Difficulties in the direct method for two baryon systems in lattice QCD

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Realistic hadron interactions in QCD
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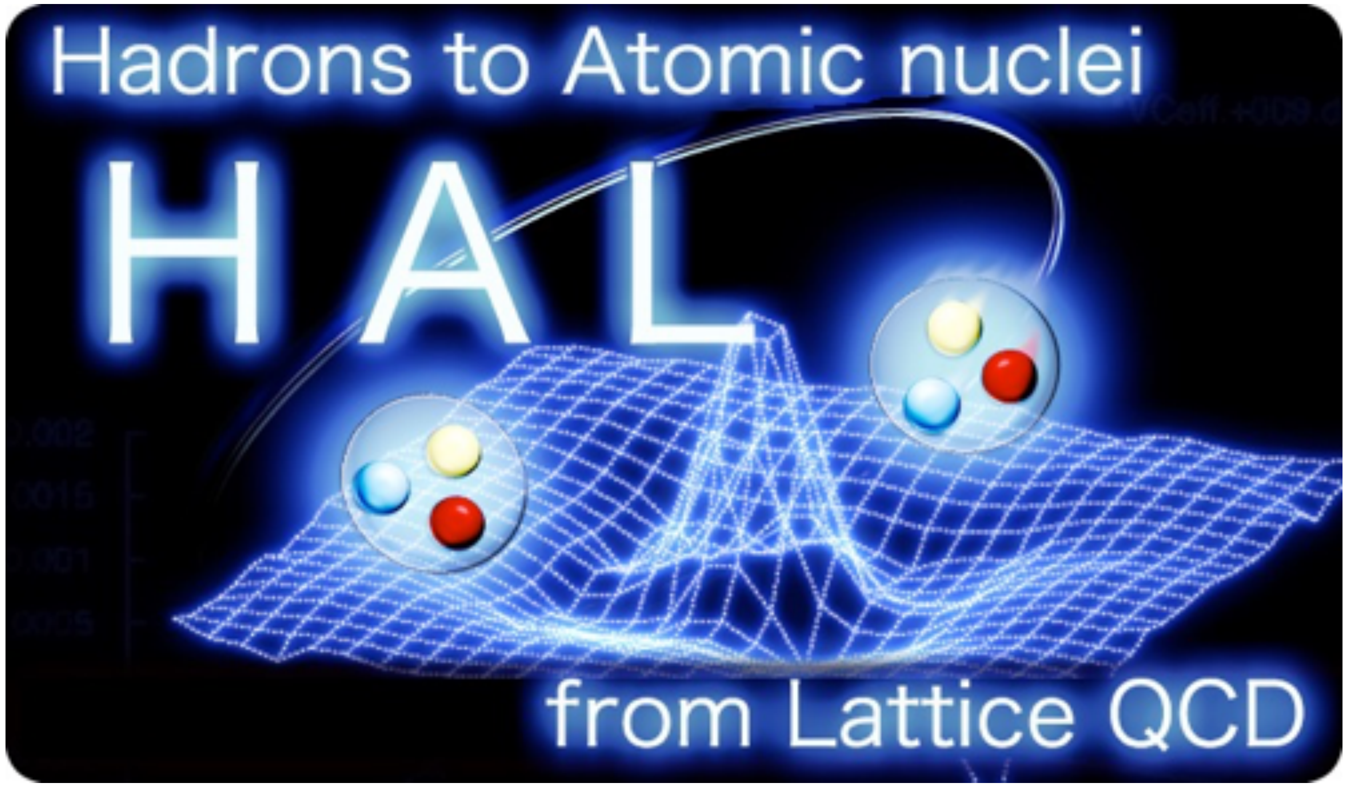
For
HAL QCD Collaboration



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Introduction
What is an issue ?

Lattice QCD methods for two-baryons

Direct method

$$G_{NN}(t) \sim e^{-E_{NN}t} \qquad t \to \infty$$

$$\Delta E = E_{NN} - 2m_N$$ → binding energy

+ finite volume formula → phase shift

Lüscher, NPB354(1991)531

Potential method **(HALQCD method)**

$$G_{NN}(r,t) = \langle N(r,t)N(0,t)\bar{N}(0)\bar{N}(0)\rangle$$ → "potential"

+ Schrödinger equation → phase shift

binding energy

Both are theoretically equivalent, but

Two nucleon systems at heavy pions

	$^1\mathrm{S}_0$ "di-neutron"	$^3\mathrm{S}_1$ "deuteron"
Direct method	bound	bound
	interactions become stronger at heavier pions	
Potential method	unbound	unbound
	interactions become weaker at heavier pions	
Nature $m_\pi \simeq 140\ \mathrm{MeV}$	unbound	bound

Both must agree.
We therefore have to identify sources of this discrepancy.

Introduction

I. Direct method

II. Mirage problem (Operator dependence)

III. Sanity check

IV. Conclusion

I. Direct method

Extraction of energy shift

Energy shift $$\underset{O(10\text{ MeV})}{\Delta E} \equiv \underset{O(2\text{ GeV})}{E_{NN}} - \underset{O(2\text{ GeV})}{2m_N}$$

large cancellation

0.5 % accuracy required

Ratio $$R(t) = \frac{G_{NN}(t)}{G_N(t)^2} \sim e^{-\Delta E t}$$

expect cancellation of both statistical and systematic errors

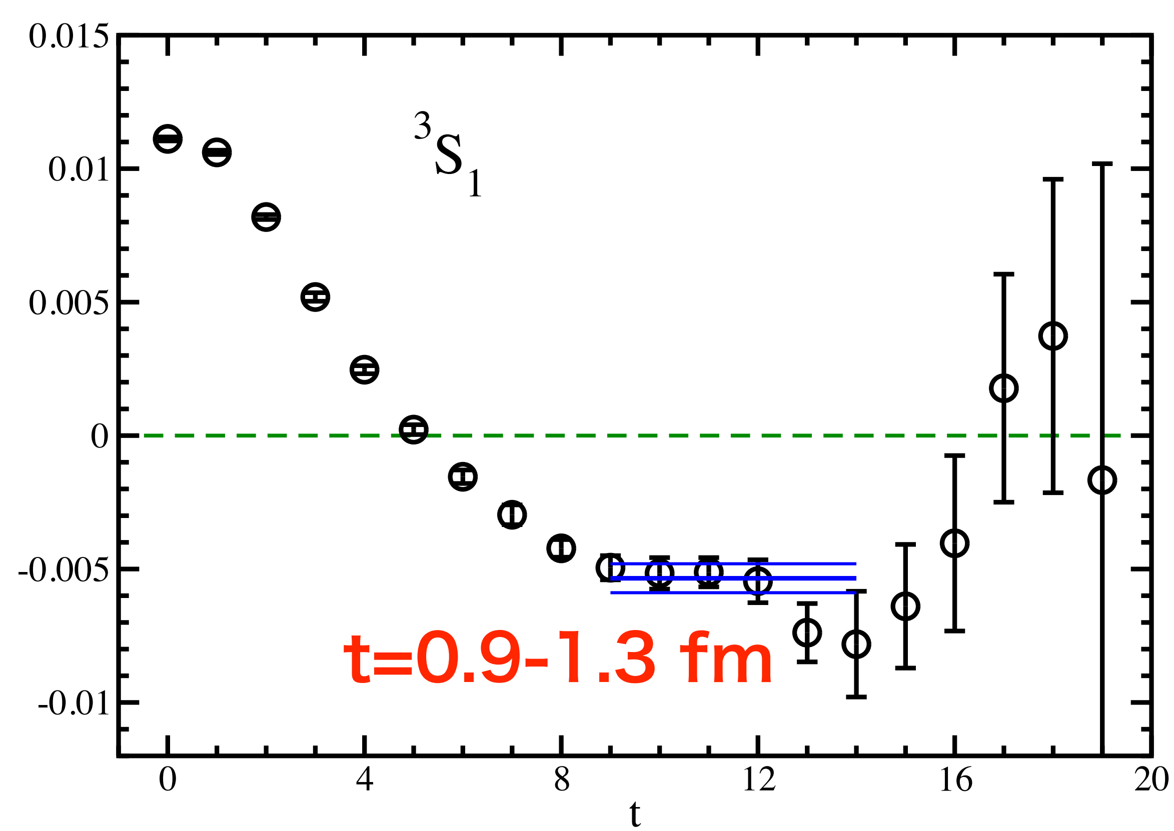
Effective energy shift

$$\Delta E(t) = \frac{1}{a} \log \frac{R(t)}{R(t+a)} \longrightarrow \Delta E, \qquad t \to \infty$$

Plateau method

We identify $\Delta E(t)$ as ΔE, if it becomes constant.

YIKU 2012: PRD86(2012)074514



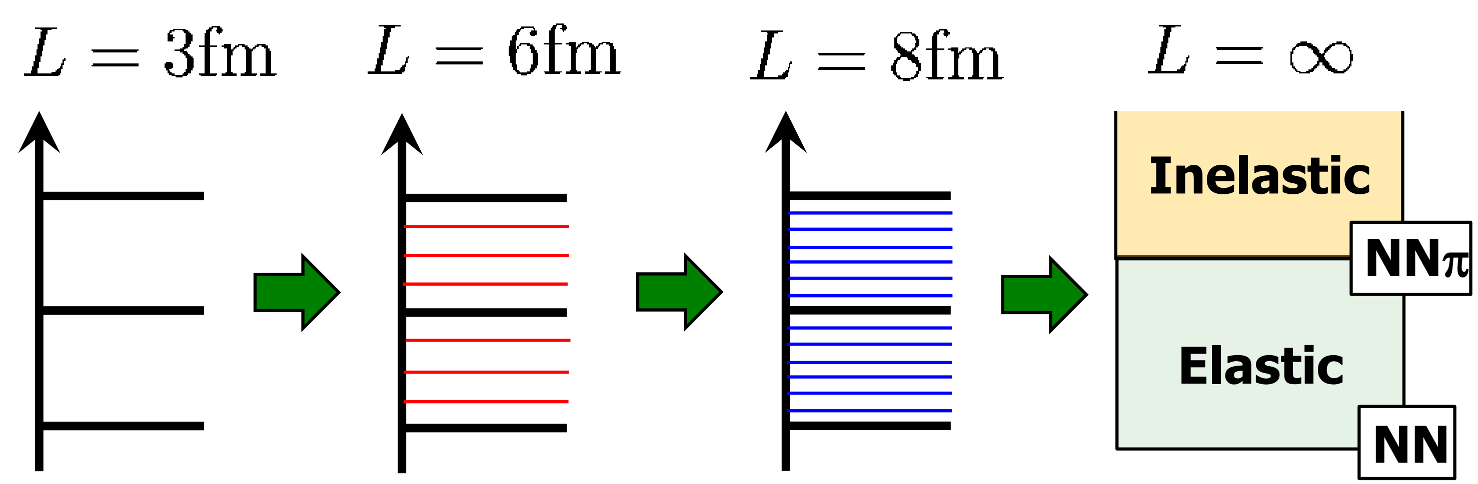
Is the plateau method reliable ?

Excitation energy $E_1 - E_0$

binding energy: very small

finite volume effect for scattering state $\simeq \dfrac{1}{m_N}\dfrac{(2\pi)^2}{L^2}$

$E_1 - E_0 \simeq 50$ MeV at $L = 4$ fm



$t \gg 1/(E_1 - E_0) \simeq 4$ fm is needed to suppress excited states.

Observing the plateau guarantees the ground state saturation even when $t \gg 1/(E_1 - E_0)$ is NOT satisfied.

claimed by Y(I)KU('11,'12,'15), NPL('12,'13,'15), CalLat('15)

Examination of the statement

Mock-up data @ $m_\pi = 0.5$ GeV, $L = 4$ fm (setup of YIKU2012)

$$R(t) = e^{-\Delta E t}\left(1 + b\, e^{-\delta E_{\rm el} t} + c\, e^{-\delta E_{\rm inel} t}\right)$$

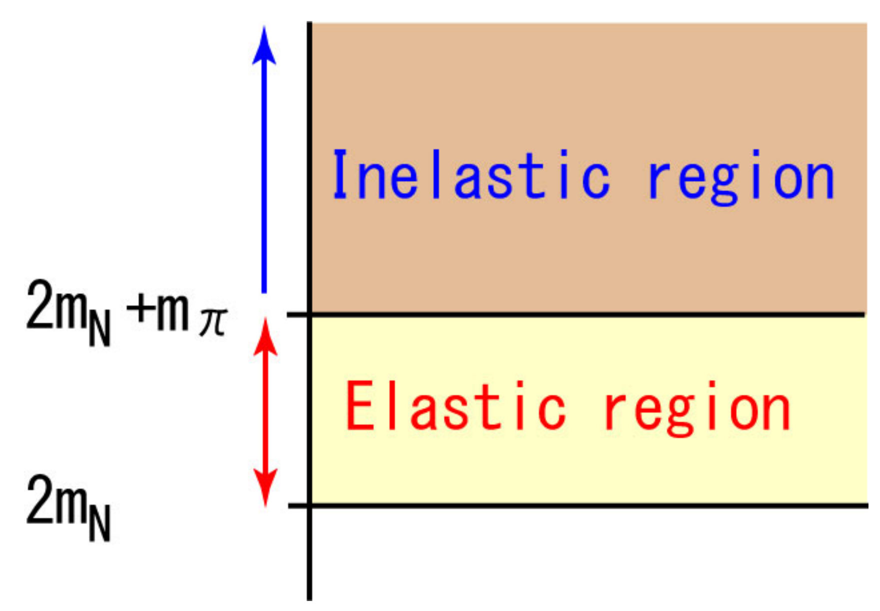
$\delta E_{\rm el} \propto \frac{1}{L^2}$ the lowest excitation energy of elastic scattering state

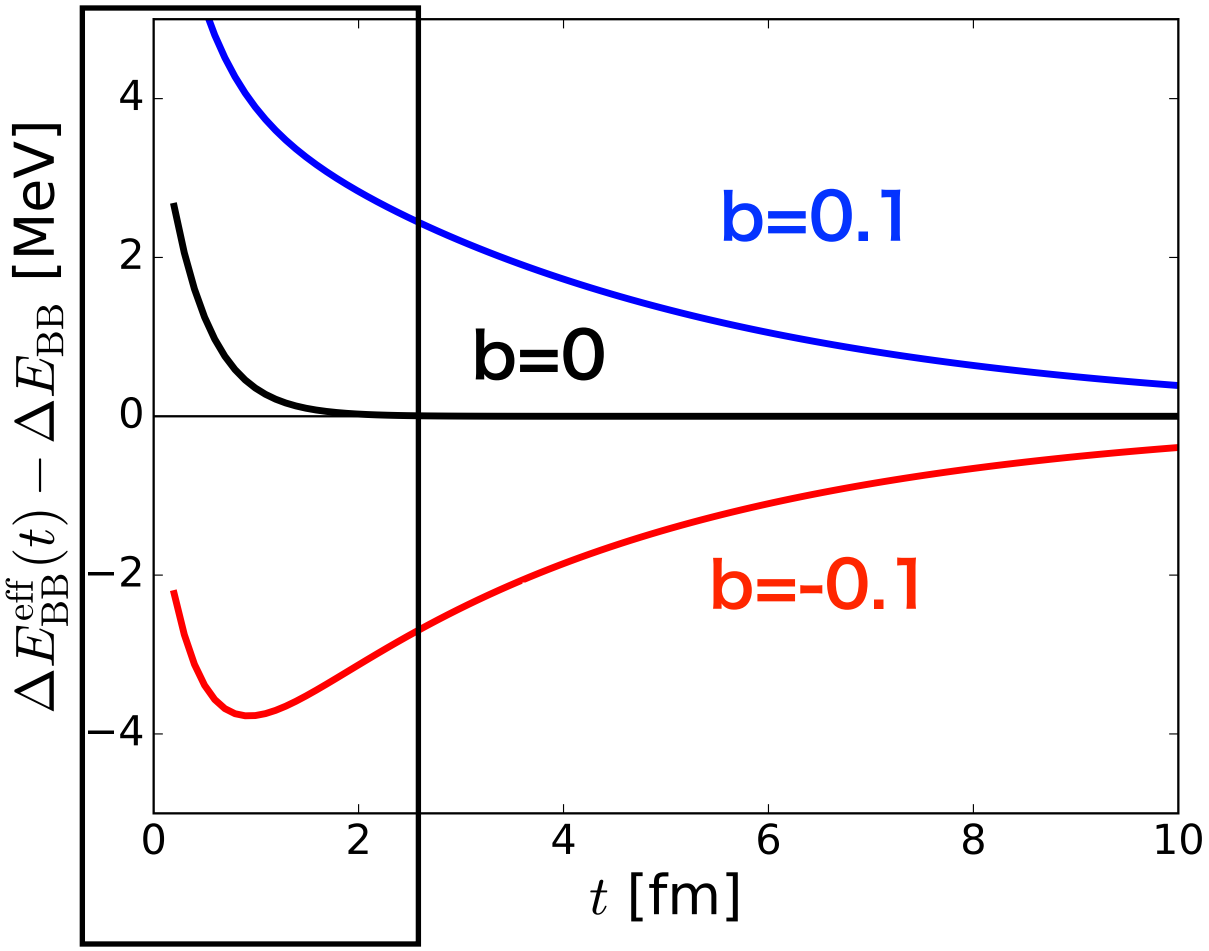
$\delta E_{\rm el} = 50$ MeV at $L \simeq 4$ fm

