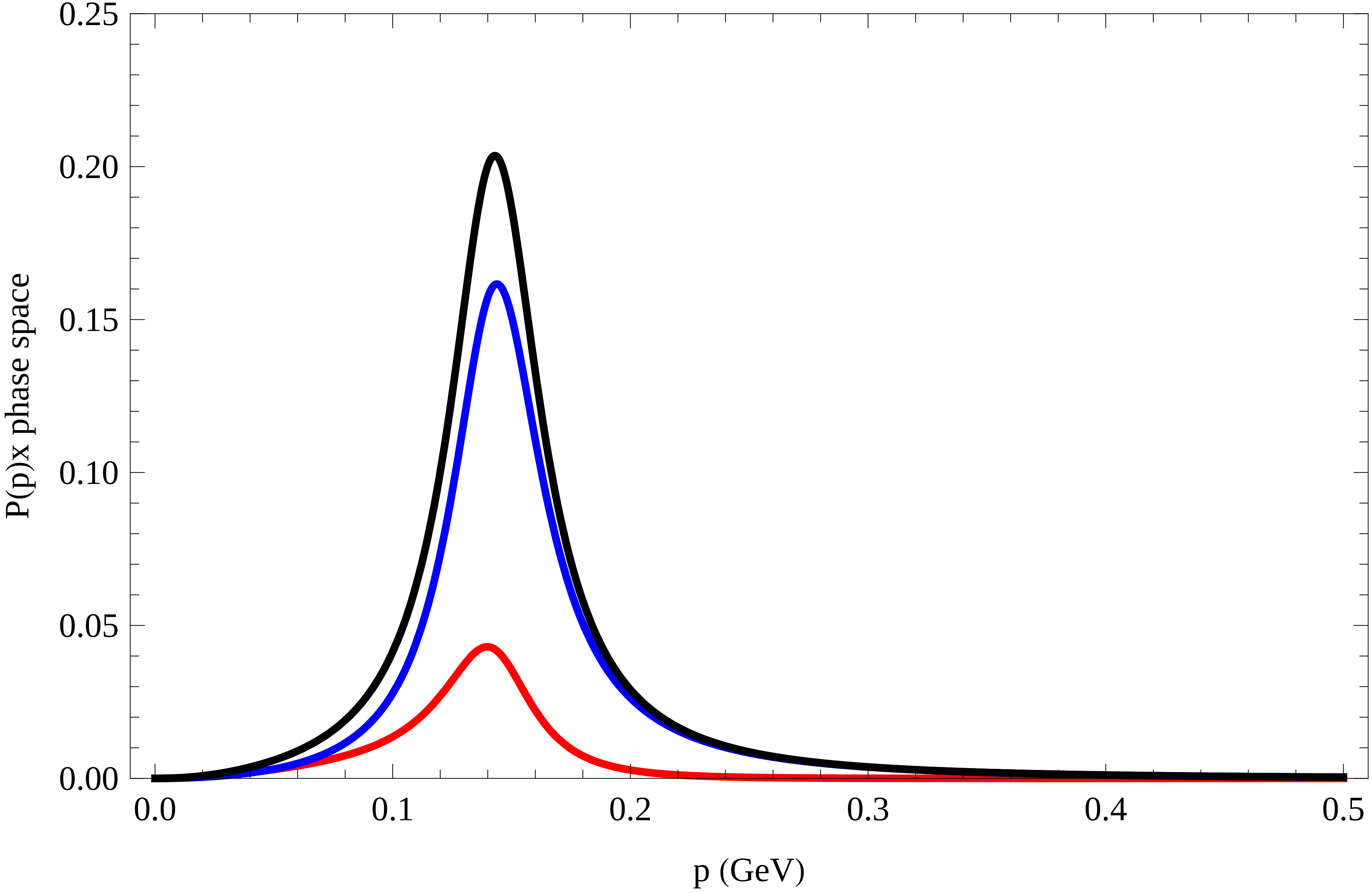
ρ_c : coalescence model

$\bar{K}N$ density matrix analogous to that for $\pi\Sigma$ h.o. w.f. with $\omega = 20.5\mathrm{MeV}$ for bound state

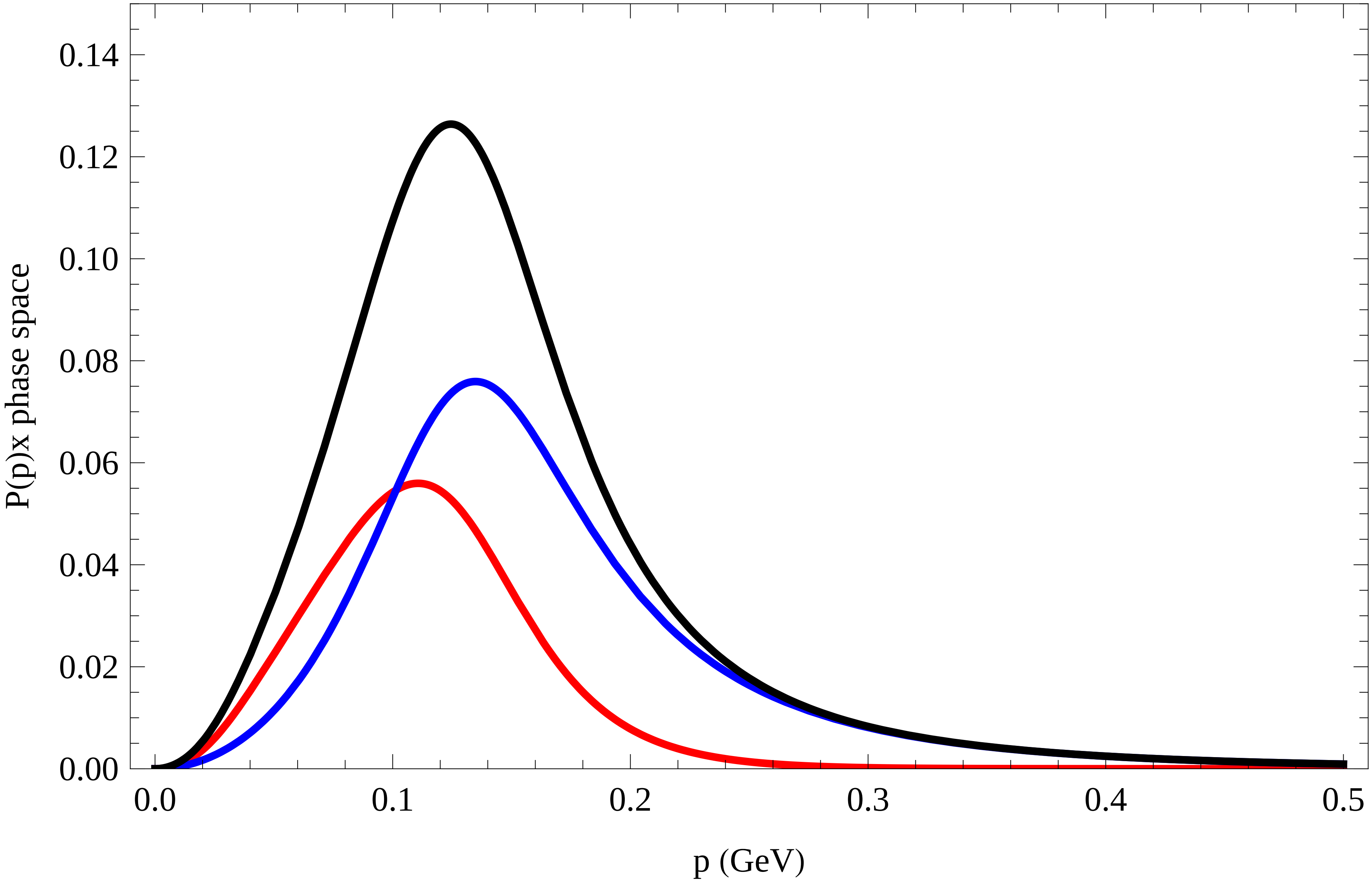
Density Matrix Parameters

$\beta(\mathrm{GeV}^{-1})$	$L(\mathrm{GeV}^{-1})$	$\alpha(\mathrm{GeV}^{-2})$	$n_{\bar{K}N}/n_{\pi\Sigma}$	ρ_c
8	565	200	2.34	0.0104
$1/\beta$		$\sqrt{4\pi\alpha}$		
$125\mathrm{MeV}$	$111\mathrm{fm}$	$9.89\mathrm{fm}$		

