Kaon-Nucleon systems in the Skyrme model

RCNP, Osaka University
Takashi Ezoe
Atsushi Hosaka

2016/11/23 Realistic hadron interactions in QCD@YITP

Contents

1. Introduction

Introduction

Kaon nucleon systems are very attractive

- Strong attraction between the anti-kaon($\bar{K}$) and the nucleon(N)

Y. Akaishi and T. Yamazaki, Phys. Rev. C **65** (2002)

- $\bar{K}$N bound state = $\Lambda(1405)$
- Few body nuclear system with $\bar{K}$ → under debate

$\bar{K}$N interaction is important
to investigate the few body systems with $\bar{K}$

Theoretical studies of $\bar{K}$N interaction

- Phenomeonlogical approach

Y. Akaishi and T. Yamazaki, Phys. Rev. **C 65** (2002) etc

- Chiral theory: based on a 4-point local interaction

T. Hyodo and W. Weise, Phys. Rev. **C 77** (2008)

K. Miyahara and T. Hyodo, Phys. Rev. C **93** (2016) etc

Investigate the KN system in the Skyrme model
where the nucleon is described as a soliton.

2. The Skyrme model

The Skyrme model 1

T.H.R. Skyrme, Nucl. Phys. **31** (1962);
Proc. Roy. Soc. A **260** (1961)

- Describe the interaction between mesons and baryons by mesons
- Baryon emerges as a soliton of meson fields.

$$\phi = \frac{1}{\sqrt{2}}\lambda_a \phi_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$U = \exp\left[i\frac{2}{F_\pi}\lambda_a \phi_a \right]$$

λ_a: Gell-Mann matrices (a = 1, 2, … ,8)

The Skyrme model 1

T.H.R. Skyrme, Nucl. Phys. **31** (1962);
Proc. Roy. Soc. A **260** (1961)

- Describe the interaction between mesons and baryons by mesons
- Baryon emerges as a soliton of meson fields.

$$\phi = \frac{1}{\sqrt{2}}\lambda_a\phi_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$U = \exp\left[i\frac{2}{F_\pi}\lambda_a\phi_a\right]$$

λ_a: Gell-Mann matrices (a = 1, 2, … ,8)

- **For SU(2)**

$$L = \underbrace{\frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)}_{\text{kinetic term}} + \underbrace{\frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2}_{\text{the Skyrme term}}$$

F_π, e: parameters

The Skyrme model 2

・Hedgehog ansatz

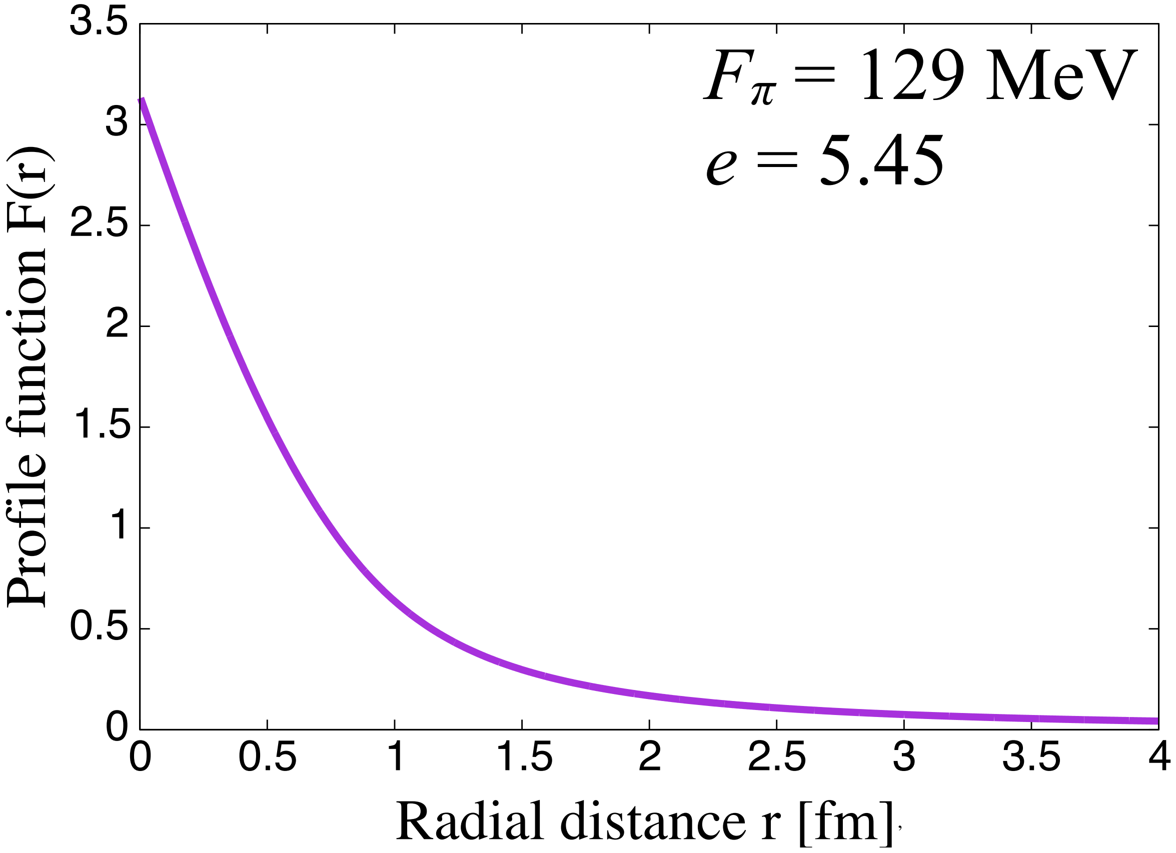
π has three degrees of freedom(π^0, π^+, π^-)