$b = \pm 0.1$ 10 % contamination $b = 0$ for a comparison

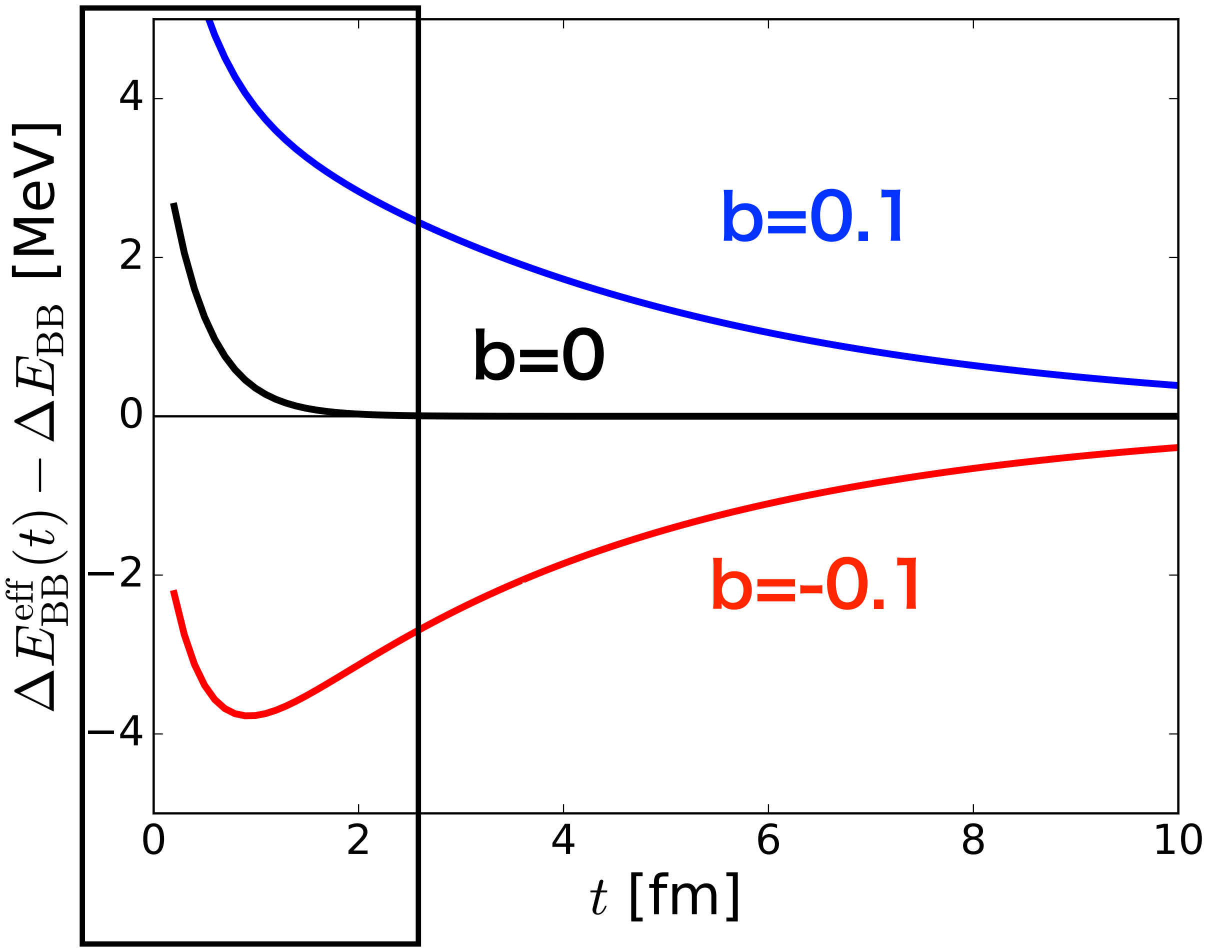
$\delta E_{\rm inel} = 500$ MeV the inelastic energy from heavy pions

$c = 0.01$ 1% contamination

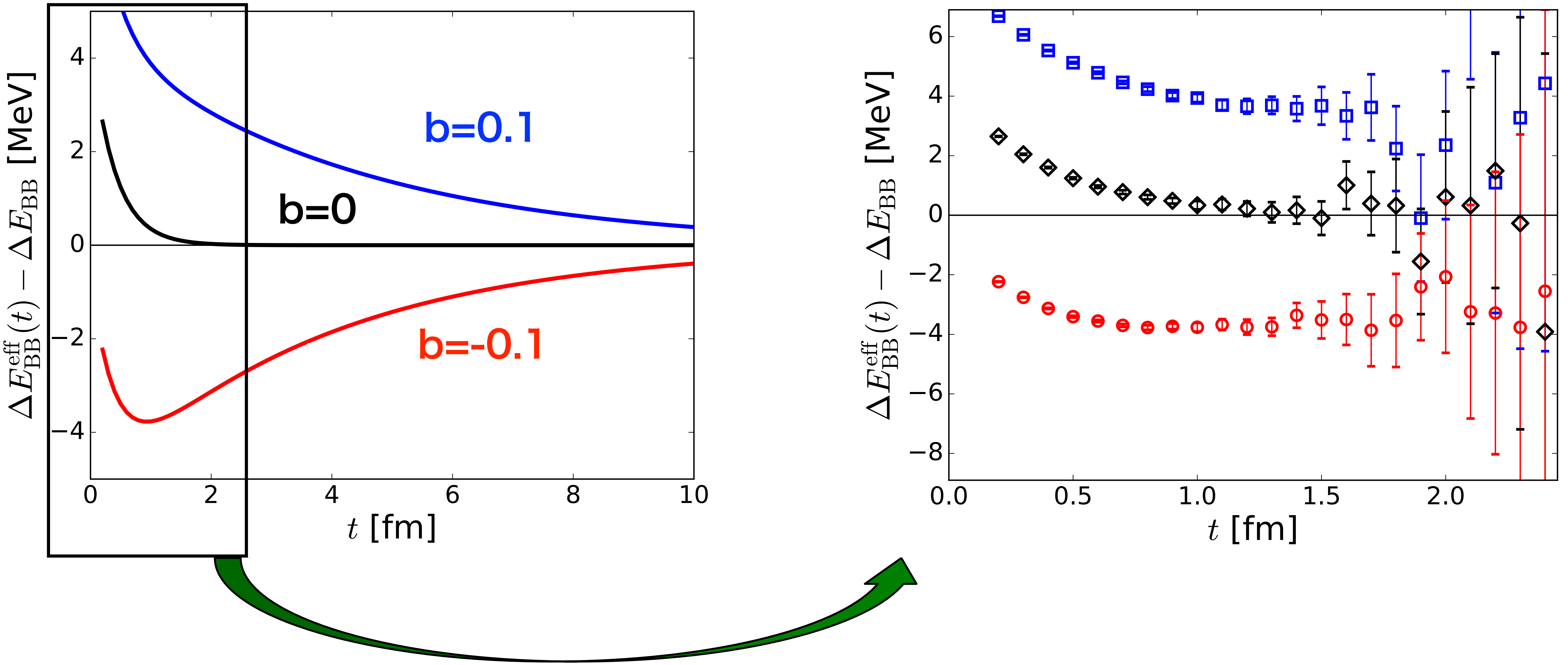




b=0.1
b=0
b=-0.1
$\Delta E^{\mathrm{eff}}_{\mathrm{BB}}(t) - \Delta E_{\mathrm{BB}}$ [MeV]
t [fm]



Zoom + increasing errors and fluctuations



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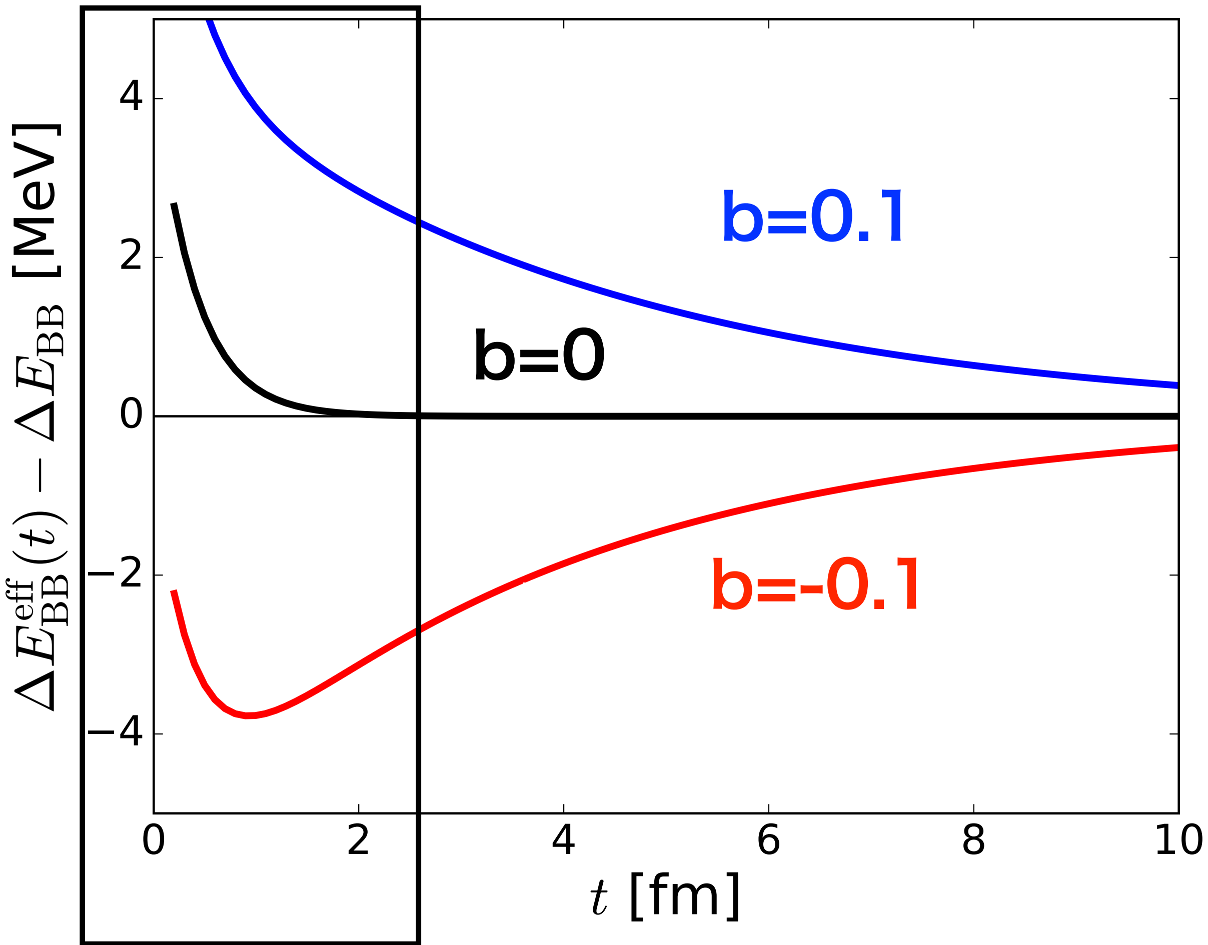
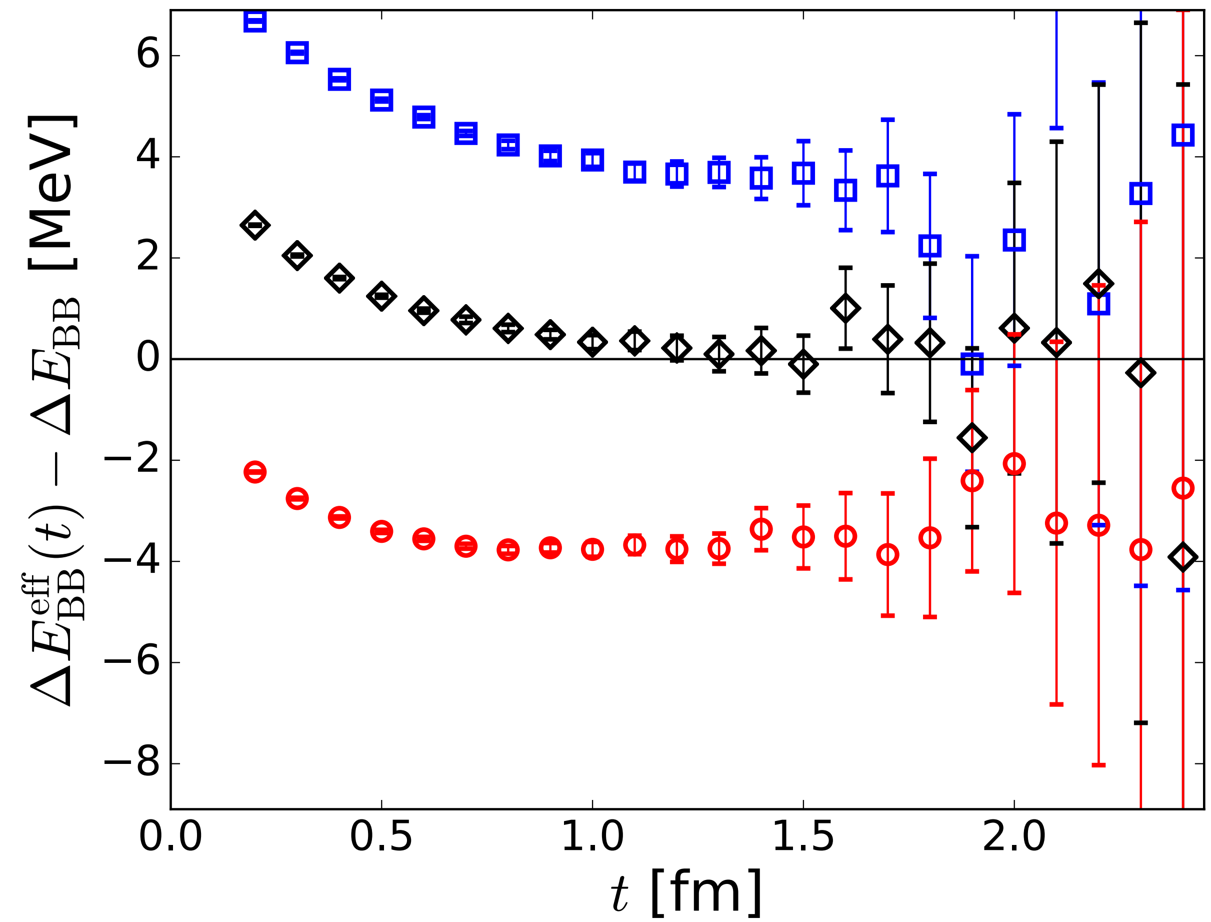
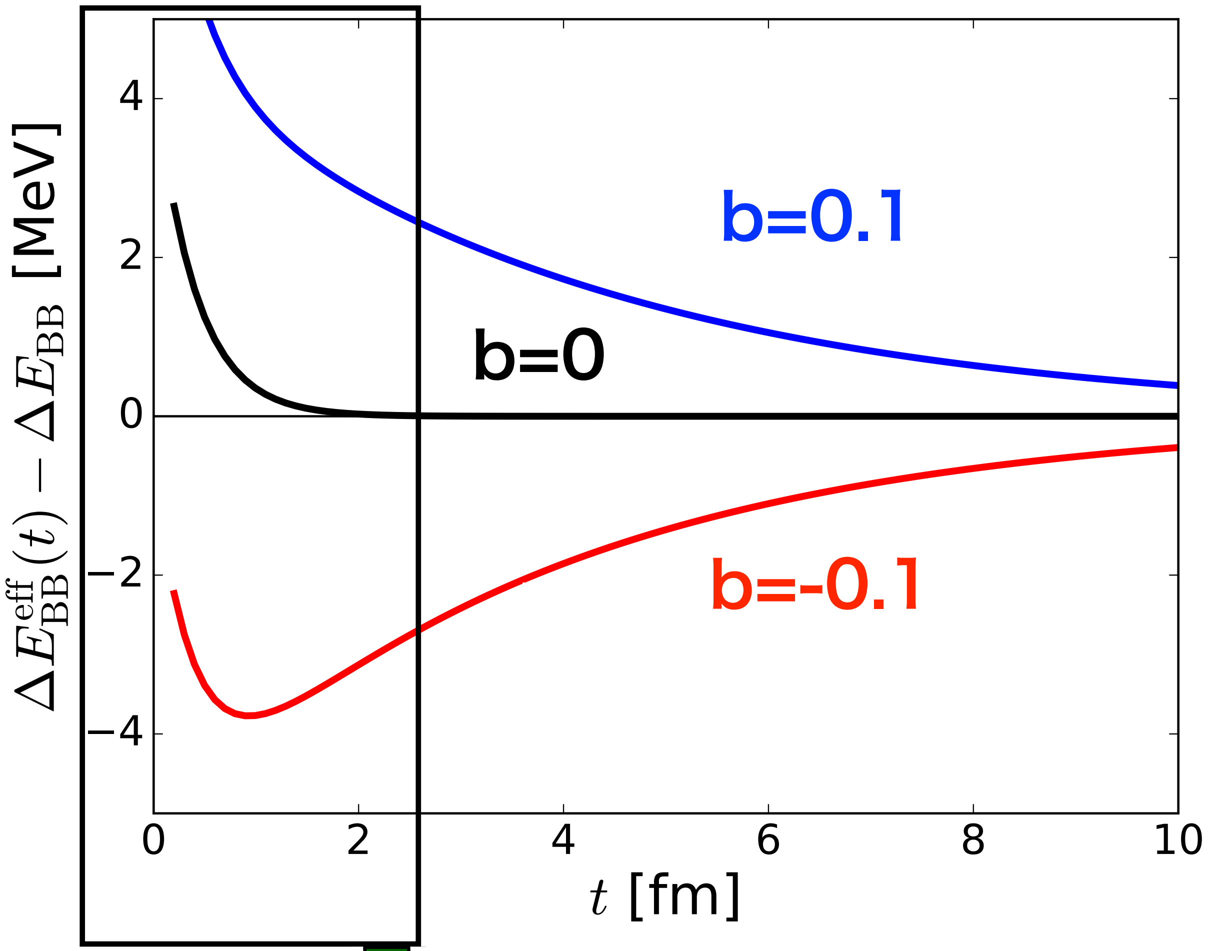
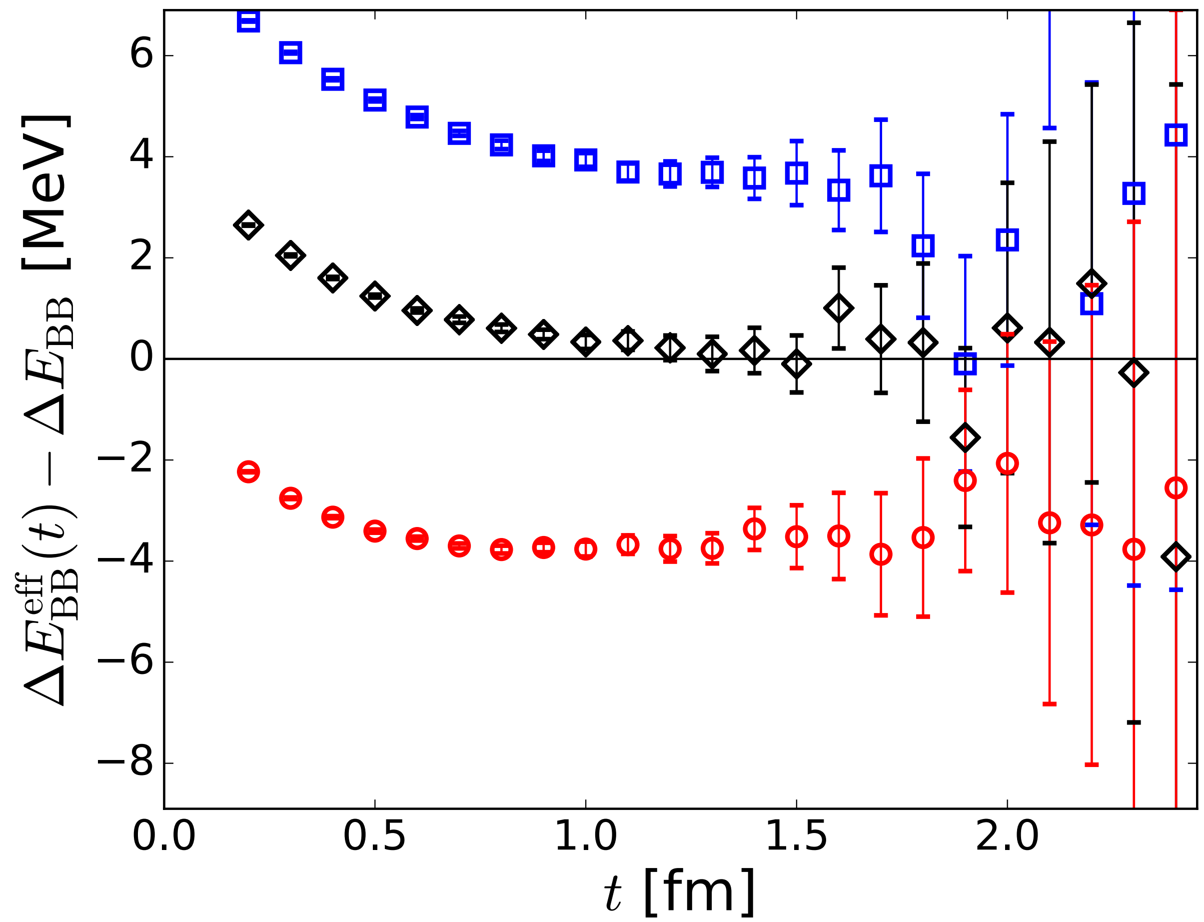