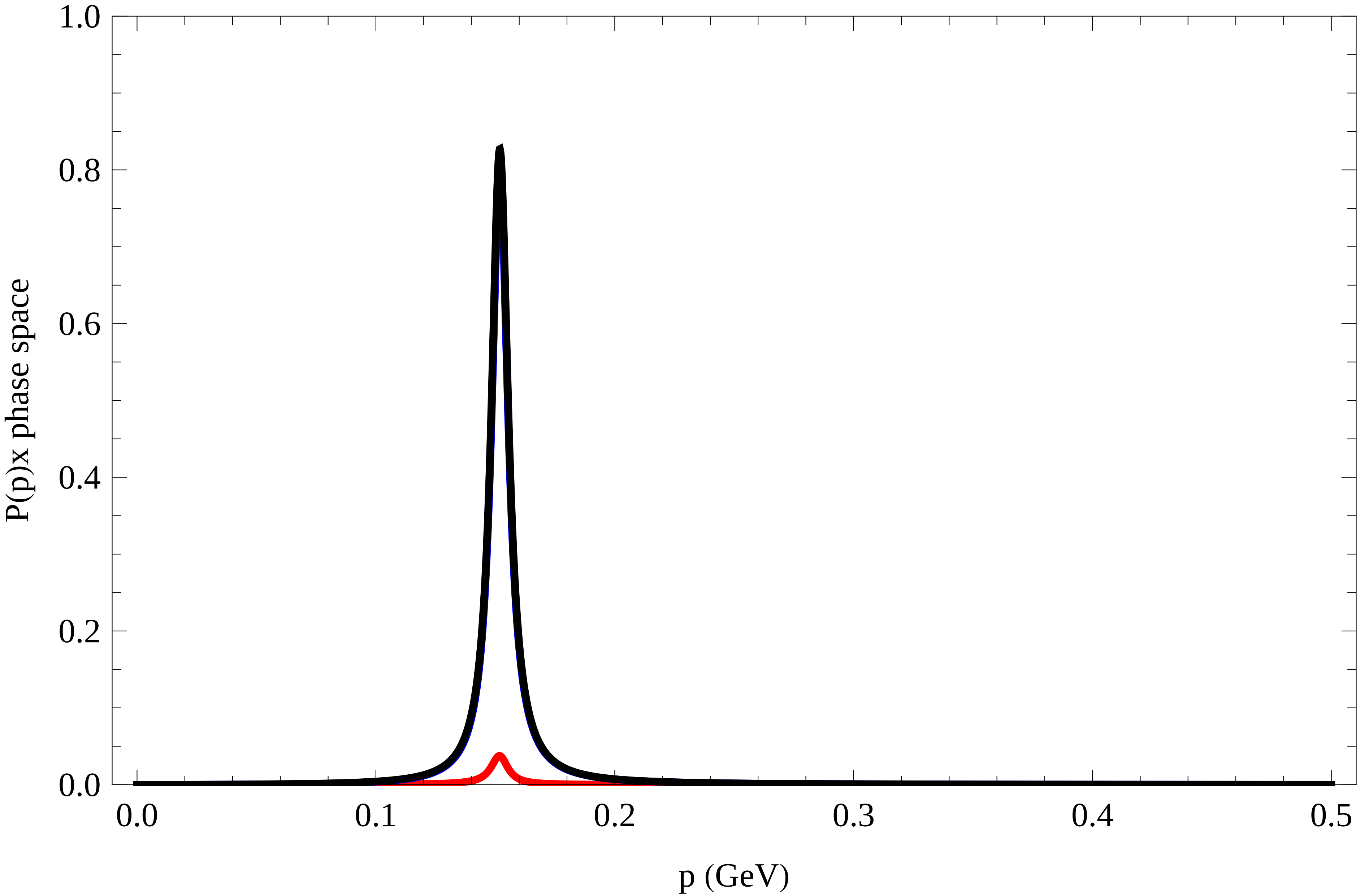
P's for "ExHIC" density matrix
($P_{ab}^{(1)}$ (red), P_c (blue), $P_{ab}^{(1)} + P_c$ (black)) $\times p^2/2\pi^2$
v.s. p for sets A, B, C



set A

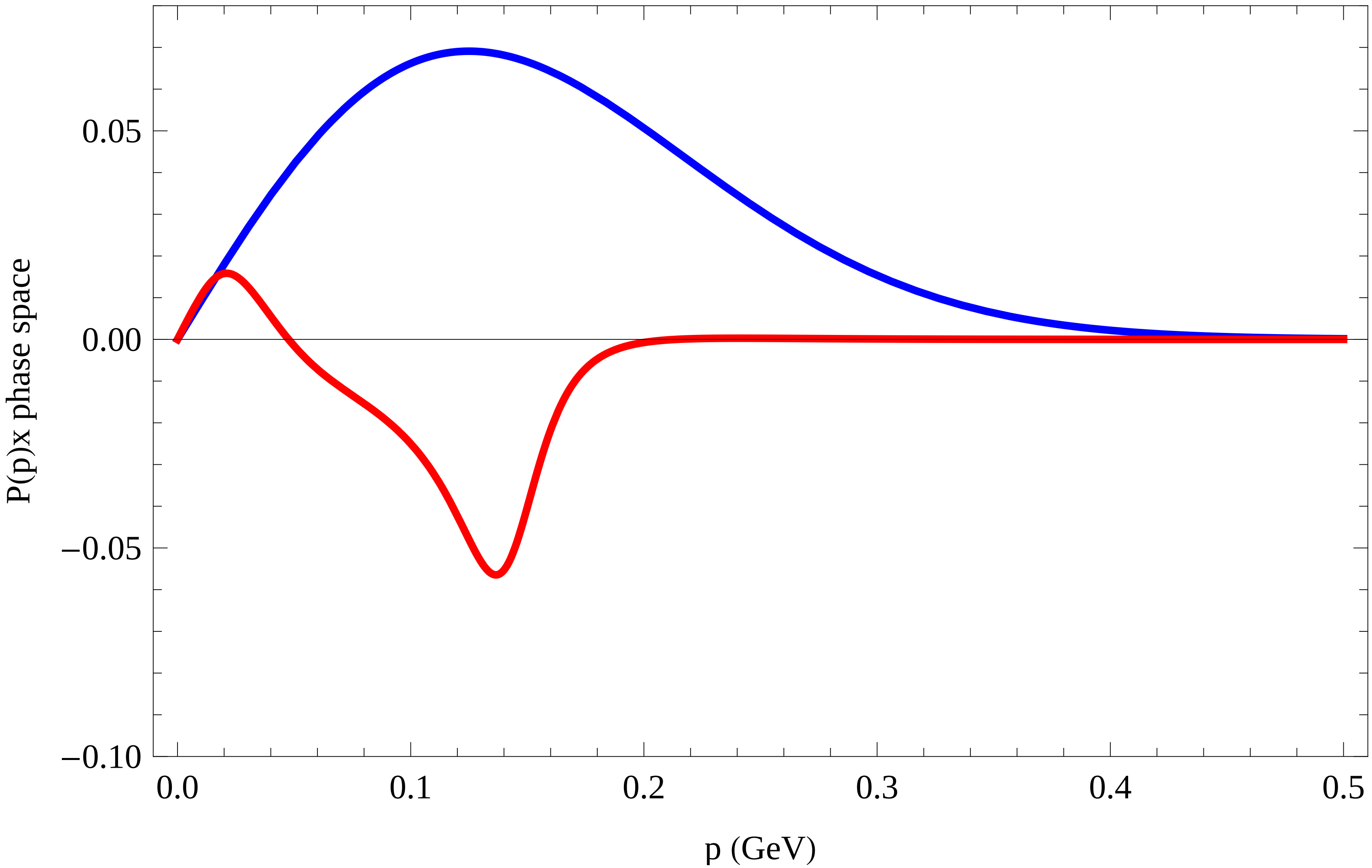

0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
P(p)x phase space
0.0
0.1
0.2
0.3
0.4
0.5
p (GeV)

