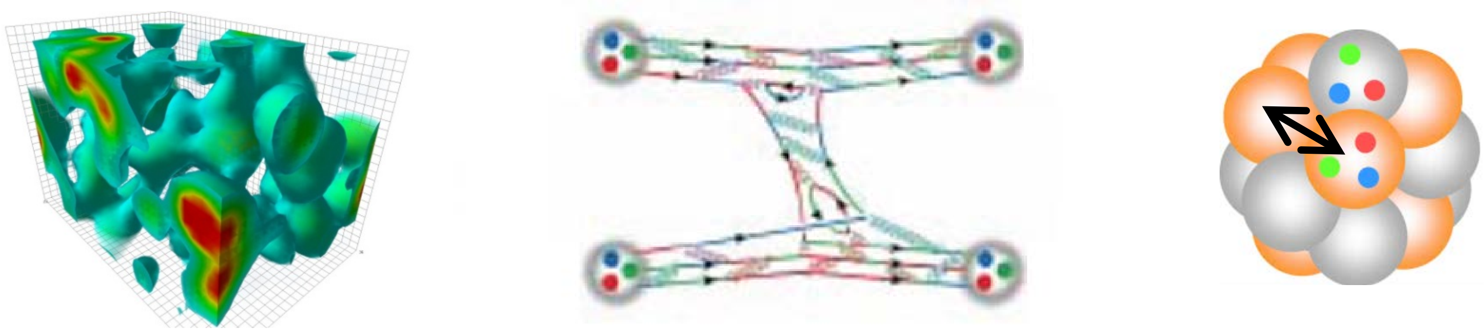
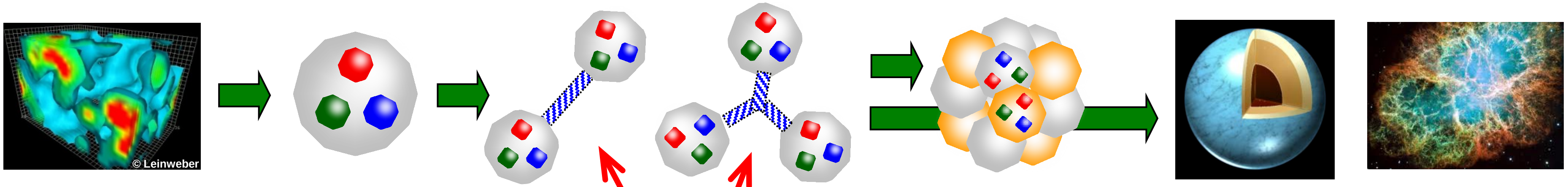
Baryon-Baryon Interactions from Lattice QCD

Takumi Doi
(Nishina Center, RIKEN)



The Odyssey from Quarks to Universe



QCD vacuum

Baryons

Nuclei

Neutron Stars / Supernovae
Nucleosynthesis

QCD

1st-principle Lattice QCD

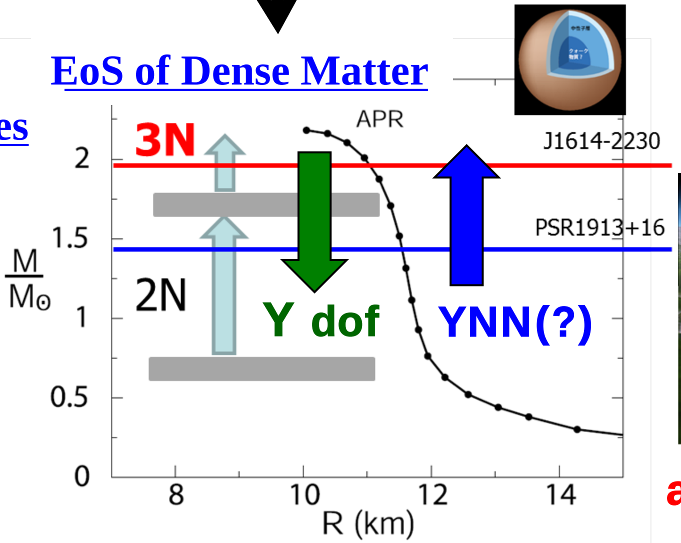
Baryon Forces

ab-initio nuclear calc.

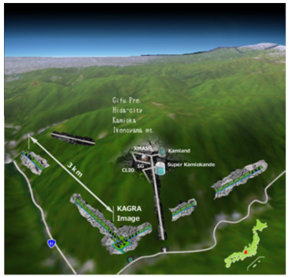
Nuclear Forces / Hyperon Forces

J-PARC

EoS of Dense Matter



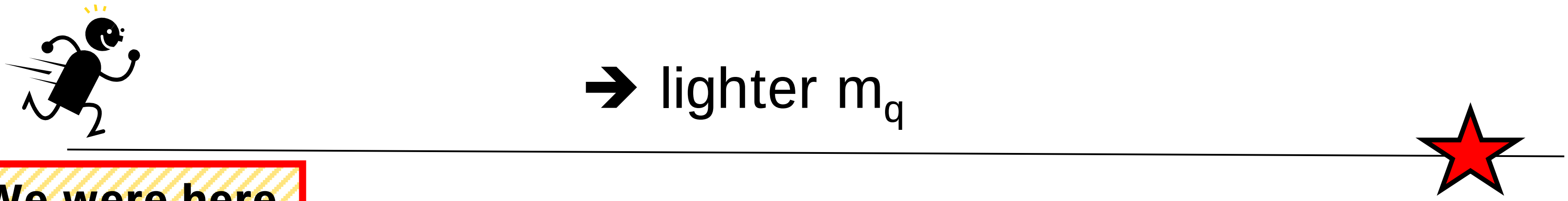
RIBF/FRIB



aLIGO/KAGRA

NS-NS merger

The Odyssey from unphysical to physical quark masses



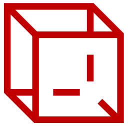
~2010

➔ lighter m_q

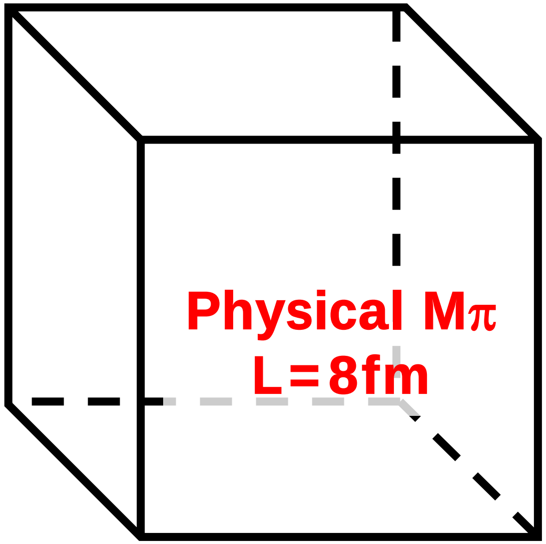
We were here

Phys. point

Mπ=0.8 GeV
L=2fm



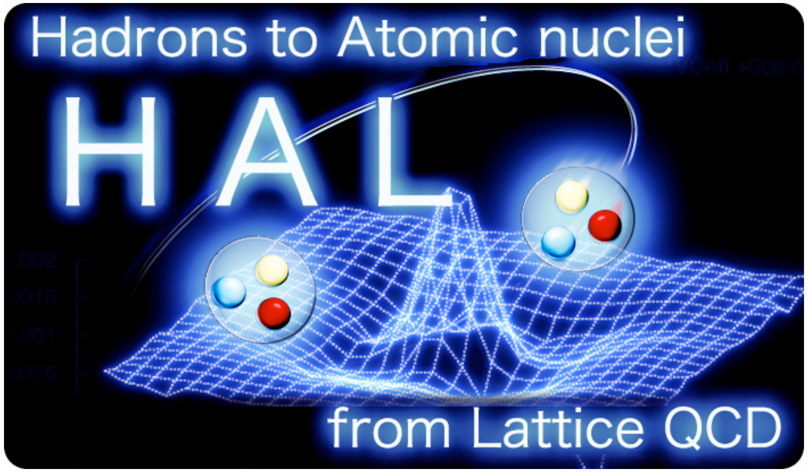
K-computer



Physical Mπ
L=8fm

HPCI Strategic Program Field 5
“The origin of matter and the universe”
FY2010-15

Hadrons to **A**tomic nuclei from **L**attice QCD
(**HAL** QCD Collaboration)

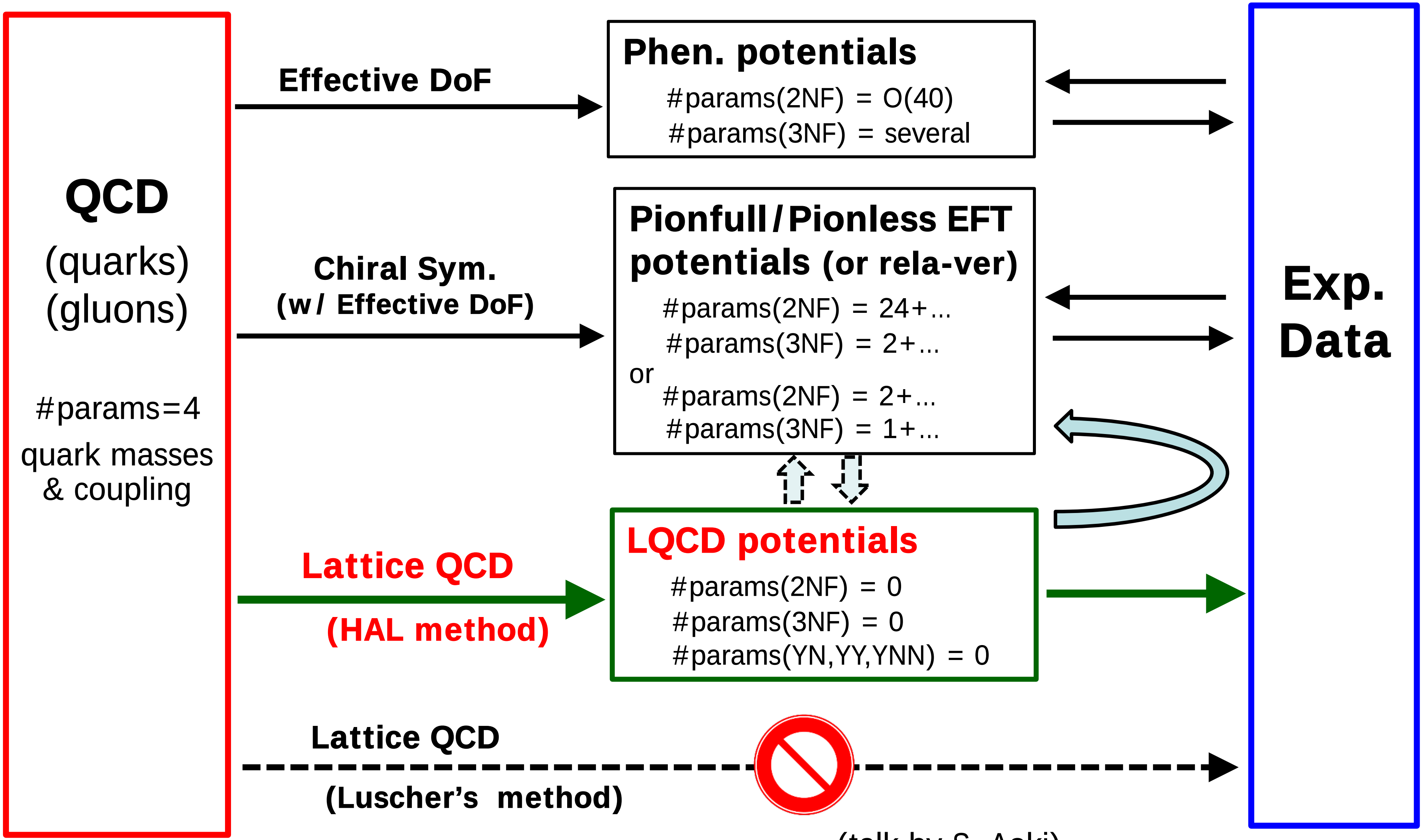


S. Aoki, D. Kawai,
T. Miyamato, K. Sasaki (YITP)
T. Doi, T. Hatsuda, T. Iritani (RIKEN)
F. Etminan (Univ. of Birjand)
S. Gongyo (Univ. of Tours)
Y. Ikeda, N. Ishii, K. Murano (RCNP)
T. Inoue (Nihon Univ.)
H. Nemura (Univ. of Tsukuba)

「20XX**年宇宙の旅**」

from Quarks to Universe

Various Theoretical methods



QCD

(quarks)
(gluons)

#params=4
quark masses
& coupling

Effective DoF

Phen. potentials

#params(2NF) = O(40)
#params(3NF) = several

Chiral Sym.
(w/ Effective DoF)

Pionfull / Pionless EFT potentials (or rela-ver)

#params(2NF) = 24+...
#params(3NF) = 2+...
or
#params(2NF) = 2+...
#params(3NF) = 1+...

Lattice QCD

(HAL method)

LQCD potentials

#params(2NF) = 0
#params(3NF) = 0
#params(YN,YY,YNN) = 0

Lattice QCD

(Luscher's method)

Exp. Data

(talk by S. Aoki)

- **Outline**
  - ~~Introduction~~
  - Theoretical framework
    - Direct method (Luscher’s method)
    - HAL QCD method
  - Challenges for multi-body systems on the lattice
  - Reliability test of LQCD methods
  - Results at heavy quark masses
  - Results at physical quark masses
  - Summary / Prospects

Interactions on the Lattice

- Direct method (Luscher's method)
  - Phase shift & B.E. from temporal correlation in finite V

M.Luscher, CMP104(1986)177
CMP105(1986)153
NPB354(1991)531

- HAL QCD method
  - "Potential" from spacial (& temporal) correlation
  - Phase shift & B.E. by solving Schrodinger eq in infinite V

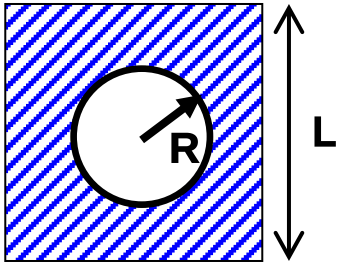
Ishii-Aoki-Hatsuda, PRL99(2007)022001, PTP123(2010)89
HAL QCD Coll., PTEP2012(2012)01A105

Luscher's formula: Scatterings on the lattice

- Consider Schrodinger eq at asymptotic region

$$(\nabla^2 + k^2)\psi_k(\boldsymbol{r}) = mV_k(\boldsymbol{r})\psi_k(\boldsymbol{r})$$

$$V_k(\boldsymbol{r}) = 0 \text{ for } r > R$$



  - (periodic) Boundary Condition in finite V ➔ constraint on energies of the system
  - Energy E ⇔ phase shift (at E)

$$k \cot \delta_{\mathbf{E}} = \frac{2}{\sqrt{\pi}L} Z_{00}(1; q^2), \quad q = \frac{kL}{2\pi}, \quad E = 2\sqrt{m^2 + k^2}$$

Large V: $\Delta E = E - 2m = -\frac{4\pi \mathbf{a}}{mL^3}\left[1 + c_1 \frac{a}{L} + c_2 \left(\frac{a}{L}\right)^2 + \mathcal{O}(\frac{1}{L^3})\right]$

- Calculate the energy spectrum of NN on (finite V) lattice
  - Temporal correlation in Euclidean time ➔ energy

$$G(t) = \langle 0|\mathcal{O}(t)\overline{\mathcal{O}}(0)|0|\rangle = \sum_n A_n e^{-E_n t} \to A_0 e^{-E_0 t} \quad (t \to \infty)$$

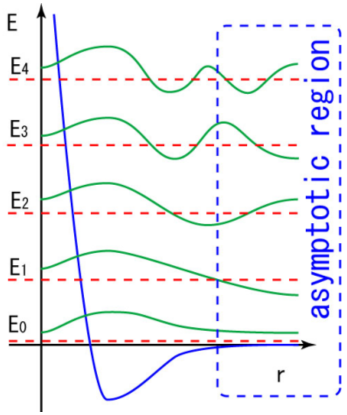
[HAL QCD method]

- “Potential” defined through phase shifts (S-matrix)

- Nambu-Bethe-Salpeter (NBS) wave function

$$\boxed{\psi(\vec{r}) = \langle 0|N(\vec{x}+\vec{r})N(\vec{x})|N(k)N(-k);W\rangle}$$

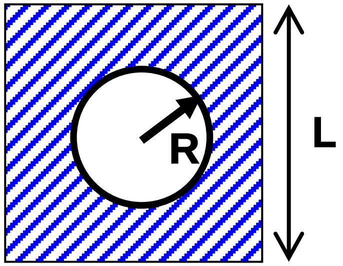
$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \qquad r > R \qquad\qquad W = 2\sqrt{m^2+k^2}$$



  - Wave function ←→ phase shifts

$$\psi(r) \simeq A\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

(below inelastic threshold)



Extended to multi-particle systems

M.Luscher, NPB354(1991)531
C.-J.Lin et al., NPB619(2001)467
CP-PACS Coll., PRD71(2005)094504

Ishizuka, Pos LAT2009 (2009) 119
Aoki-Hatsuda-Ishii PTP123(2010)89
S.Aoki et al., PRD88(2013)014036

(44)

Asymptotic form of BS wave function

[C.-J.D.Lin et al., NPB619,467(2001)]

For simplicity, we consider BS wave function of two pions

$$\psi_{\vec q}(\vec x) \equiv \langle 0 | N(\vec x)\; N(\vec 0) | N(\vec q) N(-\vec q), in \rangle$$

complete set

$$1 = \int \frac{d^3 p}{(2\pi)^3 2E_N(\vec p)} |N(\vec p)\rangle\langle N(\vec p)| + \cdots$$

$$= \int \frac{d^3 p}{(2\pi)^3 2E_N(\vec p)} \langle 0 | N(\vec x) | N(\vec p)\rangle \langle N(\vec p) | N(\vec 0) | N(\vec q) N(-\vec q), in \rangle + I(\vec x)$$

$Z^{1/2} e^{i\vec p\cdot\vec x}$

$$disc. + Z^{1/2} \frac{T(\vec p;\vec q)}{m_N^2 - (2E_N(\vec q) - E_N(\vec p))^2 + \vec p^2 - i\varepsilon}$$

$$= Z\left(e^{i\vec q\cdot\vec x} + \frac{1}{(2\pi)^3} \int \frac{d^3 p}{2E_N(\vec p)} \frac{T(\vec p;\vec q)}{4E_N(\vec q)\cdot(E_N(\vec p) - E_N(\vec q) - i\varepsilon)} e^{i\vec p\cdot\vec x} \right)$$

Integral is dominated by the on-shell contribution $E_N(\vec p) \approx E_N(\vec q)$

➔ T-matrix becomes the on-shell T-matrix

$$T^{(s-wave)}(s) = \frac{E(\vec q)}{2|\vec q|}(-i)\left(e^{2i\delta_0(s)} - 1\right)$$

$$= Z\left(e^{i\vec q\cdot\vec x} + \frac{1}{2i}\left(e^{2i\delta_0(s)} - 1\right)\frac{e^{iqr}}{qr} \right) + \cdots$$

The asymptotic form

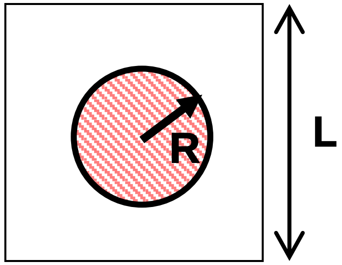
$$\boxed{\psi_{\vec q}(\vec x) = Z e^{i\delta_0(s)} \frac{\sin(qr + \delta_0(s))}{qr} + \cdots \quad \text{(s-wave)}}$$

This is analogous
to a non-rela. wave function

“Potential” as a representation of S-matrix

- Consider the wave function at “interacting region”

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi(\boldsymbol{r}'), \quad r < R$$



Probe interactions in “direct” way

- – U(r,r’): faithful to the phase shift by construction
  - U(r,r’): NOT an observable, but well defined
  - U(r,r’): E-independent, while non-local in general

Proof of Existence of E-independent potential

$V_W(\boldsymbol{r})\psi_W(\boldsymbol{r}) = (E_W - H_0)\psi_W(\boldsymbol{r})$ **[START] local but E-dep pot. (L³xL³ dof)**

- We consider the linear-indep wave functions and define

$$\mathcal{N}_{W_1 W_2} = \int d\boldsymbol{r}\,\overline{\psi_{W_1}(\boldsymbol{r})}\psi_{W_2}(\boldsymbol{r})$$

- We define the non-local potential

$$U(\boldsymbol{r},\boldsymbol{r}') = \sum_{W_1,W_2}^{W_{\rm th}} (E_{W_1} - H_0)\psi_{W_1}(\boldsymbol{r})\mathcal{N}^{-1}_{W_1 W_2}\overline{\psi_{W_2}(\boldsymbol{r}')}$$

- The above potential trivially satisfy Schrodinger eq.

$$\begin{aligned}
\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi_W(\boldsymbol{r}') &= \int d\boldsymbol{r}' \sum_{W_1,W_2}^{W_{\rm th}} (E_{W_1} - H_0)\psi_{W_1}(\boldsymbol{r})\mathcal{N}^{-1}_{W_1 W_2}\overline{\psi_{W_2}(\boldsymbol{r}')}\psi_W(\boldsymbol{r}') \\
&= \sum_{W_1,W_2}^{W_{\rm th}} (E_{W_1} - H_0)\psi_{W_1}(\boldsymbol{r})\mathcal{N}^{-1}_{W_1 W_2}\mathcal{N}_{W_2 W} \\
&= (E_W - H_0)\psi_W(\boldsymbol{r})
\end{aligned}$$

Intuitive understanding

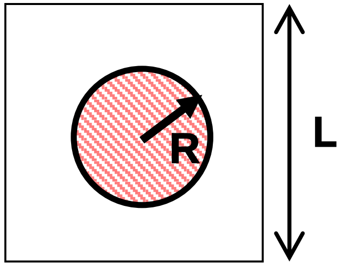
[GOAL] non-local but E-indep pot. (L³xL³ dof)

c.f. Krolikowski-Rzewuski, Nuovo Cimento, 4, 1212 (1956)

“Potential” as a representation of S-matrix

- Consider the wave function at “interacting region”

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m \int d\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}')\psi(\boldsymbol{r}'), \quad r < R$$



Probe interactions in “direct” way

- U(r,r’): faithful to the phase shift by construction
  - U(r,r’): NOT an observable, but well defined
  - U(r,r’): E-independent, while non-local in general
- Phase shifts at all E (below inelastic threshold) obtained by solving Scrodinger eq in infinite V
  - Non-locality ➔ derivative expansion

Okubo-Marshak(1958)

$$U(\vec{r}, \vec{r}') = \underset{\text{LO}}{V_c(r)} + \underset{\text{LO}}{S_{12}V_T(r)} + \underset{\text{NLO}}{\vec{L}\cdot\vec{S}V_{LS}(r)} + \underset{\text{NNLO}}{\mathcal{O}(\nabla^2)}$$

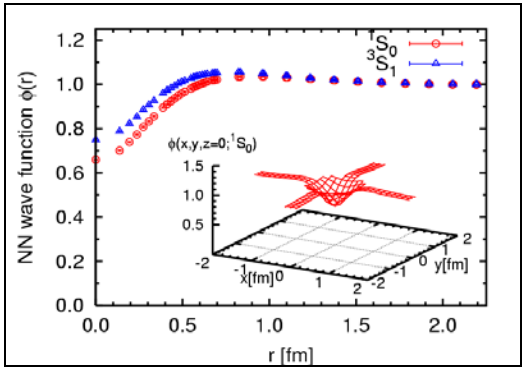
Check on convergence: K.Murano et al., PTP125(2011)1225

Control the E-dependence of phase shifts

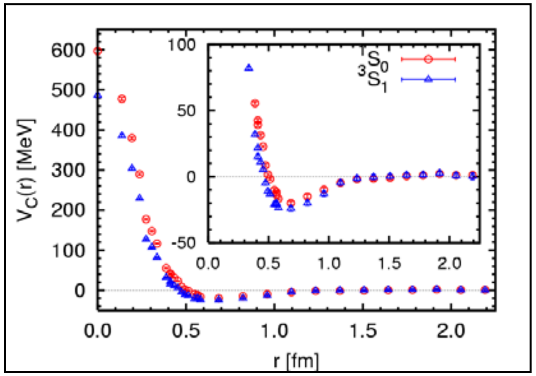
HAL QCD method

Lattice QCD

NBS wave func.