Zoom + increasing errors and fluctuations

“Plateaux” at t ~ 1 fm
but they are fake (Mirrage)





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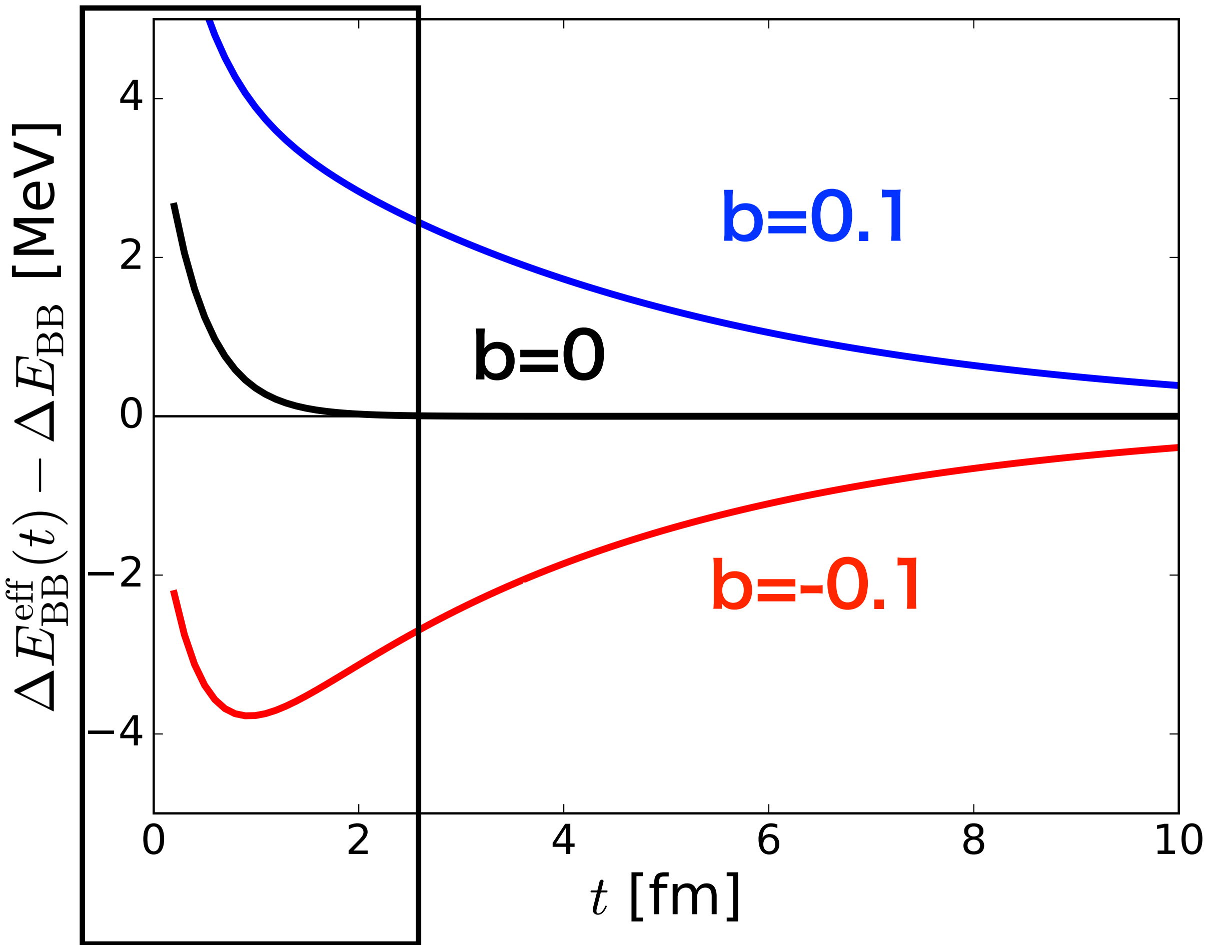
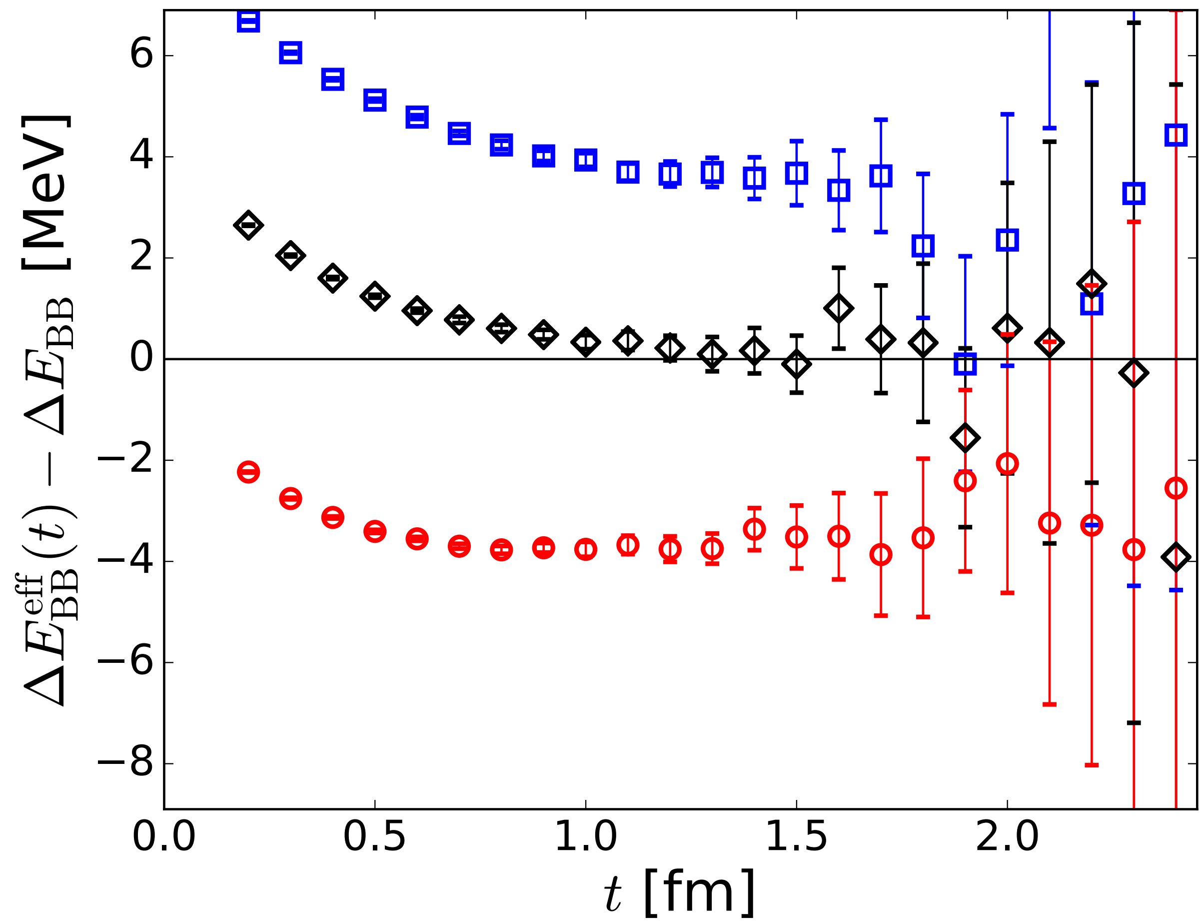
Observing the plateau guarantees the ground state saturation even when $t \gg 1/(E_1 - E_0)$ is NOT satisfied. claimed by Y(I)KU('11,'12,'15), NPL('12,'13,'15), CalLat('15)

It's a Myth !





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It’s a Myth !

The “looking for a plateau at small t” method does not work.