set B

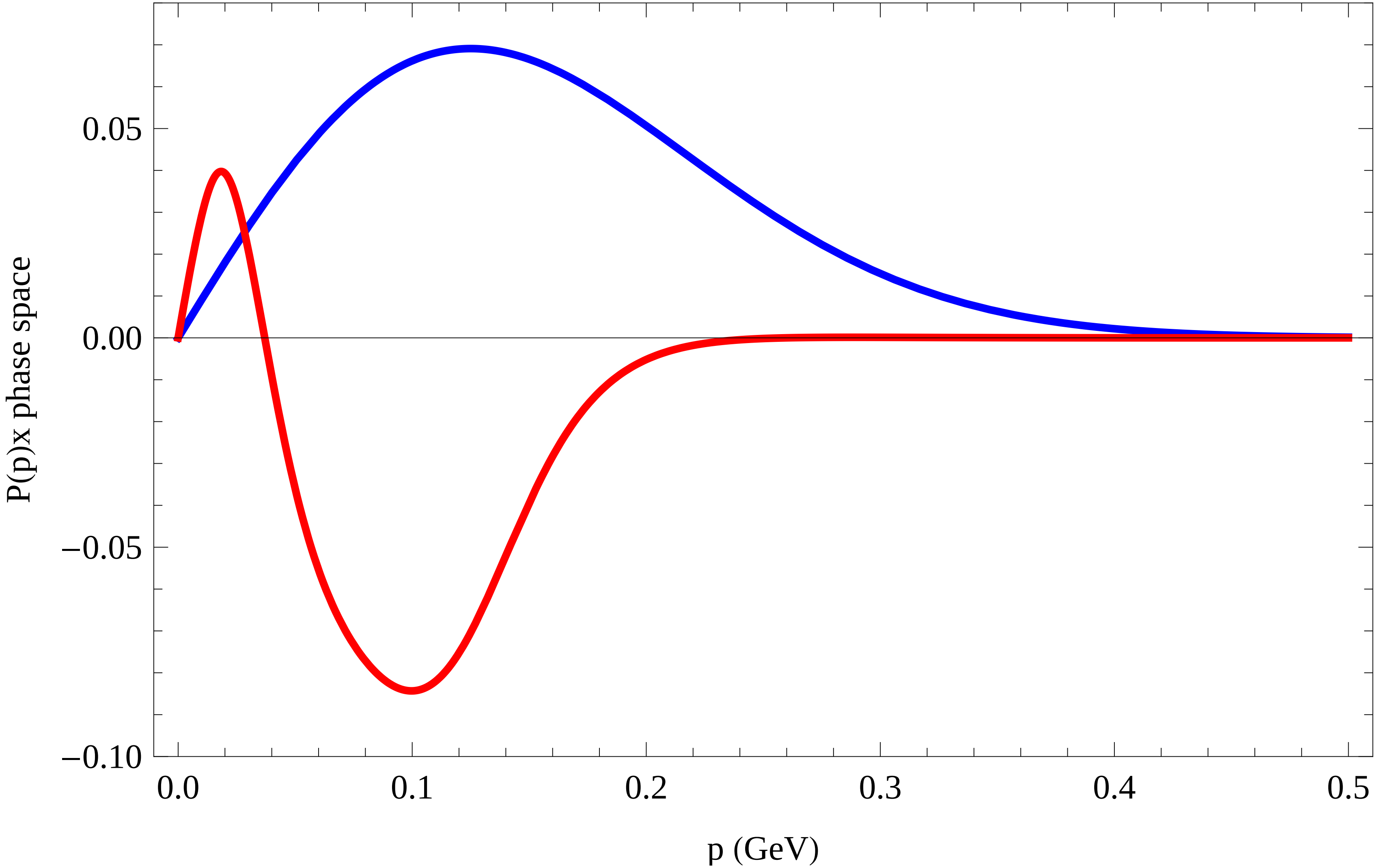


set C

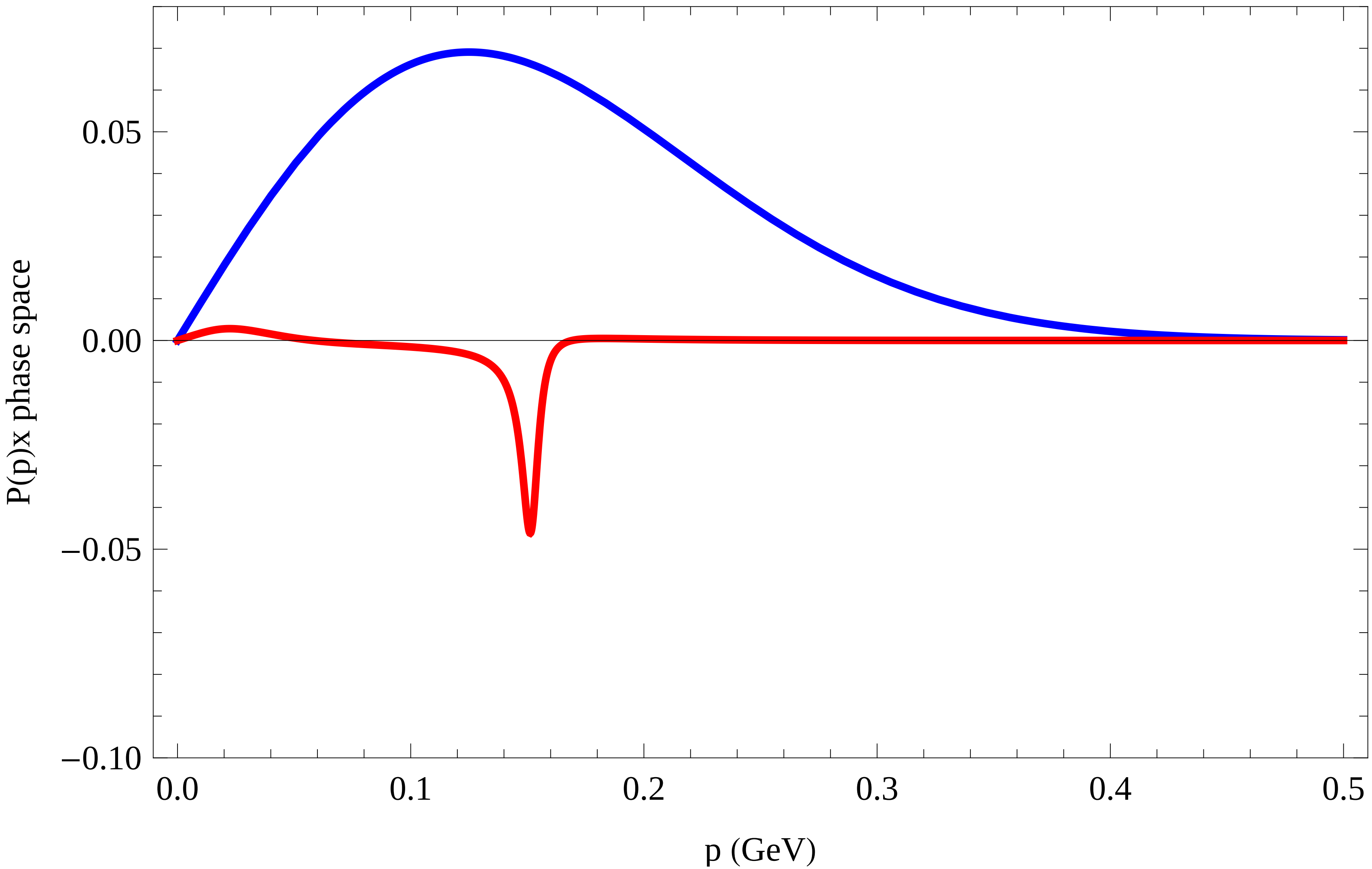
$(P_{ab}^{(0)}/100$ (blue), $P_{ab}^{(2)}$ (red)) $\times p/2\pi$ v.s. p for sets A, B, C



