Baryon-Baryon Interactions from Lattice QCD

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**The Odyssey from Quarks to Universe**

- **QCD vacuum** → **Baryons** → **Nuclei**
- **Neutron Stars / Supernovae Nucleosynthesis**

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**Baryon Forces**

- **1st-principle Lattice QCD**
- **ab-initio nuclear calc.**

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**EoS of Dense Matter**

- **3N**
- **2N**
- **Y dof**
- **YNN(?)**

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**J-PARC**

- **Materials and Life Science Experimental Facility**
- **Hadron Beam Facility**
- **Neutrino to Kamiooka**

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**RI BF/ FRI B**

- **aLIGO/KAGRA**
- **NS-NS merger**

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**Nuclear Forces / Hyperon Forces**

- **QCD vacuum**
- **Baryons**
- **Nuclei**

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The Odyssey from unphysical to physical quark masses

~2010

We were here

lighter $m_q$

$M_\pi = 0.8$ GeV
$L = 2$ fm

K-computer

HPCI Strategic Program Field 5
“The origin of matter and the universe”
FY2010-15

Phys. point

Physical $M_\pi$
$L = 8$ fm
Hadrons to Atomic nuclei from Lattice QCD (HAL QCD Collaboration)

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T. Miyamato, K. Sasaki (YITP)
T. Doi, T. Hatsuda, T. Iritani (RIKEN)
F. Etminan (Univ. of Birjand)
S. Gongyo (Univ. of Tours)
Y. Ikeda, N. Ishii, K. Murano (RCNP)
T. Inoue (Nihon Univ.)
H. Nemura (Univ. of Tsukuba)

「20xx年宇宙の旅」
from Quarks to Universe
Various Theoretical methods

**QCD**
- (quarks)
- (gluons)
  - #params=4
  - quark masses & coupling

**Effective DoF**

**Phen. potentials**
- #params(2NF) = O(40)
- #params(3NF) = several

**Pionfull/ Pionless EFT potentials (or rela-ver)**
- #params(2NF) = 24+…
- #params(3NF) = 2+…
- or
- #params(2NF) = 2+…
- #params(3NF) = 1+…

**Effective DoF**

**Chiral Sym.**
- (w/ Effective DoF)

**Lattice QCD**
- (HAL method)

**LQCD potentials**
- #params(2NF) = 0
- #params(3NF) = 0
- #params(YN,YY,YNN) = 0

**Lattice QCD**
- (Luscher’s method)

**Exp. Data**

(talk by S. Aoki)
• **Outline**

  — Introduction
  
  — **Theoretical framework**
    • Direct method (Luscher’s method)
    • HAL QCD method
  
  — Challenges for multi-body systems on the lattice
  
  — Reliability test of LQCD methods
  
  — Results at heavy quark masses
  
  — Results at physical quark masses
  
  — Summary / Prospects
Interactions on the Lattice

• Direct method (Luscher’s method)
  – Phase shift & B.E. from temporal correlation in finite V

  M. Luscher, CMP104(1986)177
  CMP105(1986)153
  NPB354(1991)531

• HAL QCD method
  – “Potential” from spacial (& temporal) correlation
  – Phase shift & B.E. by solving Schrodinger eq in infinite V

  HAL QCD Coll., PTEP2012(2012)01A105
Luscher’s formula: Scatterings on the lattice

- Consider Schrödinger eq at asymptotic region

\[(\nabla^2 + k^2)\psi_k(r) = mV_k(r)\psi_k(r)\]

\[V_k(r) = 0 \text{ for } r > R\]

- (periodic) Boundary Condition in finite V

\[\Rightarrow \text{constraint on energies of the system}\]

- Energy E \(\Longleftrightarrow\) phase shift (at E)

\[k \cot \delta_E = \frac{2}{\sqrt{\pi L}} Z_{00}(1; q^2), \quad q = \frac{kL}{2\pi}, \quad E = 2\sqrt{m^2 + k^2}\]

Large V:

\[\Delta E = E - 2m = -\frac{4\pi a}{mL^3} \left[1 + c_1 \frac{a}{L} + c_2 \left(\frac{a}{L}\right)^2 + \mathcal{O}\left(\frac{1}{L^3}\right)\right]\]

- Calculate the energy spectrum of NN on (finite V) lattice

- Temporal correlation in Euclidean time \(\Rightarrow\) energy

\[G(t) = \langle 0|\mathcal{O}(t)\bar{\mathcal{O}}(0)|0\rangle = \sum_n A_n e^{-E_n t} \to A_0 e^{-E_0 t} \quad (t \to \infty)\]
[HAL QCD method]

- “Potential” defined through phase shifts (S-matrix)
- Nambu-Bethe-Salpeter (NBS) wave function

\[
\psi(\vec{r}) = \langle 0 | N(\vec{x} + \vec{r}) N(\vec{x}) | N(k) N(-k); W \rangle
\]

\[
(\nabla^2 + k^2) \psi(\vec{r}) = 0, \quad r > R \quad W = 2\sqrt{m^2 + k^2}
\]

- Wave function ↔ phase shifts

\[
\psi(r) \approx A \frac{\sin(\frac{kr - l\pi}{2} + \delta(k))}{kr}
\]

( below inelastic threshold)

Extended to multi-particle systems

M. Luscher, NPB354(1991)531
C.-J. Lin et al., NPB619(2001)467
CP-PACS Coll., PRD71(2005)094504

Ishizuka, Pos LAT2009 (2009) 119
Aoki-Hatsuda-Ishii PTP123(2010)89
Aoki et al., PRD88(2013)014036
Asymptotic form of BS wave function

For simplicity, we consider BS wave function of two pions

\[
\psi_q(\vec{x}) = \left\langle 0 \left| N(\vec{x}) N(\vec{0}) N(\vec{q}) N(-\vec{q}), \text{in} \right. \right\rangle
\]