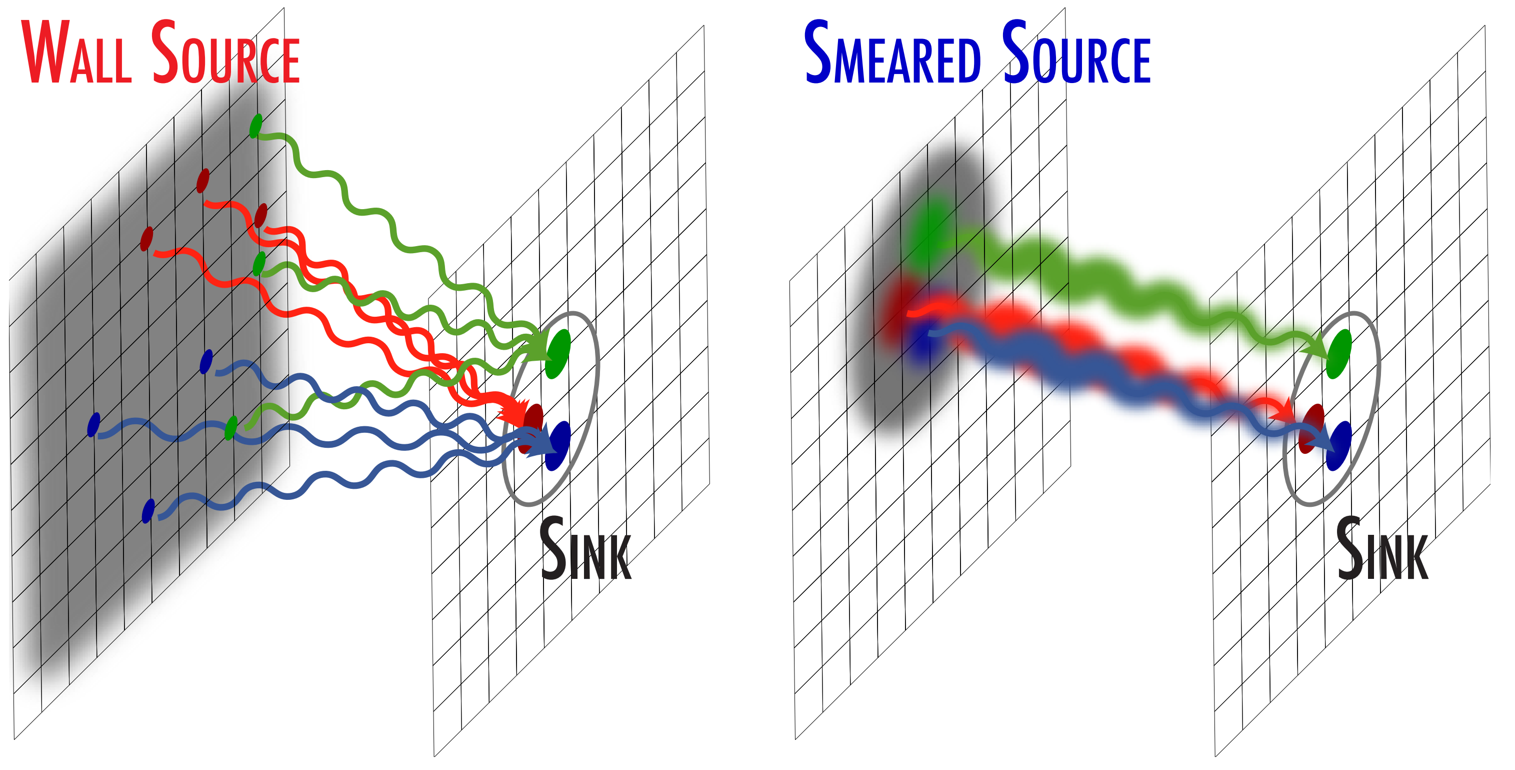
II. Mirage problem
(Operator dependence)

- Manifestation of the problem I -

T. Iritani et al. (HAL QCD), JHEP1610(2016)101 (arXiv:1607.06371)

Source operator dependence of plateaux

quark wall source vs quark smeared source



$$\sum_{\mathbf{y}} q(\mathbf{y}, t_0)$$

$$\sum_{\mathbf{y}} e^{-B|\mathbf{x}_0-\mathbf{y}|} q(\mathbf{y}, t_0)$$

b are different between the two.

Lattice setup 2+1 flavor QCD

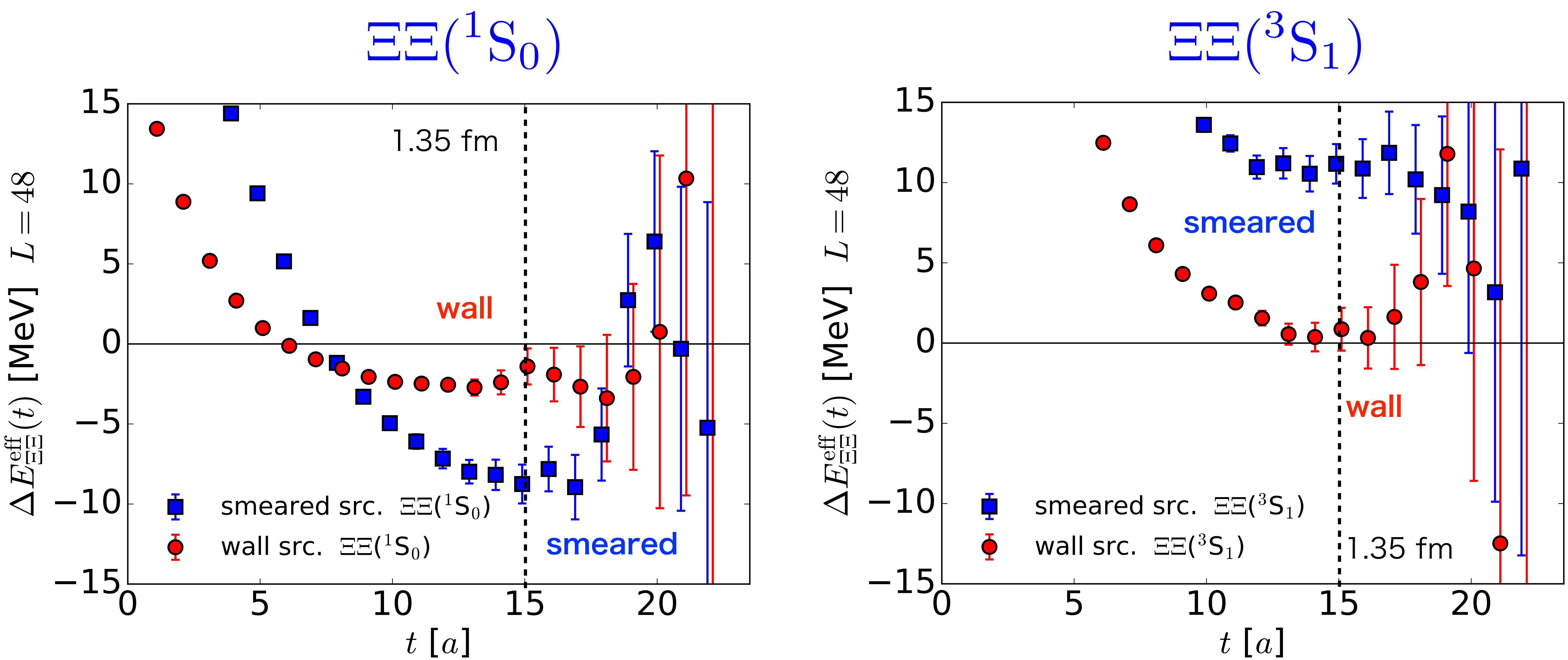
same gauge configurations of YIKU 2012

$$a = 0.09 \text{ fm } (a^{-1} = 2.2 \text{ GeV})$$

$$m_\pi = 0.51 \text{ GeV}, m_N = 1.32 \text{ GeV}, m_K = 0.62 \text{ GeV}, m_\Xi = 1.46 \text{ GeV}$$

Energy shift of ΞΞ

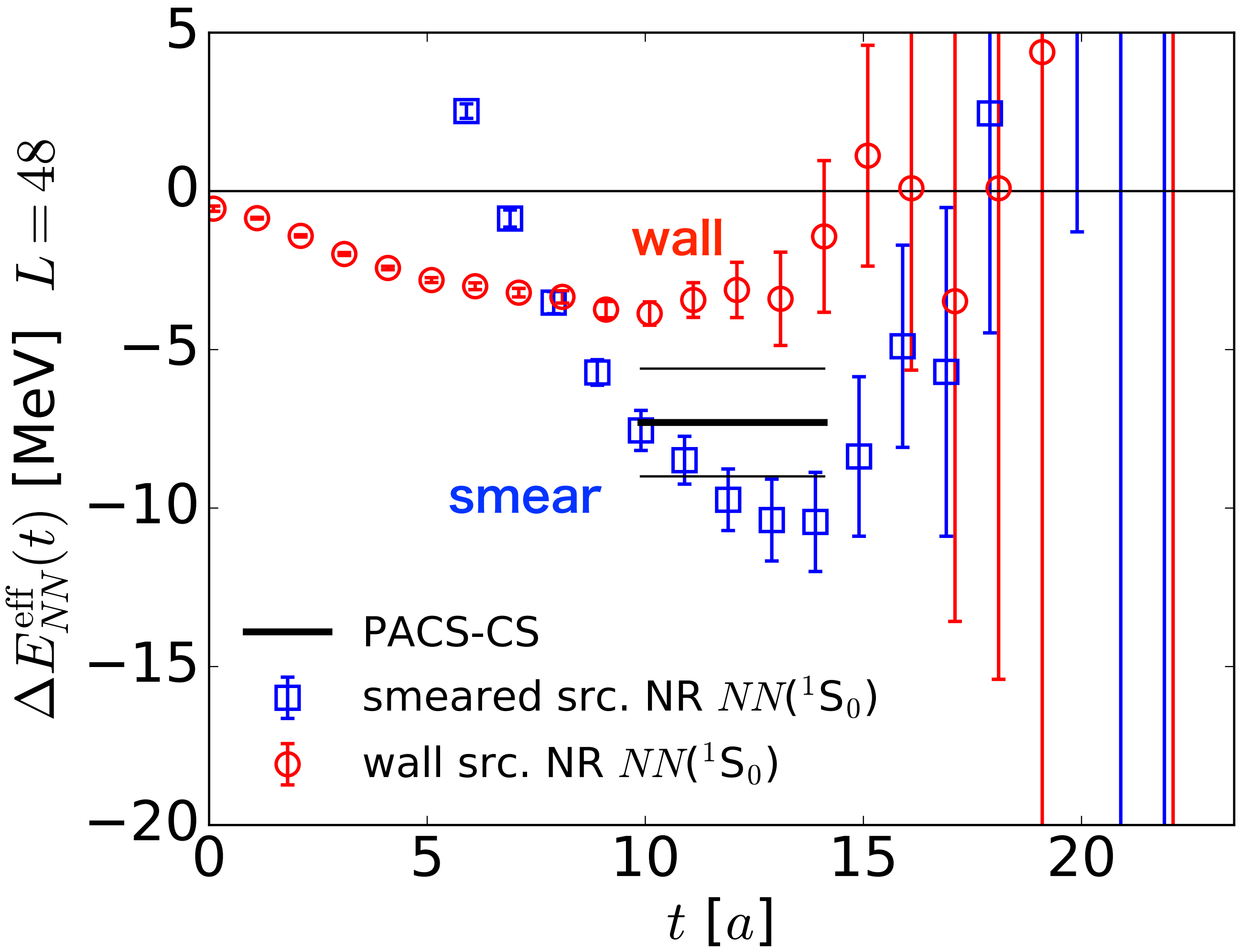
smaller statistical errors



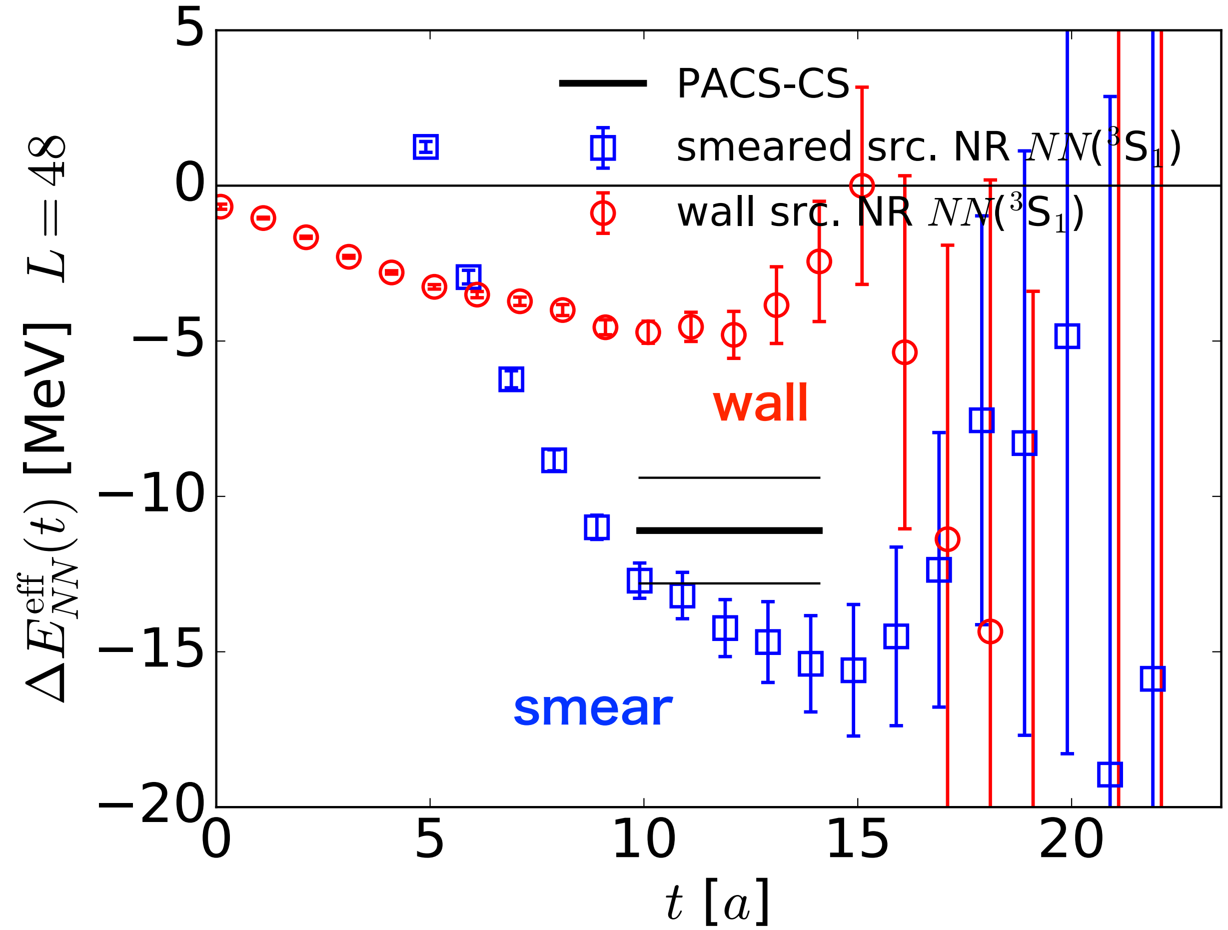
- Not surprisingly, two sources disagree.
- The potential danger becomes reality.
- Plateau-like structures around t=1-1.5 fm are by no means trustable.
- Both might agree at t > 18a, but errors are too large.

Same problem also appears for NN

$NN(^1\mathrm{S}_0)$



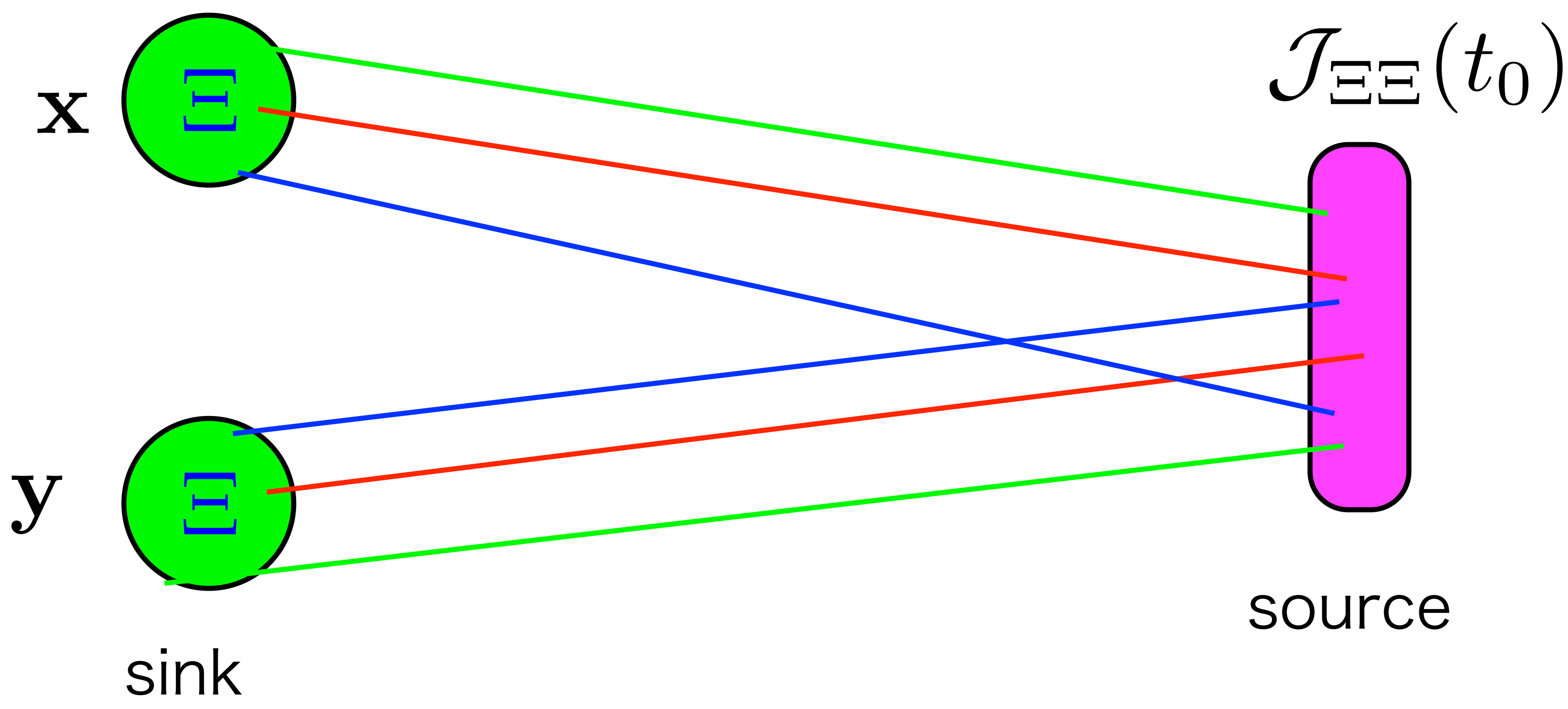
$NN(^3\mathrm{S}_1)$



With larger errors, disagreement also exists.