Lat Nuclear Force



$$\psi_{NBS}(\vec{r}) = \langle 0|N(\vec{r})N(\vec{0})|N(\vec{k})N(-\vec{k}), in\rangle$$
$$\simeq A_k \sin(kr - l\pi/2 + \delta_l(k))/(kr)$$

(at asymptotic region)

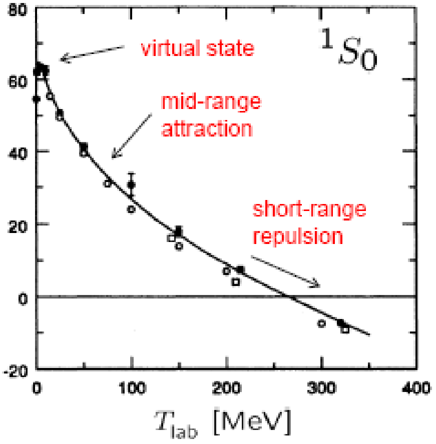
$$(k^2/m_N - H_0)\, \psi(\vec{r}) = \int d\vec{r}'\, U(\vec{r},\vec{r}')\psi(\vec{r}')$$

E-indep (& non-local) Potential:
Faithful to phase shifts

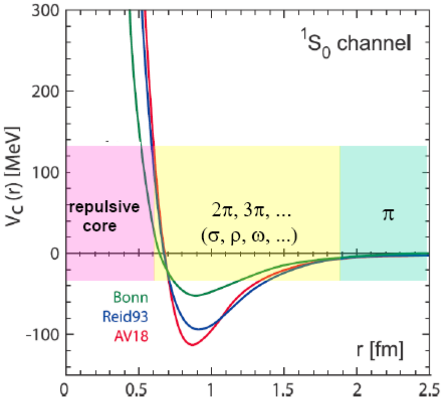
Analog to …

Scattering Exp.

Phase shifts



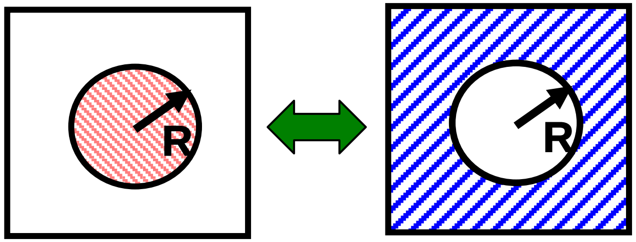
Phen. Potential



A few remarks on the Lattice Potential

- Potential is NOT an observable and is NOT unique:
  They are, however, phase-shift equivalent potentials
  - Choosing the pot. ←→ choosing the “scheme” (sink op.)

- Potential approach has some benefits:
  - Convenient to understand physics
  - Many body systems (sign problem partially avoided)
  - Finite V artifact better under control
  - Excited states better under control
    - G.S. saturation NOT necessary
    - Coupled Channel Systems



Crucial for multi-body on Lat

- **Outline**
  - Introduction
  - Theoretical framework
  - Challenges for multi-body systems on the lattice
    - Signal/Noise Issue
    - Coupled Channel Systems
    - Computational Challenge
  - Reliability test of LQCD methods
  - Results at heavy quark masses
  - Results at physical quark masses
  - Summary / Prospects

Challenges in multi-baryons on the lattice

- Signal / Noise issue

Parisi, Lepage (1989)

  - G.S. saturation by t ➔ ∞ required in LQCD

$$G(r,t) = \langle 0|\mathcal{O}(r,t)\overline{\mathcal{O}}(0)|0|\rangle = \sum_n a_n \psi_n(r) e^{-E_n t} \xrightarrow[t\to\infty]{} a_0 \psi_0(r) e^{-E_0 t}$$

  - pion

$$\frac{\text{Signal}}{\text{Noise}} \sim \frac{\langle \pi(t)\pi(0)\rangle}{\sqrt{\langle \pi\pi(t)\pi\pi(0)\rangle}} \sim \frac{\exp(-m_\pi t)}{\sqrt{\exp(-2m_\pi t)}} \sim \boxed{\text{const.}}$$

  - nucleon

$$\frac{\text{Signal}}{\text{Noise}} \sim \frac{\langle N(t)\bar{N}(0)\rangle}{\sqrt{\langle |N(t)\bar{N}(0)|^2\rangle}} \sim \frac{\exp(-m_N t)}{\sqrt{\exp(-3m_\pi t)}} \sim \boxed{\exp[-(m_N - 3/2 m_\pi)t]}$$

$$\frac{\text{Signal}}{\text{Noise}} \sim \frac{\langle N^{\mathbf{A}}(t)\bar{N}^{\mathbf{A}}(0)\rangle}{\sqrt{\langle |N^{\mathbf{A}}(t)\bar{N}^{\mathbf{A}}(0)|^2\rangle}} \sim \frac{\exp(-\mathbf{A} m_N t)}{\sqrt{\exp(-3\mathbf{A} m_\pi t)}} \sim \boxed{\exp[-\mathbf{A}(m_N - 3/2 m_\pi)t]}$$

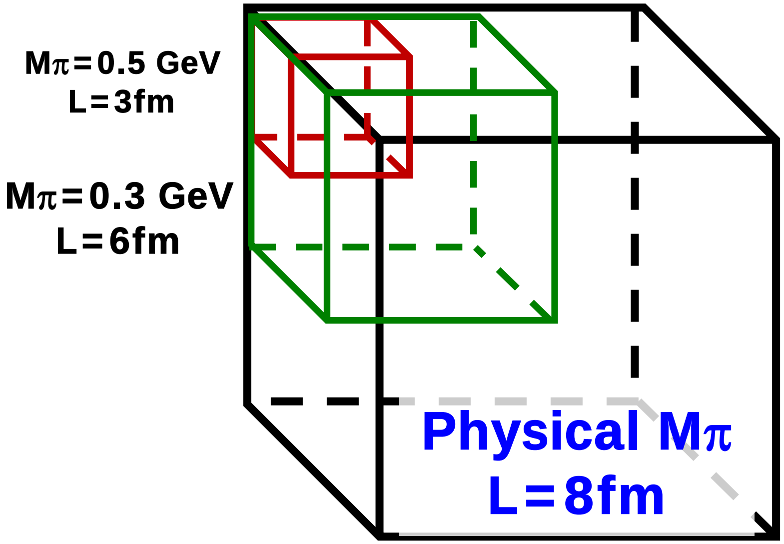
(for mass number = A)

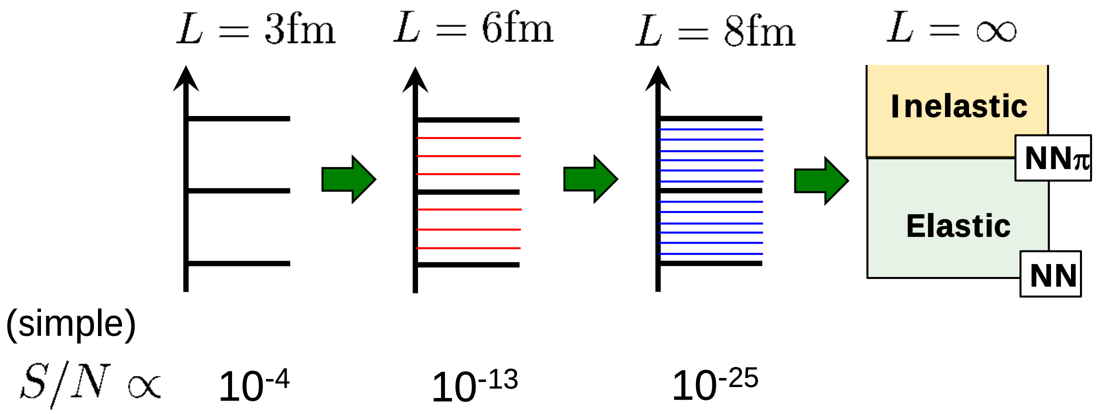
Challenges in multi-baryons on the lattice

- Excitation energy ~ binding energy or finite V effect

(very small)

$$E_1 - E_0 \simeq \frac{\vec{p}^2}{m_N} \simeq \frac{1}{m_N}\frac{(2\pi)^2}{L^2}$$





System w/o Gap

New Challenge for multi-body systems

(For both of Direct method / (old) HAL method)

Our solution

Time-dependent HAL method

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

E-indep of potential U(r,r') ➔ *(excited) scatt states share the same U(r,r')*

*They are **not contaminations**, **but signals***

Original (t-indep) HAL method

$$G_{NN}(\vec{r},t) = \langle 0|N(\vec{r},t)N(\vec{0},t)\overline{\mathcal{J}_{\rm src}(t_0)}|0\rangle$$

$$R(\boldsymbol{r},t) \equiv G_{NN}(\boldsymbol{r},t)/G_N(t)^2 = \sum_i A_{W_i}\psi_{W_i}(\boldsymbol{r})e^{-(W_i-2m)t}$$

← Many states contribute

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi_{W_0}(\boldsymbol{r}') = (E_{W_0} - H_0)\psi_{W_0}(\boldsymbol{r})$$

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi_{W_1}(\boldsymbol{r}') = (E_{W_1} - H_0)\psi_{W_1}(\boldsymbol{r})$$

...

New t-dep HAL method

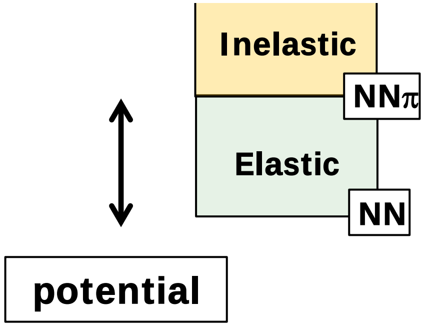
All equations can be combined as

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')R(\boldsymbol{r}',t) = \left(-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - H_0\right)R(\boldsymbol{r},t)$$

~~G.S. saturation~~ ➔ "Elastic state" saturation

[Exponential Improvement]

System w/ Gap

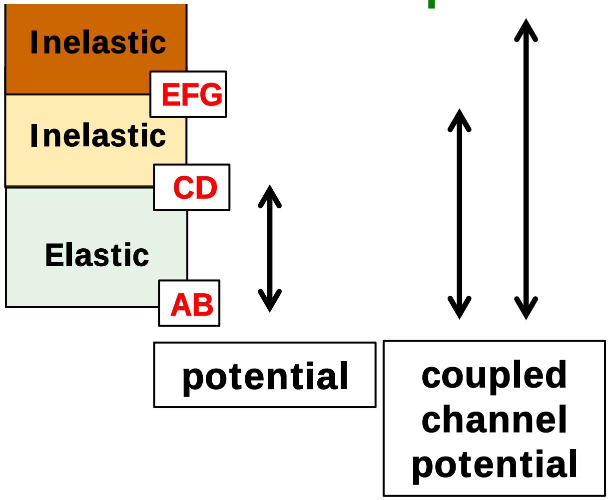


Coupled Channel systems

(beyond inelastic threshold)

- Essential in many interesting physics
  - Hyperon Forces (e.g., H-dibaryon (ΛΛ-NΞ-ΣΣ))
  - Exotic mesons, Resonances, etc. (e.g., Zc(3900))

➔ Coupled channel potentials in HAL method



$$\psi_{AB}(\boldsymbol{r},\boldsymbol{k}) = 1/\sqrt{Z_A Z_B}\cdot\langle 0|\phi_A(\boldsymbol{x}+\boldsymbol{r})\phi_B(\boldsymbol{x})|W\rangle$$

$$\psi_{CD}(\boldsymbol{r},\boldsymbol{q}) = 1/\sqrt{Z_C Z_D}\cdot\langle 0|\phi_C(\boldsymbol{x}+\boldsymbol{r})\phi_D(\boldsymbol{x})|W\rangle$$

$$W = \sqrt{m_A^2+k^2}+\sqrt{m_B^2+k^2} = \sqrt{m_C^2+q^2}+\sqrt{m_D^2+q^2}$$

$$(E_{k_i}^{AB} - H_0^{AB})\psi_{AB}(\boldsymbol{r},k_i) = \int d\boldsymbol{r}' U_{AB,AB}(\boldsymbol{r},\boldsymbol{r}')\psi_{AB}(\boldsymbol{r}',k_i) + \int d\boldsymbol{r}' U_{AB,CD}(\boldsymbol{r},\boldsymbol{r}')\psi_{CD}(\boldsymbol{r}',q_i)$$

$$(E_{q_i}^{CD} - H_0^{CD})\psi_{CD}(\boldsymbol{r},q_i) = \int d\boldsymbol{r}' U_{CD,AB}(\boldsymbol{r},\boldsymbol{r}')\psi_{AB}(\boldsymbol{r}',k_i) + \int d\boldsymbol{r}' U_{CD,CD}(\boldsymbol{r},\boldsymbol{r}')\psi_{CD}(\boldsymbol{r}',q_i)$$

S.Aoki et al. (HAL Coll.), Proc. Jpn. Acad. Ser. B87(2011)

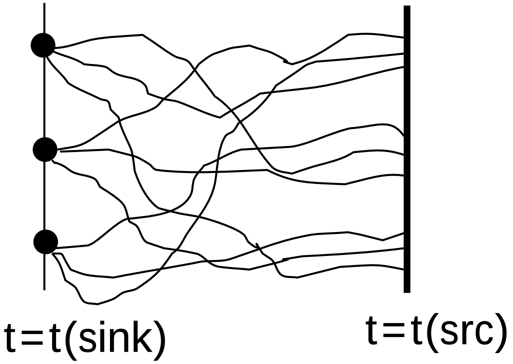
S.Aoki et al. (HAL Coll.), PRD87(2013)034512

Computational Challenge

- **Enormous comput. cost for multi-baryon correlators**
  - Wick contraction (permutations)

$$\sim [\,(\tfrac{3}{2}A)!\,]^2$$

(A: mass number)



  - color/spinor contractions

$$\sim 6^A \cdot 4^A \quad \text{or} \quad 6^A \cdot 2^A$$

See also T. Yamazaki et al., PRD81(2010)111504

  - **Unified Contraction Algorithm (UCA)** TD, M.Endres, CPC184(2013)117
    - A novel method which unifies two contractions

Permuted Sum

$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi'_1)\bar{q}(\xi'_2)\bar{q}(\xi'_3)\bar{q}(\xi'_4)\bar{q}(\xi'_5)\bar{q}(\xi'_6)(t_0)\rangle \times \mathrm{Coeff}^{2N}(\xi'_1,\cdots,\xi'_6)$$

Sum over color/spinor unified list

Drastic Speedup

$$\boxed{\times 192 \text{ for } {}^3\mathrm{H}/{}^3\mathrm{He},\ \times 20736 \text{ for } {}^4\mathrm{He},\ \times 10^{11} \text{ for } {}^8\mathrm{Be}}$$

(x add'l. speedup)

See also subsequent works: Detmold et al., PRD87(2013)114512
Gunther et al., PRD87(2013)094513

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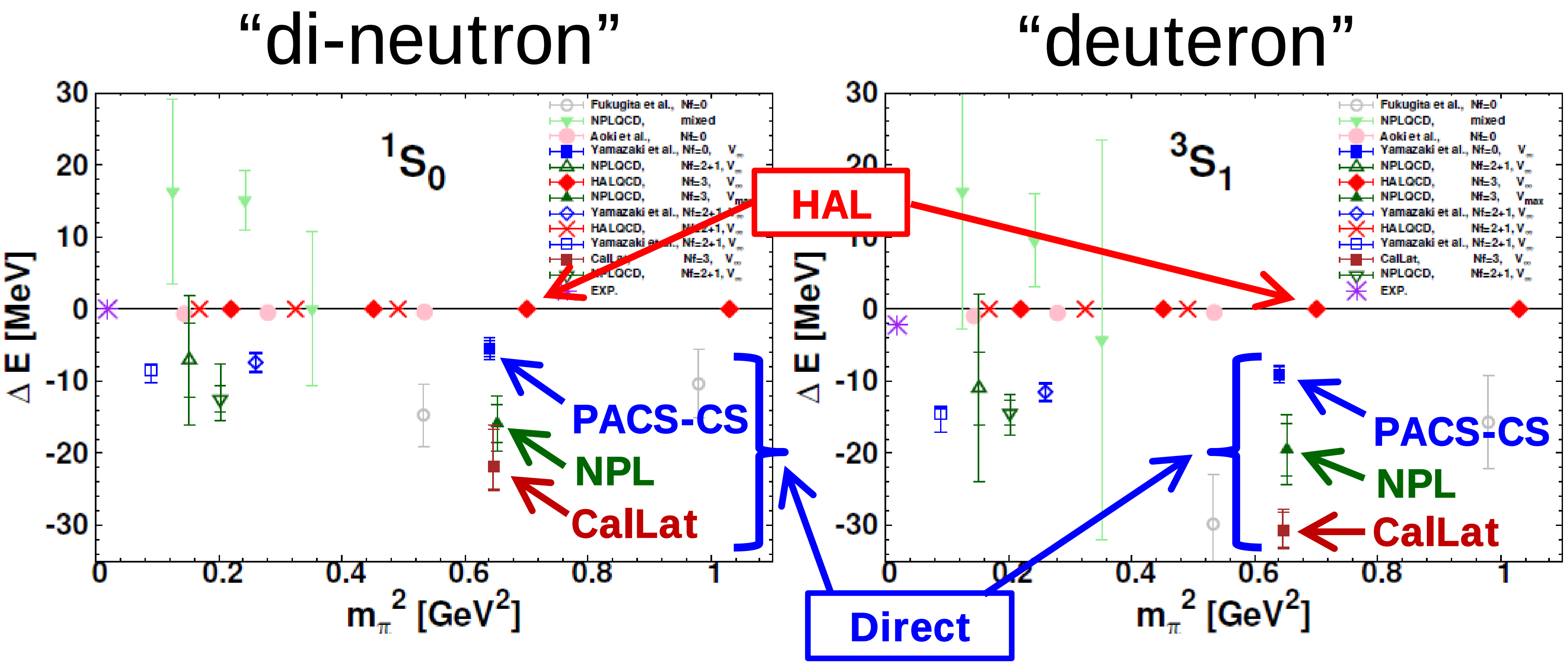
    - Coupled Channel Systems ➔ Coupled channel HAL potential
    - Computational Challenge ➔ Unified Contraction Algorithm
  - Reliability test of LQCD methods
    - Direct method & HAL method: Comparative study
  - Results at heavy quark masses ➔ Talk by S. Aoki
  - Results at physical quark masses
  - Summary / Prospects

Direct method vs HAL method

Reviewed in T.D. PoS LAT2012,009 (+ updates)



HAL method (HAL) : **unbound**

Direct method (PACS-CS (Yamazaki et al.) / NPL / CalLat): **bound**

c.f. I=2 pipi : Direct & HAL methods agree well Kurth et al., JHEP1312(2013)015

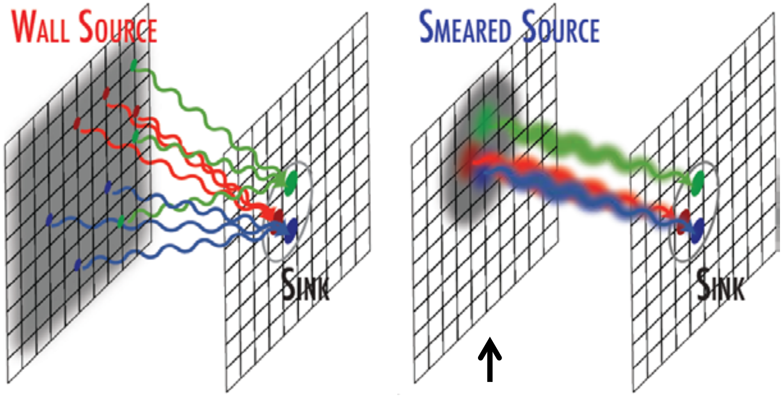
Reliability Test of LQCD methods

T. Iritani et al. (HAL), JHEP1610(2016)101

- **Employ the same config** used in previous Direct method study

YIKU2012 = T. Yamazaki et al. PRD86(2012)074514

  - **High statistics** (e.g., 48^4 smeared: x8 #stat of YIKU2012)
  - **Both of wall & smeared src setup**
    - smeared ➔ same as YIKU2012



6-quarks
w/ same center

- Nf=2+1 clover LQCD
  - $m_\pi = 0.51$GeV, $m_N = 1.32$GeV, $m_\Xi = 1.46$GeV, 1/a=2.2GeV (a=0.09fm)
  - L=2.9, 3.6, 4.3, 5.8 fm (32^3x48, 40^3x48, 48^3x48, 64^3x64)
  - NN ($^1S_0$), NN ($^3S_1$) & ΞΞ ($^1S_0$), ΞΞ ($^3S_1$)
    - N.B. ΞΞ ($^1S_0$) ~ flavor SU(3) partner of NN ($^1S_0$), but much better S/N

Check by source op. dependence

ΞΞ ($^1S_0$)

(L=4.3fm)