・two of these: the angles of the radial vector, θ, φ

・the rest: a function depending on r

⇒a special configuration called the hedgehog ansatz

Hedgehog ansatz: $U_H = \exp\left[i\boldsymbol{\tau}\cdot\hat{r}\boxed{F(r)}\right]$



minimize the mass of the soliton

with B.C.: $F(\infty) = 0,\ F(0) = \pi$

G. S. Adkins, C. R. Nappi and E. Witten,
Nucl. Phys. B **228** (1983)

The Skyrme model 3

・Quantization

The hedgehog solution is a classical field configuration

→without spin or isospin

→become a physical state by quantization

$$U_H(\boldsymbol{x}) \to U_H(t,\boldsymbol{x}) = A(t)\exp\left[i\tau_a R_{ab}(t)\hat{r}_b F(r)\right]A^\dagger(t)$$

$A(t)$: 2×2 isospin rotation matrix

$R_{ab}(t)$: 3×3 spatial rotation matrix

Baryon with $I=J$ are generated due to the symmetry which the hedgehog ansatz has

・Quantized Hamiltonian

$$H = M_{sol} + \boxed{\frac{J(J+1)}{2\Lambda}}$$

the rotation energy

M_{sol}: soliton mass

J: spin or isospin value

Λ: moment of inertia

3. Method

Method

SU(3) symmetry is broken → $m_u = m_d = 0, m_s \neq 0$

Callan-Klebanov approach (CK approach)

- Introduce the kaon as fluctuations around the hedgehog soliton
- Form a bound state of the kaon and the hedgehog soliton
- rotate the system to generate hyperons
- Follow the $1/N_c$ counting rule
-

C.G. Callan and I. Klebanov, Nucl. Phys. **B 262** (1985)

C .G.Callan, K .Hornbostel and I. Klebanov, Phys. Lett. **B 202** (1988)

Our approach

- Rotate the hedgehog soliton to generate the nucleon
- Introduce the kaon as fluctuations around the nucleon
- describe kaon-nucleon systems
- Violate the $1/N_c$ counting rule
-

Method

SU(3) symmetry is broken → $m_u = m_d = 0, m_s \neq 0$

Callan-Klebanov approach (CK approach)

- Introduce the kaon as fluctuations around the hedgehog soliton
- Form a bound state of the kaon and the hedgehog soliton
- rotate the system to generate hyperons
- Follow the $1/N_c$ counting rule
- Projection after variation, The strong coupling

C.G. Callan and I. Klebanov, Nucl. Phys. **B 262** (1985)
C .G.Callan, K .Hornbostel and I. Klebanov, Phys. Lett. **B 202** (1988)

Our approach

- Rotate the hedgehog soliton to generate the nucleon
- Introduce the kaon as fluctuations around the nucleon
- describe kaon-nucleon systems
- Violate the $1/N_c$ counting rule
- Variation after projection, The weak coupling

T. Ezoe. and A. Hosaka Phys. Rev. D **94**, 034022 (2016)

Lagrangian and ansatz

- **Extension to the SU(3) Skyrme model**

$$\begin{aligned} L \quad &= \quad \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 \\ &\quad\quad + L_{SB} + L_{WZ} \end{aligned}$$

- **Ansatz**

$$U = \begin{cases} A\left(t\right)\sqrt{U_\pi}U_K\sqrt{U_\pi}A^\dagger\left(t\right) & \text{: Callan-Klebanov ansatz} \\ A\left(t\right)\sqrt{U_\pi}A^\dagger\left(t\right)U_K A\left(t\right)\sqrt{U_\pi}A^\dagger\left(t\right) & \textbf{: Our ansatz} \end{cases}$$

$$U_\pi = \begin{pmatrix} U_H & 0 \\ 0 & 1 \end{pmatrix}$$

Hedgehog ansatz
(2×2 matrix)

$$U_K = \exp\left[i\frac{2}{F_\pi}\lambda_a K_a\right], \quad a = 3, 4, 5, 6$$

$$\lambda_a K_a = \sqrt{2}\begin{pmatrix} 0_{2\times 2} & K \\ K^\dagger & 0 \end{pmatrix} \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad K^\dagger = \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix}$$