**complete set**

\[
1 = \int \frac{d^3p}{(2\pi)^3 2E_N(\vec{p})} |N(\vec{p})\rangle \langle N(\vec{p})| + \cdots
\]

\[
= \int \frac{d^3p}{(2\pi)^3 2E_N(\vec{p})} \left\langle 0 \left| N(\vec{x}) \right. \right\rangle \langle N(\vec{p})| N(\vec{0}) \langle N(\vec{q}) N(-\vec{q}), \text{in} \rangle + I(\vec{x})
\]

\[
Z^{1/2} e^{i\vec{p} \cdot \vec{x}}
\]

\[
disc. + Z^{1/2} \frac{T(\vec{p}; \vec{q})}{m^2_N - (2E_N(\vec{q}) - E_N(\vec{p}))^2 + \vec{p}^2 - i\epsilon}
\]

\[
= Z \left( e^{i\vec{p} \cdot \vec{x}} + \frac{1}{(2\pi)^3} \int \frac{d^3p}{2E_N(\vec{p})} \frac{T(\vec{p}; \vec{q})}{4E_N(\vec{q}) \cdot (E_N(\vec{p}) - E_N(\vec{q}) - i\epsilon)} e^{i\vec{p} \cdot \vec{x}} \right)
\]

Integral is dominated by the on-shell contribution \( E_N(\vec{p}) \approx E_N(\vec{q}) \)

\( \Rightarrow \) T-matrix becomes the on-shell T-matrix

\[
T^{(s\text{-wave})}(s) = \frac{E(\vec{q})}{2|\vec{q}|} (-i) \left( e^{2i\delta_0(s)} - 1 \right)
\]

The asymptotic form

\[
\psi_q(\vec{x}) = Z e^{i\delta(s)} \frac{\sin(qr + \delta_0(s))}{qr} + \cdots \text{ (s-wave)}
\]

This is analogous to a non-rela. wave function
“Potential” as a representation of S-matrix

- Consider the wave function at “interacting region”
  \[(\nabla^2 + k^2)\psi(r) = m \int dr' U(r, r')\psi(r'), \quad r < R\]

  * **Probe interactions in “direct” way**

  - \(U(r, r')\): faithful to the phase shift by construction
    - \(U(r, r')\): **NOT** an observable, but well defined
    - \(U(r, r')\): E-independent, while non-local in general
Proof of Existence of E-independent potential

\[ V_W(r)\psi_W(r) = (E_W - H_0)\psi_W(r) \]  

[START] local but E-dep pot. (L^3 x L^3 dof)

- We consider the linear-indep wave functions and define
  
  \[ N_{W_1W_2} = \int dr \overline{\psi_{W_1}(r)}\psi_{W_2}(r) \]

- We define the non-local potential

  \[ U(r, r') = \sum_{W_1, W_2}^{W_{th}} (E_{W_1} - H_0)\psi_{W_1}(r)N_{W_1W_2}^{-1}\overline{\psi_{W_2}(r')}\psi_{W_2}(r') \]

- The above potential trivially satisfy Schrodinger eq.

\[ \int dr' U(r, r')\psi_W(r') = \int dr' \sum_{W_1, W_2}^{W_{th}} (E_{W_1} - H_0)\psi_{W_1}(r)N_{W_1W_2}^{-1}\overline{\psi_{W_2}(r')}\psi_{W_2}(r') \psi_W(r') \]

\[ = \sum_{W_1, W_2}^{W_{th}} (E_{W_1} - H_0)\psi_{W_1}(r)N_{W_1W_2}^{-1}N_{W_2W} \]

\[ = (E_W - H_0)\psi_W(r) \]

[GOAL] non-local but E-indep pot. (L^3 x L^3 dof)

“Potential” as a representation of S-matrix

• Consider the wave function at “interacting region”

\[(\nabla^2 + k^2)\psi(r) = m \int dr' U(r, r') \psi(r'), \quad r < R\]

Probes interactions in “direct” way

– U(r, r'): faithful to the phase shift by construction
  - U(r, r'): **NOT** an observable, but well defined
  - U(r, r'): E-independent, while non-local in general

– Phase shifts at **all** E (below inelastic threshold) obtained by solving Schrodinger eq in infinite V

– Non-locality \(\rightarrow\) derivative expansion

\[U(\vec{r}, \vec{r}') = V_c(r) + S_{12} V_T(r) + \vec{L} \cdot \vec{S} V_{LS}(r) + O(\nabla^2)\]

LO LO NLO NNLO

Check on convergence: K. Murano et al., PTP125(2011)1225

Control the E-dependence of phase shifts
HAL QCD method

NBS wave func.

\[ \psi_{NBS}(\vec{r}) = \langle 0 | N(\vec{r}) N(\vec{0}) | N(\vec{k}) N(-\vec{k}), i\eta \rangle \]
\[ \approx A_k \sin(kr - l\pi/2 + \delta_l(k))/(kr) \]
(at asymptotic region)

Lat Nuclear Force

\[ \frac{(k^2/m_N - H_0)}{\psi(\vec{r})} = \int d\vec{r}' U(\vec{r}, \vec{r}') \psi(\vec{r}') \]

E-indep (& non-local) Potential: Faithful to phase shifts

Analog to …

Phase shifts

virtual state
mid-range attraction
short-range repulsion

Phen. Potential

\[ V_{\text{C}(0)}(\text{MeV}) \]

\[ 0 \]

\[ T_{\text{lab}} \text{ [MeV]} \]

\[ 0 \]

\[ 2\pi, 3\pi, ... \]

\[ (\pi, \rho, \omega, ...) \]

\[ \pi \]

Bonn

Bonn

Bonn

Bonn

Bonn

Bonn

AV18

AV18

AV18

AV18

AV18
A few remarks on the Lattice Potential

• Potential is NOT an observable and is NOT unique: They are, however, phase-shift equivalent potentials
  – Choosing the pot. ↔ choosing the “scheme” (sink op.)