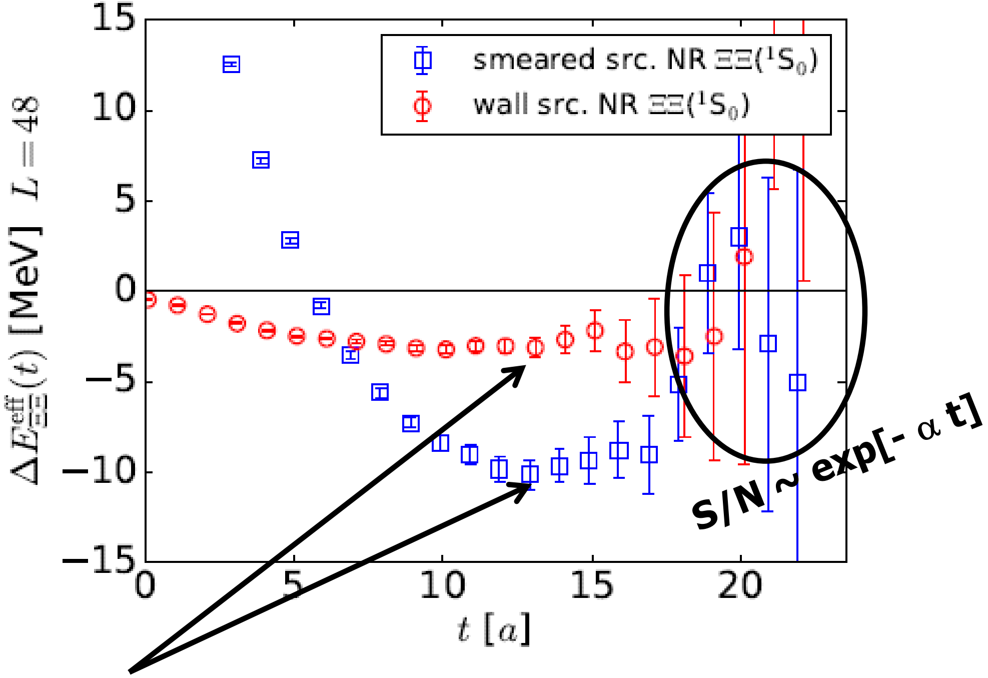
- **Examine the consistency** between **smeared** & **wall** source

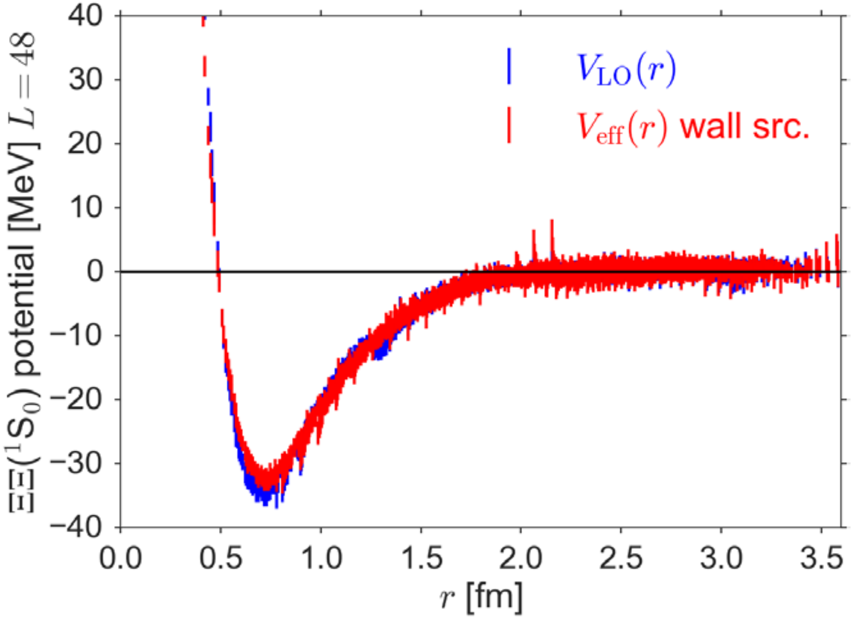
Luscher's method

(ΔE ➔ phase shift)

HAL method

(V(r) ➔ phase shift)





Inconsistent "signal" (red (wall) vs blue (smeared))

➔ cannot judge which (or neither) is reliable

$V^{\rm eff}$(r) from wall &

$V^{\rm LO}$(r) from wall+smeared

are consistent

FAILED

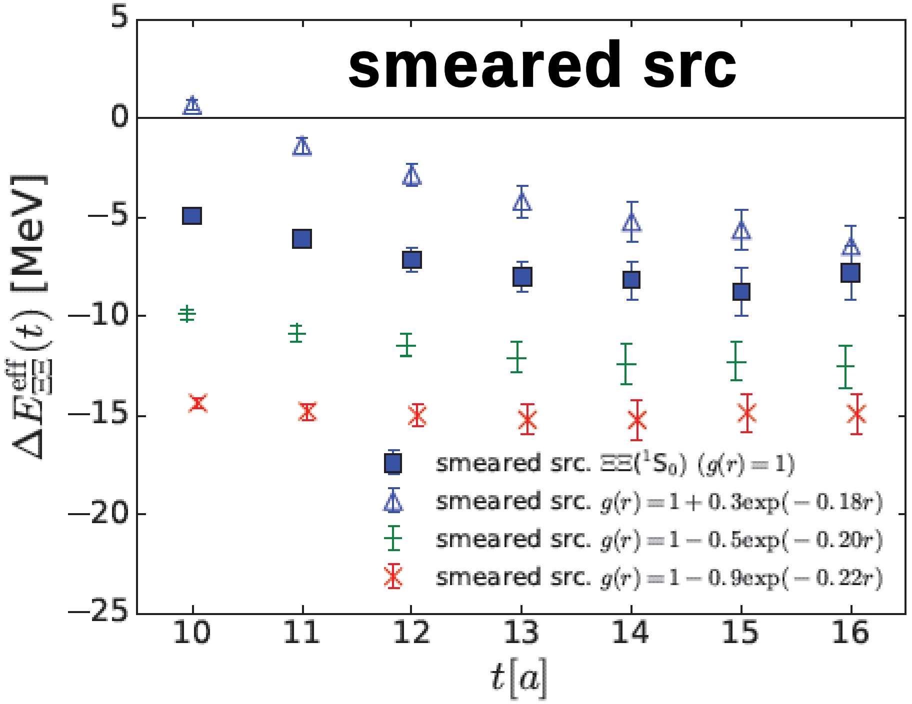
PASSED

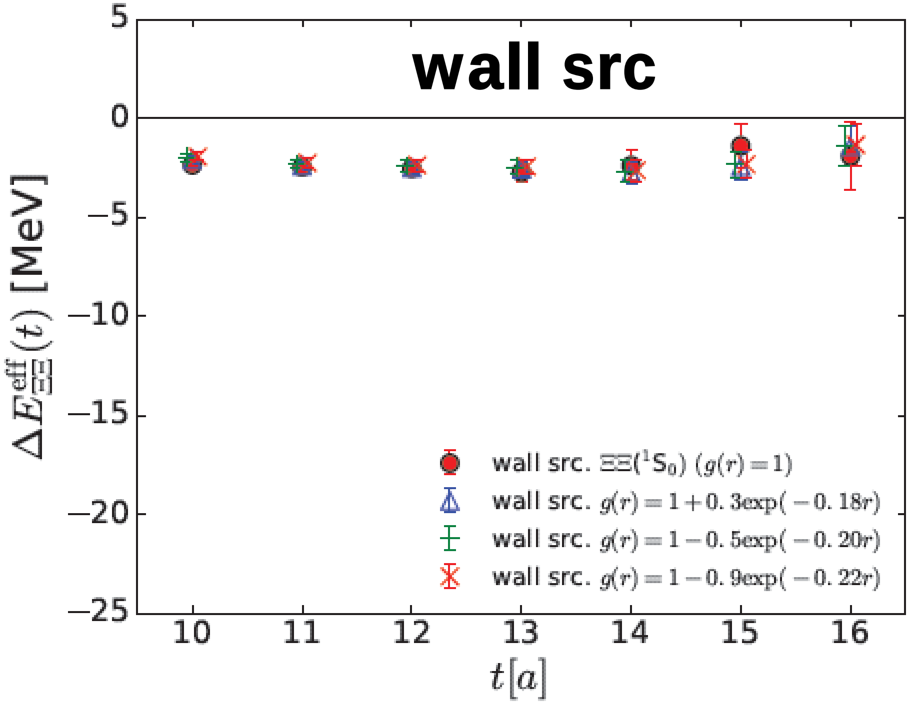
Check by sink op. dependence (for direct method)

Generalized Direct method (by generalized sink projection)

$$\tilde{R}^{(f)}(t) = \sum_{\vec{r}} f(\vec{r}) R(\vec{r}, t) = \sum_{\vec{r}} f(\vec{r}) \sum_{\vec{x}} \langle 0 | B(\vec{r} + \vec{x}, t) B(\vec{x}, t) \overline{\mathcal{J}_{\rm src}(0)} | 0 \rangle / \{G_B(t)\}^2$$

c.f. standard Direct method ←→ f(r)=1





Many inconsistent "plateaux"

➔ **Predictive power is LOST**

"smeared is better" is too naive

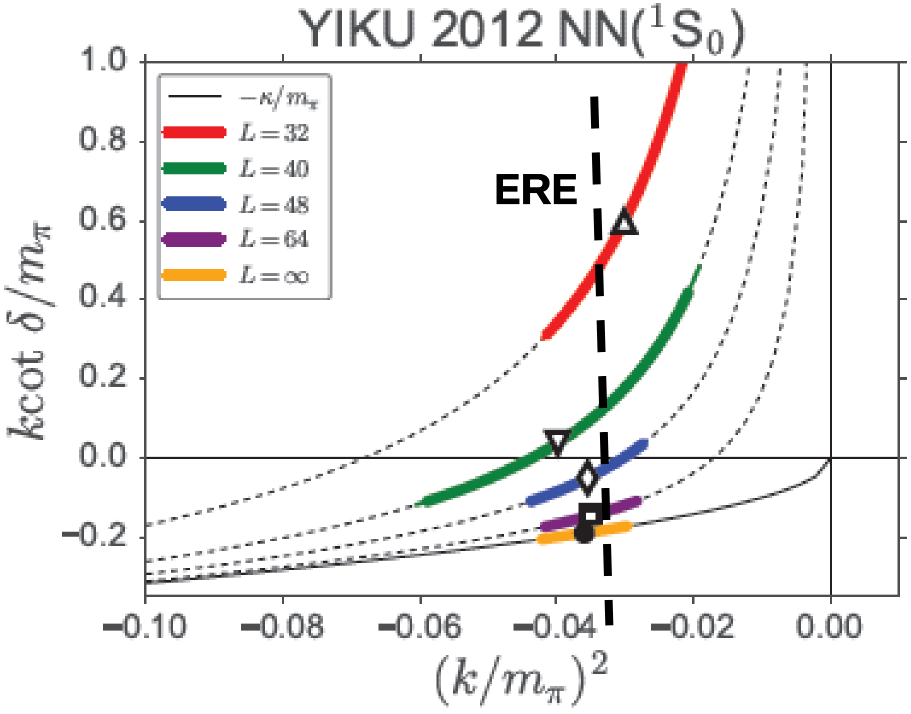
"Sanity Check" for results from direct method

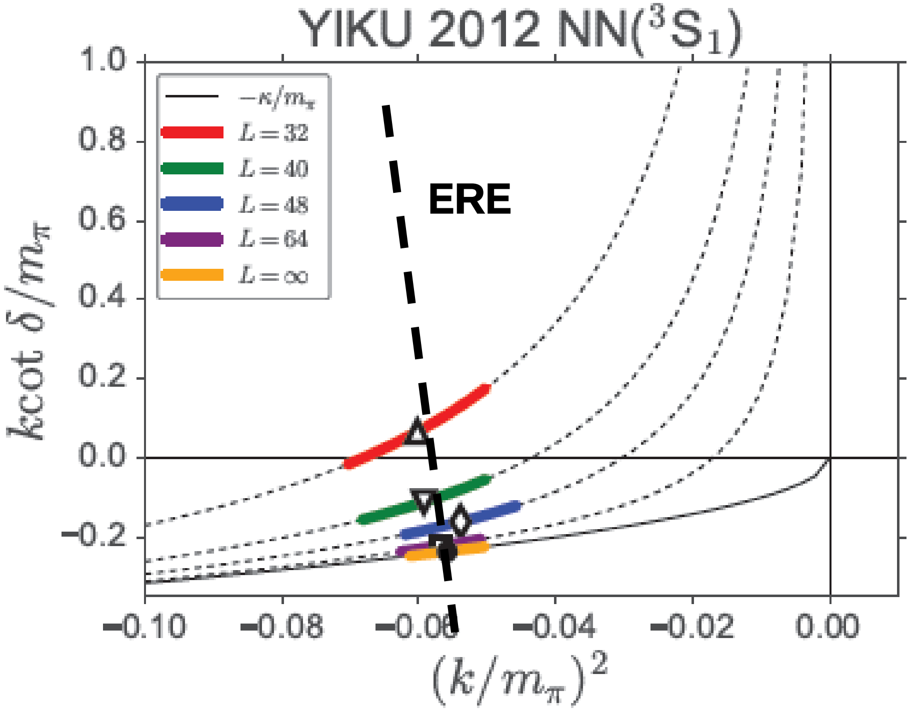
Aoki-Doi-Iritani, arXiv:1610.09763

$$\text{ERE:}\quad k\cot\delta(k) = \frac{1}{\mathbf{a}} + \frac{1}{2}\mathbf{r}\,k^2 + \cdots$$

If we examine the data from

T. Yamazaki et al. PRD86(2012)074514





singular behaviors

$$1/a \simeq -\infty$$

$$r \simeq -\infty$$

Manifestation of problem
in the direct method

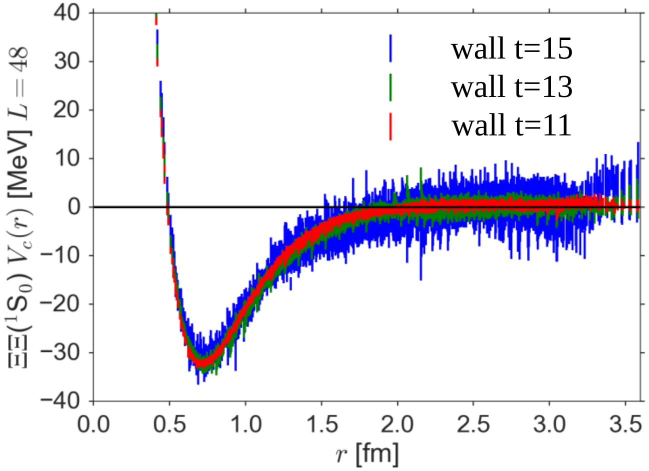
Check Table for NN (direct method)

	single baryon	double baryon				**Overall Verdict**
	plateau check	mirage plateau	src-dep check	sink-dep check	Effective Range expansion check	
YKU 2011	○	×	△	**Not checked**	×	**False**
YIKU 2012	○	×	×	×	×	**False**
YIKU 2015	○	×	**Not checked**	**Not checked**	×	**False**
NPL 2012	○	×	**Not checked**	**Not checked**	×	**False**
NPL 2013	○	×	**Not checked**	**Not checked**	△	**False**
NPL 2015	△	×	**Not checked**	**Not checked**	×	**False**

Anatomy of the direct method from HAL QCD potential

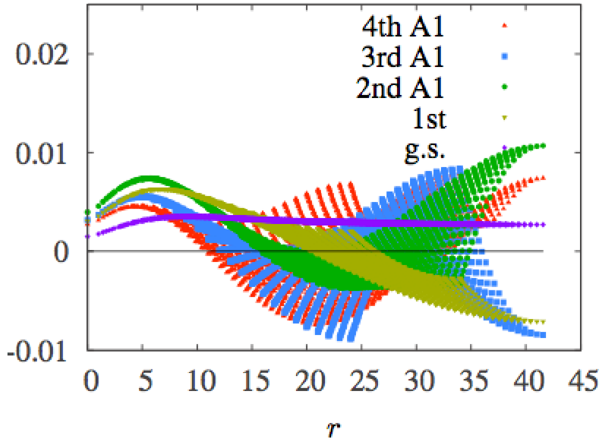
Understand the origin of “fake plateaux”

Potential



Solve Schrodinger eq.
in Finite V

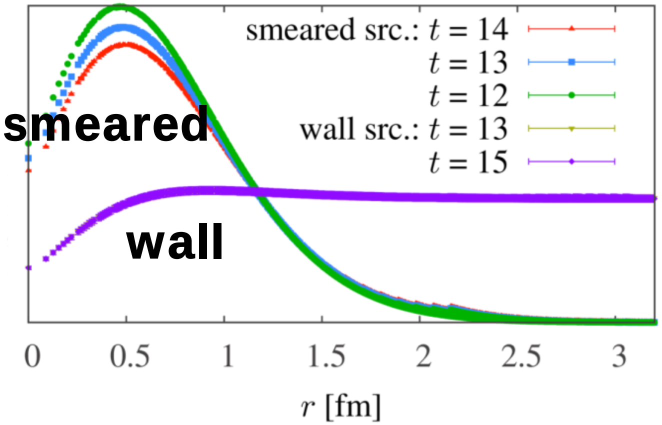
Eigen-wave functions



Eigen-energies

n-th A1	ΔE_n [MeV]
0	-2.58(1)
1	52.49(2)
2	112.08(2)
3	169.78(2)
4	224.73(1)

NBS correlator R(r,t)



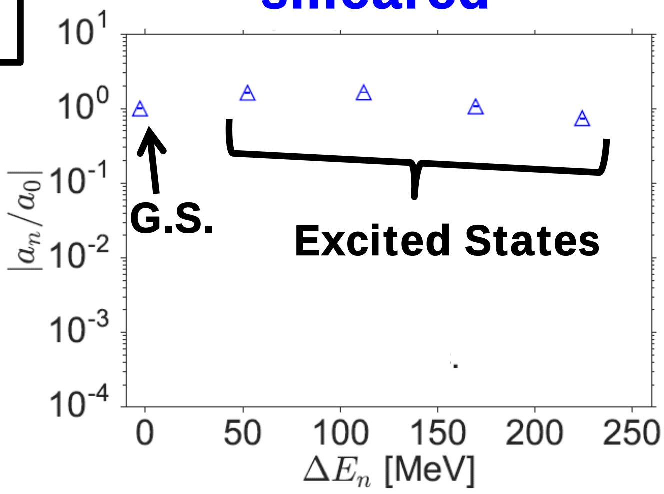
Decompose NBS correlator
to each eigenstates

NBS correlator R(r,t)

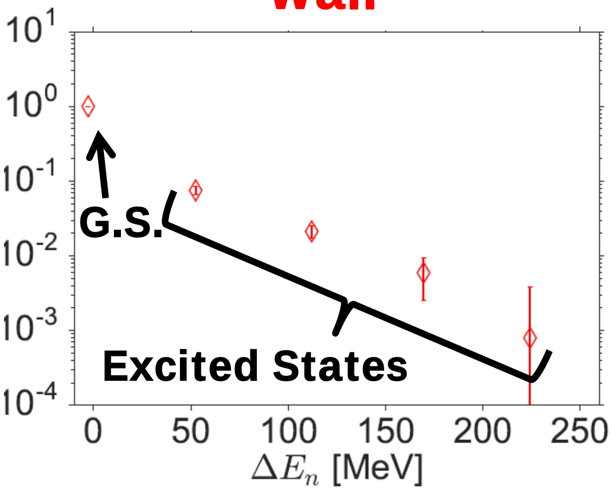
Contribution from each (excited) states (@ t=0)

Decompose NBS correlator to each eigenstates

smeared



wall



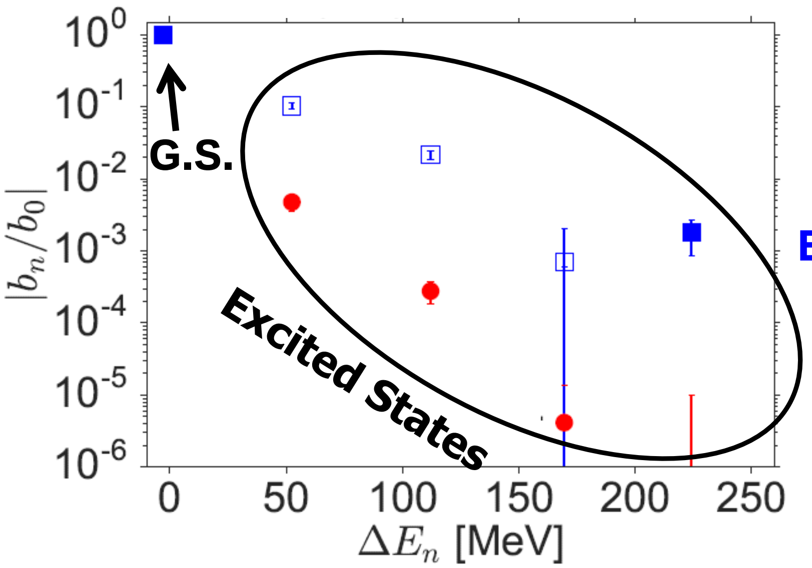
excited states NOT suppressed

excited states suppressed

Temporal-correlator $R(t) = \Sigma_r R(r,t)$

(R(t) w/ smeared has been used in Luscher's method)

Contribution from each (excited) states (@ t=0)

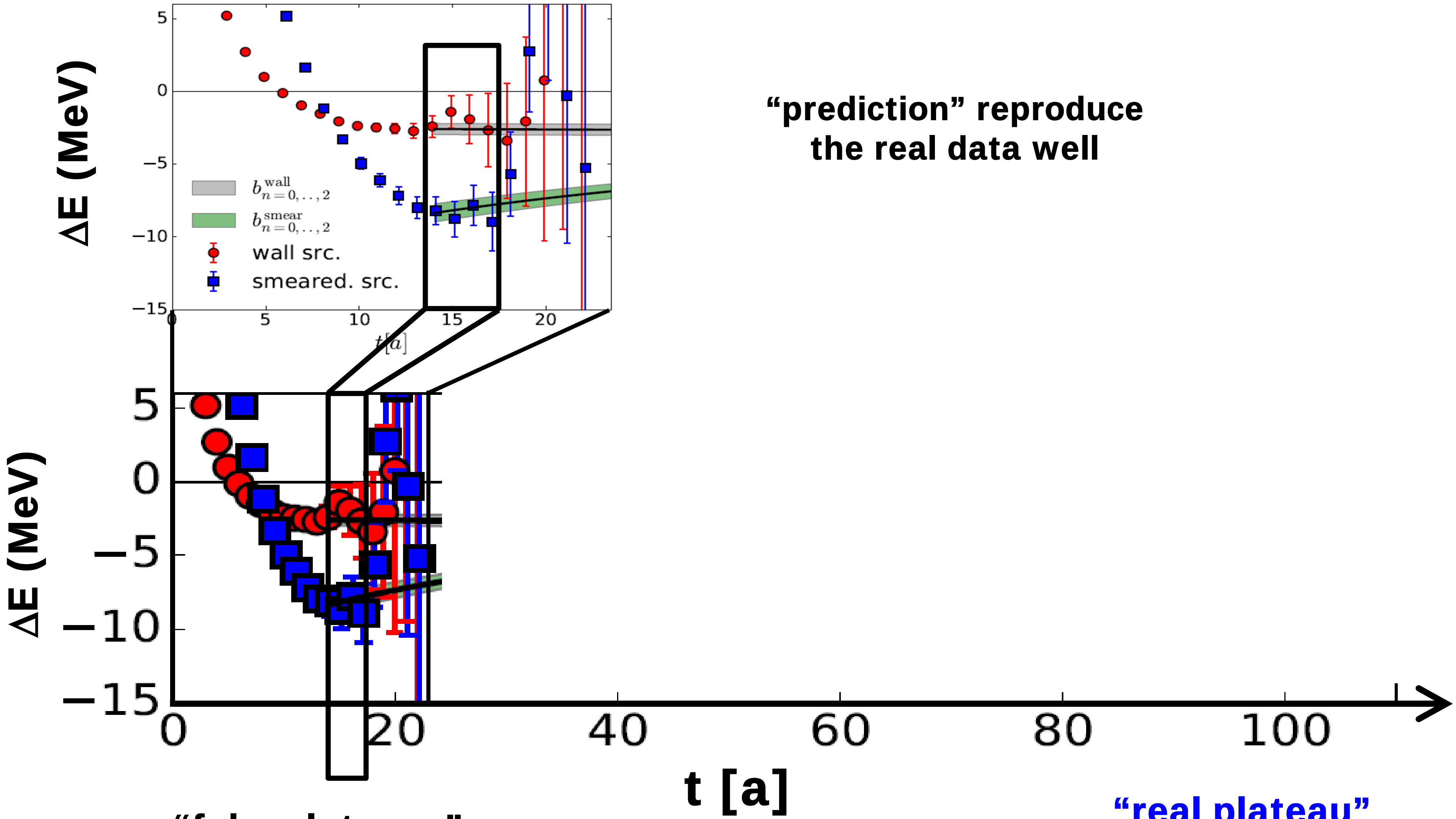


Blue: smeared

Red: wall

Understand the origin of "fake plateaux"