In addition, we may have

Sink 2-baryon operator dependence of plateaux



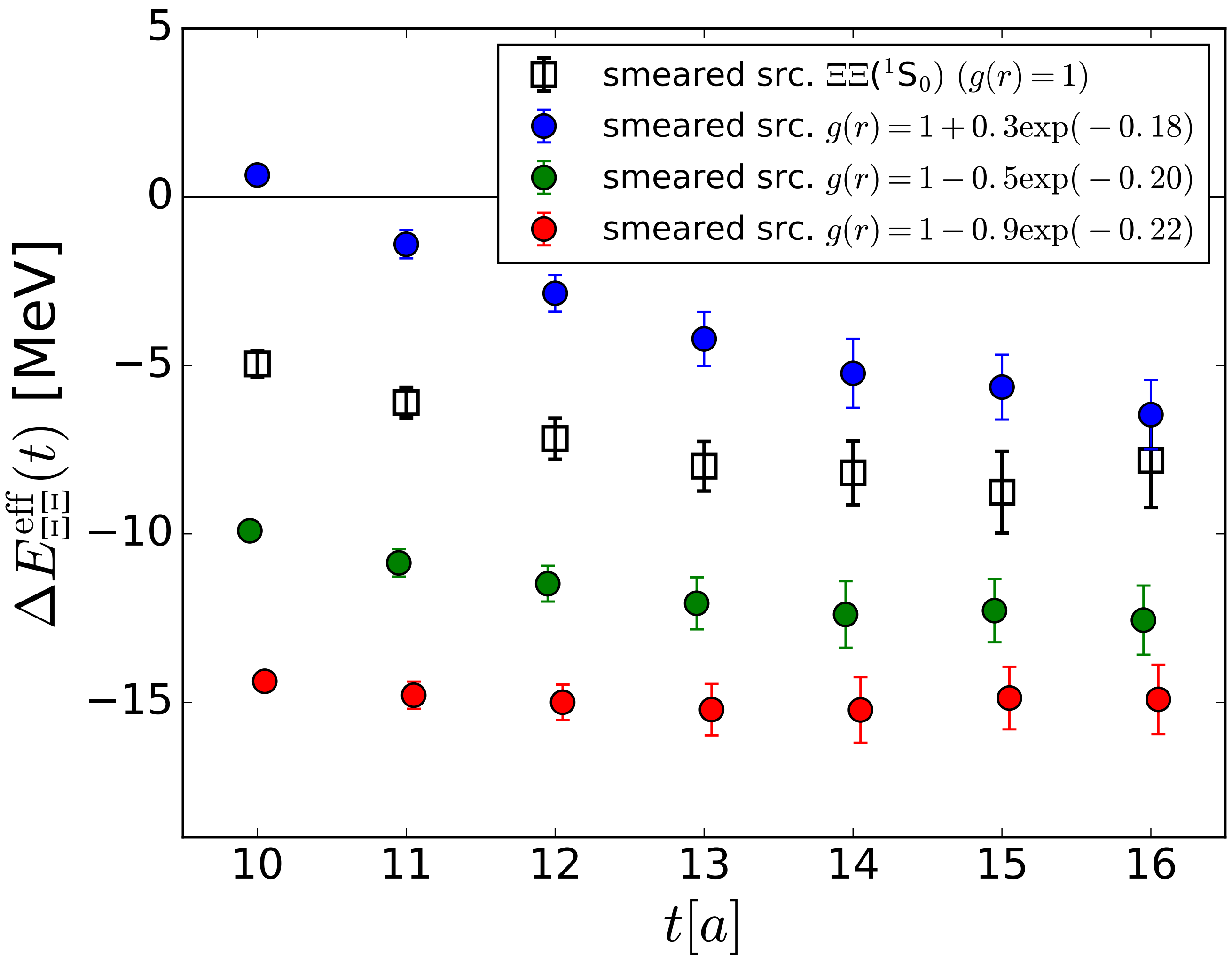
$$G_{\Xi\Xi}(t) = \sum_{\mathbf{x},\mathbf{y}} g(|\mathbf{x}-\mathbf{y}|)\langle \Xi(\mathbf{x},t)\Xi(\mathbf{y},t)\mathcal{J}_{\Xi\Xi}(t_0)\rangle$$

$$g(r) = 1 \ : \ \text{standrad sink operator}$$

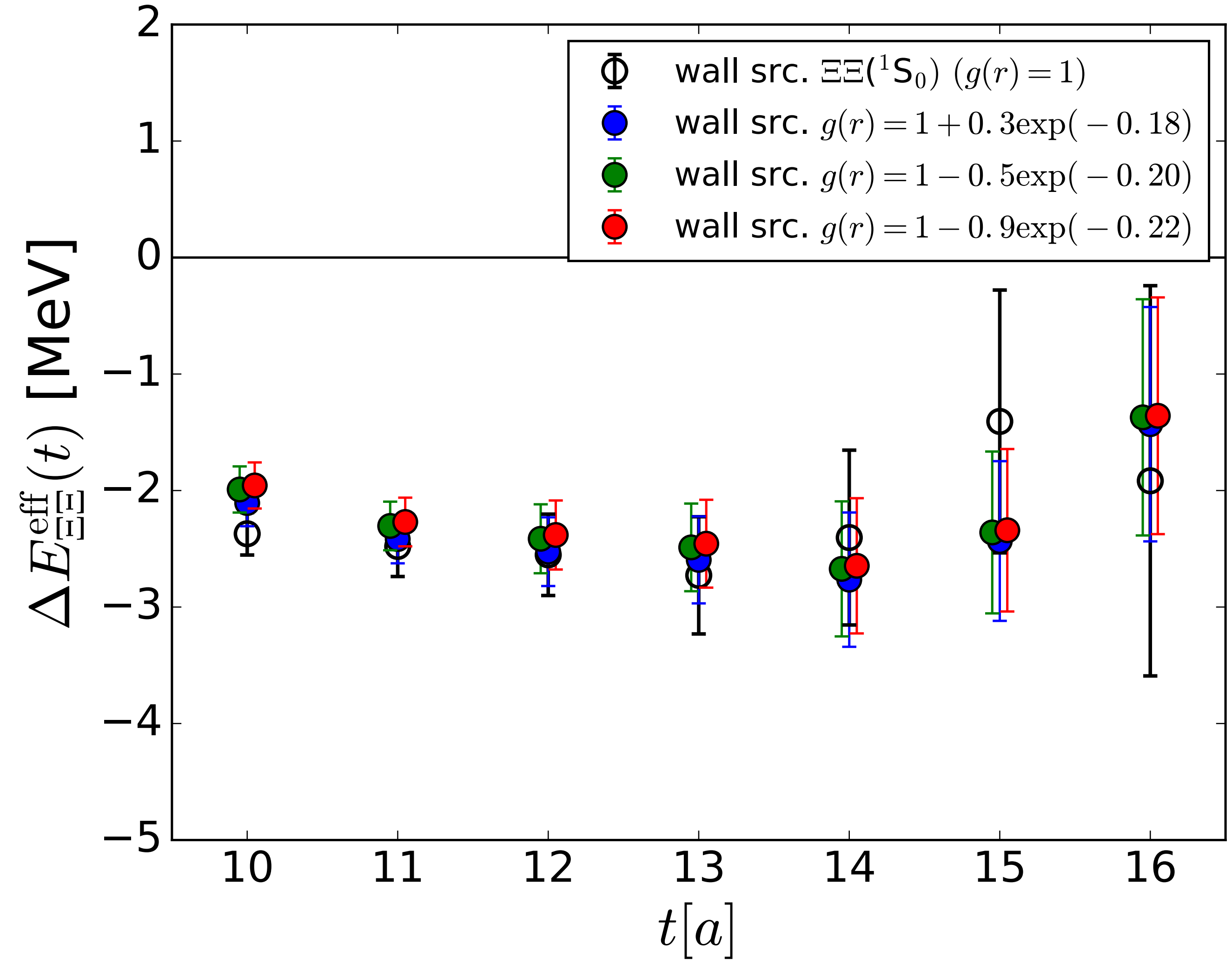
$$g(r) = 1 + A\exp(-Br) \ : \ \text{generalized sink operator}$$

The true plateau must NOT dependent on g(r).

Smeared source



Wall source



- smeared source is very sensitive to g(r).
  - Sometimes deeper and more stable.
  - one can produce an arbitrary value (within a certain range) by g(r).
- Wall source is insensitive to g(r).

- Dangers of fake plateaux exit in principle for the direct method.

- Problem becomes manifest in the strong source/sink operator dependences of plateau values in YIKU 2012.

- Are there any symptoms in other results ?

    - Study of source dependences requires additional simulations.

    - need simpler and easier check

III. Sanity check

- Manifestation of the problem II -

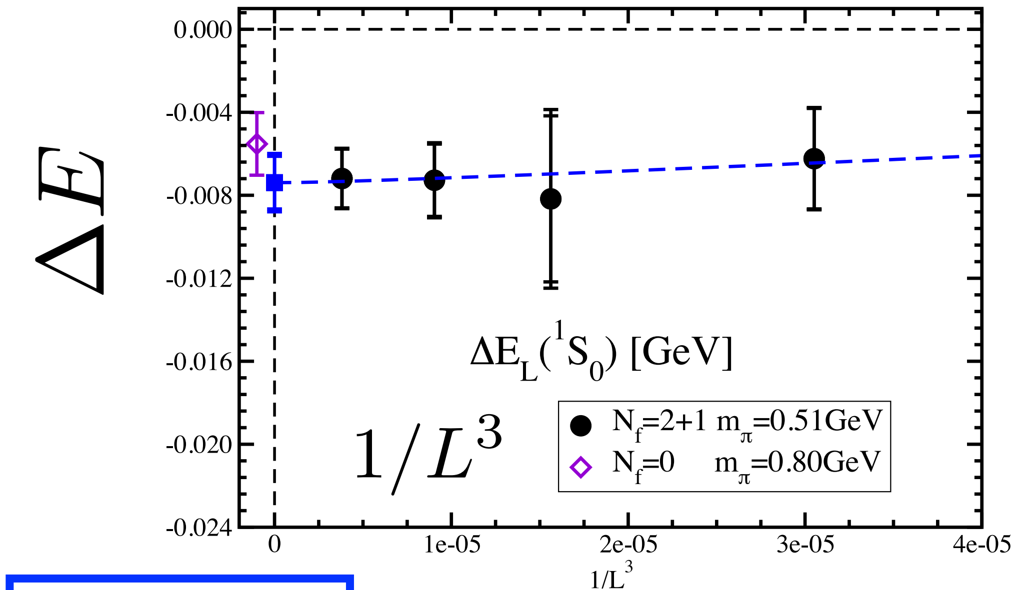
S. Aoki, T. Doi, T. Iritani, PoS(Lattice2016) 109 (aiXiv:1610:09763)

Finite volume formula

Lüscher, NPB354(1991)531

Direct method

YIKU2012

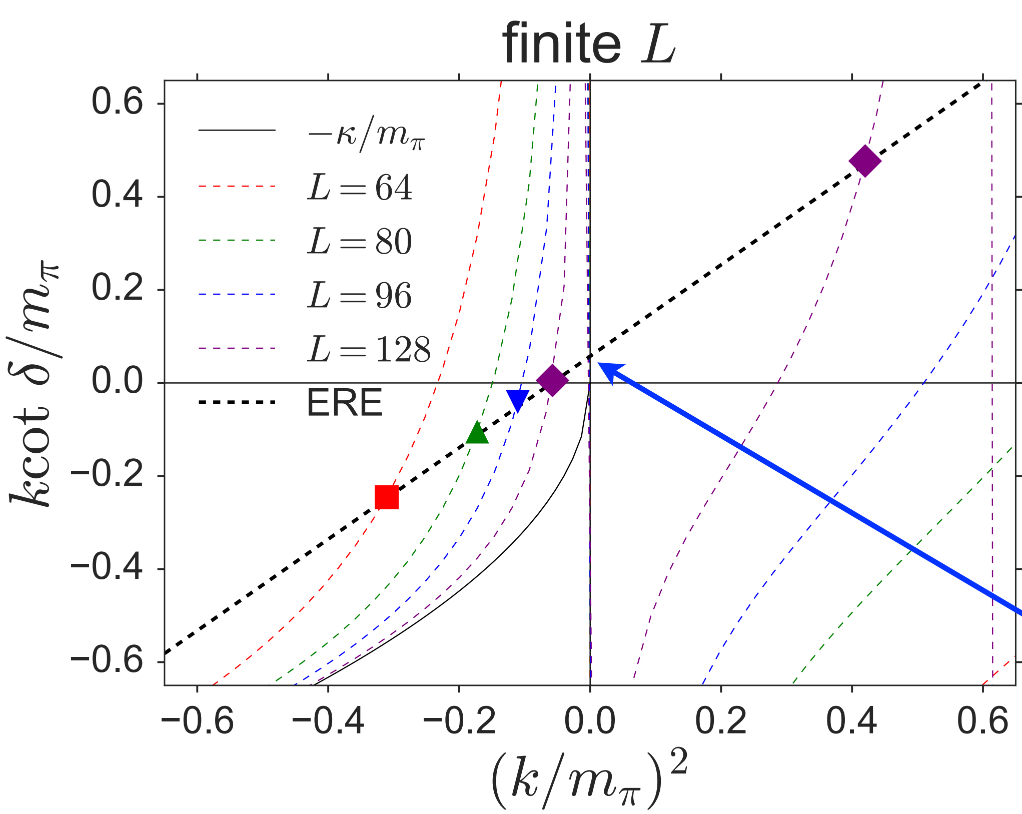


$$\Delta E = 2\sqrt{k^2 + m_N^2} - 2m_N, \quad q = \frac{kL}{2\pi}$$

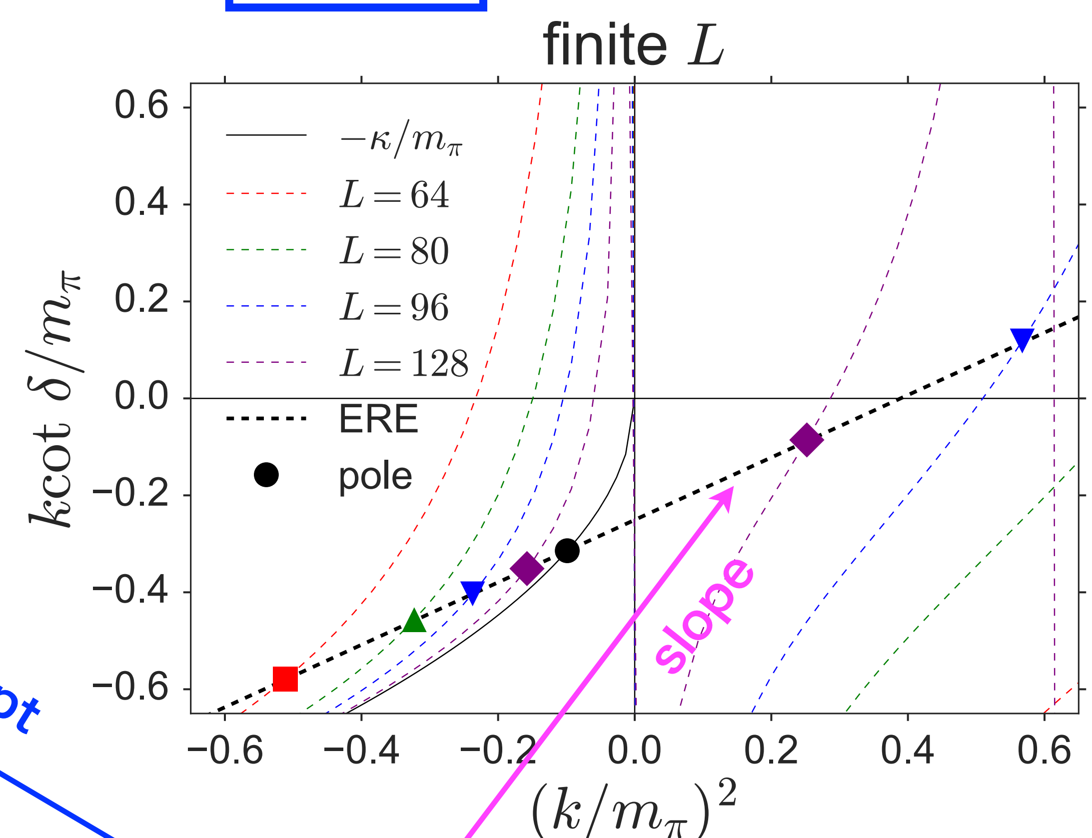
$$\boxed{k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\vec{n}^2 - q^2}}$$