• Potential approach has some benefits:
  – Convenient to understand physics
  – Many body systems (sign problem partially avoided)
  – Finite V artifact better under control
  – Excited states better under control
    • G.S. saturation NOT necessary
    • Coupled Channel Systems

Crucial for multi-body on Lat
• **Outline**
  
  – Introduction
  
  – Theoretical framework
  
  – Challenges for multi-body systems on the lattice
    • Signal/Noise Issue
    • Coupled Channel Systems
    • Computational Challenge
  
  – Reliability test of LQCD methods
  
  – Results at heavy quark masses
  
  – Results at physical quark masses
  
  – Summary / Prospects
Challenges in multi-baryons on the lattice

- **Signal / Noise issue**
  - G.S. saturation by $t \rightarrow \infty$ required in LQCD

\[
G(r, t) = \langle 0 | \mathcal{O}(r, t) \bar{\mathcal{O}}(0) | 0 \rangle = \sum_n \alpha_n \psi_n(r) e^{-E_n t} \xrightarrow{t \rightarrow \infty} \alpha_0 \psi_0(r) e^{-E_0 t}
\]

- **pion**

\[
\text{Signal/Noise} \sim \frac{\langle \pi(t)\pi(0) \rangle}{\sqrt{\langle \pi\pi(t)\pi\pi(0) \rangle}} \sim \frac{\exp(-m_\pi t)}{\sqrt{\exp(-2m_\pi t)}} \sim \text{const.}
\]

- **nucleon**

\[
\text{Signal/Noise} \sim \frac{\langle N(t)\bar{N}(0) \rangle}{\sqrt{\langle |N(t)\bar{N}(0)|^2 \rangle}} \sim \frac{\exp(-m_N t)}{\sqrt{\exp(-3m_\pi t)}} \sim \exp[-(m_N - 3/2m_\pi) t]
\]

\[
\text{Signal/Noise} \sim \frac{\langle N^A(t)\bar{N}^A(0) \rangle}{\sqrt{\langle |N^A(t)\bar{N}^A(0)|^2 \rangle}} \sim \frac{\exp(-A m_N t)}{\sqrt{\exp(-3A m_\pi t)}} \sim \exp[-A(m_N - 3/2m_\pi) t]
\]

(for mass number = $A$)
Challenges in multi-baryons on the lattice

- Excitation energy ~ binding energy or finite V effect

\[ E_1 - E_0 \approx \frac{\vec{p}^2}{m_N} \approx \frac{1}{m_N} \frac{(2\pi)^2}{L^2} \] (very small)

**Physical** \( M_\pi = 0.5 \text{ GeV} \) \( L = 8 \text{ fm} \)

\( M_\pi = 0.3 \text{ GeV} \) \( L = 6 \text{ fm} \)

\( M_\pi = 0.5 \text{ GeV} \) \( L = 3 \text{ fm} \)

**System w/o Gap**

New Challenge for multi-body systems

(For both of Direct method / (old) HAL method)

\[ S/N \propto 10^{-4} \quad 10^{-13} \quad 10^{-25} \]
Time-dependent HAL method

E-indep of potential $U(r,r') \Rightarrow$ (excited) scatt states share the same $U(r,r')$
They are not contaminations, but signals

Original (t-indep) HAL method

$$G_{NN}(\vec{r}, t) = \langle 0 | N(\vec{r}, t) N(\vec{0}, t) \mathcal{J}_{src}(t_0) | 0 \rangle$$
$$R(r, t) \equiv G_{NN}(r, t)/G_N(t)^2 = \sum_i A_{W_i} \psi_{W_i}(r) e^{-(W_i-2m)t}$$
$$\int dr' U(r, r') \psi_{W_0}(r') = (E_{W_0} - H_0) \psi_{W_0}(r)$$
$$\int dr' U(r, r') \psi_{W_1}(r') = (E_{W_1} - H_0) \psi_{W_1}(r)$$

New t-dep HAL method

All equations can be combined as

$$\int dr' U(r, r') R(r', t) = \left( -\frac{\partial}{\partial t} + \frac{1}{4m} \frac{\partial^2}{\partial t^2} - H_0 \right) R(r, t)$$

G.S. saturation $\Rightarrow$ “Elastic state” saturation

[Exponential Improvement]
Coupled Channel systems
(beyond inelastic threshold)

• Essential in many interesting physics
  – Hyperon Forces (e.g., H-dibaryon (ΛΛ-NΞ-ΣΣ))
  – Exotic mesons, Resonances, etc. (e.g., Zc(3900))

→ Coupled channel potentials in HAL method

\[
\psi_{AB}(r, k) = \frac{1}{\sqrt{Z_A Z_B}} \langle 0 | \phi_A(x + r) \phi_B(x) | W \rangle \\
\psi_{CD}(r, q) = \frac{1}{\sqrt{Z_C Z_D}} \langle 0 | \phi_C(x + r) \phi_D(x) | W \rangle \\
W = \sqrt{m_A^2 + k^2} + \sqrt{m_B^2 + k^2} = \sqrt{m_C^2 + q^2} + \sqrt{m_D^2 + q^2}
\]

\[
(E_{ki}^{AB} - H_0^{AB}) \psi_{AB}(r, k_i) = \int dr' U_{AB,AB}(r, r') \psi_{AB}(r', k_i) + \int dr' U_{AB,CD}(r, r') \psi_{CD}(r', q_i)
\]

\[
(E_{qi}^{CD} - H_0^{CD}) \psi_{CD}(r, q_i) = \int dr' U_{CD,AB}(r, r') \psi_{AB}(r', k_i) + \int dr' U_{CD,CD}(r, r') \psi_{CD}(r', q_i)
\]