We are now ready to "predict" the behavior of m(eff) of ΔE at any "t"



"prediction" reproduce
the real data well

"fake plateaux"
at t ~ 1fm

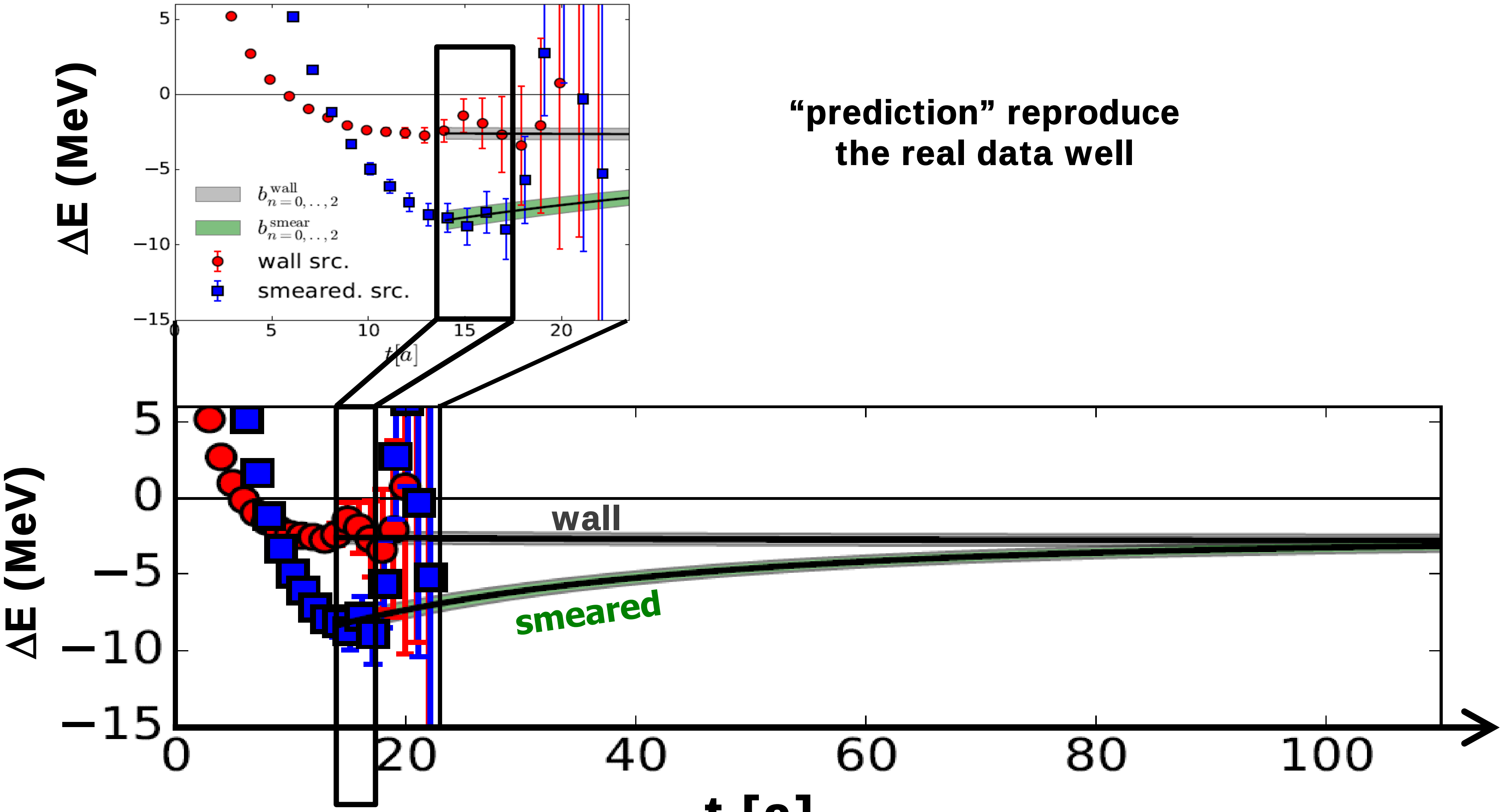
HAL method is crucial !

"real plateau"
at t ~ 10fm

(E_1-E_0=50MeV)

Understand the origin of “fake plateaux”

We are now ready to “predict” the behavior of m(eff) of ΔE at any “t”



“prediction” reproduce
the real data well

“fake plateaux”
at t ~ 1fm

HAL method is crucial !

“real plateau”
at t ~ 10fm

(E_1-E_0=50MeV)

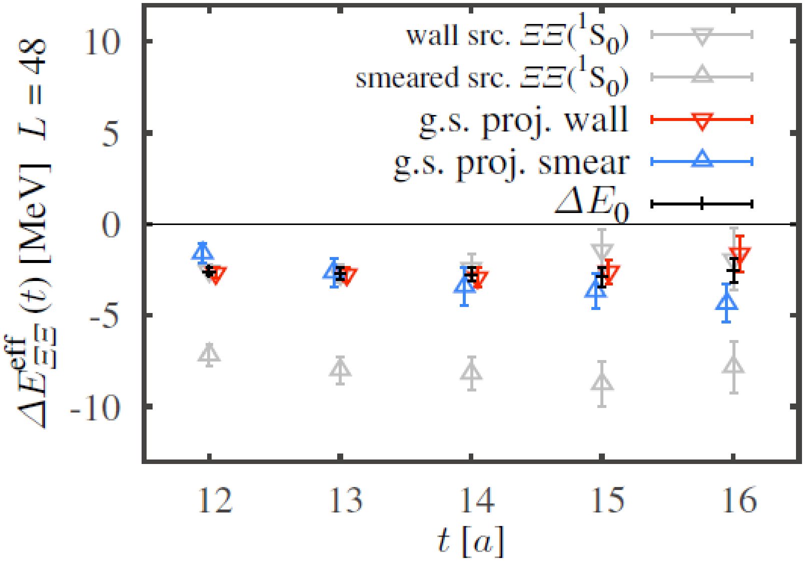
Direct method “educated by HAL method”

Generalized Direct method (by generalized sink projection)

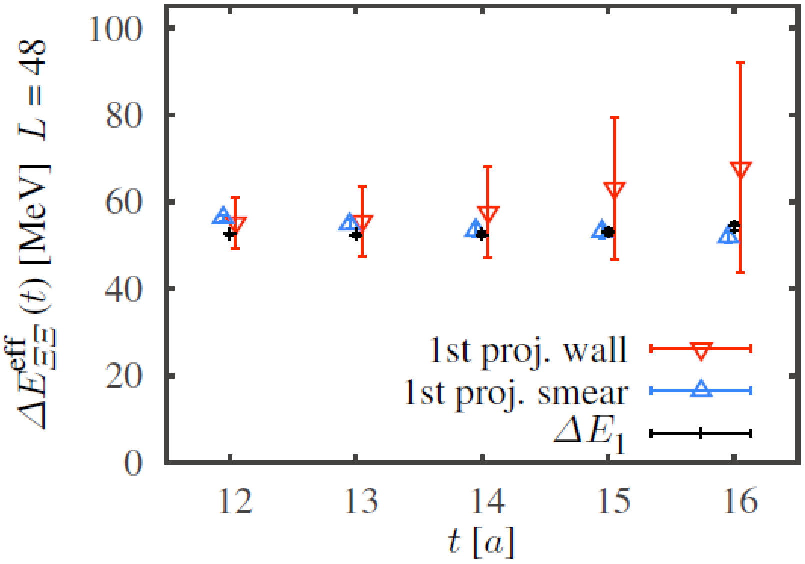
$$\tilde{R}^{(f)}(t) = \sum_{\vec{r}} f(\vec{r}) R(\vec{r},t) = \sum_{\vec{r}} f(\vec{r}) \sum_{\vec{x}} \langle 0|B(\vec{r}+\vec{x},t)B(\vec{x},t)\overline{\mathcal{J}_{\rm src}(0)}|0\rangle / \{G_B(t)\}^2$$

f(r) 🡐 eigen-wave func from HAL potential at finite V

f(r) = $\psi^{+}_{\rm G.S.}$(r)



f(r) = $\psi^{+}_{\rm 1st.}$(r)



ΔE : Direct (wall / smeared) = Potential (wall / smeared)

Direct method has (useful) ~~predictive power~~ postdictive power

Variational method could be helpful for direct method

- **Outline**
  - – Introduction
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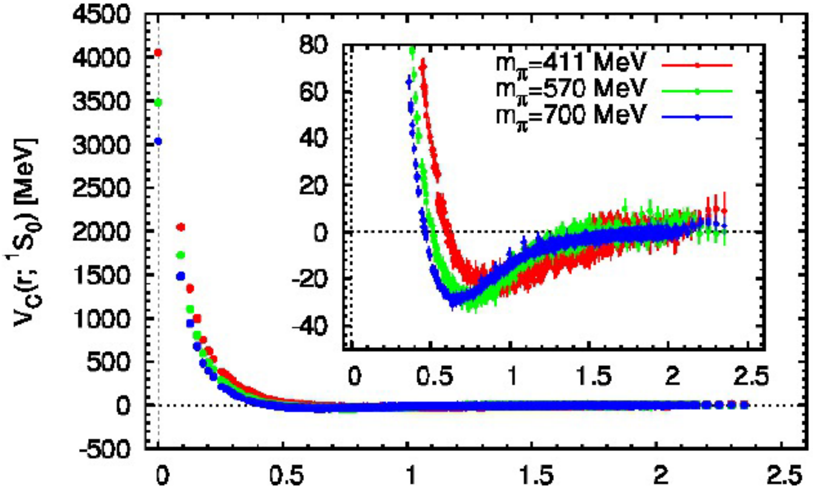
NN-forces (P=(+) channel)

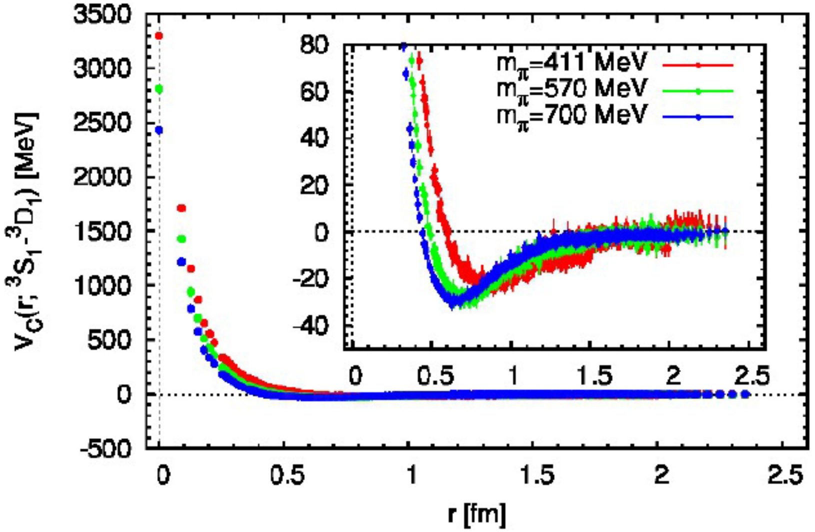
($m\pi$=0.41-0.70 GeV)

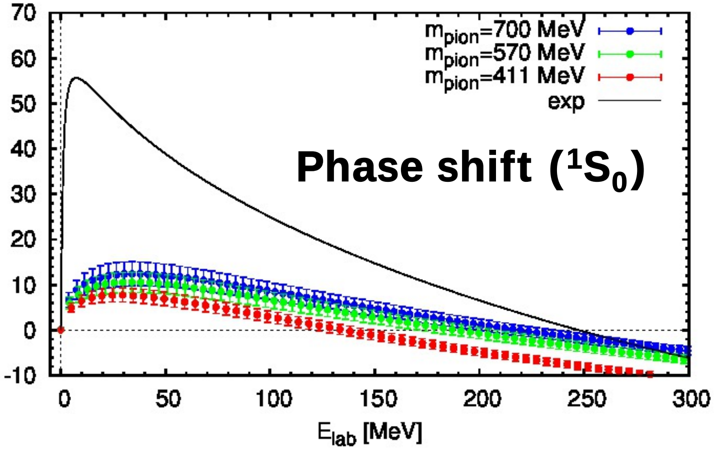
Central in $^1S_0$



$^3S_1$-$^3D_1$ channel

Central

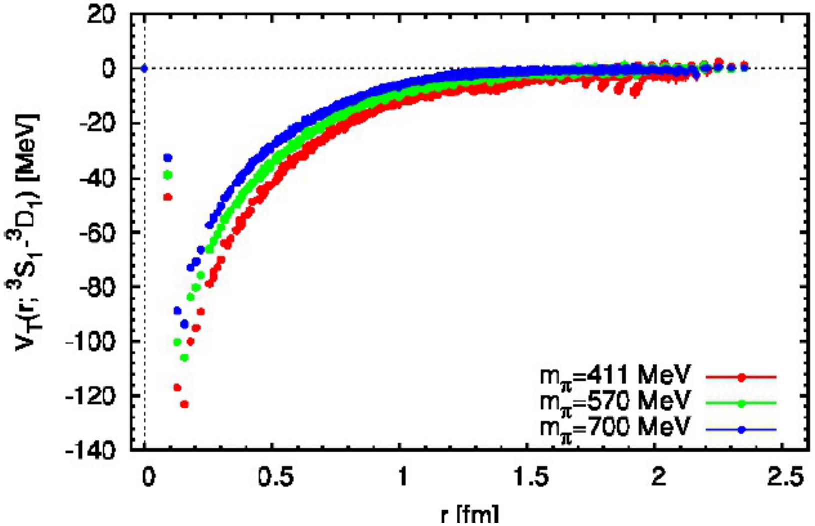




Phase shift ($^1S_0$)

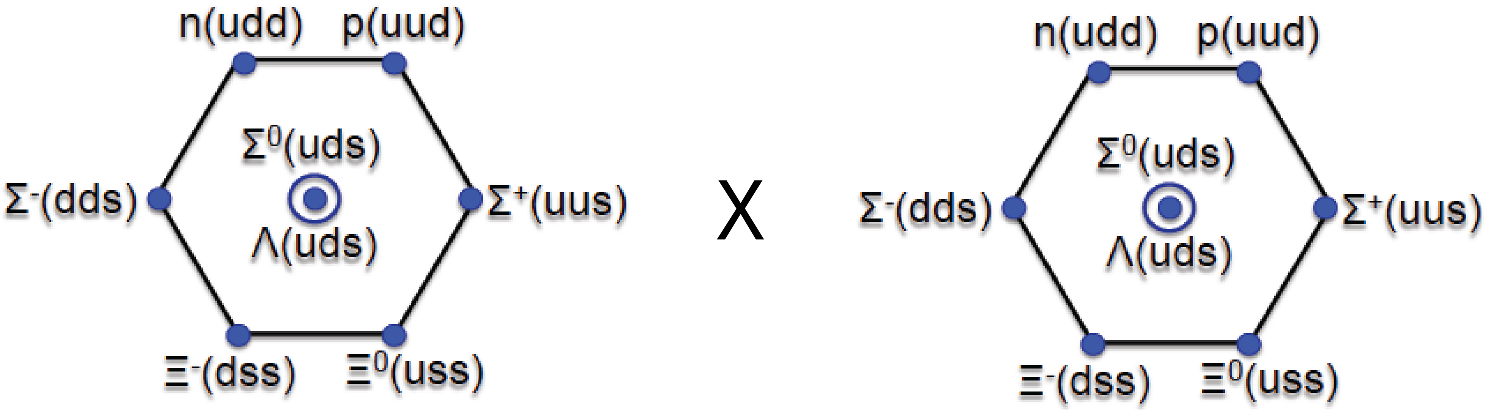
Attractive, Unbound

Tensor



N.Ishii, PoS(CD12)(2013)025

Hyperon Forces



$$8 \times 8 = \underset{\textbf{symmetric}}{\underline{27 + 8s + 1}} + \underset{\textbf{anti-symmetric}}{\underline{10^* + 10 + 8a}}$$

NN channel

SU(3) broken point:

H. Nemura et al., PLB673(2009)136

K. Sasaki et al., PTEP2015(2015)113B01

SU(3) symmetric point:

SU(3) study

BB potentials

a=0.12fm, L=3.9fm,
m(PS)= 0.47-1.2GeV

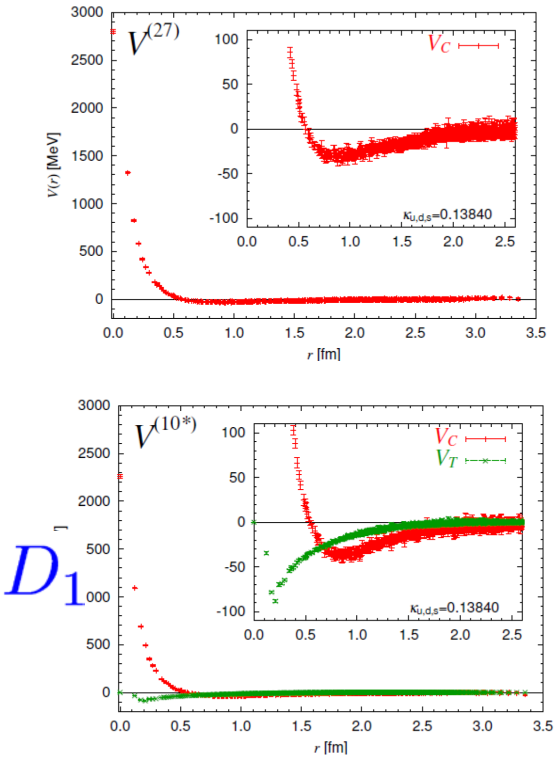
T.Inoue et al. (HAL.), NPA881(2012)28

NN sector

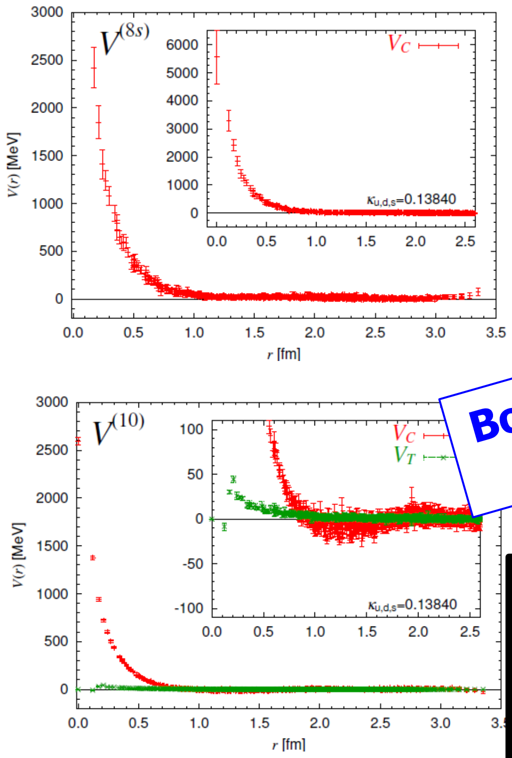
YN / YY sector

$^1S_0$

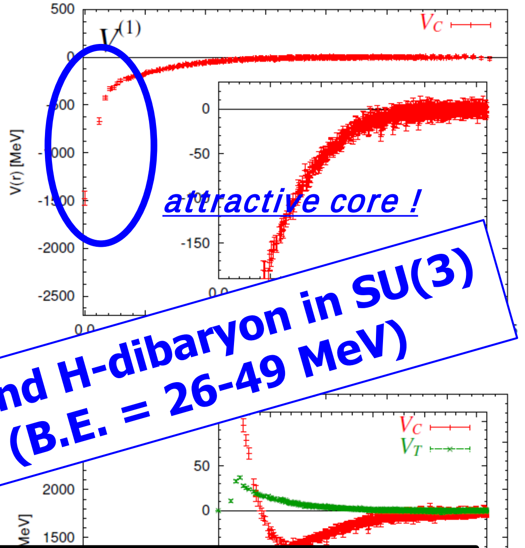
$^3S_1 - ^3D_1$



27,10*:
Same as NN

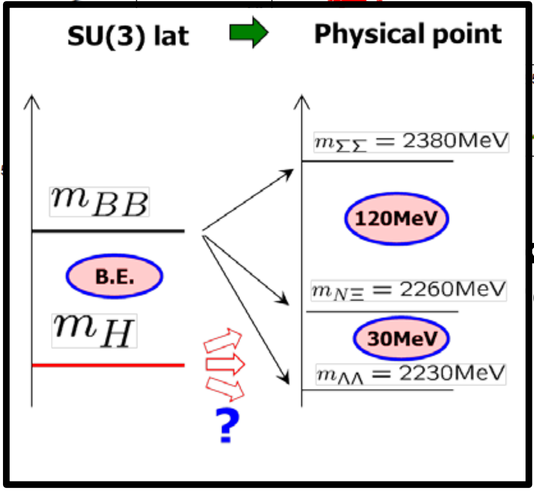


8s,10:
strong repulsive core



Bound H-dibaryon in SU(3)
(B.E. = 26-49 MeV)

ket
e



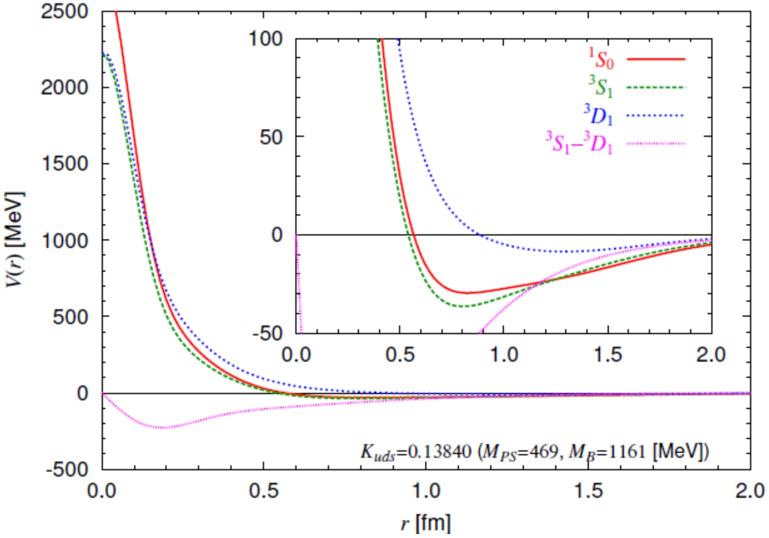
NN : unbound ($^1S_0$, $^3S_1$-$^3D_1$)

Repulsive core
← Pauli principle !