$\delta(k)$:scattering phase shift

unbound



bound



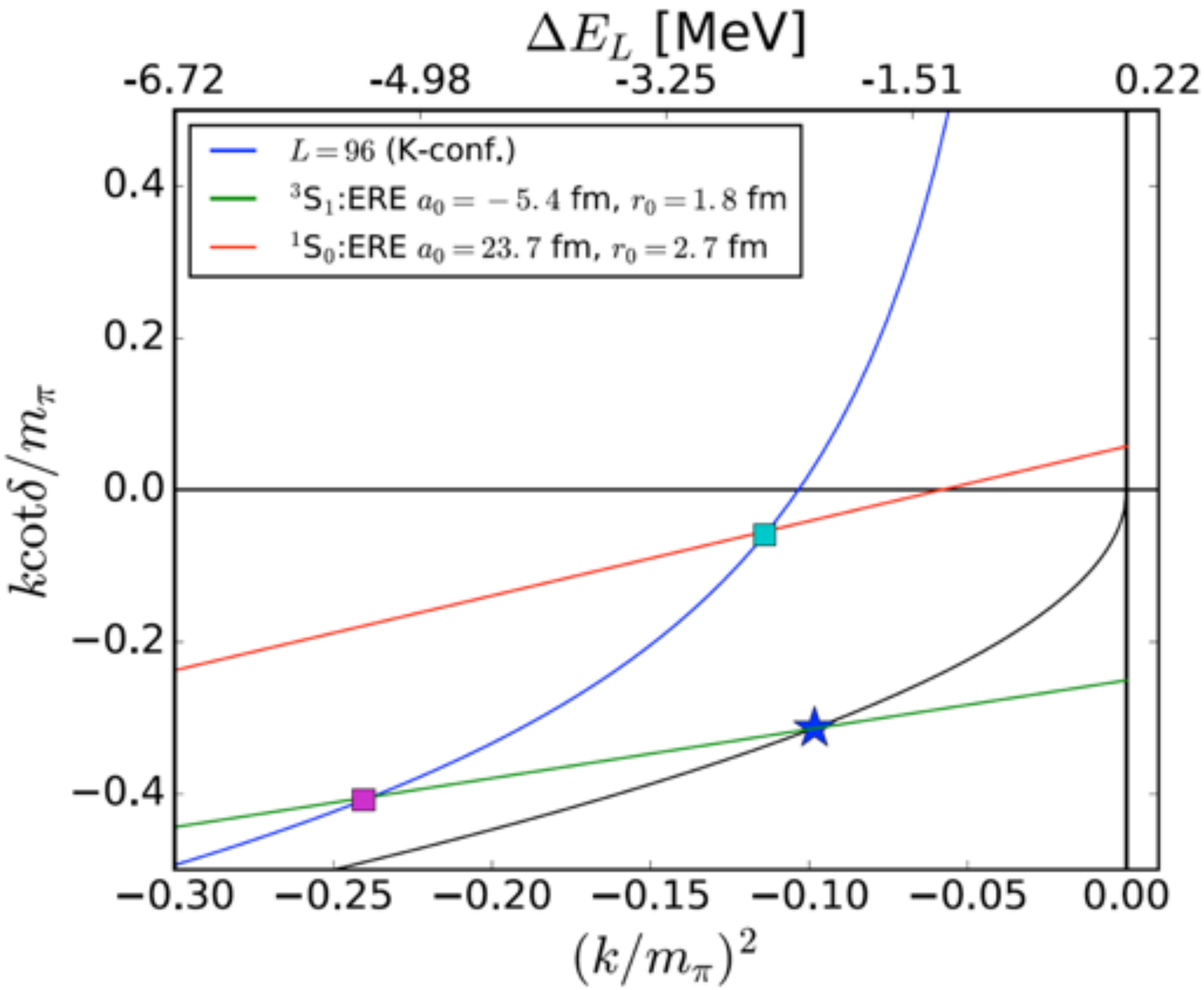
intercept

slope

Effective Range Expansion (ERE)

$$k \cot \delta(k) = \frac{1}{a} + \frac{1}{2} r k^2 + \cdots$$

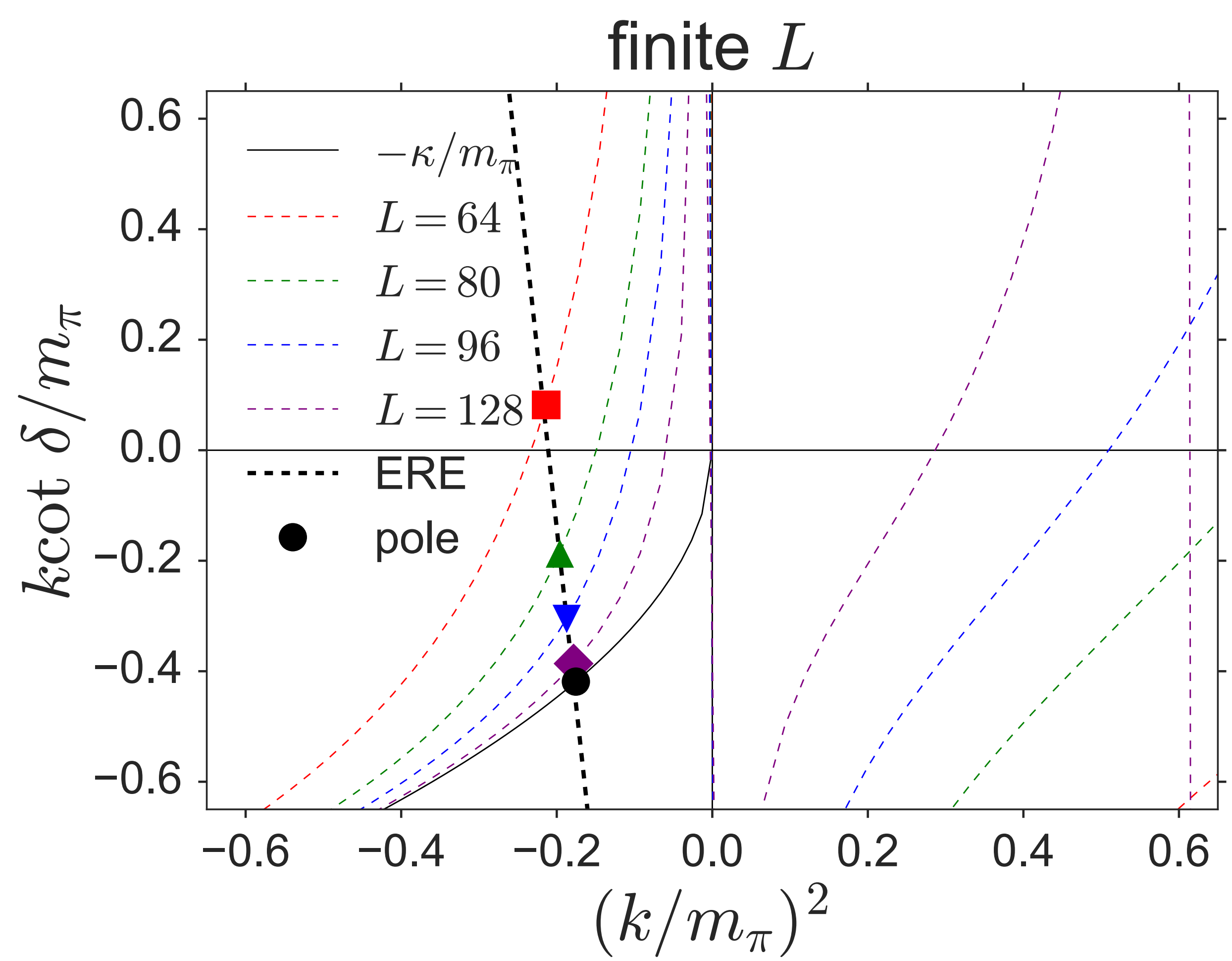
ERE at physical pion mass



Instead, a behavior shown below indicates the problem in lattice QCD data.

$$1/a \simeq -\infty, \quad r \simeq -\infty$$

"Sanity Check"



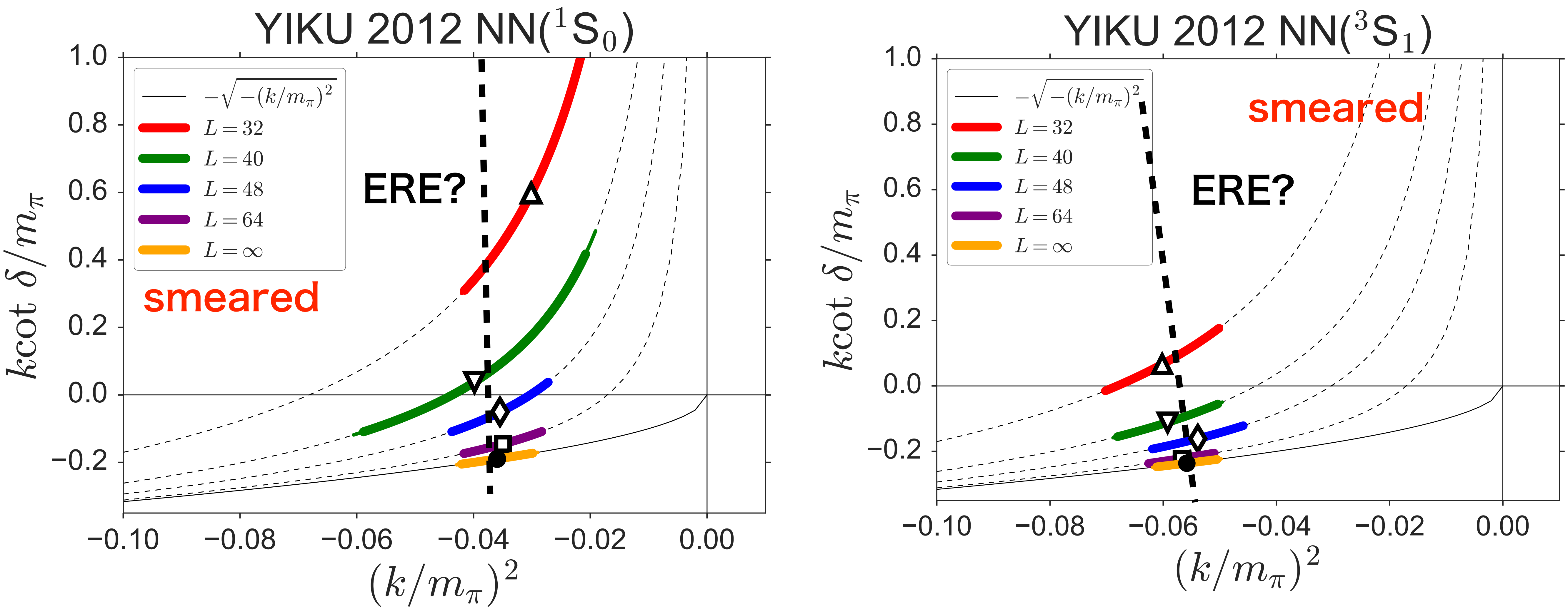
YIKU2012

Yamazaki et al. PRD86(2012)074514

$m_\pi = 0.51$ GeV, $L = 2.9 - 5.8$ fm

$\Delta E_{NN}(^1\mathrm{S}_0) = -7.4(1.3)(0.6)$ MeV

$\Delta E_{NN}(^3\mathrm{S}_1) = -11.5(1.1)(0.6)$ MeV



singular behaviors

ΔE is almost independent on L, while it is shallow bound state.

"Not Sanity"

IV. Conclusion

The direct method gives no reliable result for two(or more)-baryon systems so far, since systematic errors due to contaminations from excited (elastic) states are not under control.

Do not be misled.