S. Aoki et al. (HAL Coll.), PRD 87(2013) 034512


**Computational Challenge**

- Enormous comput. cost for multi-baryon correlators
  - Wick contraction (permutations)
    \[ \sim \left( \frac{3}{2} A \right)! \] \( (A: \text{ mass number}) \)
  - color/spinor contractions
    \[ \sim 6^A \cdot 4^A \text{ or } 6^A \cdot 2^A \]
  - Unified Contraction Algorithm (UCA)
    - A novel method which unifies two contractions

\[ \Pi^{2N} \simeq \langle qqqqqq(t) \bar{q}(\xi'_1) \bar{q}(\xi'_2) \bar{q}(\xi'_3) \bar{q}(\xi'_4) \bar{q}(\xi'_5) \bar{q}(\xi'_6)(t_0) \rangle \times \text{Coeff}^{2N}(\xi'_1, \cdots, \xi'_6) \]

**Drastic Speedup**

\[ \times 192 \text{ for } ^3\text{H}/^3\text{He}, \times 20736 \text{ for } ^4\text{He}, \times 10^{11} \text{ for } ^8\text{Be} \] (x add’l. speedup)

See also subsequent works: Detmold et al., PRD87(2013)114512
Gunther et al., PRD87(2013)094513
• Outline
  – Introduction
  – Theoretical framework
  – Challenges for multi-body systems on the lattice
    • Signal/Noise Issue ➔ Time-dependent HAL method
    • Coupled Channel Systems ➔ Coupled channel HAL potential
    • Computational Challenge ➔ Unified Contraction Algorithm
  – Reliability test of LQCD methods
    • Direct method & HAL method: Comparative study ➔ Talk by S. Aoki
  – Results at heavy quark masses
  – Results at physical quark masses
  – Summary / Prospects
Direct method vs HAL method

Reviewed in T.D. PoS LAT2012,009 (+ updates)

HAL method (HAL): unbound
Direct method (PACS-CS (Yamazaki et al.)/ NPL/ CalLat): bound

c.f. I=2 pipi: Direct & HAL methods agree well Kurth et al., JHEP1312(2013)015
**Reliability Test of LQCD methods**

T. Iritani et al. (HAL), JHEP1610(2016)101

- **Employ the same config** used in previous Direct method study
  
  YIKU2012 = T. Yamazaki et al. PRD86(2012)074514

  - **High statistics** (e.g., $48^4$ smeared: $x8$ #stat of YIKU2012)

  - **Both of wall & smeared src setup**
    - smeared $\rightarrow$ same as YIKU2012

- **Nf=2+1 clover LQCD**
  
  - $m_\pi = 0.51\text{GeV}$, $m_N = 1.32\text{GeV}$, $m_\Xi = 1.46\text{GeV}$, $1/a=2.2\text{GeV}$ ($a=0.09\text{fm}$)
  
  - $L=2.9, 3.6, 4.3, 5.8$ fm ($32^3 \times 48, 40^3 \times 48, 48^3 \times 48, 64^3 \times 64$)

  - NN ($^1S_0$), NN ($^3S_1$) & $\Xi \Xi$ ($^1S_0$), $\Xi \Xi$ ($^3S_1$)
    - N.B. $\Xi \Xi$ ($^1S_0$) $\sim$ flavor SU(3) partner of NN ($^1S_0$), but much better S/N
Check by source op. dependence

- Examine the consistency between smeared & wall source

**Luscher’s method**
$(\Delta E \rightarrow \text{phase shift})$

**HAL method**
$(V(r) \rightarrow \text{phase shift})$

Inconsistent “signal” (red (wall) vs blue (smeared))
$\Rightarrow$ cannot judge which (or neither) is reliable

$\Xi \Xi (1S_0)$
$L=4.3\text{fm}$

**FAILED**

$V_{\text{eff}}(r)$ from wall & $V^{\text{LO}}(r)$ from wall+smeared are consistent

**PASSED**

- Examine the consistency between smeared & wall source
Check by sink op. dependence (for direct method)

**Generalized Direct method** (by generalized sink projection)

\[
\tilde{R}(f)(t) = \sum_{\vec{r}} f(\vec{r}) R(\vec{r}, t) = \sum_{\vec{r}} f(\vec{r}) \sum_{\vec{x}} \langle 0 | B(\vec{r} + \vec{x}, t) B(\vec{x}, t) J_{\text{src}}(0) | 0 \rangle / \{ G_B(t) \}^2
\]

c.f. standard Direct method \( \Leftrightarrow f(r) = 1 \)

Many inconsistent “plateaux”

\[ \rightarrow \text{Predictive power is LOST} \]

“smeared is better” is too naive
“Sanity Check” for results from direct method

Aoki-Doi-Iritani, arXiv:1610.09763

ERE: \[ k \cot \delta(k) = \frac{1}{a} + \frac{1}{2} r k^2 + \ldots \]

If we examine the data from

T. Yamazaki et al. PRD86(2012)074514

manifestation of problem

in the direct method
<table>
<thead>
<tr>
<th></th>
<th>single baryon</th>
<th>double baryon</th>
<th>Overall Verdict</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>plateau check</td>
<td>mirage plateau</td>
<td></td>
</tr>
<tr>
<td><strong>YIKU 2011</strong></td>
<td>○</td>
<td>×</td>
<td>False</td>
</tr>
<tr>
<td><strong>YIKU 2012</strong></td>
<td>○</td>
<td>×</td>
<td>False</td>
</tr>
<tr>
<td><strong>YIKU 2015</strong></td>
<td>○</td>
<td>×</td>
<td>False</td>
</tr>
<tr>
<td><strong>NPL 2012</strong></td>
<td>○</td>
<td>×</td>
<td>False</td>
</tr>
<tr>
<td><strong>NPL 2013</strong></td>
<td>○</td>
<td>×</td>
<td>False</td>
</tr>
<tr>
<td><strong>NPL 2015</strong></td>
<td>△</td>
<td>×</td>
<td>False</td>
</tr>
</tbody>
</table>
Anatomy of the direct method from HAL QCD potential
Understand the origin of “fake plateaux”