M.Oka et al., NPA464(1987)700

From LQCD to Nuclei / Neutron Star

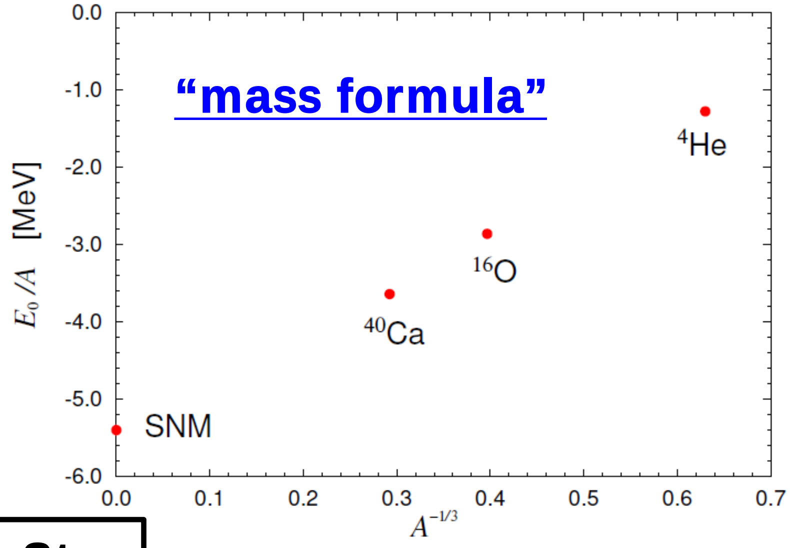
Lat NN forces



(SU(3), m(PS)=0.47GeV)

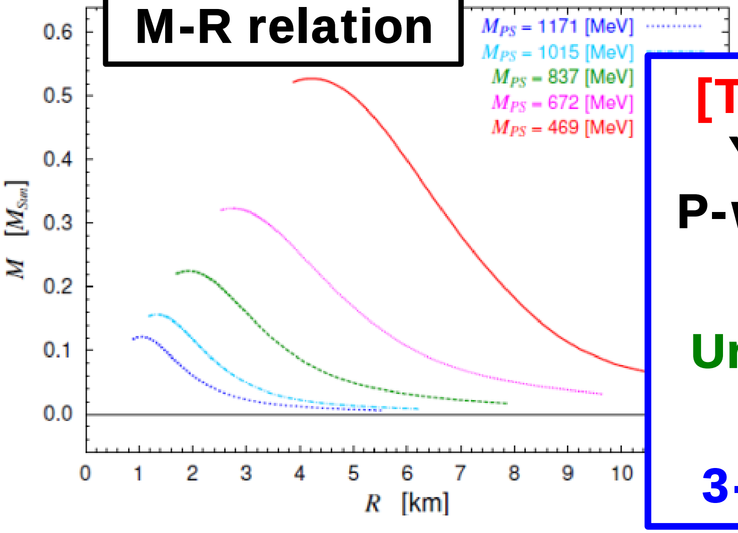
BHF

B.E. of medium-heavy nuclei



BHF & TOV

Neutron Star
M-R relation



[To be included]
YN / YY forces
P-wave / LS forces

[LQCD]
Unphysical mass

[Missing]
3-baryon forces

T.Inoue et al. (HAL Coll.) PRL111(2013)112503

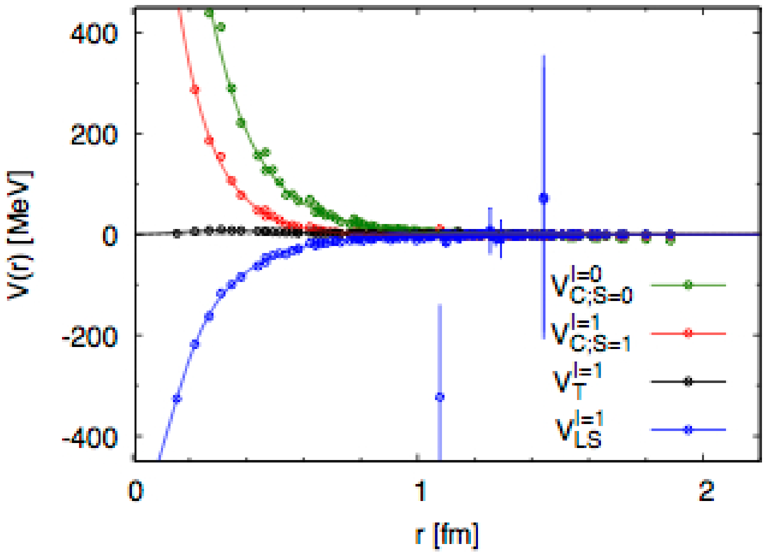
T.Inoue et al. (HAL Coll.), PRC91(2015)011001

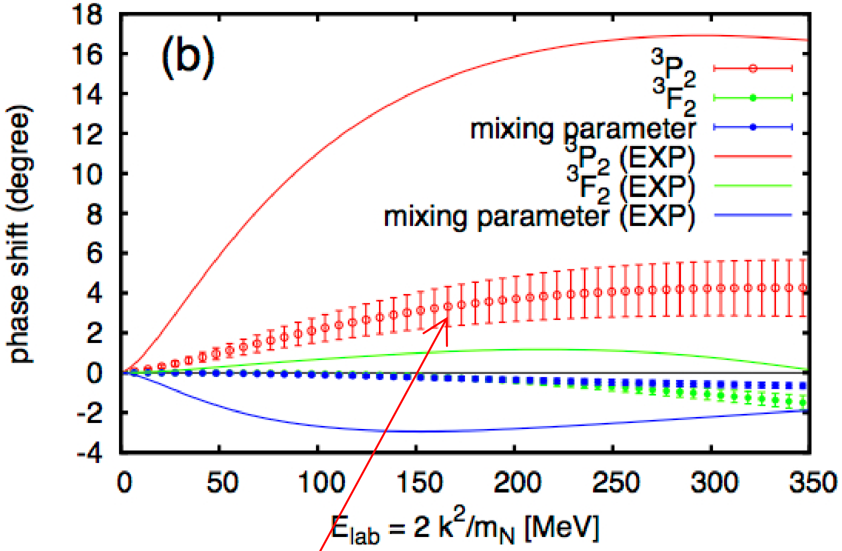
NN-forces in P=(−) channel (mπ=1.1 GeV)

- Central, tensor & LS forces

$$^1P_1,\ ^3P_0,\ ^3P_1,\ ^3P_2\text{–}^3F_2$$

Phase shifts





Superfluidity $^3P_2$ in neutron star

←→ neutrino cooling

Attractive in $^3P_2$

←→ observation of Cas A NS

Qualitatively good, but strength is weak
(We also observe potentials glow by lighter mass)

K.Murano et al., PLB735(2014)19

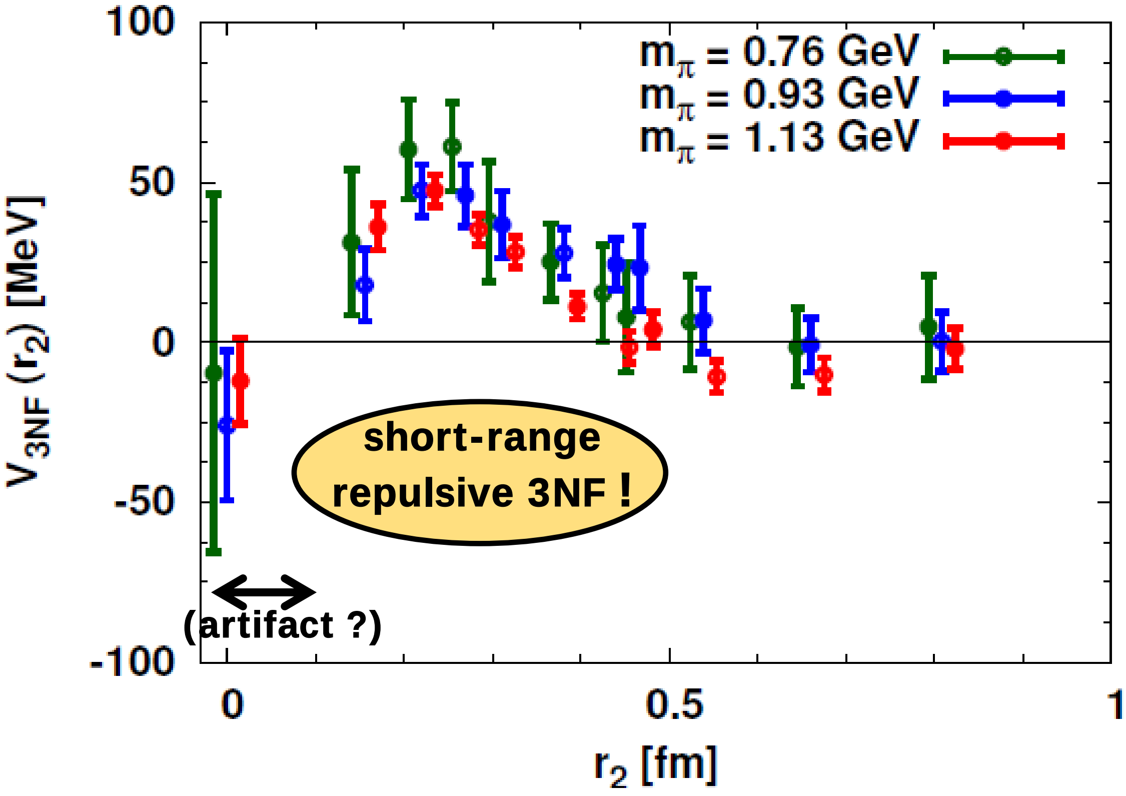
c.f. CalLat Coll. 1508.00886

3N-forces (3NF)

($N_f=2$, $m_\pi=0.76\text{-}1.1$ GeV)

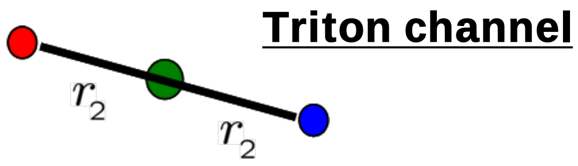
T.D. et al. (HAL QCD Coll.) PTP127(2012)723

+ t-dep method updates etc.

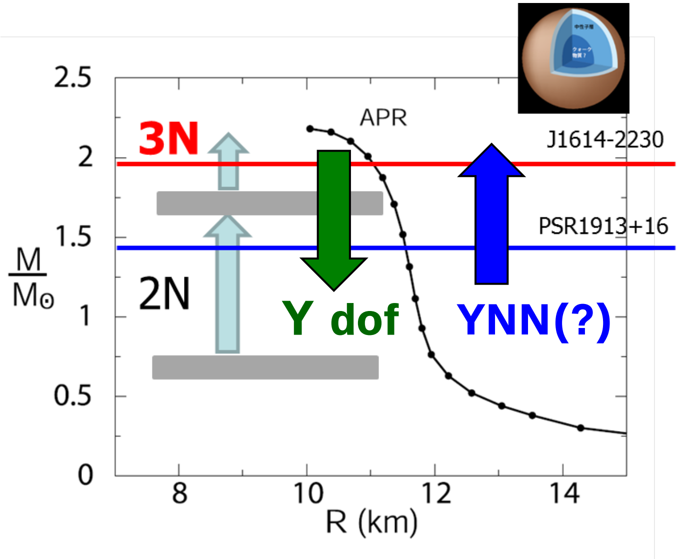


short-range repulsive 3NF !

(artifact ?)



Triton channel



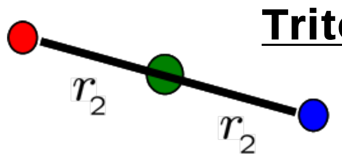
Unified Contraction Algorithm (UCA) is crucial (x192 speedup)

How about other geometries ?

How about YNN, YYN, YYY ?

How about lighter quark masses ?

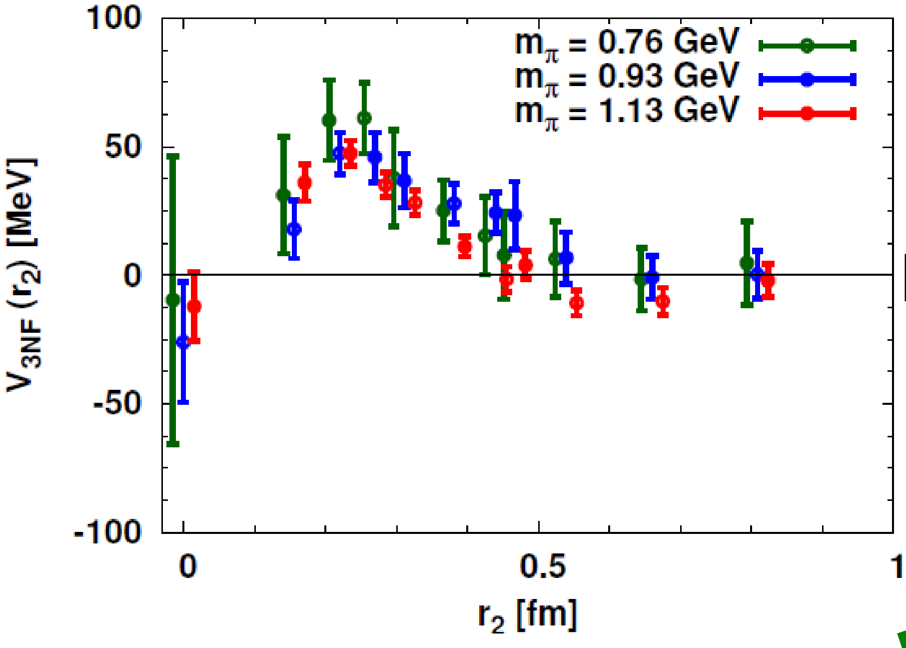
3N-forces (3NF)

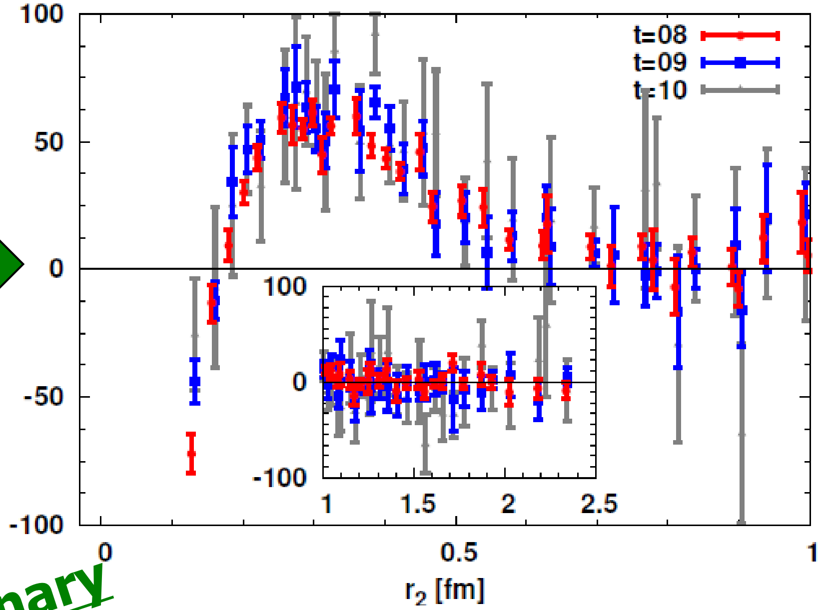


Triton channel

Nf=2, mπ=0.76-1.1 GeV

Nf=2+1, mπ=0.51 GeV





Preliminary

Magnitude of 3NF is similar for all masses
Range of 3NF tend to get longer (?) for m(pi)=0.5GeV

Kernel: ~50% efficiency achieved !

- **Outline**
  - – Introduction
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  - – Reliability test of LQCD methods
  - – Results at heavy quark masses
  - – Results at (almost) physical quark masses
    - Nuclear forces and Hyperon forces
    - Impact on dense matter
  - – Summary / Prospects

- Baryon Forces from LQCD — Ishii-Aoki-Hatsuda (2007)
- Exponentially better S/N — Ishii et al. (2012)
- Coupled channel systems — Aoki et al. (2011,13)

[Theory] = HAL QCD method

Baryon Interactions at Physical Point

[Hardware]

= K-computer [10PFlops]

+ FX100 [1PFlops] @ RIKEN
+ HA-PACS [1PFlops] @ Tsukuba

- HPCI Field 5 “Origin of Matter and Universe”

[Software]

= Unified Contraction Algorithm

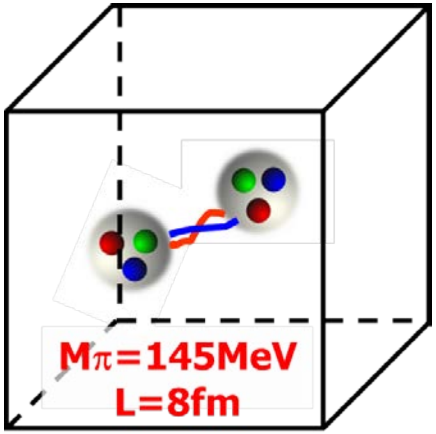
- Exponential speedup — Doi-Endres (2013)

$^3\mathrm{H}/^3\mathrm{He}$: $\times 192$
$^4\mathrm{He}$: $\times 20736$
$^8\mathrm{Be}$: $\times 10^{11}$

Setup of Lattice QCD

- **Nf = 2+1 full QCD**
  - Clover fermion + Iwasaki gauge action
  - Non-perturbatively O(a)-improved
  - APE-Stout smearing (α=0.1, n_{stout}=6)
  - m(pi) ~= 145 MeV, m(K) ~= 525 MeV
  - #traj ~= 2000 generated

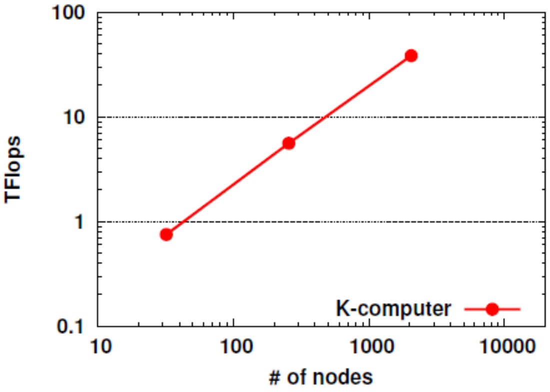
K.I. Ishikawa et al., PoS LAT2015, 075



96^4 box
(a~= 0.085fm)

- **Measurement**
  - Wall source w/ Coulomb gauge
  - Efficient implementation of UCA
  - Block solver for multiple RHS
  - K-computer @ 2048 node (x 8core/node)
    - ~25% efficiency (~65 TFlops sustained)
  - Calc to increase #stat in progress
  - All results preliminary

Weak scaling
(NBS calc part, w/o solver, w/o IO)



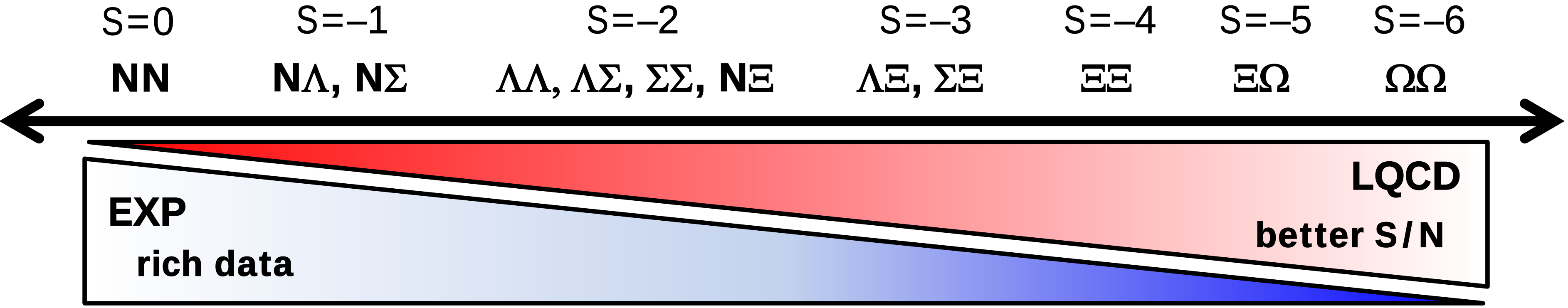
Target of Interactions

- NN/YN/YY for central/tensor forces in P=(+) (S, D-waves)

Central **Tensor**

$$U(\vec{r},\vec{r'}) = V_C(r) + S_{12}V_T(r) + \vec{L}\cdot\vec{S}V_{LS}(r) + \mathcal{O}(\nabla^2)$$

LO **LO** **NLO** **NNLO** (derivative expansion)

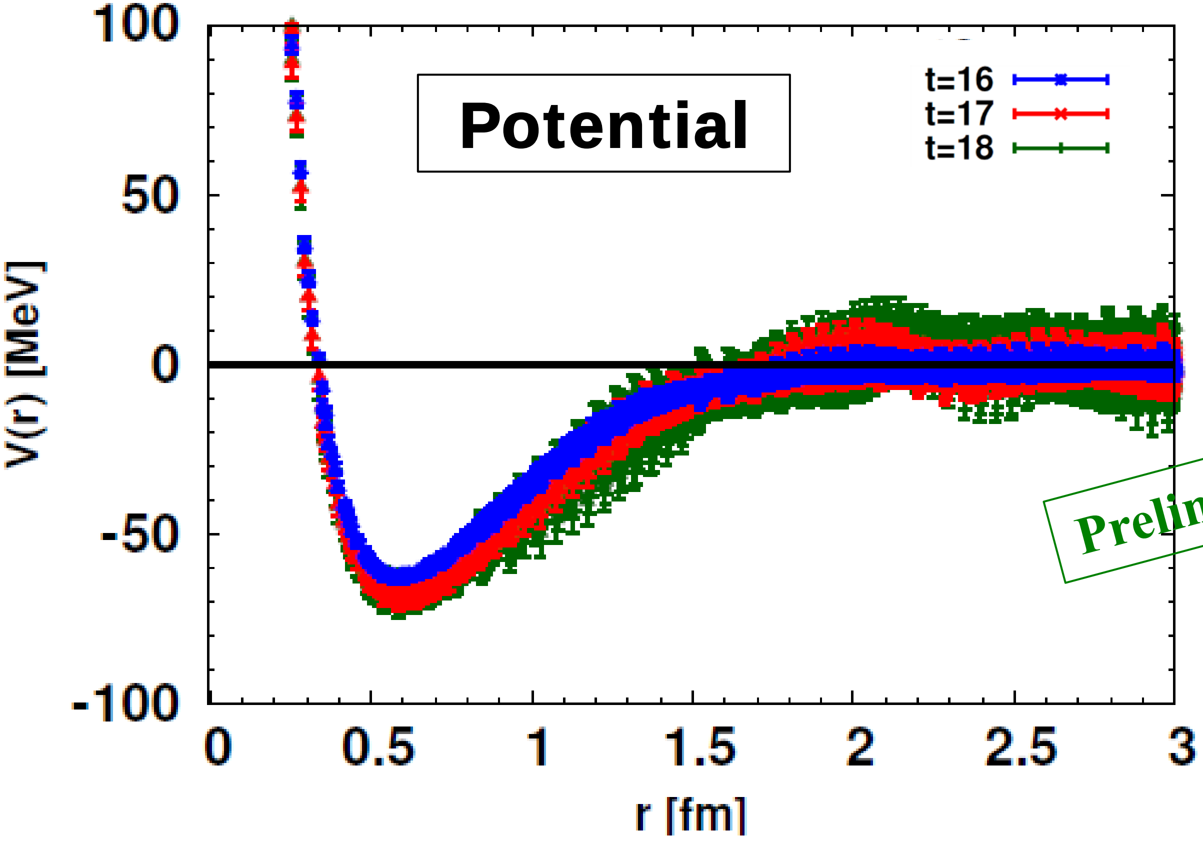


Hyperon in neutron star and EoS ? Exotic states ?