Check Table for NN

	single baryon	double baryon				
	plateau check	mirage plateau	src-dep check	sink-dep check	Effective Range expansion check	**Overall Verdict**
YKU 2011	○	×	△	**Not checked**	×	**False**
YIKU 2012	○	×	×	×	×	**False**
YIKU 2015	○	×	**Not checked**	**Not checked**	×	**False**
NPL 2012	○	×	**Not checked**	**Not checked**	×	**False**
NPL 2013	○	×	**Not checked**	**Not checked**	△	**False**
NPL 2015	△	×	**Not checked**	**Not checked**	×	**False**

HALQCD potential method ? → T. Doi’s talk on Nov. 23

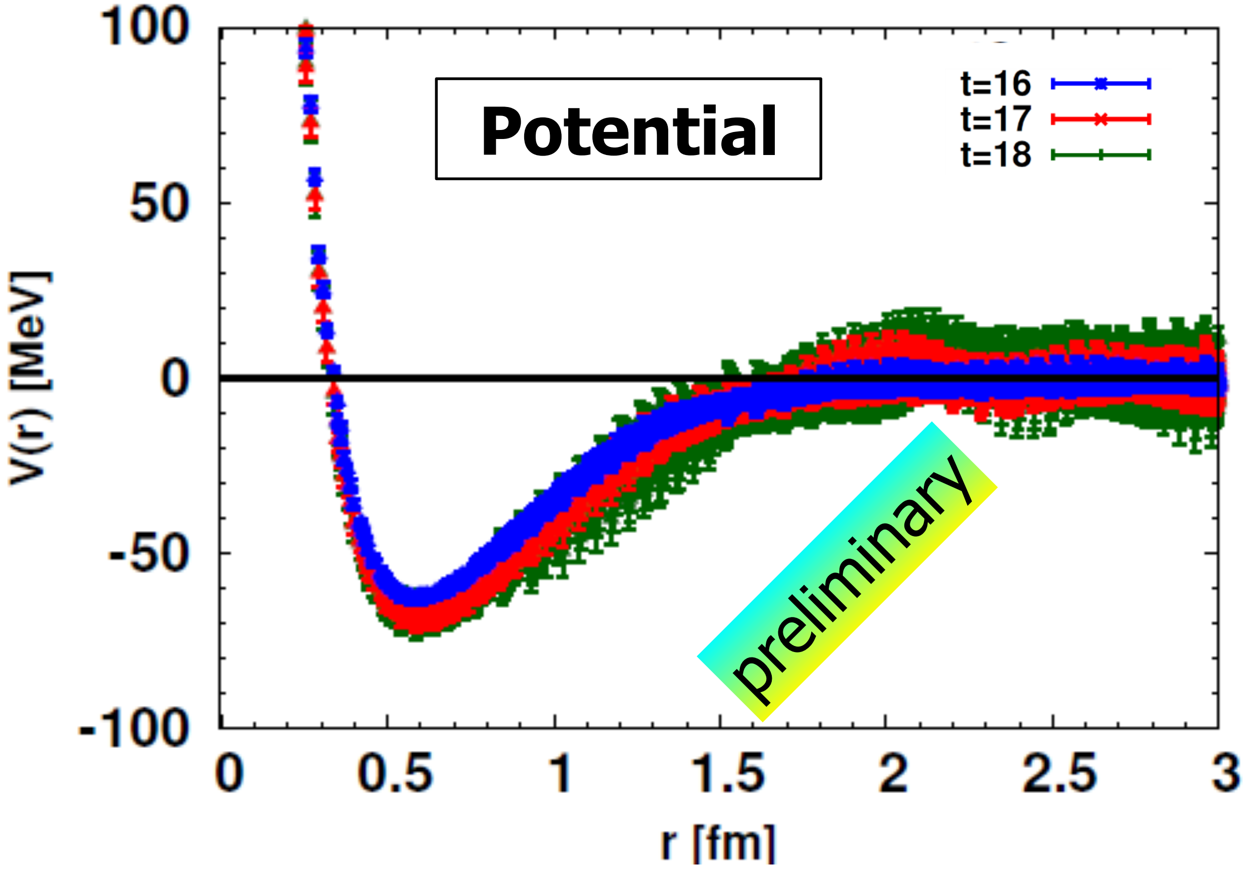
Potentials at physical pion

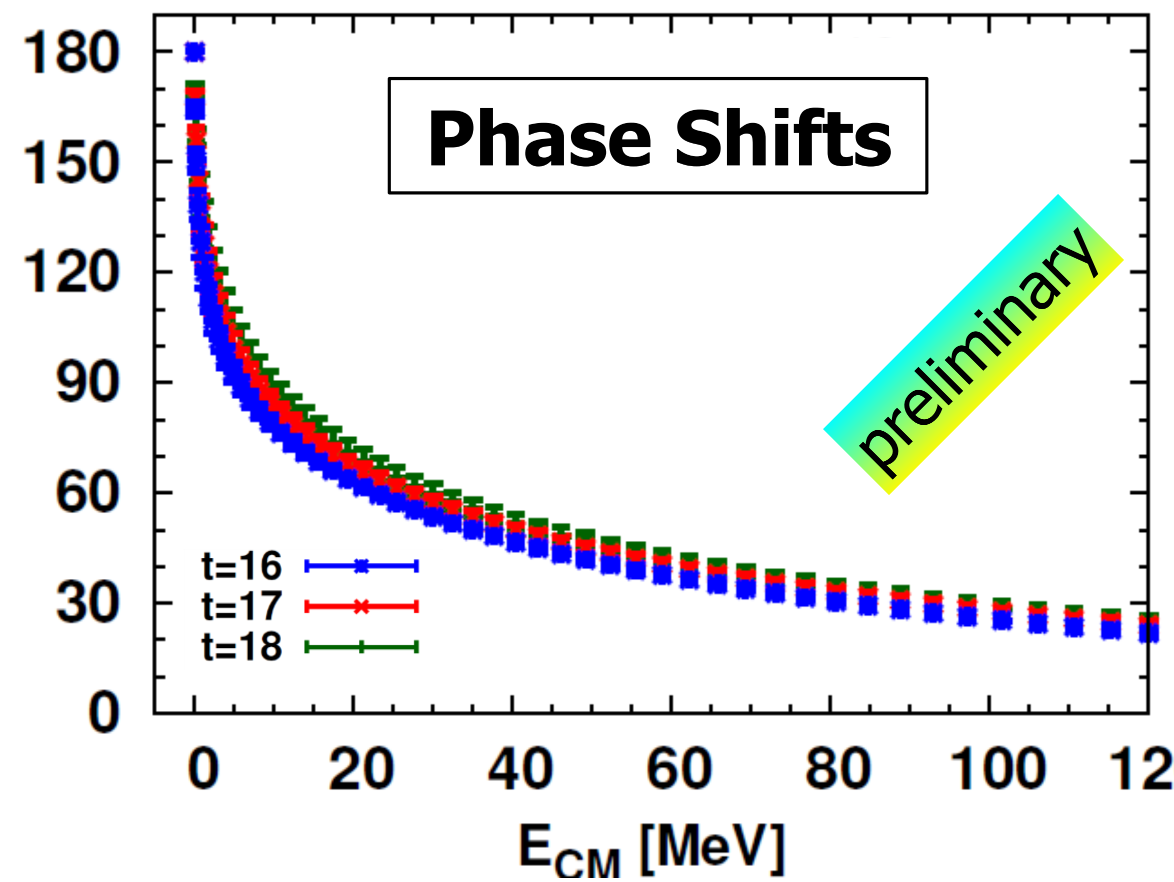
S. Gongyo

K-computer [10PFlops]

2+1 flavor QCD, $m_\pi \simeq 145$ MeV, $a \simeq 0.085$ fm, $L \simeq 8$ fm

$\Omega\Omega$ potential



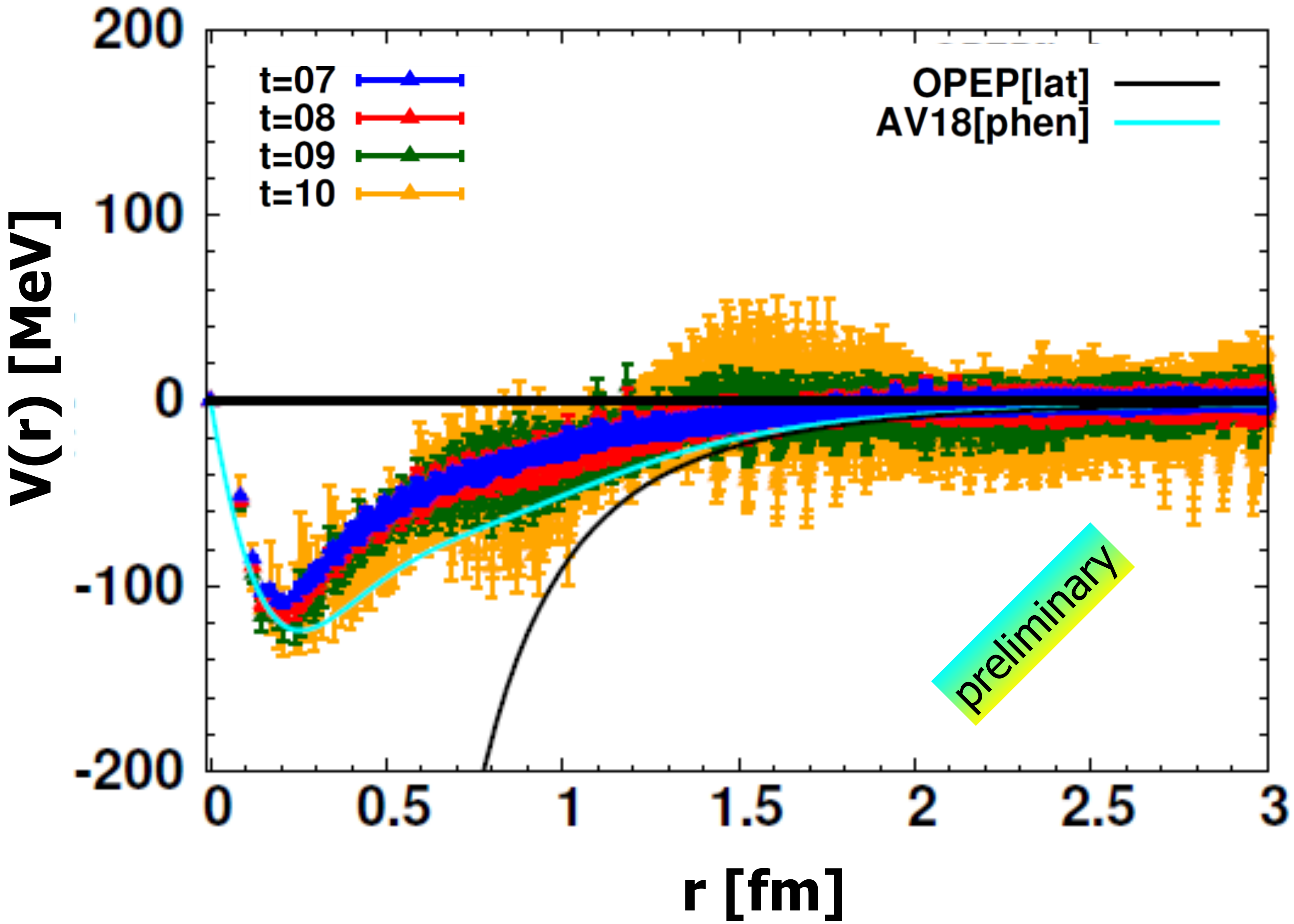


Strong attraction → Vicinity of bound/unbound (~ unitary limit)

The most strange dibaryon ?

$NN(^3S_1)$ tensor potential

T. Doi



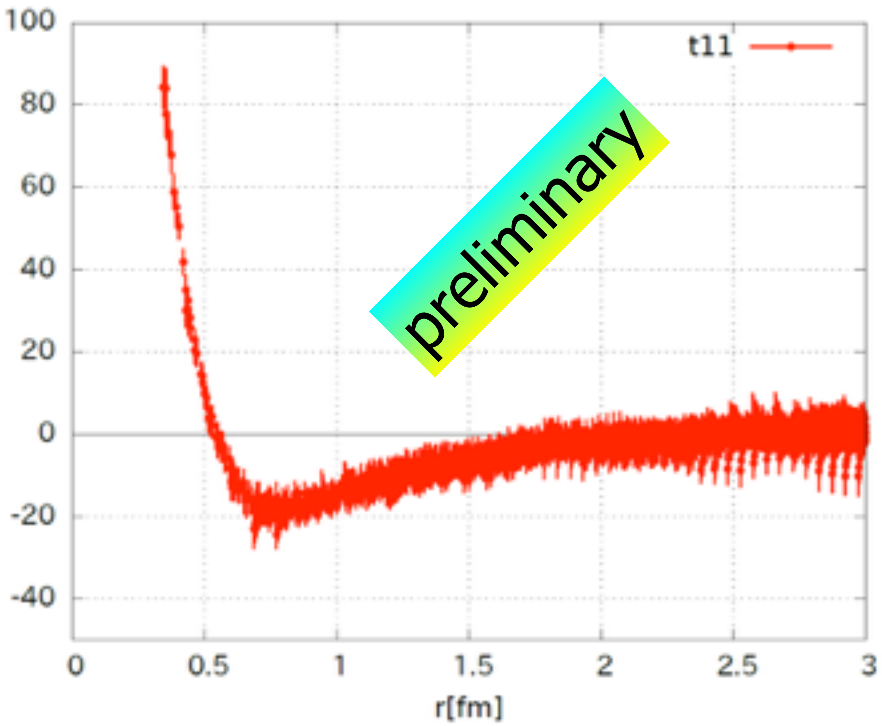
Qualitatively similar tail to one pion exchange potential (OPEP)

reduction of errors is definitely needed.

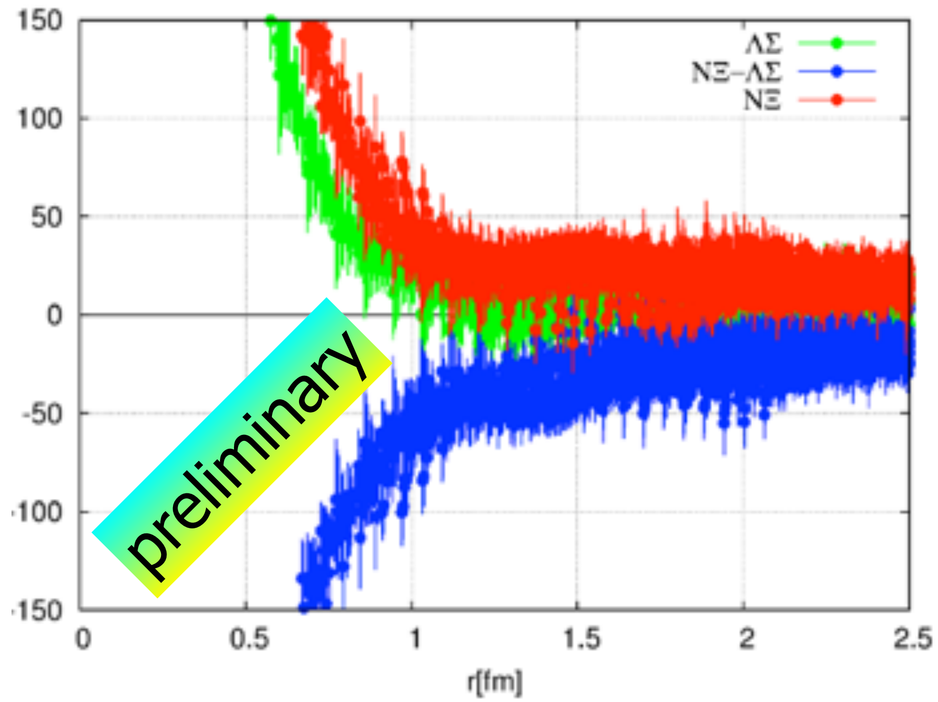
$N\Xi$ potentials

K. Sasaki

$N\Xi(I=0,{}^3\mathrm{S}_1)$



$N\Xi-\Lambda\Sigma(I=1,{}^1\mathrm{S}_0)$



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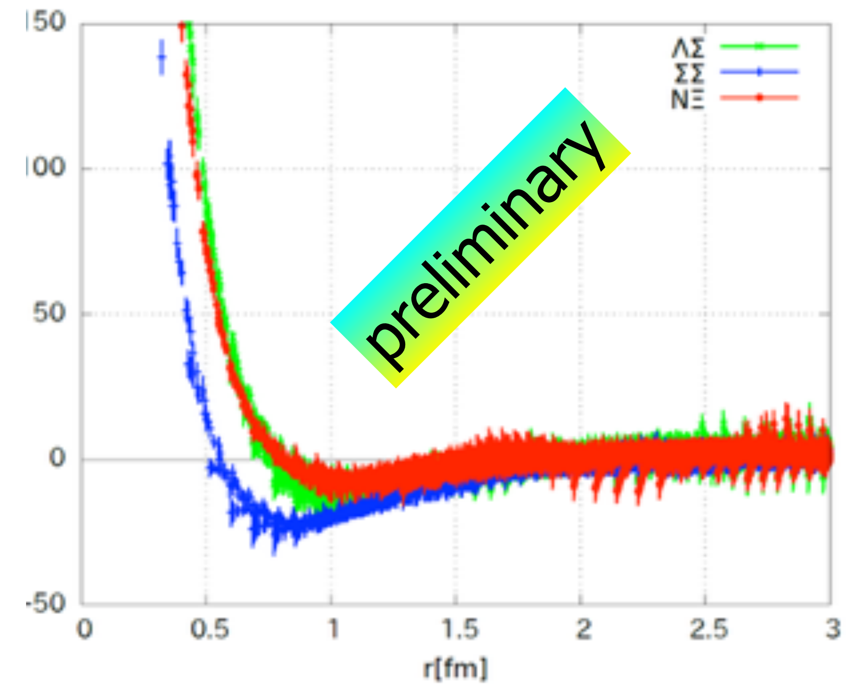
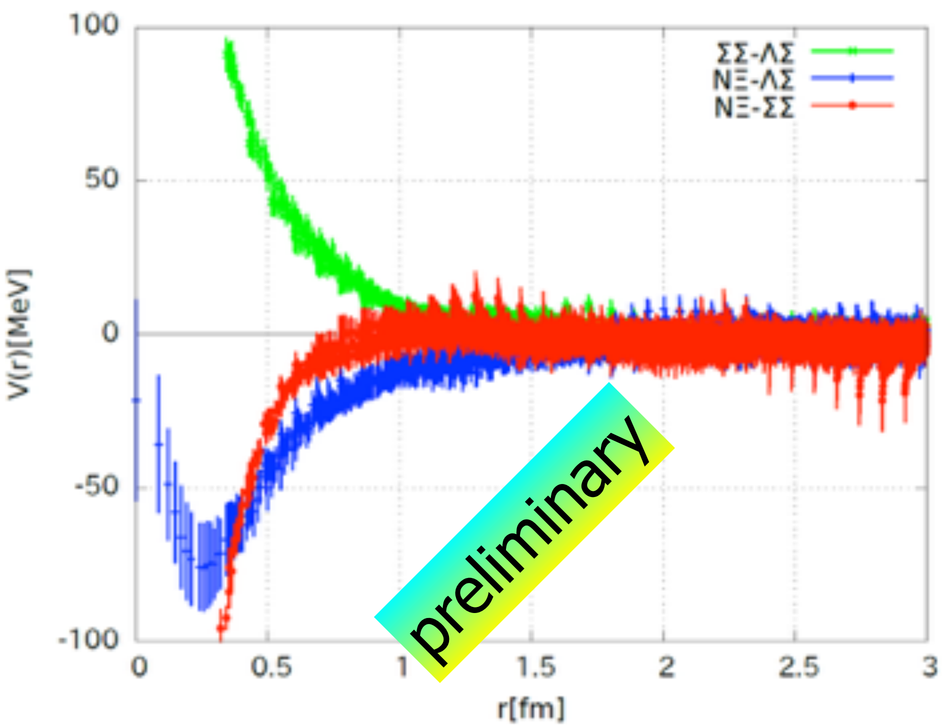