**Potential**

![Potential graph]

-wall $t=15$
-wall $t=13$
-wall $t=11$

**Solve Schrodinger eq. in Finite V**

**Eigen-wave functions**

![Eigen-wave functions graph]

**Eigen-energies**

<table>
<thead>
<tr>
<th>$n$-th A1</th>
<th>$\Delta E_n$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2.58(1)</td>
</tr>
<tr>
<td>1</td>
<td>52.49(2)</td>
</tr>
<tr>
<td>2</td>
<td>112.08(2)</td>
</tr>
<tr>
<td>3</td>
<td>169.78(2)</td>
</tr>
<tr>
<td>4</td>
<td>224.73(1)</td>
</tr>
</tbody>
</table>

**NBS correlator $R(r,t)$**

![NBS correlator graph]

-smeared src.: $t=14$
-smeared src.: $t=13$
-smeared src.: $t=12$
-wall src.: $t=13$
-wall src.: $t=15$

**Decompose NBS correlator to each eigenstates**

-r [fm]

0 0.5 1 1.5 2 2.5 3 3.5
NBS correlator $R(r,t)$

Contribution from each (excited) states (@ $t=0$)

Decompose NBS correlator to each eigenstates

Temporal-correlator $R(t) = \sum_r R(r,t)$

(R(t) w/ smeared has been used in Luscher’s method)

Contribution from each (excited) states (@ $t=0$)

Excited States

- G.S.
- Excited States

Excited states NOT suppressed

Excited states suppressed

Blue: smeared

Red: wall
Understand the origin of “fake plateaux”

We are now ready to “predict” the behavior of $m_{\text{eff}}$ of $\Delta E$ at any “t”

“prediction” reproduce the real data well

$\Delta E$ (MeV)

$t [a]$ (E$_1$-E$_0$=50MeV)

“fake plateaux” at $t \sim 1$fm

“real plateau” at $t \sim 10$fm

HAL method is crucial!
Understand the origin of "fake plateaux"

We are now ready to "predict" the behavior of \( m(\text{eff}) \) of \( \Delta E \) at any "t"

"prediction" reproduce the real data well

HAL method is crucial!

"fake plateaux" at \( t \sim 1\text{fm} \)

"real plateau" at \( t \sim 10\text{fm} \)

\((E_1-E_0=50\text{MeV})\)
Direct method “educated by HAL method”

**Generalized Direct method** (by generalized sink projection)

\[
\tilde{R}^{(f)}(t) = \sum_{\vec{r}} f(\vec{r}) R(\vec{r}, t) = \sum_{\vec{r}} f(\vec{r}) \sum_{\vec{x}} \langle 0| B(\vec{r} + \vec{x}, t) B(\vec{x}, t) \mathcal{J}_{\text{src}}(0)|0\rangle / \{G_B(t)\}^2
\]

\[f(r) \leftrightarrow \text{eigen-wave func from HAL potential at finite V}\]

\[f(r) = \psi^{+_{\text{g.s.}}}(r)\]

\[f(r) = \psi^{+_{\text{1st.}}}(r)\]

\[\Delta E : \text{Direct (wall/ smeared) = Potential (wall/ smeared)}\]

Direct method has (useful) **predictive** power (postdictive power)

Variational method could be helpful for direct method
• **Outline**
  – Introduction
  – Theoretical framework
  – Challenges for multi-body systems on the lattice
  – Reliability test of LQCD methods
  – Results at heavy quark masses with **HAL QCD method**
  – Results at physical quark masses
  – Summary / Prospects
**NN-forces (P= (+) channel)**

(m$_\pi$=0.41-0.70 GeV)

- Central in $^1S_0$
- $^3S_1$-$^3D_1$ channel

- Phase shift ($^1S_0$)
- Attractive, Unbound

N. Ishii, PoS(CD12)(2013)025
Hyperon Forces

\[ 8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a \]

**SU(3) broken point:**
H. Nemura et al., PLB673(2009)136
K. Sasaki et al., PTEP2015(2015)113B01

**SU(3) symmetric point:**
Repulsive core is well-motivated by Pauli principle.

\[ a = 0.12 \text{fm}, \; L = 3.9 \text{fm}, \; m(\text{PS}) = 0.47-1.2 \text{GeV} \]

**SU(3) study**

**BB potentials**

\[ ^1S_0 \]

\[ ^3S_1-^3D_1 \]

\[ 27,10^*: \text{Same as NN} \]

\[ 8s,10: \text{Strong repulsive core} \]

\[ \text{attractive core!} \]

Bound H-dibaryon in SU(3) (B.E. = 26-49 MeV)

**NN sector**

\[ V^{(27)} \]

\[ V^{(8s)} \]

\[ V^{(10^*)} \]

\[ V^{(10)} \]

**YN/YY sector**

\[ V^{(27)} \]

\[ V^{(8s)} \]

\[ V^{(10^*)} \]

\[ V^{(10)} \]

**SU(3) lat \rightarrow Physical point**

\[ m_{\Sigma\Sigma} = 2380 \text{MeV} \]

\[ m_{BB} = 120 \text{MeV} \]

\[ m_{\Xi\Xi} = 2260 \text{MeV} \]

\[ m_H = 30 \text{MeV} \]

\[ m_{\Lambda\Lambda} = 2230 \text{MeV} \]

NN: unbound \( ^1S_0, ^3S_1-^3D_1 \)

\[ \text{Repulsive core} \leftarrow \text{Pauli principle!} \]
From LQCD to Nuclei / Neutron Star

Lat NN forces

B.E. of medium-heavy nuclei

(SU(3), m(PS)=0.47GeV)