Hyperon forces provide precious predictions

$\Omega\Omega$ system ($^1S_0$)

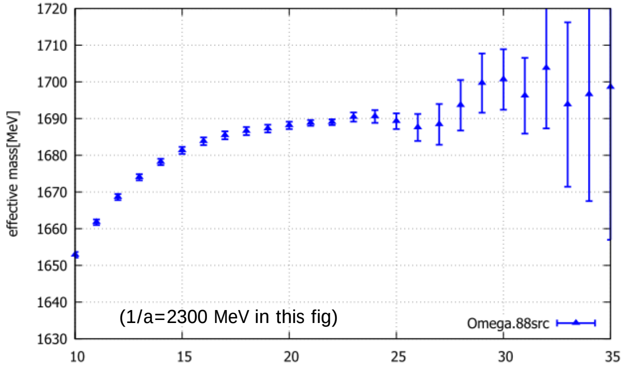
(The "most strange" dibaryon system)



Preliminary

(400conf x 4rot x 44/48src)

m(eff) for single Ω

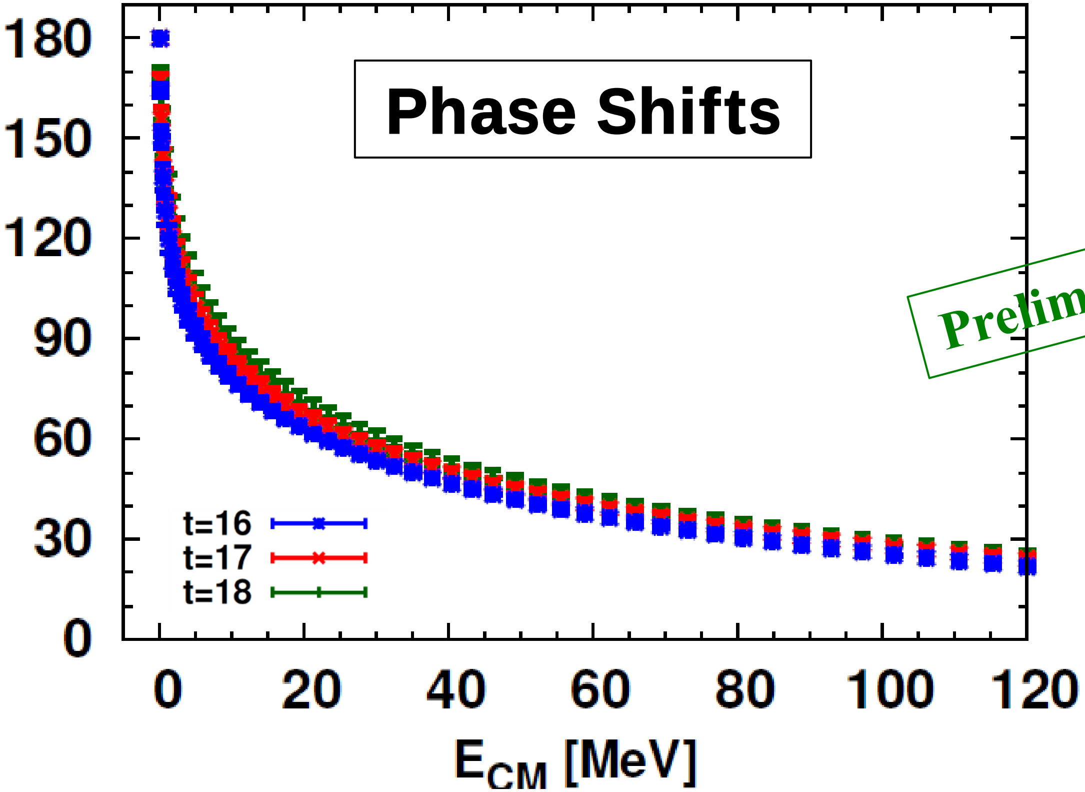


t = 18 : ~0.2-0.3% sys error

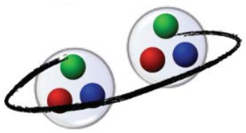
[S. Gongyo / K. Sasaki]

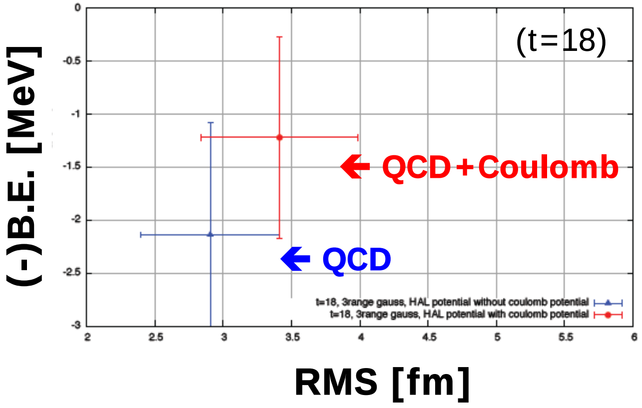
ΩΩ system ($^1S_0$)

(The "most strange" dibaryon system)



Preliminary





Strong Attraction

➔ Vicinity of bound / unbound

[~ Unitary limit]

⟺ ΩΩ correlation in HIC exp.

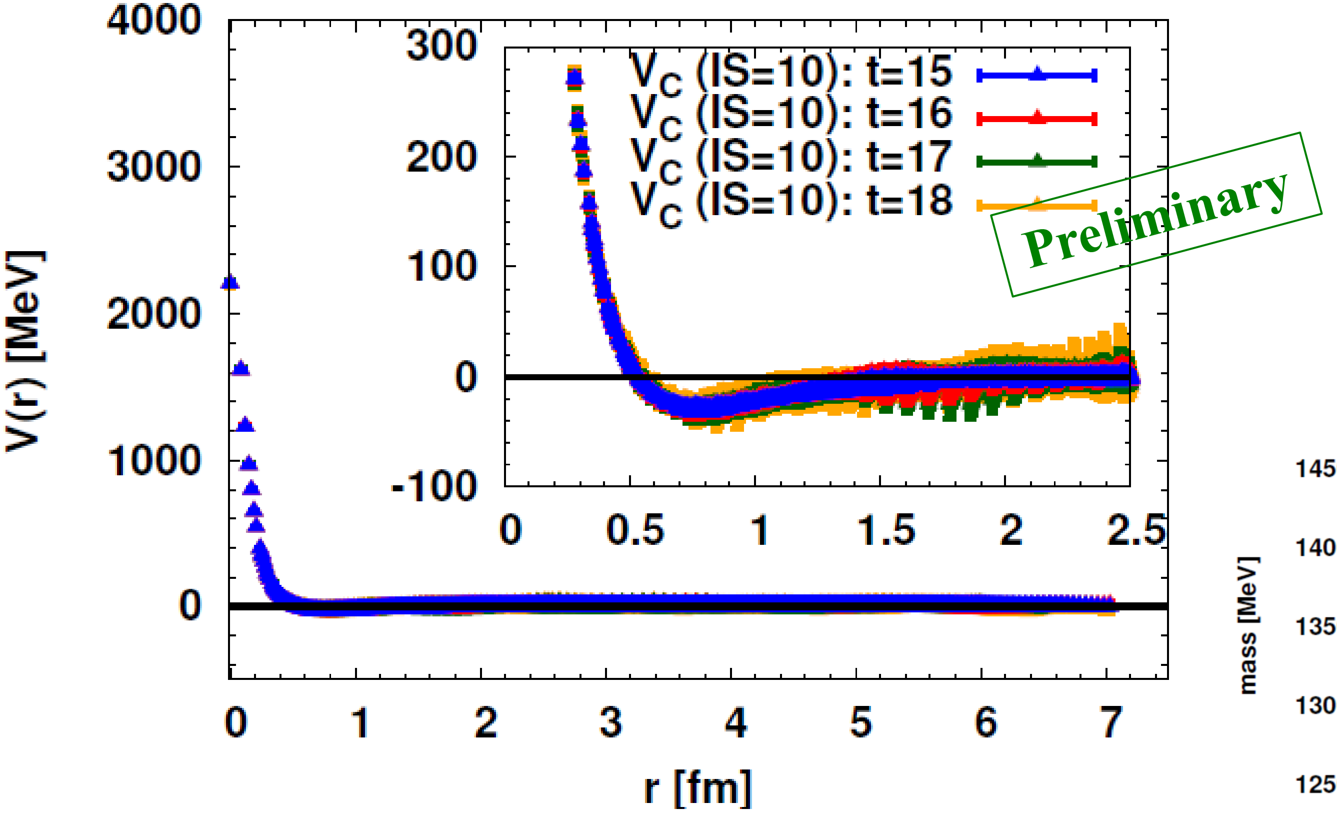
ΞΞ system (S= –4)

ΞΞ system ($^1S_0$)

Flavor SU(3)-partner of dineutron ⇨

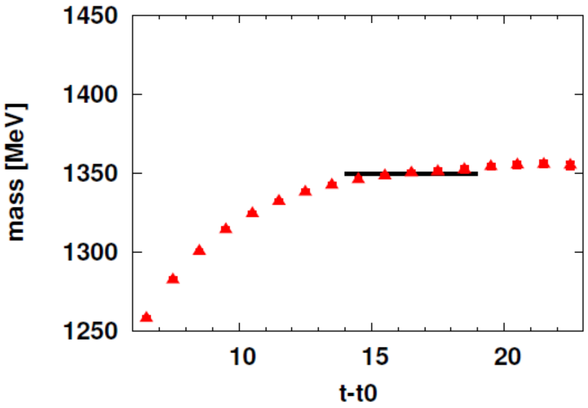
- “Doorway” to NN-forces
- Bound by SU(3) breaking ?

Potential



(400conf x 4rot x 48src)

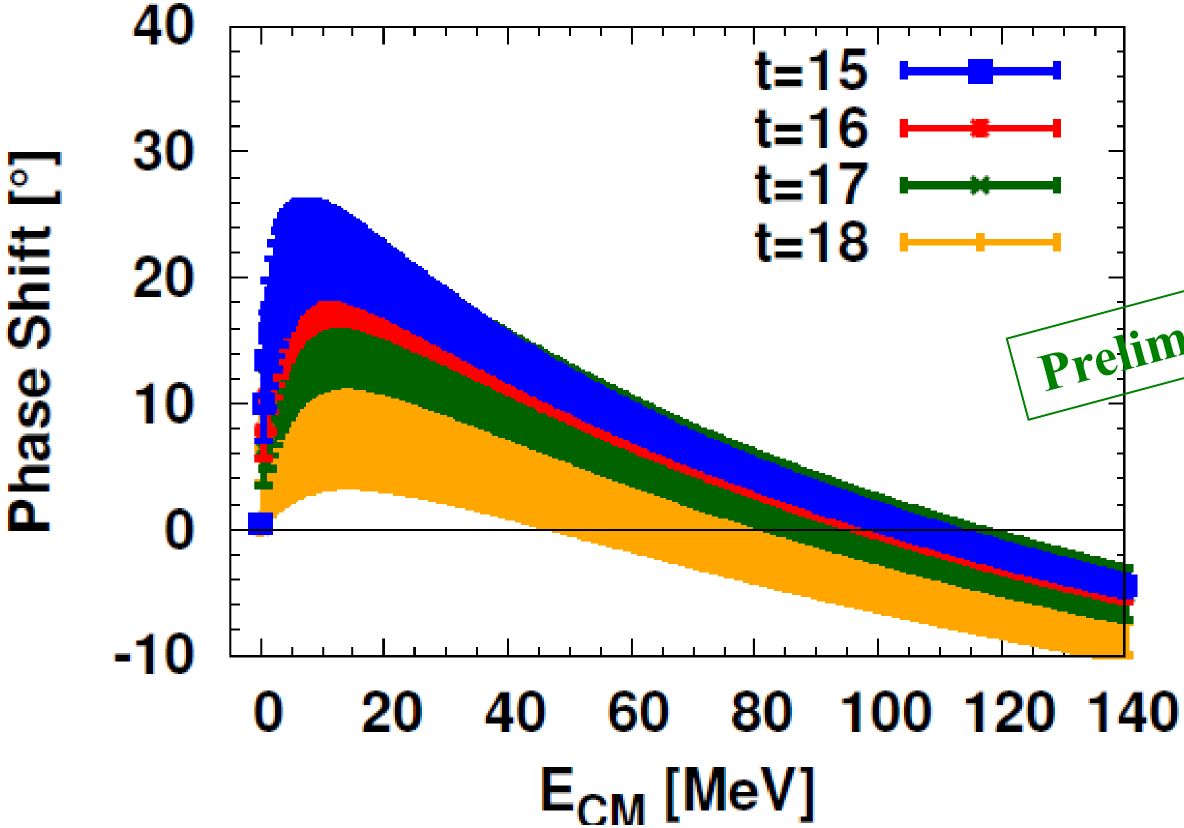
m(eff) for single Ξ



t = 14-18 : ~0.3-1% sys error

$\Xi\Xi$ system ($^1S_0$)

Phase Shifts



Preliminary

Strong Attraction

yet Unbound

⟺ **$\Xi\Xi$ correlation in HIC**

(2-gauss + 2-OBEP fit)

(400conf x 4rot x 48src)

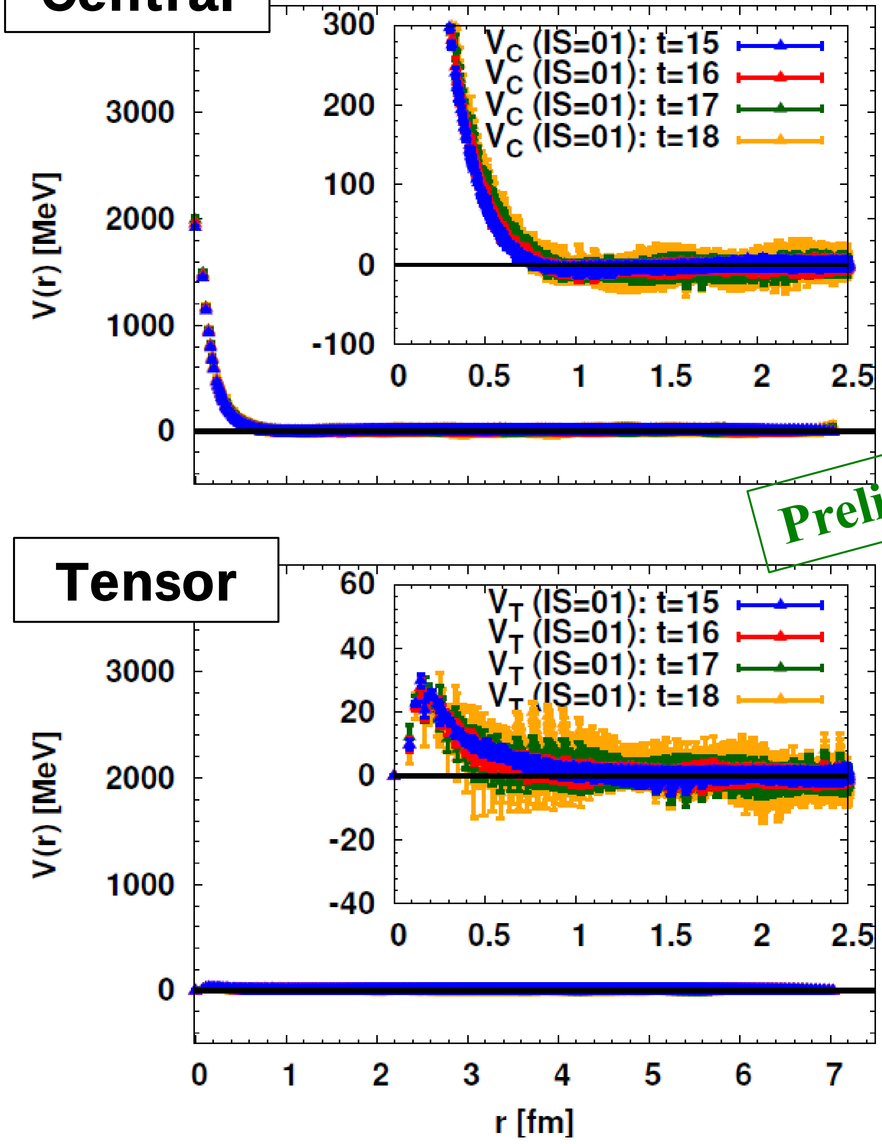
(t-dependence will be checked again w/ larger #stat)

ΞΞ system ($^3S_1$-$^3D_1$)

Potentials

Central

Tensor



Preliminary

10plet ⇔ unique w/ hyperon DoF

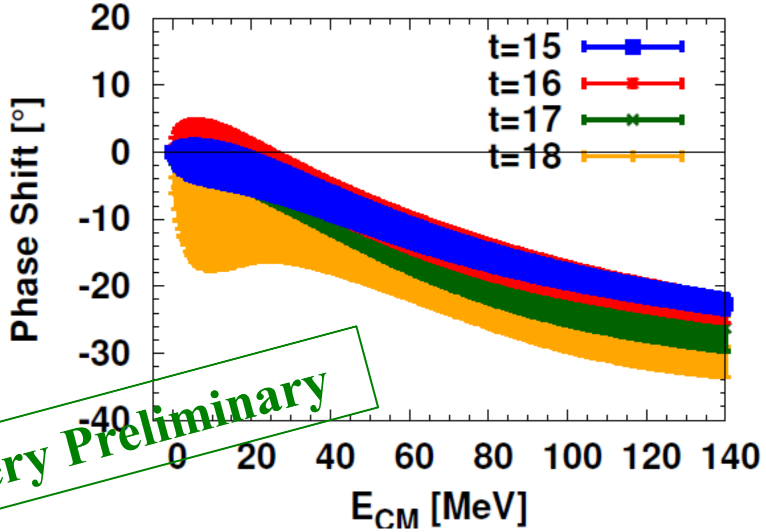
Flavor SU(3)-partner of Σ^- n

⇨ • Σ^- in neutron star ?

Central: Strong Repulsion

Tensor: Weak

Phase Shifts (effective $^3S_1$)



Very Preliminary

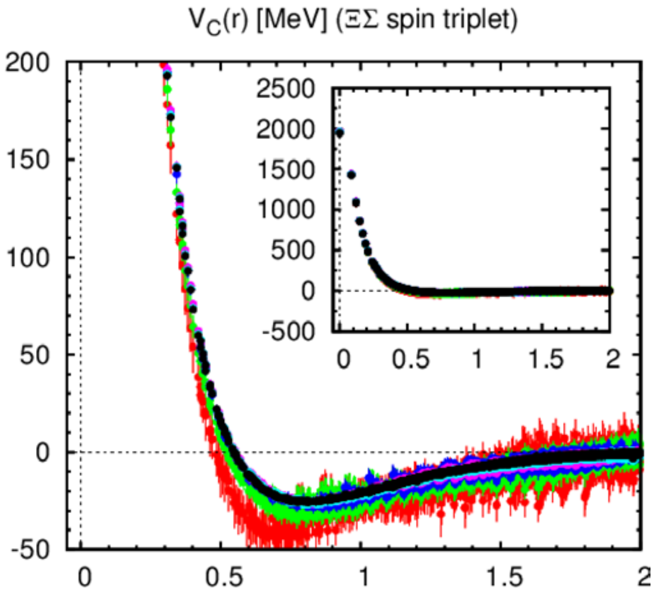
S= –3 systems

- ΞΣ (I=3/2)
  - $^1S_0$ ∼ 27-plet
⇔ NN($^1S_0$) + SU(3) breaking
  - $^3S_1$-$^3D_1$ ∼10*-plet
⇔ NN($^3S_1$-$^3D_1$) + SU(3) breaking
- ΞΛ–ΞΣ (I=1/2) : coupled channel
  - $^1S_0$ ∼ 27-plet & 8s-plet
  - $^3S_1$-$^3D_1$ ∼10-plet & 8a-plet

ΞΣ(I=3/2, spin triplet)

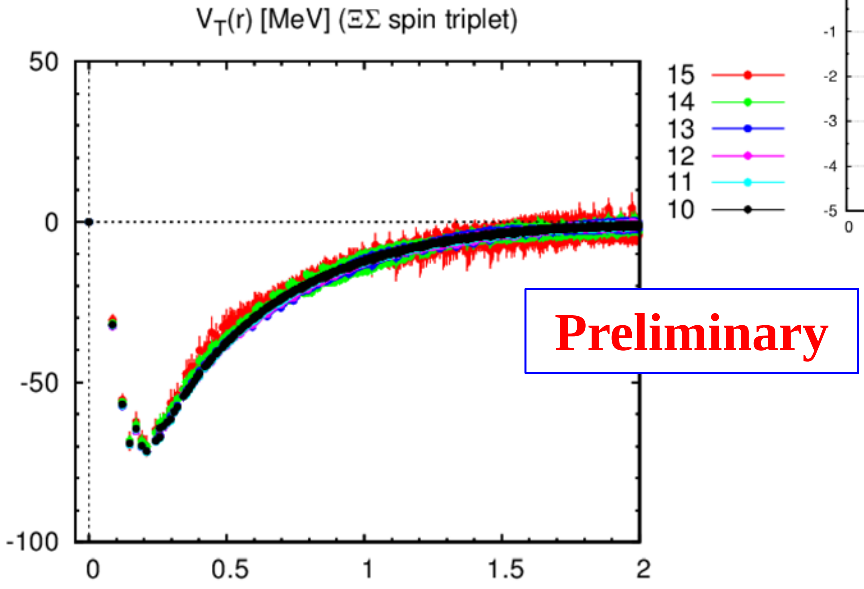
(bar) phase shifts & mixing

Central

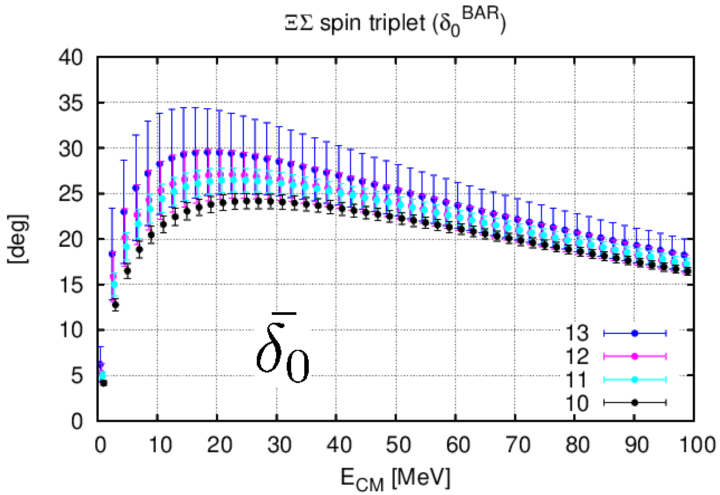


$V_C(r)$ [MeV] (ΞΣ spin triplet)

Tensor

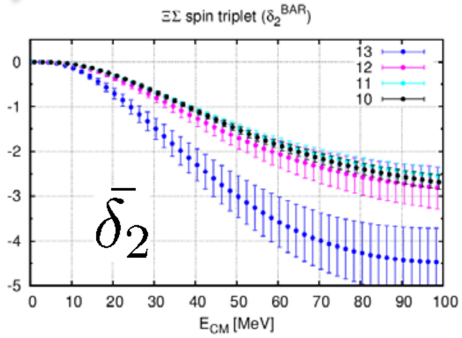


$V_T(r)$ [MeV] (ΞΣ spin triplet)

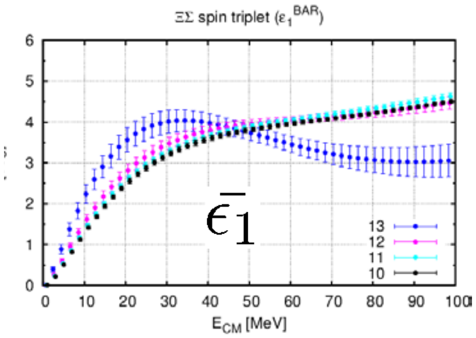


ΞΣ spin triplet (δ_0^{BAR})

unbound



ΞΣ spin triplet (δ_2^{BAR})



ΞΣ spin triplet (ε_1^{BAR})

N.B. t-dep should be checked;
single m_B has ~0.3-3% sys @ t=10-14

(200conf x 4rot x 48src)

[N. Ishii]

S= –2 systems

- **$\Lambda\Lambda$–$N\Xi$–$\Sigma\Sigma$ ($^1S_0$)**
  - **H-dibaryon channel**

- **$N\Xi$ interactions**
  - **Ξ-hypernuclei**
  - **Ξ in neutron star ?**

… and many more interactions !