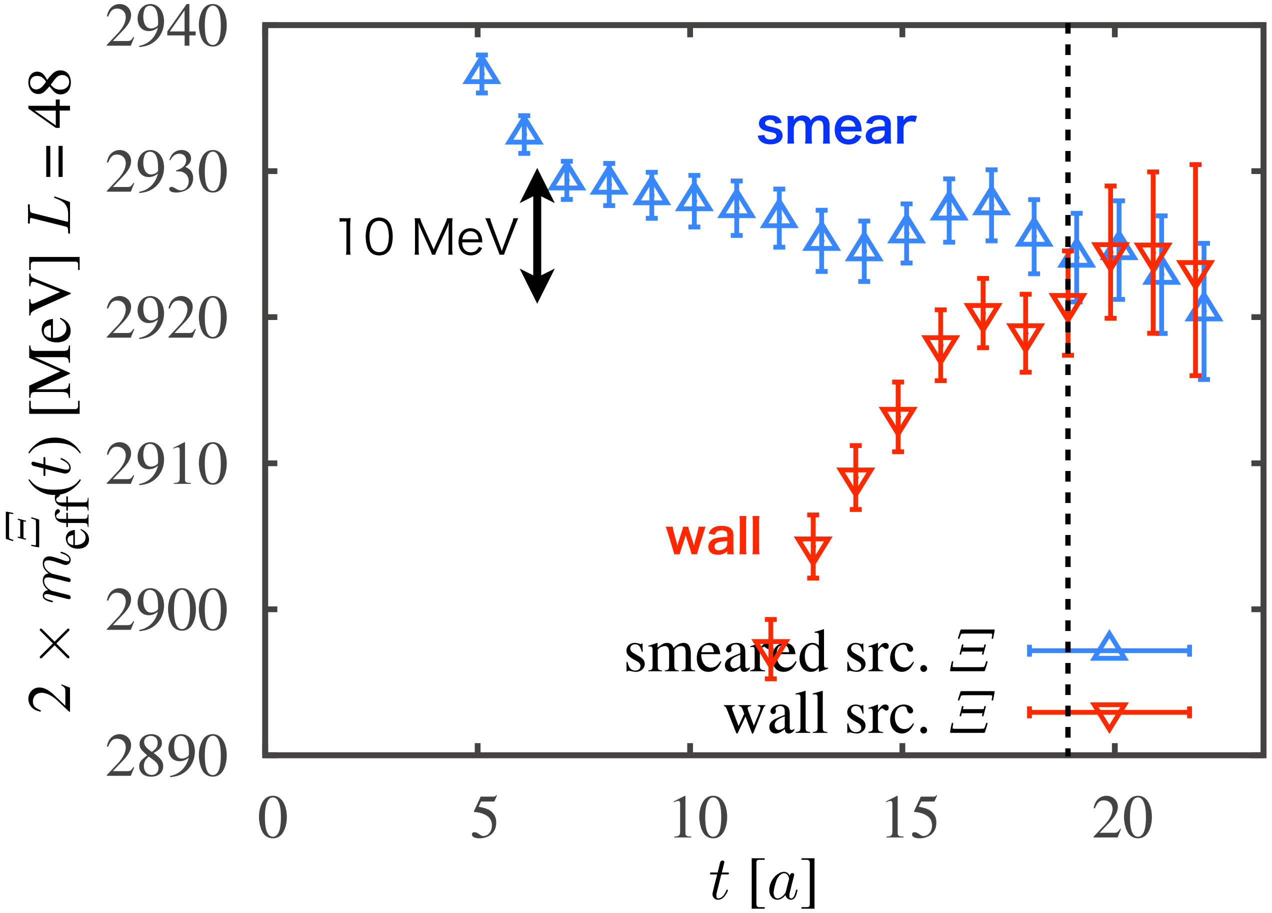


Is the interaction net attractive ?

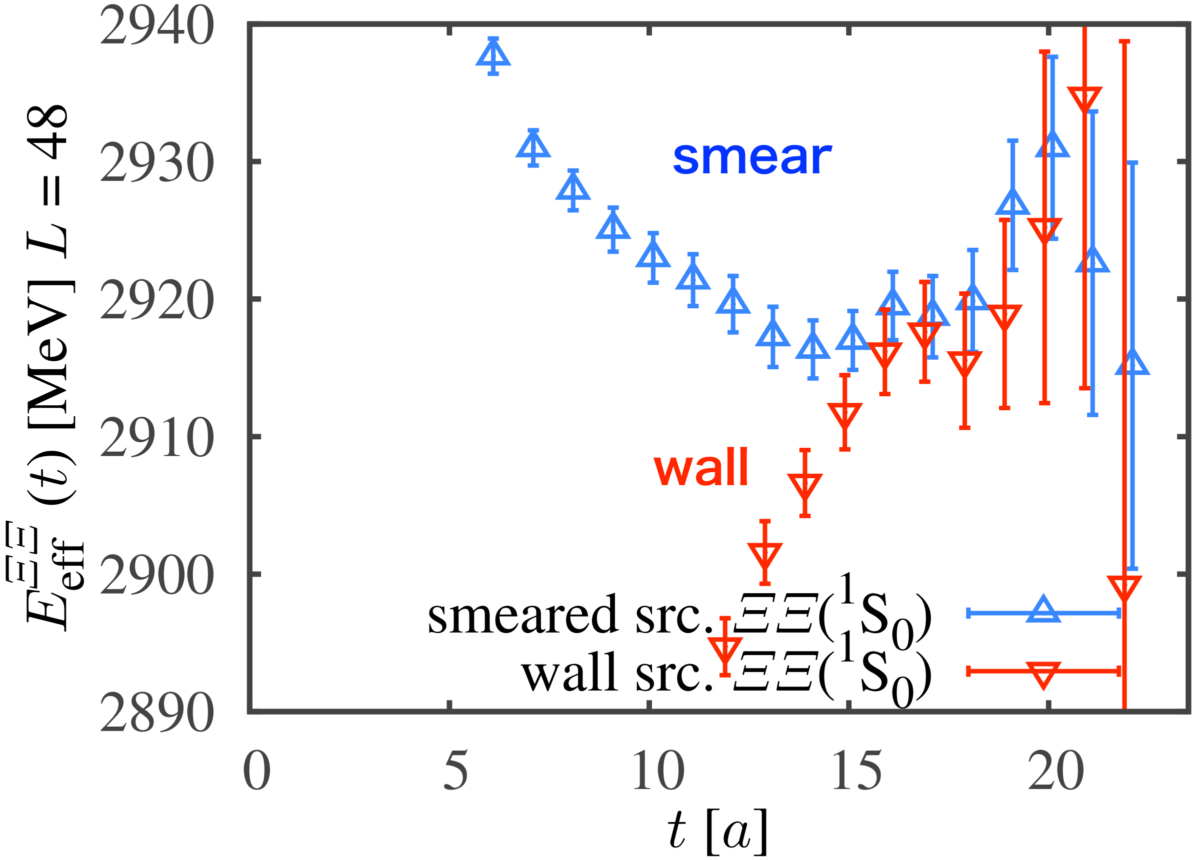
Stay tuned !

Numerator and denominator

$2m_{\Xi}$



$E_{\Xi\Xi}(^3\mathrm{S}_1)$



Smeared source looks better for the single baryon,
but it still keeps changing in the fine scale.

Method relies on cancellation of systematics

Numerator and denominator

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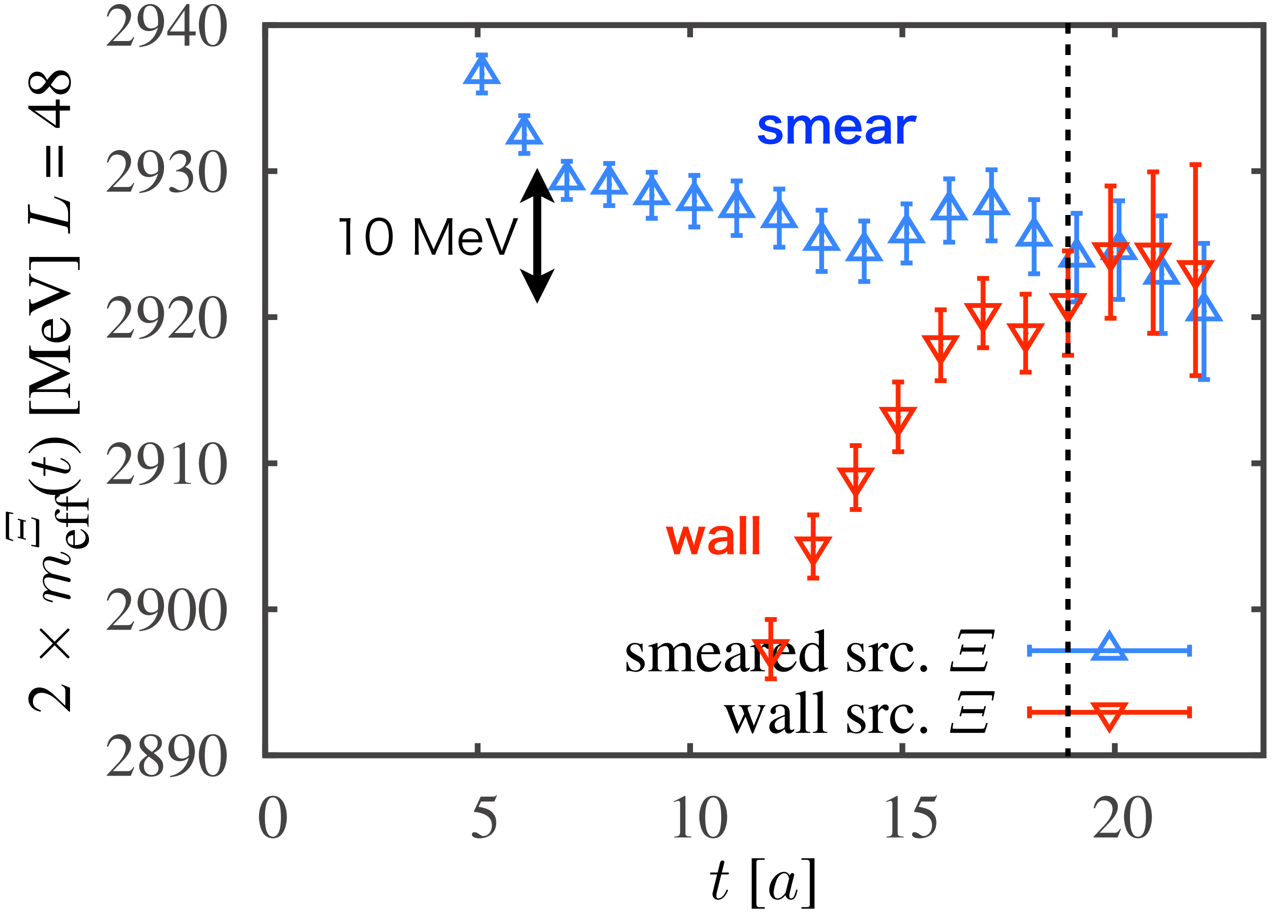

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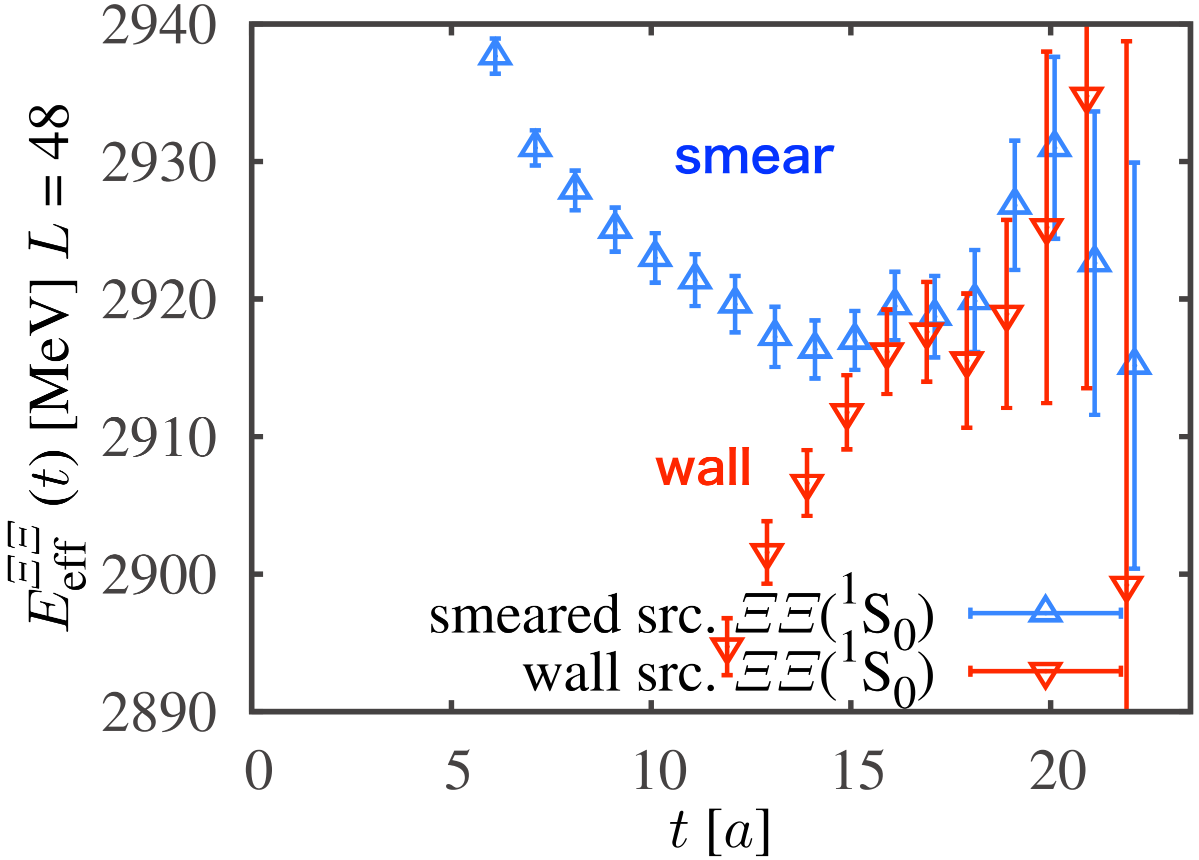

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