BHF & TOV

Neutron Star M-R relation

[To be included]
YN/YY forces
P-wave/LS forces
[LQCD]
Unphysical mass
[Missing]
3-baryon forces

T.Inoue et al. (HAL Coll.) PRL111(2013)112503
T.Inoue et al. (HAL Coll.), PRC91(2015)011001
NN-forces in $P=(-)$ channel \( (m_\pi=1.1 \text{ GeV}) \)

- Central, tensor & LS forces

\[ 1P_1, 3P_0, 3P_1, 3P_2-3F_2 \]

Superfluidity $^3P_2$ in neutron star
\[ \leftrightarrow \text{ neutrino cooling} \]

\[ \leftrightarrow \text{ observation of Cas A NS} \]

K. Murano et al., PLB735(2014)19

c.f. CalLat Coll. 1508.00886

Phase shifts

Qualitatively good, but strength is weak
(We also observe potentials glow by lighter mass)
3N-forces (3NF) \[(N_f=2, \ m_\pi=0.76-1.1 \ GeV)\]

T.D. et al. (HAL QCD Coll.) PTP127(2012)723
+ t-dep method updates etc.

How about other geometries?

3N-forces (3NF)

Unified Contraction Algorithm (UCA) is crucial (x192 speedup)

How about YNN, YYN, YYY?

How about lighter quark masses?

short-range repulsive 3NF!

Triton channel

(artifact ?)
3N-forces (3NF)

**Nf=2, m\(\pi\)=0.76-1.1 GeV**

- Magnitude of 3NF is similar for all masses
- Range of 3NF tend to get longer (?) for m(\(\pi\))=0.51 GeV

**Nf=2+1, m\(\pi\)=0.51 GeV**

Kernel: ~50% efficiency achieved!
• **Outline**
  – Introduction
  – Theoretical framework
  – Challenges for multi-body systems on the lattice
  – Reliability test of LQCD methods
  – Results at heavy quark masses
  – **Results at (almost) physical quark masses**
    • Nuclear forces and Hyperon forces
    • Impact on dense matter
  – Summary / Prospects
• Baryon Forces from LQCD
  • Exponentially better S/N
  • Coupled channel systems

[Theory] = HAL QCD method

Baryon Interactions at Physical Point

[Hardware] = K-computer [10 PFlops]
  + FX100 [1 PFlops] @ RIKEN
  + HA-PACS [1 PFlops] @ Tsukuba

[Software] = Unified Contraction Algorithm
  • Exponential speedup

\[ ^3\text{H} / ^3\text{He} : \times 192 \]
\[ ^4\text{He} : \times 20736 \]
\[ ^8\text{Be} : \times 10^{11} \]
Setup of Lattice QCD

- **Nf = 2+1 full QCD**
  - Clover fermion + Iwasaki gauge action
  - Non-perturbatively O(a)-improved
  - APE-Stout smearing ($\alpha=0.1$, $n_{\text{stout}}=6$)
  - $m(\pi) \approx 145$ MeV, $m(K) \approx 525$ MeV
  - #traj $\approx 2000$ generated

K.I. Ishikawa et al., PoS LAT2015, 075

- **Measurement**
  - Wall source w/ Coulomb gauge
  - Efficient implementation of UCA
  - Block solver for multiple RHS
  - K-computer @ 2048 node (x 8core/node)
    - ~25% efficiency (~65 TFlops sustained)
  - Calc to increase #stat in progress
  - All results preliminary

**Weak scaling**
(NBS calc part, w/o solver, w/o IO)

96$^4$ box
(a$\approx 0.085$fm)
Target of Interactions

- **NN/YN/YY** for central/tensor forces in $P=(+)$ (S, D-waves)

\[
U(r, r') = V_c(r) + S_{12} V_T(r) + L \cdot \vec{S} V_{LS}(r) + \mathcal{O}(\nabla^2)
\]

**Central**  **Tensor**

LO  LO  NLO  NNLO  (derivative expansion)

<table>
<thead>
<tr>
<th>$S$</th>
<th>NN</th>
<th>NΛ, NΣ</th>
<th>ΛΛ, ΛΣ, ΣΣ, ΝΞ</th>
<th>ΛΞ, ΣΞ</th>
<th>ΞΞ</th>
<th>ΞΩ</th>
<th>ΩΩ</th>
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<td>ΛΛ, ΛΣ, ΣΣ, ΝΞ</td>
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<td>ΞΞ</td>
<td>ΞΩ</td>
<td>ΩΩ</td>
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<tr>
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<td>NN</td>
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<td>ΞΞ</td>
<td>ΞΩ</td>
<td>ΩΩ</td>
</tr>
<tr>
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<td>NN</td>
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<td>ΞΞ</td>
<td>ΞΩ</td>
<td>ΩΩ</td>
</tr>
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<td>NN</td>
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<td>ΞΞ</td>
<td>ΞΩ</td>
<td>ΩΩ</td>
</tr>
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<td>ΞΩ</td>
<td>ΩΩ</td>
</tr>
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<td>NN</td>
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<td>ΞΞ</td>
<td>ΞΩ</td>
<td>ΩΩ</td>
</tr>
</tbody>
</table>

**EXP rich data**

LQCD  better S/ N

Hyperon in neutron star and EoS?  Exotic states?

Hyperon forces provide precious predictions
\[ \Omega \Omega \text{ system } (^{1}S_{0}) \]

The "most strange" dibaryon system

Potential

\[
V(r) \text{ [MeV]}
\]

\[
r \text{ [fm]}
\]

\[ t = 16, t = 17, t = 18 \]

(400conf x 4rot x 44/48src)

Preliminary

\[ m(\text{eff}) \text{ for single } \Omega \]

\[ t = 18 : \sim 0.2-0.3\% \text{ sys error} \]

[S. Gongyo / K. Sasaki]
**Phase Shifts**

**\( \Omega \Omega \) system \(^{1S_0}\) (The “most strange” dibaryon system)

**Strong Attraction**

→ Vicinity of bound/ unbound

[~ Unitary limit]

⇔ \( \Omega \Omega \) correlation in HIC exp.
system \((S = -4)\)
Flavor SU(3)-partner of dineutron \( \Rightarrow \)

- “Doorway” to NN-forces
- Bound by SU(3) breaking?