➔ K. Sasaki's talk

S= –1 systems

←→ strangeness nuclear physics (Λ-hypernuclei @ J-PARC)

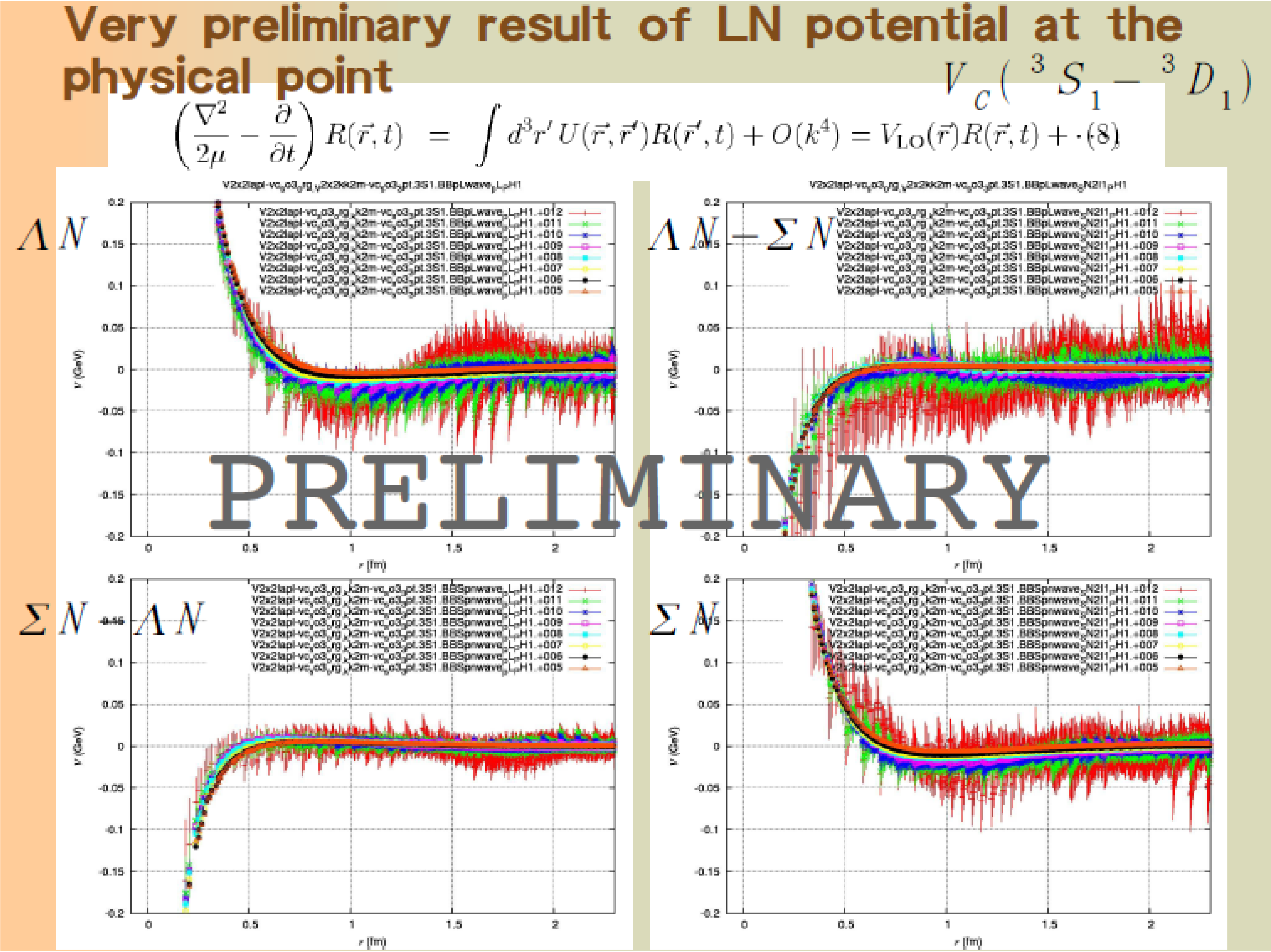
Λ should (?) appear in the core of Neutron Star

←→ Huge impact on EoS of high dense matter

- **ΛN–ΣN (I=1/2) : coupled channel**
  - **$^1S_0$ ∼ 27-plet & 8s-plet**
  - **$^3S_1$-$^3D_1$ ∼10*-plet & 8a-plet**
- **ΣN (I=3/2)**
  - **$^1S_0$ ∼ 27-plet**
    ⇔ NN($^1S_0$) + SU(3) breaking
  - **$^3S_1$-$^3D_1$ ∼10-plet**

ΛN–ΣN Vc potential in $^3S_1$–$^3D_1$

[H. Nemura]



Very preliminary result of LN potential at the physical point

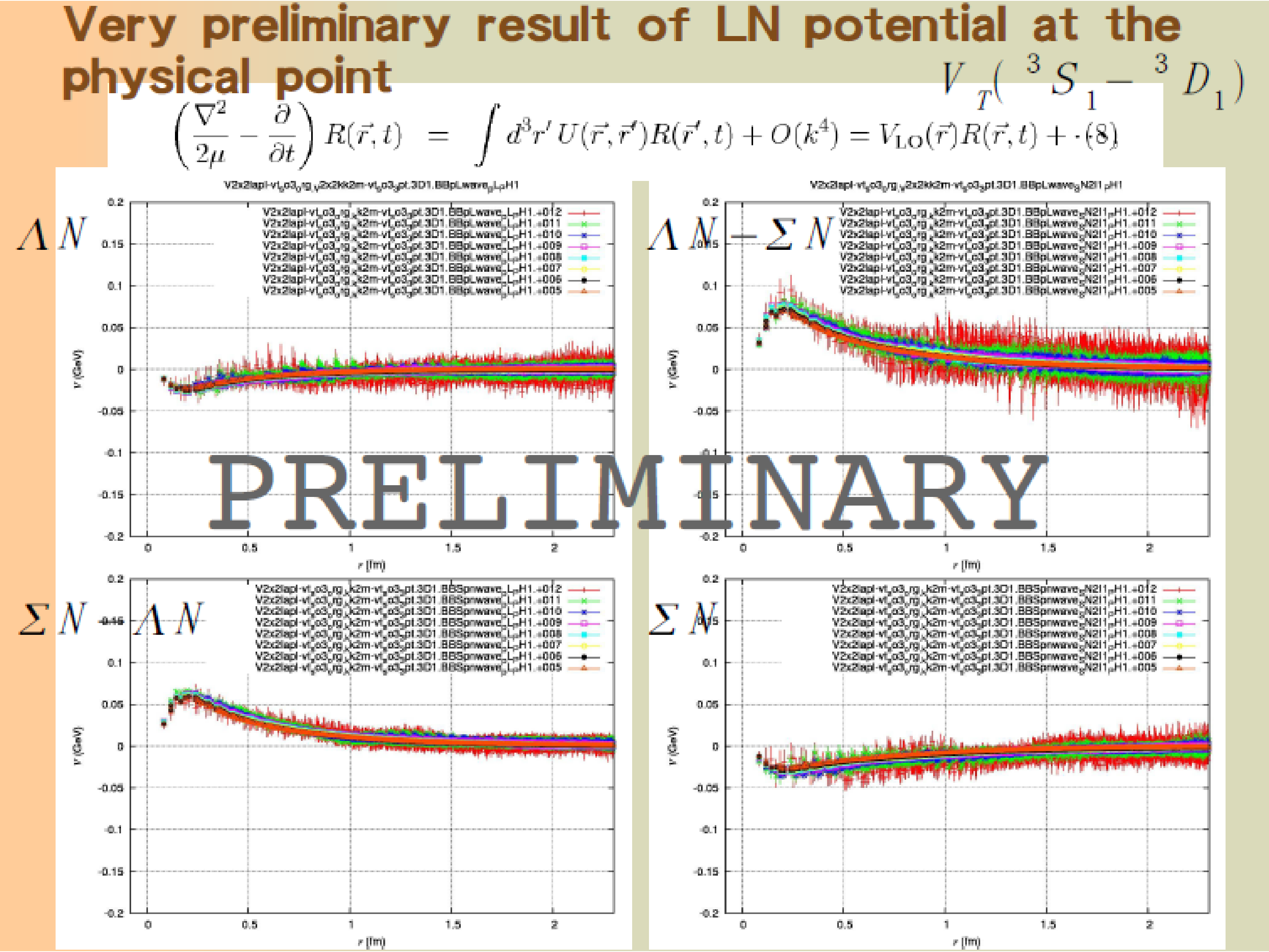
$V_C(^3S_1 - ^3D_1)$

$$\left(\frac{\nabla^2}{2\mu}-\frac{\partial}{\partial t}\right)R(\vec{r},t) = \int d^3r'\,U(\vec{r},\vec{r}')R(\vec{r}',t)+O(k^4)=V_{\rm LO}(\vec{r})R(\vec{r},t)+\cdot(8)$$

(200conf x 4rot x 52src)

ΛN–ΣN Vt potential in $^3S_1$–$^3D_1$

[H. Nemura]



Very preliminary result of LN potential at the physical point

$V_T(^3S_1 - {}^3D_1)$

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r' U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdot(8)$$

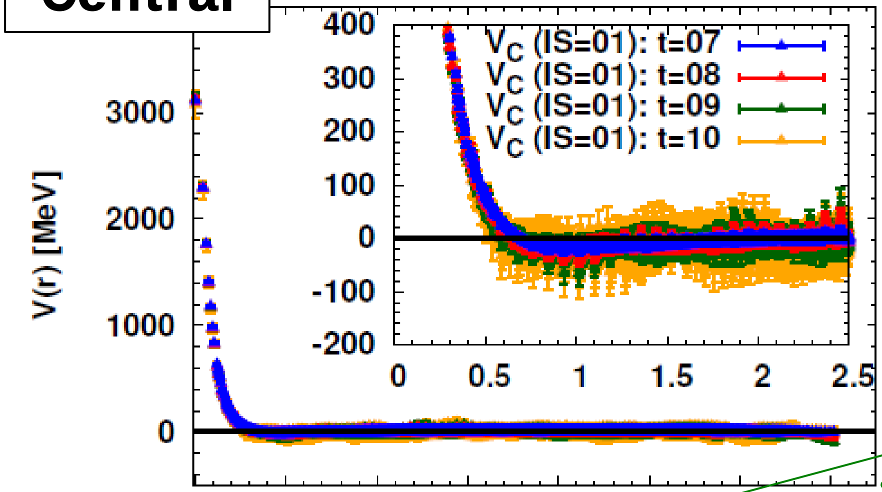
(200conf x 4rot x 52src)

NN system (S = 0)

NN system ($^3S_1$-$^3D_1$)

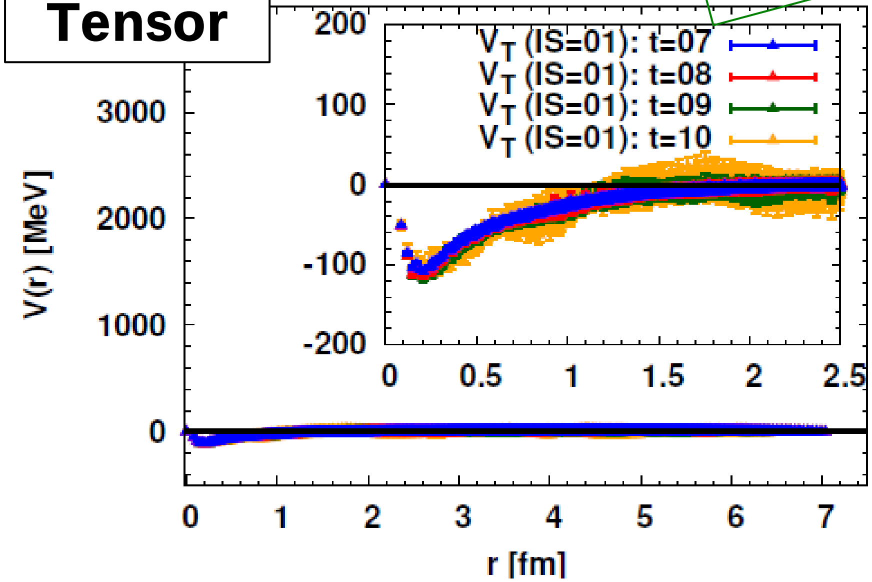
Potentials

Central

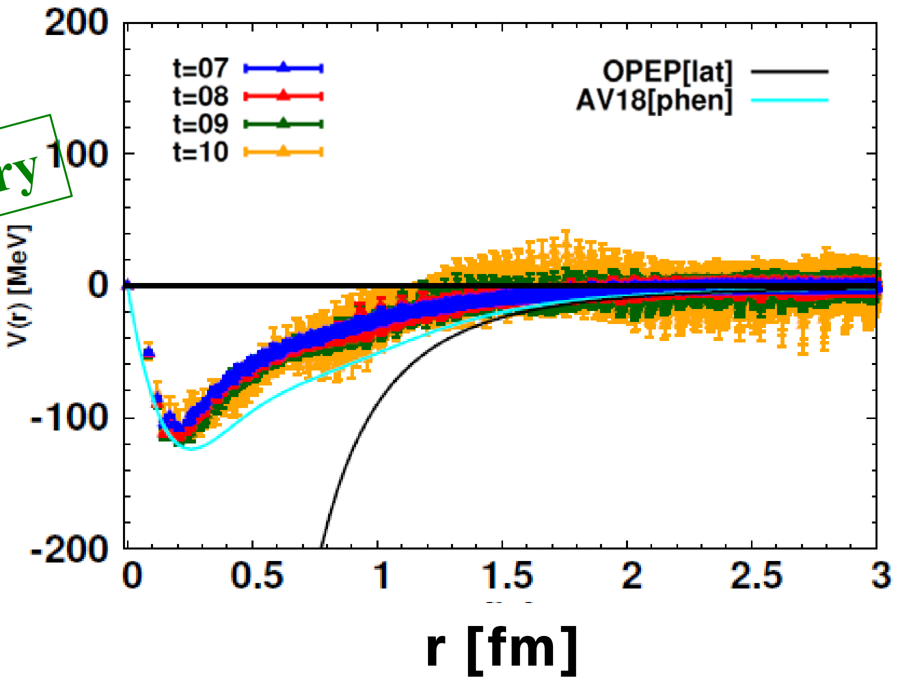


Preliminary

Tensor



- **Vc: repulsive core**
  + long-range attraction
- **Vt: strong tensor force !**

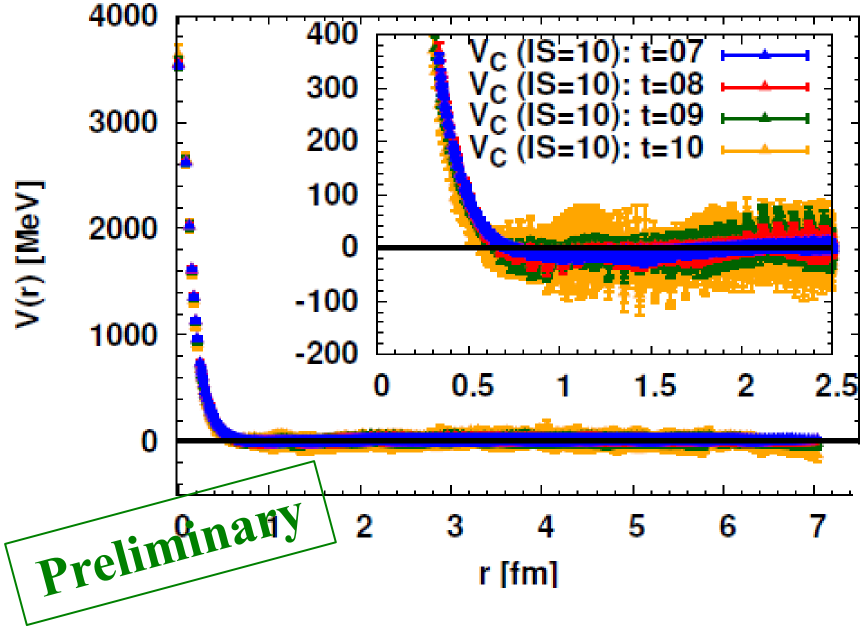


m(eff) for single N: ~2-4% sys err for t = 8-10

(400conf x 4rot x 48src)

NN system ($^1S_0$)

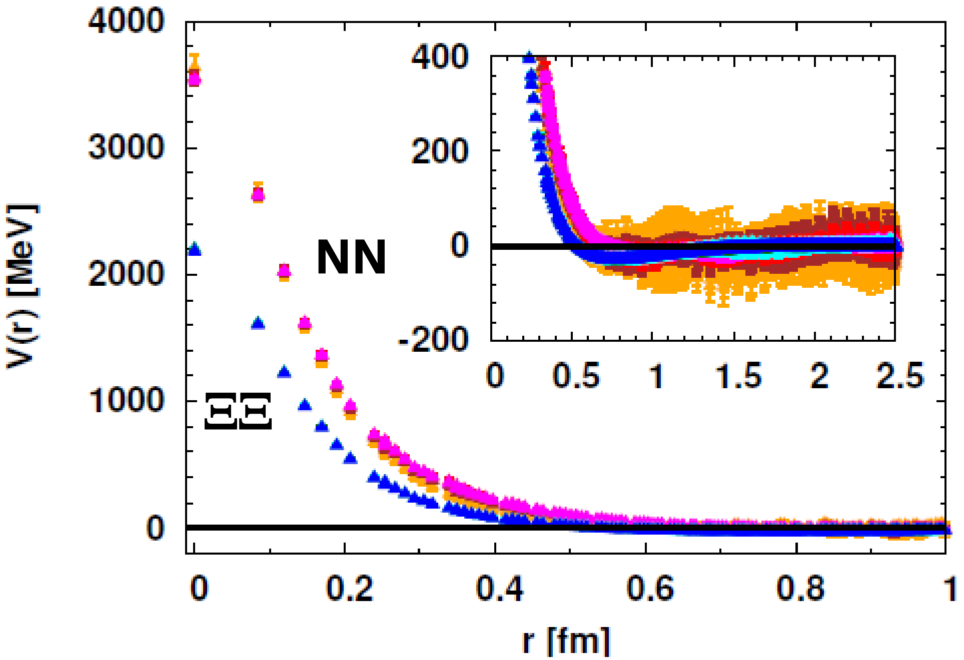
Potentials



- **Vc: repulsive core + long-range attraction**

The effect of SU(3)f breaking

NN($^1S_0$) and ΞΞ($^1S_0$) : 27-plet



Repulsive core enhanced
for lighter quark mass ? ←→ OGE ?