set A

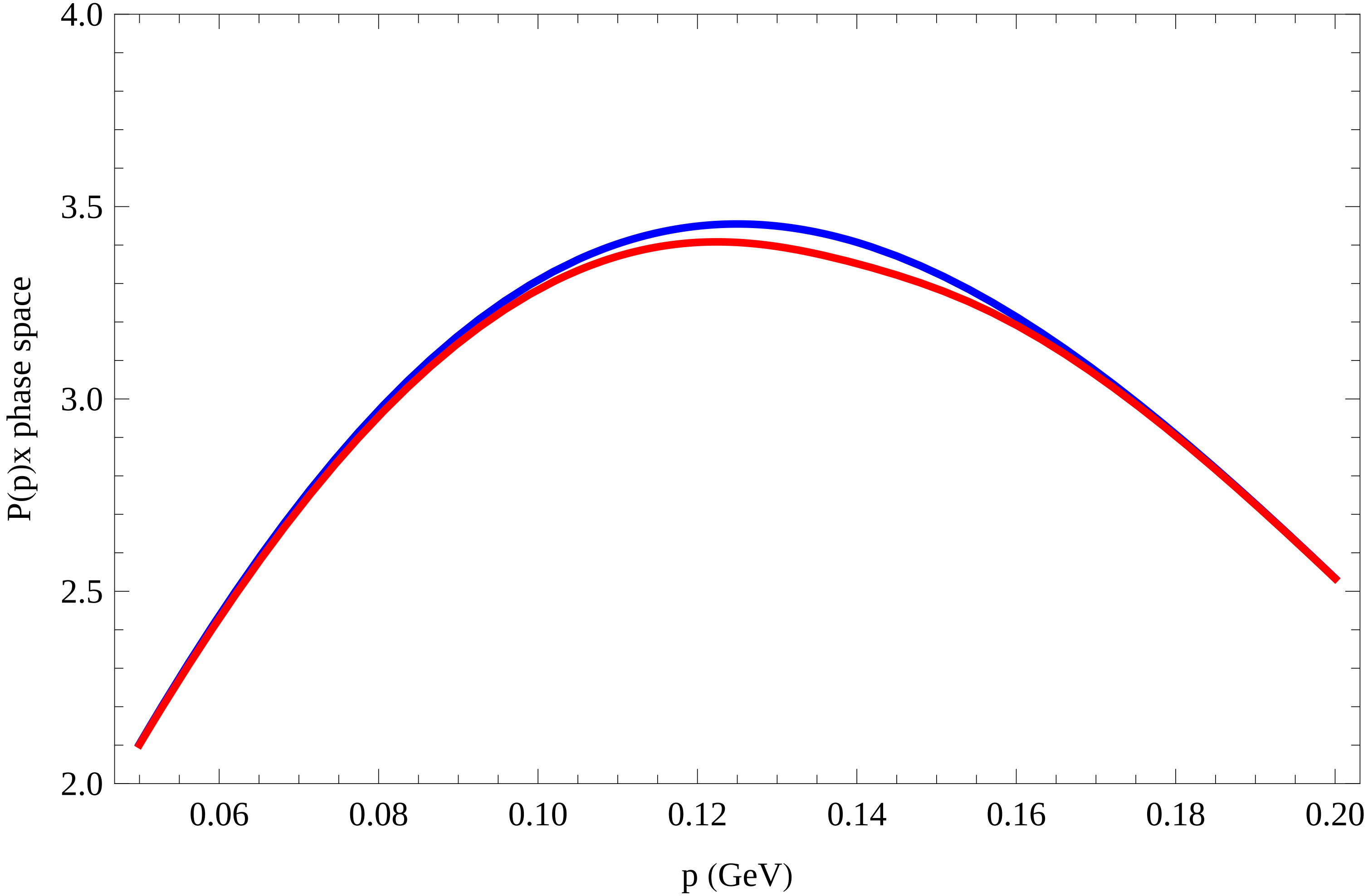


set B

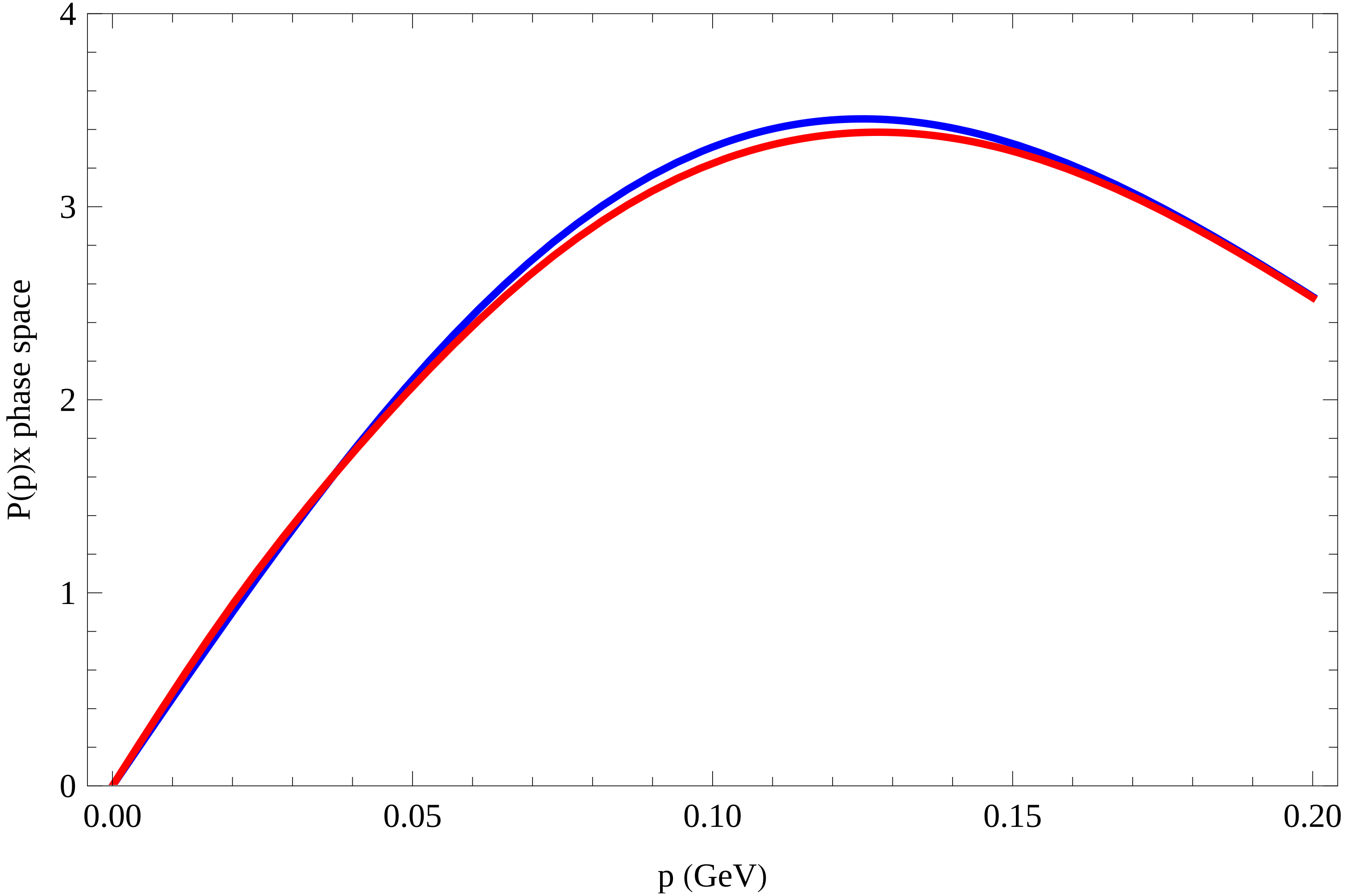


set C

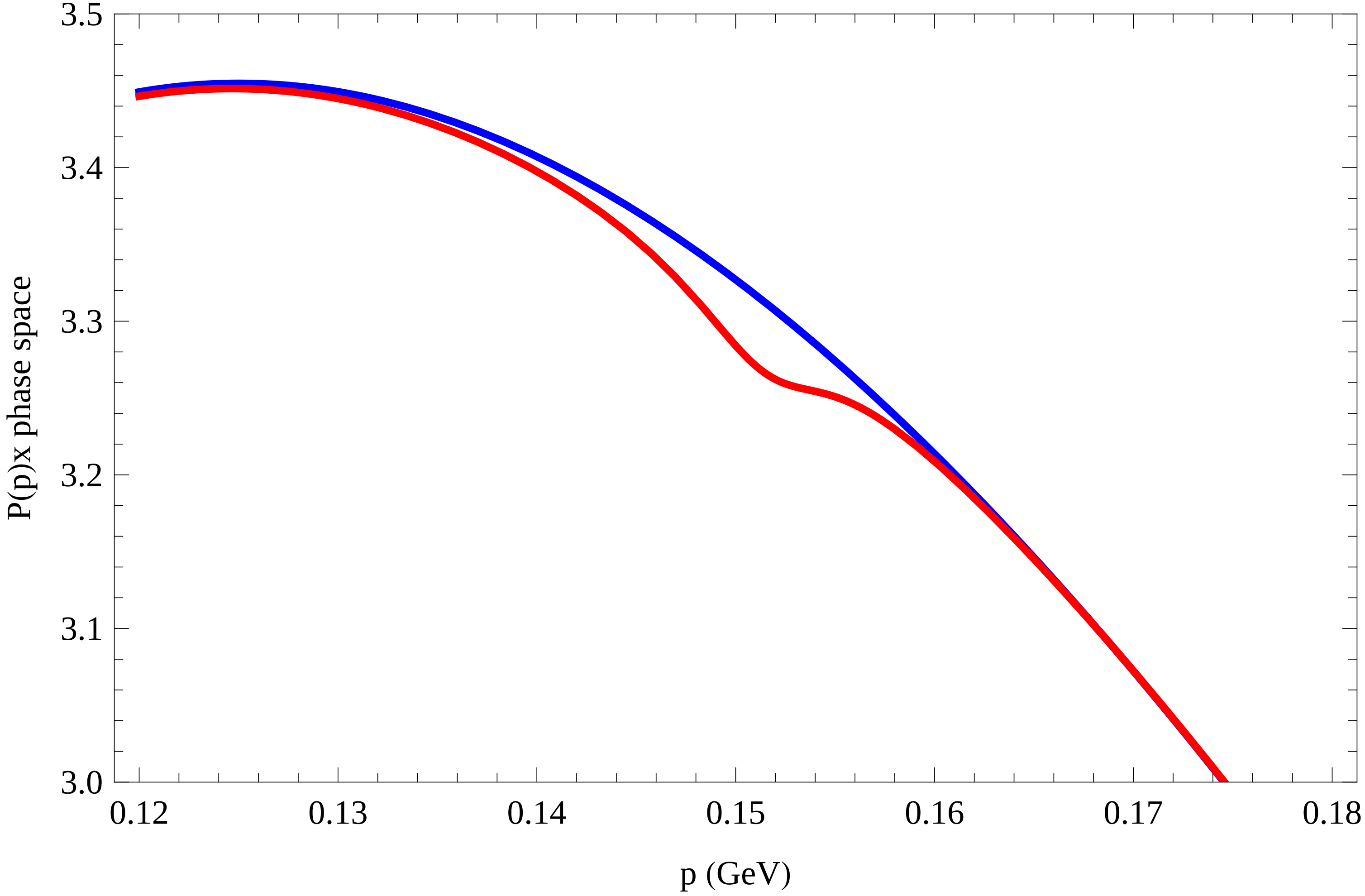
$(P_{ab}^{(0)}$ (blue), $P_{ab}^{(0)} + P_{ab}^{(2)}$ (red)) $\times p/2\pi$
v.s. p for sets A, B, C