**Potential**

(400conf x 4rot x 48src)

\[ m(\text{eff}) \text{ for single } \Xi \]

\( t = 14-18 : \sim 0.3-1\% \text{ sys error} \)
\[ \Xi \Xi \text{ system } (^{1}S_{0}) \]

**Phase Shifts**

Preliminary

Strong Attraction yet Unbound

(2-gauss + 2-OBEP fit) 
(400conf x 4rot x 48src) 
(t-dependence will be checked again w/ larger #stat)
The \(3S_1-3D_1\) system

### Potentials

**Central**

- \(V_C(IS=01): t=15\)
- \(V_C(IS=01): t=16\)
- \(V_C(IS=01): t=17\)
- \(V_C(IS=01): t=18\)

**Tensor**

- Preliminary data

### Phase Shifts

- Medium: Strong Repulsion
- Tensor: Weak

10plet ⇔ unique w/ hyperon DoF

Flavor SU(3)-partner of \(\Sigma^- n\)

\(\Sigma^-\) in neutron star?

Central: Strong Repulsion

Tensor: Weak

Phase Shifts (effective \(3S_1\))

Very Preliminary
S= –3 systems

• \( \Xi \Sigma \) (l=3/2)
  • \( ^1S_0 \) \( \sim \) 27-plet
    \( \Leftrightarrow \) NN\((^1S_0)\) + SU(3) breaking
  • \( ^3S_1^{-3}D_1 \) \( \sim \) 10*-plet
    \( \Leftrightarrow \) NN\((^3S_1^{-3}D_1)\) + SU(3) breaking

• \( \Xi \Lambda - \Xi \Sigma \) (l=1/2) : coupled channel
  • \( ^1S_0 \) \( \sim \) 27-plet & 8s-plet
  • \( ^3S_1^{-3}D_1 \) \( \sim \) 10-plet & 8a-plet
\( \Xi \Sigma (I=3/2, \text{spin triplet}) \)

Central Tensor phase shifts & mixing

\( V_C(r) \) [MeV] (\( \Xi \Sigma \) spin triplet)

\( V_T(r) \) [MeV] (\( \Xi \Sigma \) spin triplet)

N.B. t-dep should be checked; single \( m_B \) has \( \sim 0.3-3\% \) sys @ \( t=10-14 \)

Unbound

Preliminary

(200conf x 4rot x 48src) [N. Ishii]
S = −2 systems

- $\Lambda\Lambda - N\Xi - \Sigma\Sigma (^{1}S_{0})$
  - H-dibaryon channel

- $N\Xi$ interactions
  - $\Xi$-hypernuclei
  - $\Xi$ in neutron star?

... and many more interactions!

⇒ K. Sasaki’s talk
**S=−1 systems**

↔ ↔ strangeness nuclear physics (Λ-hypernuclei @ J-PARC)

Λ should (?) appear in the core of Neutron Star

↔ ↔ Huge impact on EoS of high dense matter

- **ΛN−ΣN (I=1/2) : coupled channel**
  - $^1S_0$ $\sim$ 27-plet & 8s-plet
  - $^3S_1^*−^3D_1$ $\sim$ 10*-plet & 8a-plet

- **ΣN (I=3/2)**
  - $^1S_0$ $\sim$ 27-plet
    ↔ NN($^1S_0$) + SU(3) breaking
  - $^3S_1^*−^3D_1$ $\sim$ 10-plet
$\Lambda N - \Sigma N$ Vc potential in $^3S_1 - ^3D_1$ [H. Nemura]

Very preliminary result of LN potential at the physical point

\[
\left( \frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} \right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{lo}(\vec{r}) R(\vec{r}, t) + \cdots (8)
\]

PRELIMINARY
\[ \Lambda N - \Sigma N \text{ Vt potential in } ^3S_1 - ^3D_1 \] [H. Nemura]

Very preliminary result of LN potential at the physical point

\[
\left( \frac{\nabla^2}{\mu} - \frac{\partial}{\partial t} \right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, r') R(\vec{r}', t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \cdot (8)
\]
NN system \((S = 0)\)
NN system ($^{3S_{1/2}}-^{3D_{1/2}}$)

**Potentials**

- **Central**
  - $V_C$: repulsive core + long-range attraction

- **Tensor**
  - $V_T$: strong tensor force!

Preliminary

$m(\text{eff})$ for single N: ~2-4% sys err for $t = 8-10$

(400conf x 4rot x 48src)
NN system \( ^{1S_0} \)

**Potentials**

- \( V_c \): repulsive core + long-range attraction

The effect of SU(3) \( f \) breaking

\( \text{NN}(^{1S_0}) \) and \( \Xi\Xi(^{1S_0}) \) : 27-plet

Repulsive core enhanced for lighter quark mass? \( \leftrightarrow \) OGE?