N.B. Sys error in NN may be underestimated

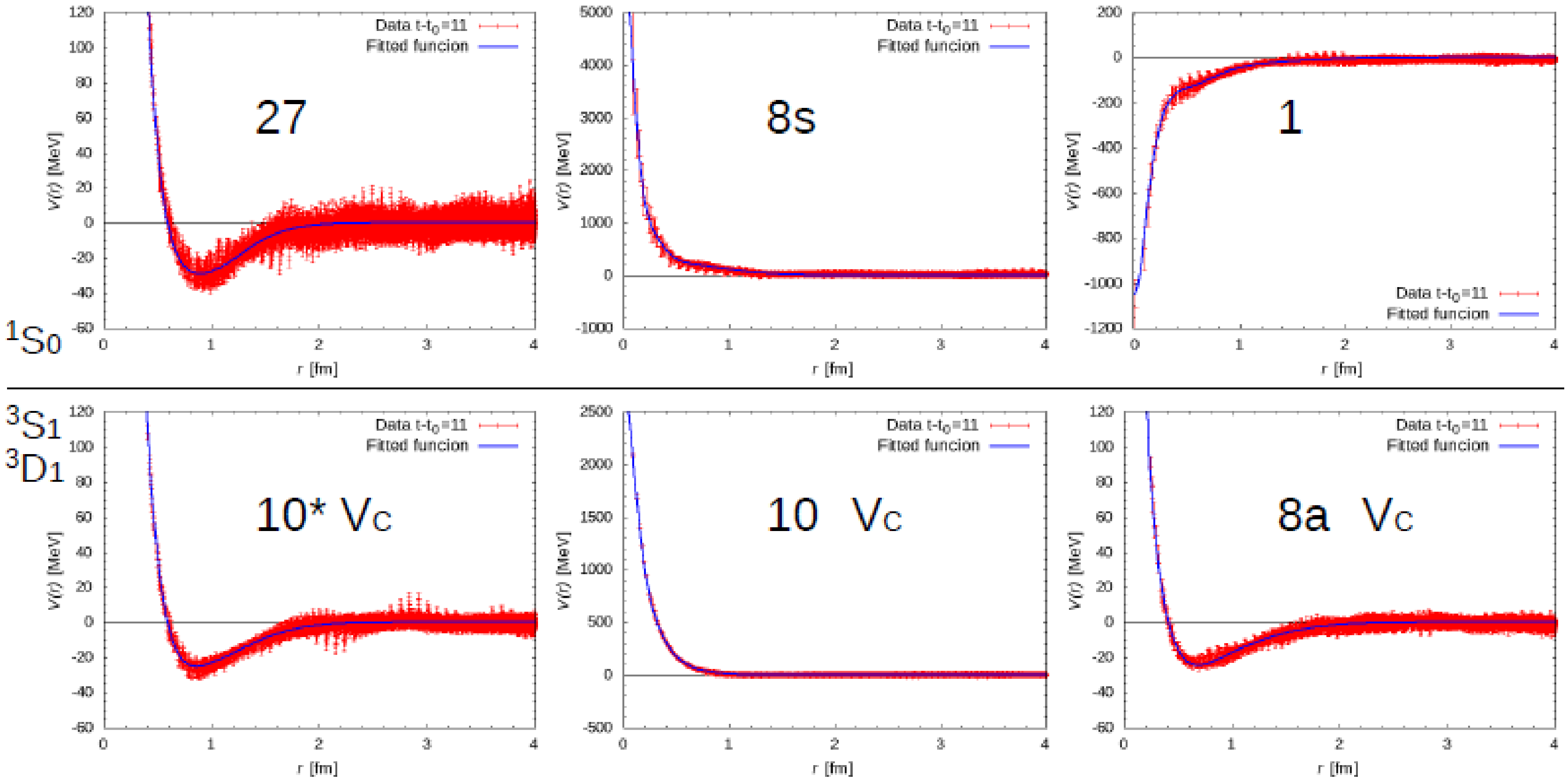
(400conf x 4rot x 48src)

Impact on dense matter

S=-2 interactions suitable to grasp whole NN/YN/YY interactions

Central Force in Irrep-base (diagonal)

$$8 \times 8 = \underbrace{27 + 8s + 1}_{^1S_0} + \underbrace{10^* + 10 + 8a}_{^3S_1\ ,\ ^3D_1}$$



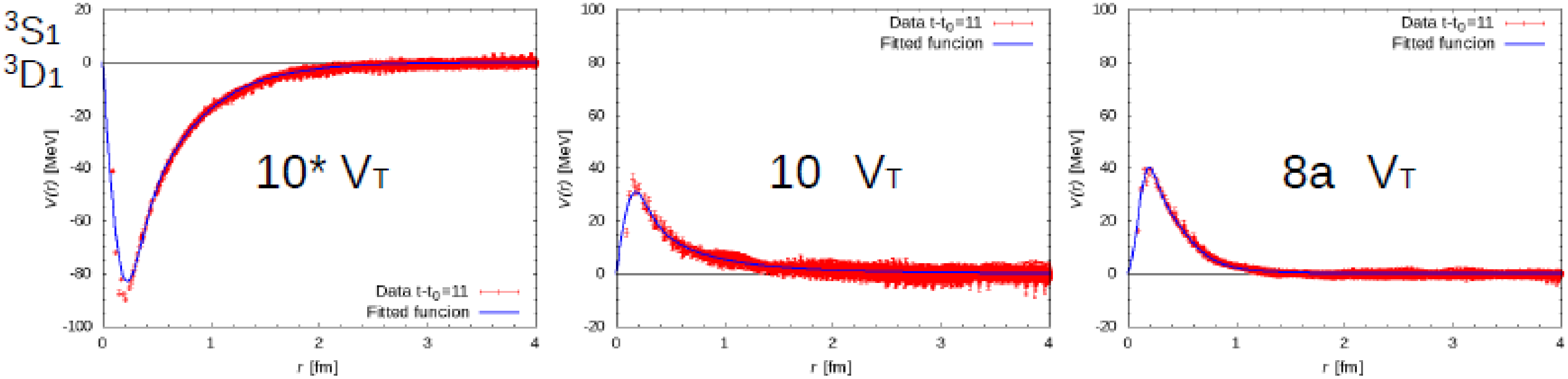
(off-diagonal component is small)

[K. Sasaki]

S=-2 interactions suitable to grasp whole NN/YN/YY interactions

Tensor Force in Irrep-base (diagonal)

$$8 \times 8 = \underbrace{27 + 8s + 1}_{{}^1S_0} + \underbrace{10^* + 10 + 8a}_{{}^3S_1\,,\ {}^3D_1}$$



➔ **We calculate single-particle energy of hyperon in nuclear matter w/ LQCD baryon forces**

(off-diagonal component neglected)

We fit by

$$V(r) = a_1\, e^{-a_2 r^2} + a_3\, e^{-a_4 r^2} + a_5 \left[\left(1 - e^{-a_6 r^2}\right) \frac{e^{-a_7 r}}{r} \right]^2 \qquad \text{(central)}$$

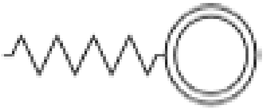
$$V(r) = a_1 \left(1 - e^{-a_2 r^2}\right)^2 \left(1 + \frac{3}{a_3 r} + \frac{3}{(a_3 r)^2}\right) \frac{e^{-a_3 r}}{r} + a_4 \left(1 - e^{-a_5 r^2}\right)^2 \left(1 + \frac{3}{a_6 r} + \frac{3}{(a_6 r)^2}\right) \frac{e^{-a_6 r}}{r} \qquad \text{(tensor)}$$

Brueckner-Hartree-Fock LOBT

- Hyperon single-particle potential

M. Baldo, G.F. Burgio, H.-J. Schulze,
Phys. Rev. C58, 3688 (1998)

$$U_Y(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \le k_F} \langle k k' | G^{SLJ}_{(YN)(YN)}\left(e_Y(k)+e_N(k')\right) | k k' \rangle$$



$$^{2S+1}L_J = {}^1S_0\,,\ {}^3S_1\,,\ {}^3D_1\,,\ \Big|\ {}^1P_1\,,\ {}^3P_J\ \cdots$$

in our study

limitation

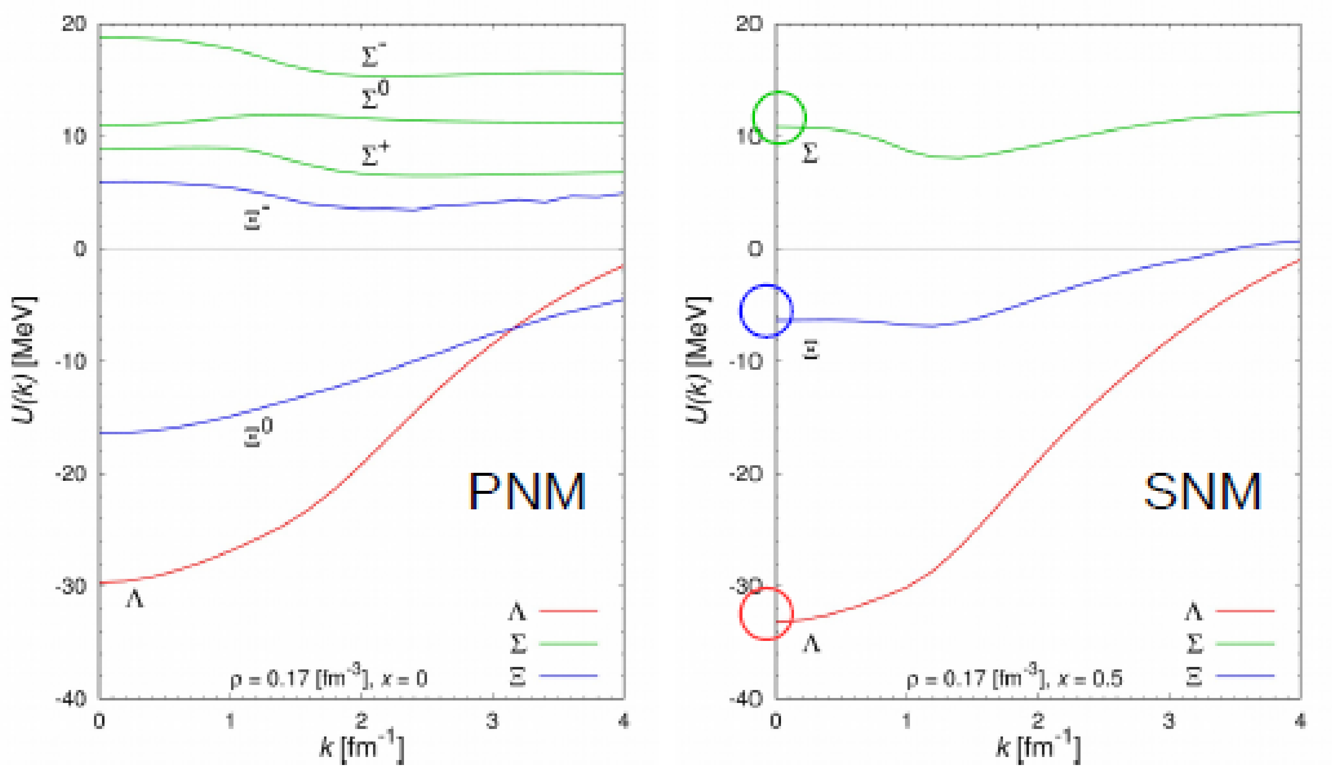
- YN G-matrix using $V^{\mathrm{LQCD}}_{S=-1}$, $M^{\mathrm{Phys}}_{N,Y}$, $U^{\mathrm{AV18,BHF}}_{n,p}$ and, U^{LQCD}_{Y}

$$Q=0 \begin{pmatrix} G^{SLJ}_{(\Lambda n)(\Lambda n)} & G_{(\Lambda n)(\Sigma^0 n)} & G_{(\Lambda n)(\Sigma^- p)} \\ G_{(\Sigma^0 n)(\Lambda n)} & G_{(\Sigma^0 n)(\Sigma^0 n)} & G_{(\Sigma^0 n)(\Sigma^- p)} \\ G_{(\Sigma^- p)(\Lambda n)} & G_{(\Sigma^- p)(\Sigma^0 n)} & G_{(\Sigma^- p)(\Sigma^- p)} \end{pmatrix} \quad Q=+1 \begin{pmatrix} G^{SLJ}_{(\Lambda p)(\Lambda p)} & G_{(\Lambda p)(\Sigma^0 p)} & G_{(\Lambda p)(\Sigma^+ n)} \\ G_{(\Sigma^0 p)(\Lambda p)} & G_{(\Sigma^0 p)(\Sigma^0 p)} & G_{(\Sigma^0 p)(\Sigma^+ n)} \\ G_{(\Sigma^+ n)(\Lambda p)} & G_{(\Sigma^+ n)(\Sigma^0 p)} & G_{(\Sigma^+ n)(\Sigma^+ n)} \end{pmatrix}$$

$$Q=-1 \quad G^{SLJ}_{(\Sigma^- n)(\Sigma^- n)} \qquad Q=+2 \quad G^{SLJ}_{(\Sigma^+ p)(\Sigma^+ p)}$$

[T. Inoue]

Hyperon single-particle potentials



$@\rho = 0.17\,[\mathrm{fm}^{-3}]$

Preliminary

- obtained by using YN,YY forces form QCD.
- Results are compatible with experimental suggestion.

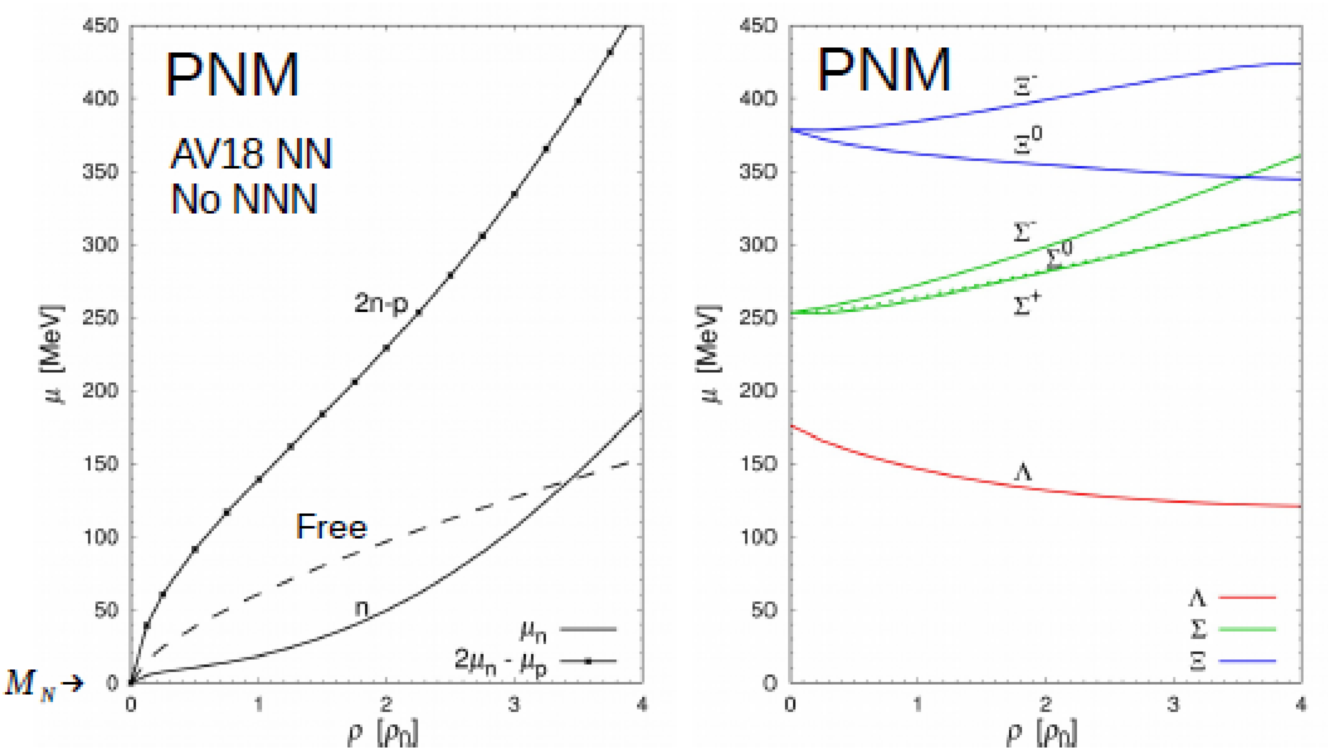
$$U_\Lambda^{\mathrm{Exp}}(0) \simeq -30\,, \quad U_\Xi(0)^{\mathrm{Exp}} \simeq -10\,, \quad U_\Sigma^{\mathrm{Exp}}(0) \geq +20 \quad [\mathrm{MeV}]$$

attraction attraction small repulsion

1

[T. Inoue]

Chemical potentials



S-wave YN only

Preliminary

- Density dependence of chemical pot. of n and Y in PNM.

$$\mu_n(\rho) = \frac{k_F^2}{2M} + U_n(\rho; k_F), \quad \mu_Y(\rho) = M_Y - M_N + U_Y(\rho; 0)$$

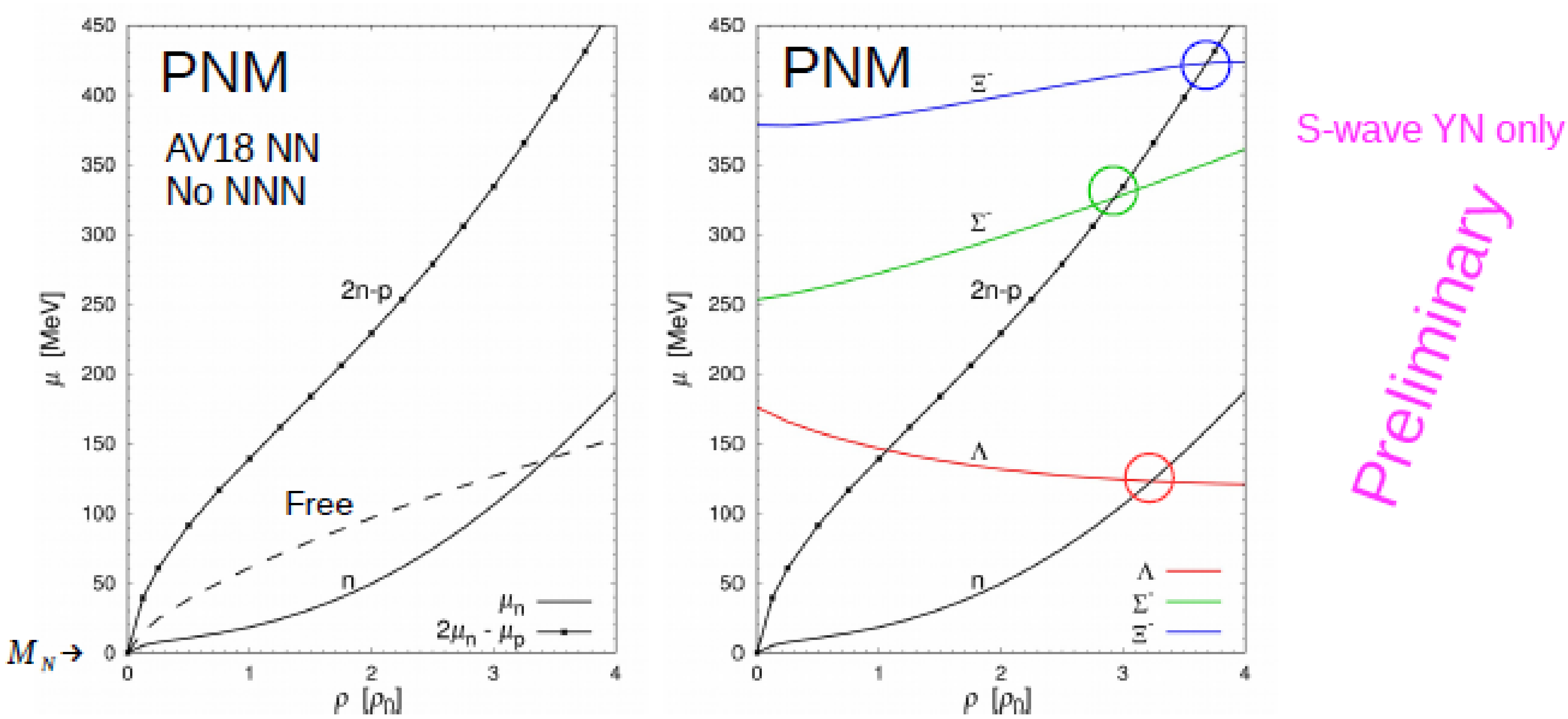
- Hyperon appear as $n \rightarrow Y^0$ if $\mu_n > \mu_{Y^0}$

$nn \rightarrow pY^-$ if $2\mu_n > \mu_p + \mu_{Y^-}$

2

[T. Inoue]

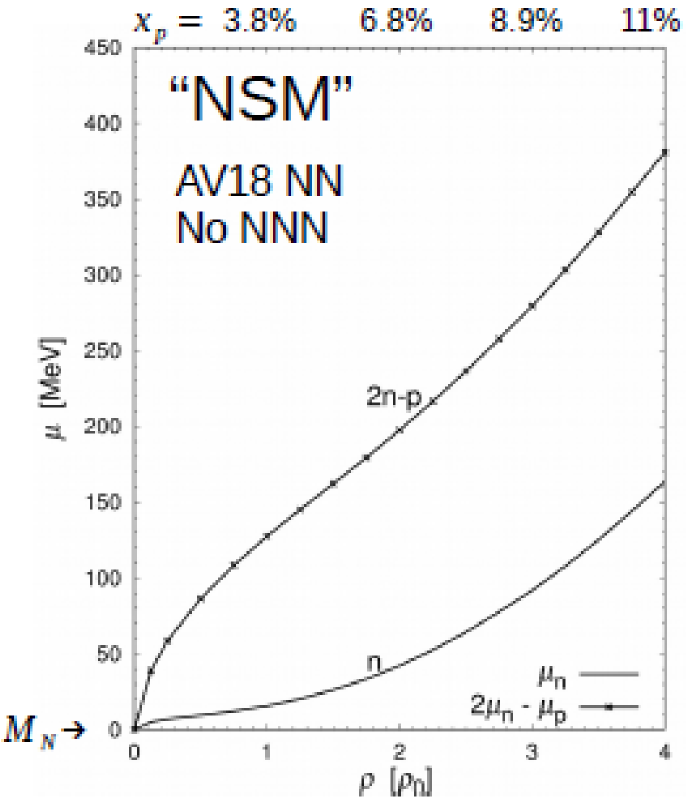
Hyperon onset (just for a demonstration)

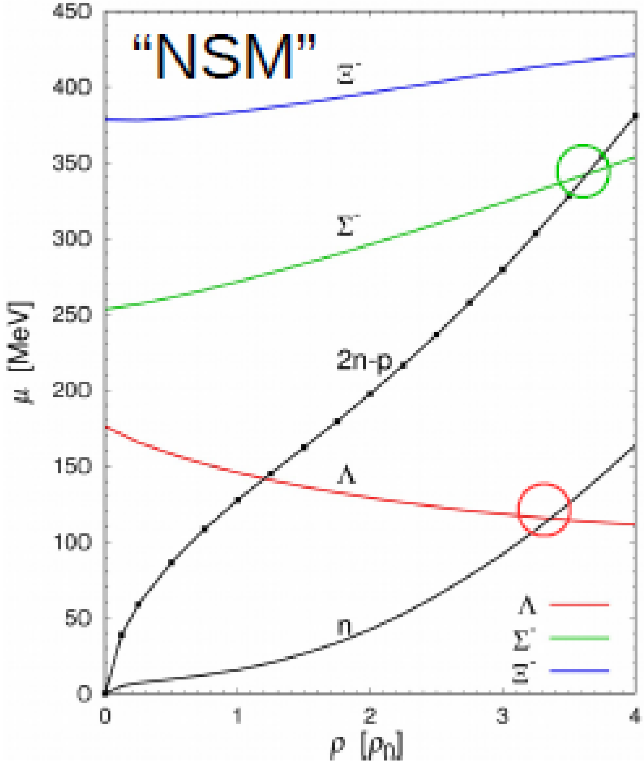


- First, Σ^- appear at 2.9 ρ_0. Next, Λ appear at 3.3 ρ_0.
  - NS matter is not PNM especially at high density.
  - We should compare with more sophisticated μ_n and μ_p.
  - P-wave YN force may be important at high density.

[T. Inoue]

Hyperon onset (just for a demonstration)





S-wave YN only

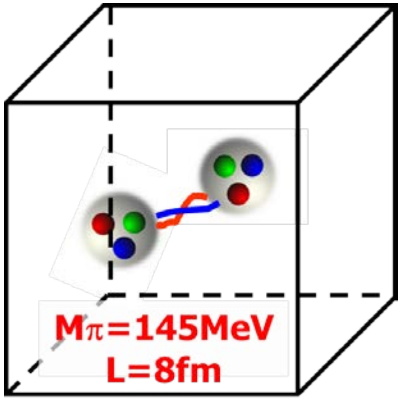
Preliminary

- "NSM" is matter w/ n, p, e, μ under β-eq and Q=0.

[T. Inoue]

Summary

- Hadron forces: Bridge between particle/nuclear/astro-physics
- HAL QCD method crucial for a reliable calculation
  - Direct method suffers from excited state contaminations
- The 1st LQCD for Baryon Interactions at ~ phys. point
  - m(pi) ~= 145 MeV, L ~= 8fm, 1/a ~= 2.3GeV
  - Central/Tensor forces for NN/YN/YY in P=(+) channel



Nuclear Physics from LQCD
New Era is dawning !

- Prospects
  - Exascale computing Era ~ 2020
  - LS-forces, P=(-) channel, 3-baryon forces, etc., & EoS