set A



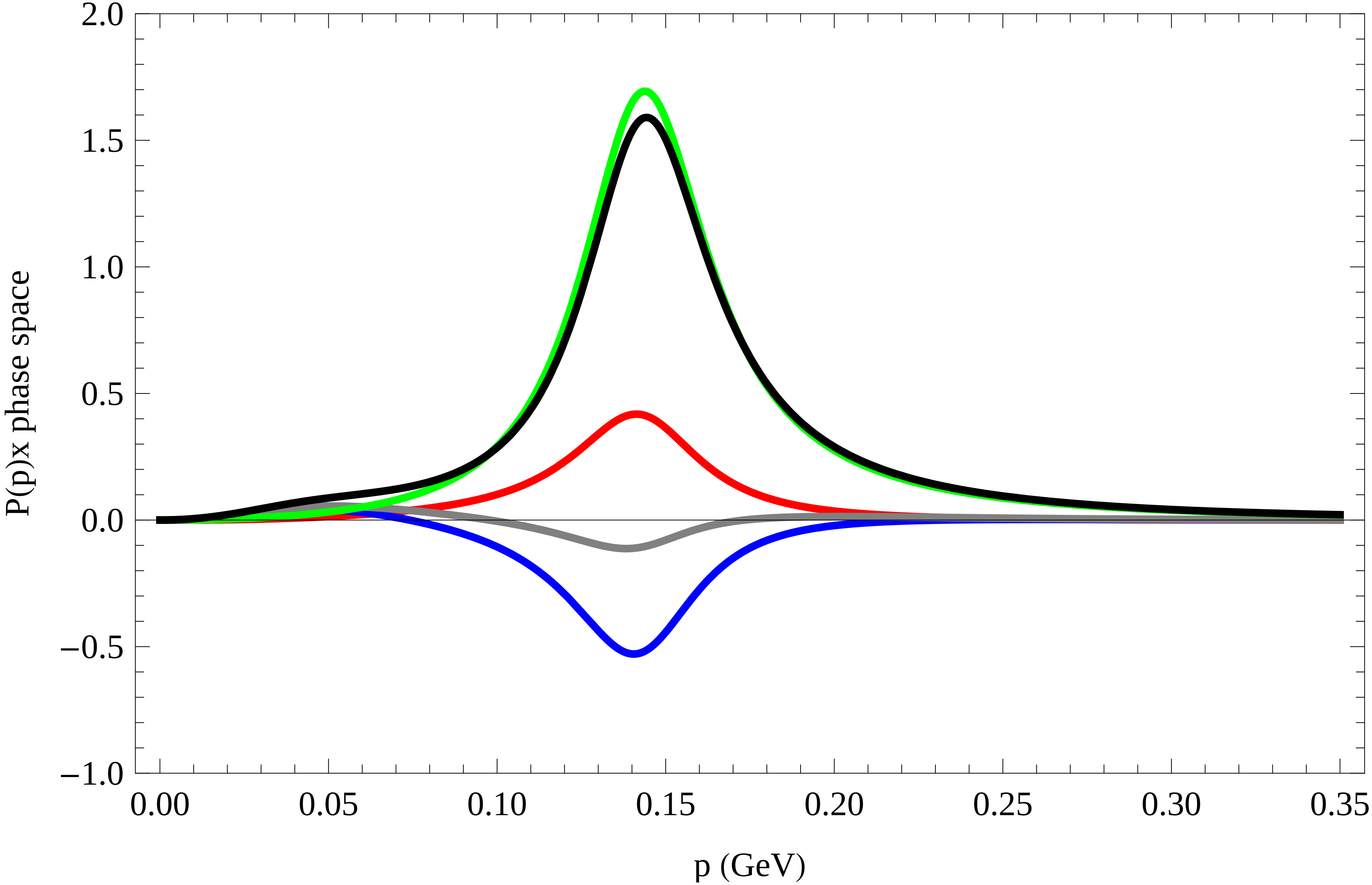
set B



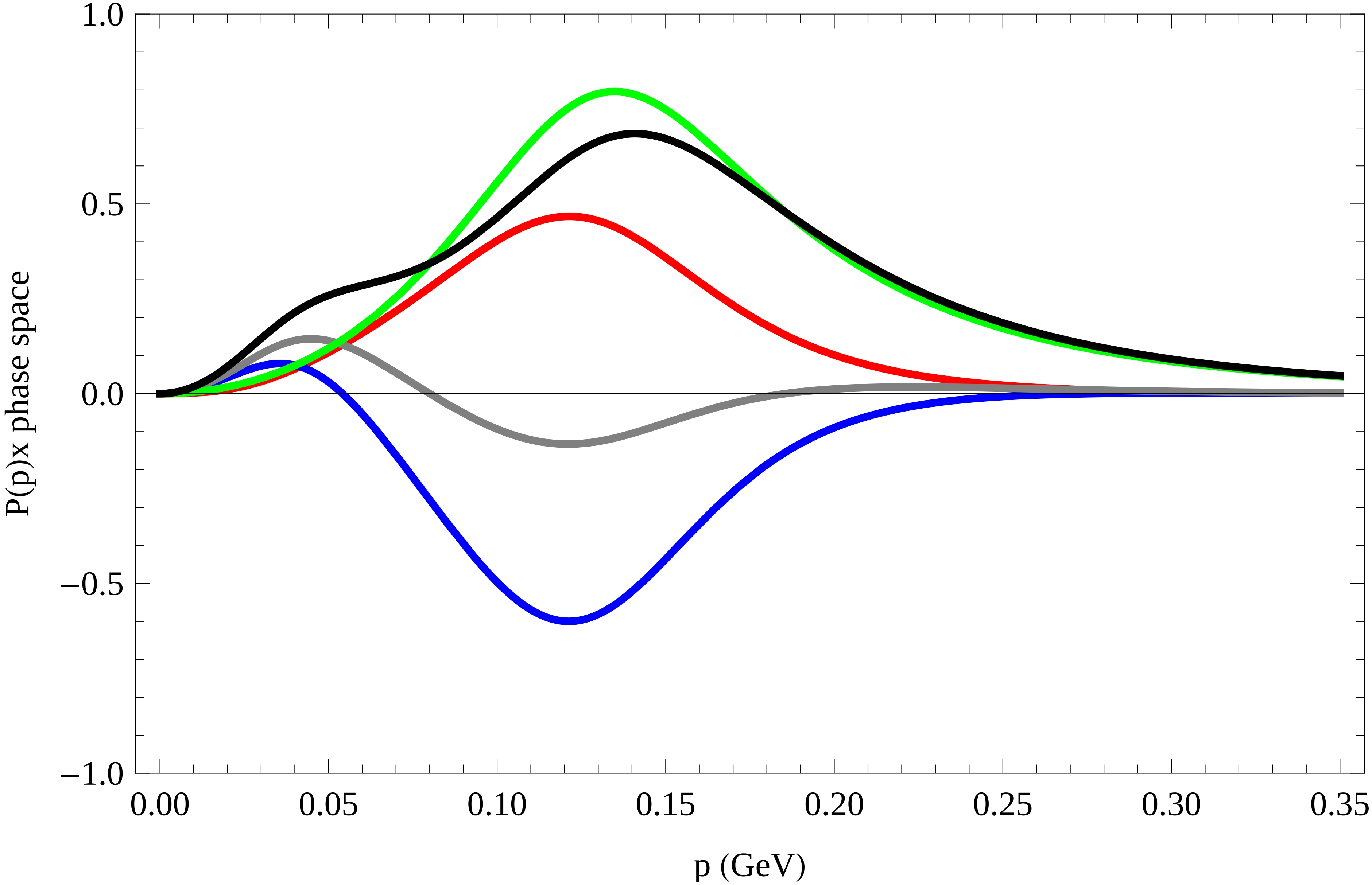
3.5
3.4
3.3
3.2
3.1
3.0
P(p)x phase space
0.12
0.13
0.14
0.15
0.16
0.17
0.18
p (GeV)

set C

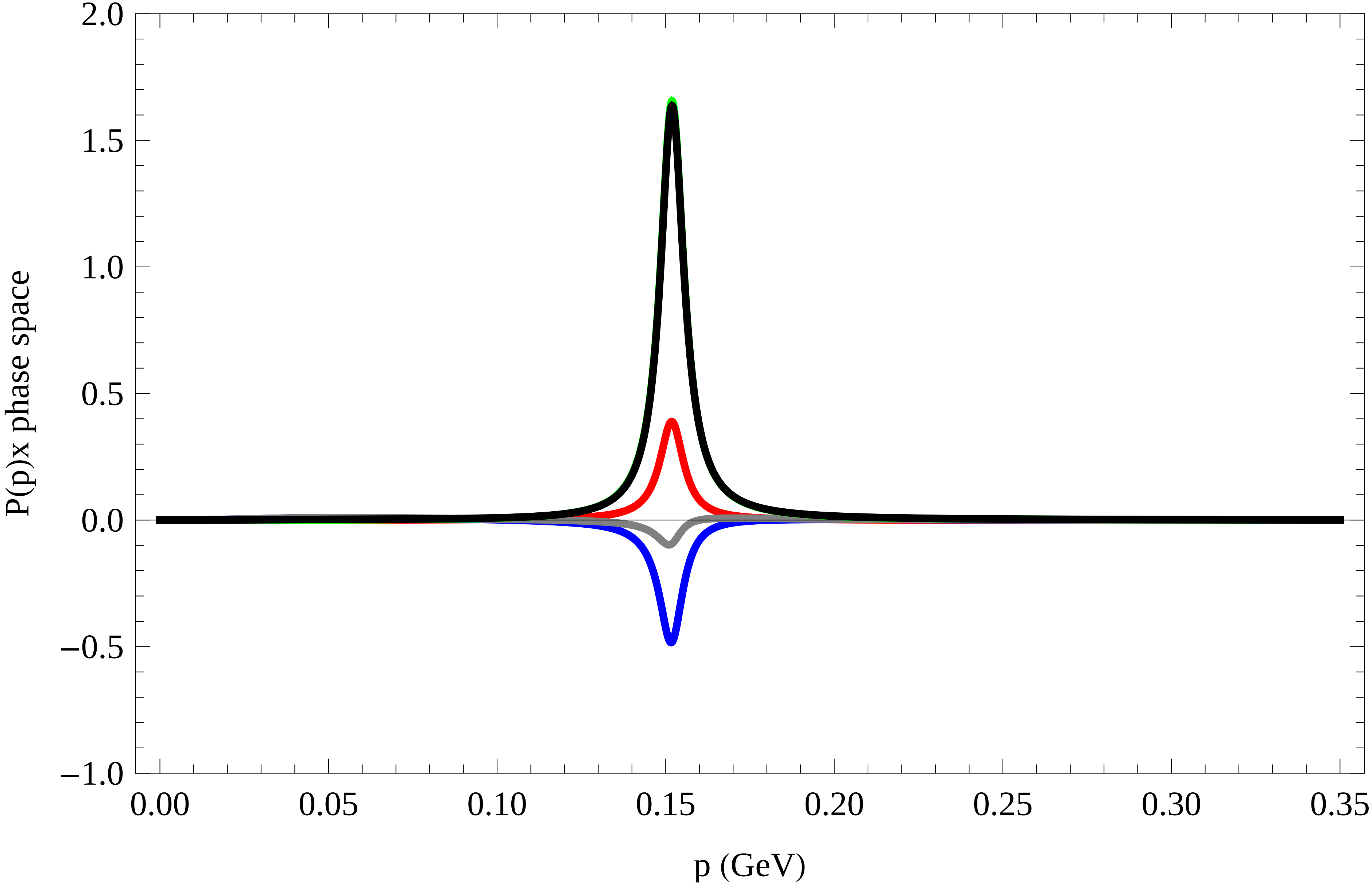
P's for spherically symmetric density matrix
($P_{ab}^{(1)}$ (red), $P_{ab}^{(2)}$ (blue), $P_{ab}^{(1)} + P_{ab}^{(2)}$ (gray),
P_c (green), $P_{ab}^{(1)} + P_{ab}^{(2)} + P_c$ (black)) $\times p^2/2\pi^2$ for sets A, B, C