N.B. Sys error in NN may be underestimated

\( (400\text{conf} \times 4\text{rot} \times 48\text{src}) \)
Impact on dense matter
S=-2 interactions suitable to grasp whole NN/YN/YY interactions

Central Force in Irrep-base (diagonal)

\[ 8 \times 8 = \frac{27 + 8s + 1}{^{1}\text{S}_0} + \frac{10^* + 10 + 8a}{^{3}\text{S}_1, ^3\text{D}_1} \]

(off-diagonal component is small)
S=-2 interactions suitable to grasp whole NN/YN/YY interactions

Tensor Force in Irrep-base (diagonal)

\[ 8 \times 8 = \frac{27 + 8s + 1}{^1S_0} + \frac{10^* + 10 + 8a}{^3S_1, ^3D_1} \]

\[ 10^* \mathbf{V}_T \]
\[ 10 \mathbf{V}_T \]
\[ 8a \mathbf{V}_T \]

\[ \text{We fit by (central)} \]

\[ V(r) = a_1 e^{-a_2 r^2} + a_3 e^{-a_4 r^2} + a_5 \left[ \left( 1 - e^{-a_6 r^2} \right) \frac{e^{-a_7 r^2}}{r} \right]^2 \]

\[ \text{We fit by (tensor)} \]

\[ V(r) = a_1 \left( 1 - e^{-a_2 r^2} \right)^2 \left( 1 + \frac{3}{a_3 r} + \frac{3}{(a_3 r)^2} \right) \frac{e^{-a_4 r^2}}{r} + a_4 \left( 1 - e^{-a_5 r^2} \right)^2 \left( 1 + \frac{3}{a_6 r} + \frac{3}{(a_6 r)^2} \right) \frac{e^{-a_7 r^2}}{r} \]

\[ \rightarrow \text{We calculate single-particle energy of hyperon in nuclear matter w/ LQCD baryon forces} \]

(off-diagonal component neglected)
Brueckner-Hartree-Fock

• Hyperon single-particle potential

\[ U_Y(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \leq k_F} \langle kk' | G^{SLJ}_{(YN)(YN)}(e_Y(k) + e_N(k')) | kk' \rangle \]

\[ 2^{S+1} L_J = \begin{array}{c}
1 \ S_0, \ 3 \ S_1, \ 3 \ D_1, \\
in our study \end{array} \left| \begin{array}{c}
1 \ P_1, \ 3 \ P_J, \ \ldots \\
\text{limitation}
\end{array} \right. \]

• YN G-matrix using \( V_{S=-1}^{LQCD} \), \( M_{N,Y}^{\text{Phys}} \), \( U_{n,p}^{\text{AV18,BHF}} \) and, \( U_Y^{LQCD} \)

\[ Q=0 \begin{pmatrix}
G_{(\Delta n)(\Delta n)}^{SLJ} & G_{(\Delta n)(\Sigma^0 n)}^{SLJ} & G_{(\Delta n)(\Sigma^- p)}^{SLJ} \\
G_{(\Sigma^0 n)(\Delta n)} & G_{(\Sigma^0 n)(\Sigma^0 n)} & G_{(\Sigma^0 n)(\Sigma^- p)} \\
G_{(\Sigma^- p)(\Delta n)} & G_{(\Sigma^- p)(\Sigma^0 n)} & G_{(\Sigma^- p)(\Sigma^- p)}
\end{pmatrix} \]

\[ Q=+1 \begin{pmatrix}
G_{(\Lambda p)(\Lambda p)}^{SLJ} & G_{(\Lambda p)(\Sigma^0 p)}^{SLJ} & G_{(\Lambda p)(\Sigma^- n)}^{SLJ} \\
G_{(\Sigma^0 p)(\Lambda p)} & G_{(\Sigma^0 p)(\Sigma^0 p)} & G_{(\Sigma^0 p)(\Sigma^- n)} \\
G_{(\Sigma^- n)(\Lambda p)} & G_{(\Sigma^- n)(\Sigma^0 p)} & G_{(\Sigma^- n)(\Sigma^- n)}
\end{pmatrix} \]

\[ Q=-1 \begin{pmatrix}
G_{(\Sigma^- n)(\Sigma^- n)}^{SLJ}
\end{pmatrix} \]

\[ Q = +2 \begin{pmatrix}
G_{(\Sigma^+ p)(\Sigma^+ p)}^{SLJ}
\end{pmatrix} \]
Hyperon single-particle potentials

- obtained by using YN, YY forces from QCD.
- Results are compatible with experimental suggestion.

\[ U_\Lambda^{\text{Exp}}(0) \approx -30, \quad U_\Xi(0)^{\text{Exp}} \approx -10, \quad U_\Sigma^{\text{Exp}}(0) \geq +20 \quad \text{[MeV]} \]

attraction
attraction small
repulsion
Chemical potentials

- Density dependence of chemical pot. of \( n \) and \( Y \) in PNM.
  \[
  \mu_n(\rho) = \frac{k_F^2}{2M} + U_n(\rho; k_F), \quad \mu_Y(\rho) = M_Y - M_N + U_Y(\rho; 0)
  \]
- Hyperon appear as \( n \to Y^0 \) if \( \mu_n > \mu_{Y^0} \)
  \[
  nn \to pY^- \quad \text{if} \quad 2\mu_n > \mu_p + \mu_{Y^-}
  \]

[ T. Inoue ]
Hyperon onset (just for a demonstration)

- First, \(\Sigma^-\) appear at 2.9 \(\rho_0\). Next, \(\Lambda\) appear at 3.3 \(\rho_0\).
- NS matter is not PNM especially at high density.
- We should compare with more sophisticated \(\mu_n\) and \(\mu_p\).
- P-wave YN force may be important at high density.
Hyperon onset (just for a demonstration)

- “NSM” is matter w/ n, p, e, μ under β-eq and $Q=0$.
Summary

- Hadron forces: Bridge between particle/nuclear/astro-physics
- HAL QCD method crucial for a reliable calculation
  - Direct method suffers from excited state contaminations
- The 1st LQCD for Baryon Interactions at ~ phys. point
  - $m(\pi) \sim 145 \text{ MeV}$, $L \sim 8 \text{ fm}$, $1/a \sim 2.3 \text{ GeV}$
  - Central/Tensor forces for NN/YN/YY in $P=(+)$ channel

Prospects

- Exascale computing Era ~ 2020
- LS-forces, $P=(-)$ channel, 3-baryon forces, etc., & EoS