2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
P(p)x phase space
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
p (GeV)

set A


1.0
0.5
0.0
−0.5
−1.0
P(p)x phase space
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
p (GeV)

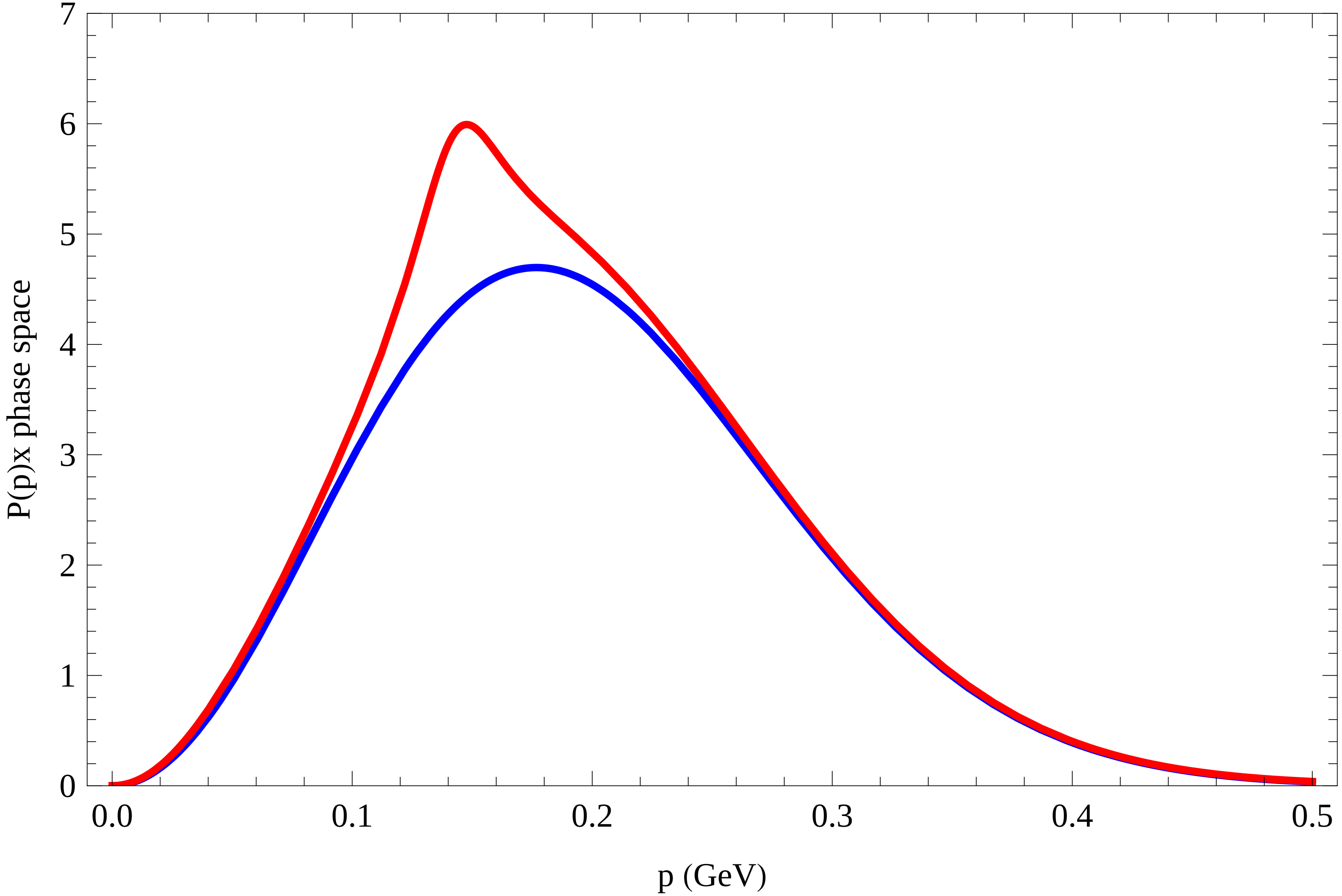
set B



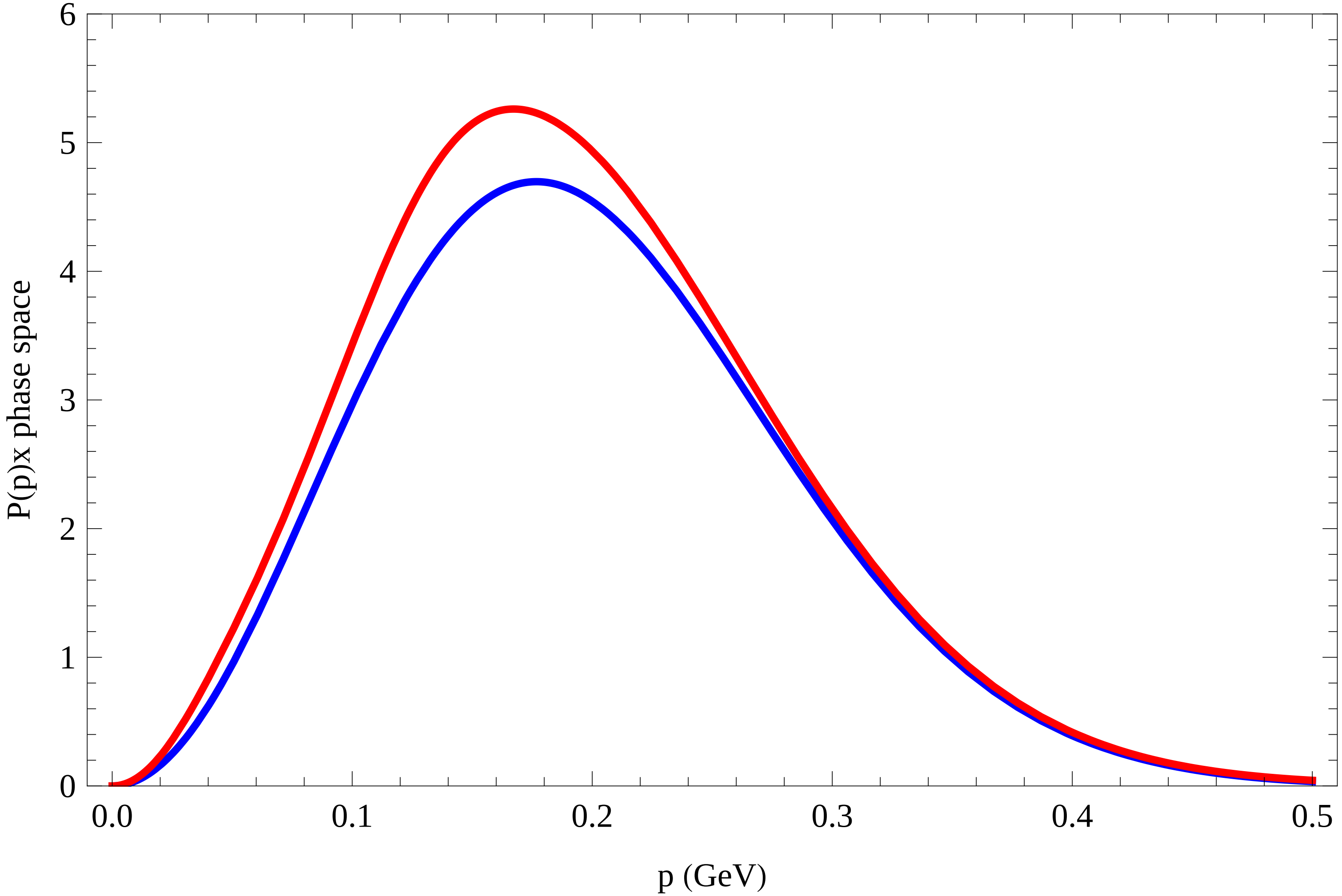
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
P(p)x phase space
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
p (GeV)

set C

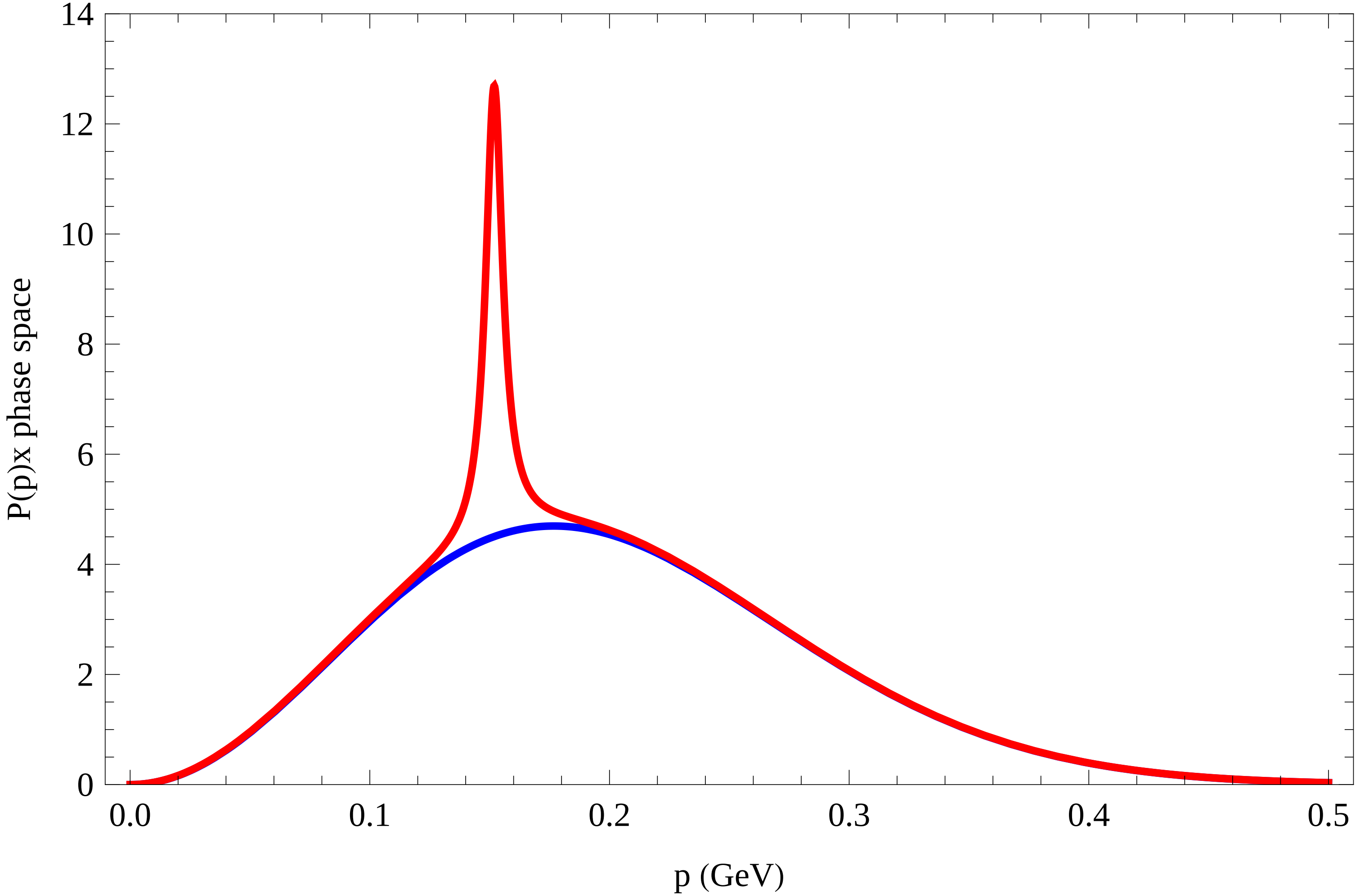
$(P_{ab}^{(0)}$ (blue), $P_{ab} + P_c$ (red)) $\times p^2/2\pi^2$
v.s. p for sets A, B, C



set A



set B



set C

Integrated production probabilities

$$\Pi = \int \frac{d\mathbf{p}}{(2\pi)^3} P(\mathbf{p})$$

sum rules

$$\Pi_{ab}^{(1)} + \Pi_{ab}^{(2)} = 0, \quad \Pi_c = \rho_c$$

$\Pi_{ab}^{(1)}/\rho_c$'s for "ExHIC" and spherical density matrices

set	"ExHIC"	spherical
A	0.274	0.22
B	0.644	0.46
C	0.060	0.046

“Production probability” using φ_r

$$\Pi^r = \Pi^r_c + \Pi^r_{ab},$$
$$\Pi^r_c = \rho_c < c|\varphi_r >^2,$$
$$\Pi^r_{ab} = \iint \frac{d\mathbf{k}d\mathbf{k}'}{(2\pi)^6} < -\mathbf{k}|\varphi_r > \rho(\mathbf{k},\mathbf{k}') < \mathbf{k}'|\varphi_r >$$
$$= < c|\varphi_r >^2 \iint \frac{d\mathbf{k}d\mathbf{k}'}{(2\pi)^6} \frac{g^2 v(k)v(k')\rho(\mathbf{k},\mathbf{k}')}{(\mathcal{E}_r - E_k)(\mathcal{E}_r - E_{k'})}$$

Π^r/ρ_c's for “ExHIC” density matrix

set	Π^r_c/ρ_c	Π^r_{ab}/ρ_c	Π^r/ρ_c
A	$1.157 - i0.116$	$-0.047 + i0.041$	$1.110 - i0.075$
B	$1.758 - i0.383$	$-0.096 - i0.011$	$1.665 - i0.398$
C	$1.024 - i0.019$	$-0.006 + i0.013$	$1.018 - i0.006$

P^r/ρ_c's for spherical density matrix

set	Π_c^r/ρ_c	Π_{ab}^r/ρ_c	Π^r/ρ_c
A	$1.157 - i0.116$	$-0.051 + i0.002$	$1.106 - i0.114$
B	$1.758 - i0.383$	$-0.056 - i0.041$	$1.705 - i0.428$
C	$1.024 - i0.019$	$-0.011 + i0.005$	$1.013 - i0.014$

5. Summary and discussions

Summary

• coalescence model for resonance particle production is formulated as experimentally observed

• Lee-type model is used for generating resonance in scattering system

• invariant mass spectrum for two particle scattering system is calculated

• completeness of scattering states lead to sum rules for production probabilities

• results depend strongly on density matrix describing particle source (collision complex)

• careful analysis needed to extract production probability from invariant mass spectrum

• use of resonance wave function gives complex production probability : real part tends to overestimate it

How can we extract the production probability of a resonance particle from experiments ?

- measure the invariant mass spectrum $P(\mathbf{p})$ for relevant region of $\mathbf{p}$
- subtract the background $P_{ab}^{(0)}(\mathbf{p})$
- integrate over relevant region of $\mathbf{p}$ to obtain $\Pi_{ab}^{(1)} + \Pi_{ab}^{(2)} + \Pi_c = \Pi_c$

Comment on Koonin-Pratt formula

$$C(\mathbf{p}) = \int d\mathbf{r}\,\tilde{\rho}(\mathbf{r})|\varphi_{\mathbf{p}}^{-}(\mathbf{r})|^2$$

to be compared with

$$P_{ab}(\mathbf{p}) = \int d\mathbf{r} \int d\mathbf{r}'\,\varphi_{\mathbf{p}}^{-\,*}(\mathbf{r})\tilde{\rho}(\mathbf{r},\mathbf{r}')\varphi_{\mathbf{p}}(\mathbf{r}')$$

P_{ab} reduces to C for $\tilde{\rho}(\mathbf{r},\mathbf{r}') \approx \delta(\mathbf{r}-\mathbf{r}')\tilde{\rho}(\mathbf{r})$, which is the high temperature approximation for thermal density matrix.