Lagrangian and ansatz

- **Extension to the SU(3) Skyrme model**

$$L = \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 + L_{SB} + L_{WZ}$$

- **Ansatz**

the kaon around the hedgehog soliton

$$U = \begin{cases} A\left(t\right)\boxed{\sqrt{U_\pi}U_K\sqrt{U_\pi}}A^\dagger\left(t\right) & \text{: Callan-Klebanov ansatz} \\ \boxed{A\left(t\right)\sqrt{U_\pi}A^\dagger\left(t\right)}U_K\boxed{A\left(t\right)\sqrt{U_\pi}A^\dagger\left(t\right)} & \textbf{: Our ansatz} \end{cases}$$

the kaon **around the rotating hedgehog soliton**

Derivation 1

・Substitute our ansatz for the Lagrangian

Ansatz

$$U = A(t)\sqrt{U_\pi}A^\dagger(t)U_K A(t)\sqrt{U_\pi}A^\dagger(t)$$

$$U_K = \exp\left[i\frac{2}{F_\pi}\lambda_a K_a\right], \quad a = 3,4,5,6 \qquad U_\pi = \begin{pmatrix} U_H & 0 \\ 0 & 1 \end{pmatrix}$$

$$\lambda_a K_a = \sqrt{2}\begin{pmatrix} 0_{2\times 2} & K \\ K^\dagger & 0 \end{pmatrix} \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad K^\dagger = \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix}$$

Lagrangian

$$L \quad = \quad \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 + L_{SB} + L_{WZ}$$

・Expand U_K up to second order of the kaon field K

Obtaining Lagrangian

$$L = L_{SU(2)} + L_{KN}$$

$$L_{SU(2)} = \frac{1}{16}{F_\pi}^2 \mathrm{tr}\left[\partial_\mu \tilde{U}^\dagger \partial^\mu \tilde{U}\right] + \frac{1}{32e^2}\mathrm{tr}\left[\partial_\mu \tilde{U}\tilde{U}^\dagger, \partial_\nu \tilde{U}\tilde{U}^\dagger\right]^2$$

$$\begin{aligned} L_{KN} \;\; = \;\; & \left(D_\mu K\right)^\dagger D^\mu K - K^\dagger a_\mu^\dagger a^\mu K - m_K^2 K^\dagger K \\ & + \frac{1}{(eF_\pi)^2}\left\{ -K^\dagger K \mathrm{tr}\left[\partial_\mu \tilde{U}\tilde{U}^\dagger, \partial_\nu \tilde{U}\tilde{U}^\dagger\right]^2 - 2\left(D_\mu K\right)^\dagger D_\nu K \mathrm{tr}\left(a^\mu a^\nu\right) \right. \\ & \qquad \left. - \frac{1}{2}\left(D_\mu K\right)^\dagger D^\mu K \mathrm{tr}\left(\partial_\nu \tilde{U}^\dagger \partial^\nu \tilde{U}\right) + 6\left(D_\nu K\right)^\dagger \left[a^\nu, a^\mu\right] D_\mu K \right\} \\ & + \frac{3i}{F_\pi^2} B^\mu \left[\left(D_\mu K\right)^\dagger K - K^\dagger \left(D_\mu K\right)\right] \end{aligned}$$

$$\tilde{U} = A(t) U_H A^\dagger(t), \quad \tilde{\xi} = A(t)\sqrt{U_H} A^\dagger(t) \qquad D_\mu K = \partial_\mu K + v_\mu K$$

$$v_\mu = \frac{1}{2}\left(\tilde{\xi}^\dagger \partial_\mu \tilde{\xi} + \tilde{\xi}\partial_\mu \tilde{\xi}^\dagger\right)$$

$$a_\mu = \frac{1}{2}\left(\tilde{\xi}^\dagger \partial_\mu \tilde{\xi} - \tilde{\xi}\partial_\mu \tilde{\xi}^\dagger\right)$$

$$B^\mu = -\frac{\varepsilon^{\mu\nu\alpha\beta}}{24\pi^2}\mathrm{tr}\left[\left(U_H^\dagger \partial_\nu U_H\right)\left(U_H^\dagger \partial_\alpha U_H\right)\left(U_H^\dagger \partial_\beta U_H\right)\right]$$

G. S. Adkins, C. R. Nappi and E. Witten,
Nucl. Phys. B **228** (1983)

Derivation 2

・Decompose the kaon filed

Spatial wave function

$$\begin{pmatrix} K^{+} \\ K^{0} \end{pmatrix} = \psi_I K\left(t, \boldsymbol{r}\right) \rightarrow \underline{\psi_I}\ \underline{K\left(\boldsymbol{r}\right)}\, e^{-iEt}$$

Isospin wave function

・Expand the $K(r)$ by the spherical harmonics

$$K\left(\boldsymbol{r}\right) = \sum_{l,m} C_{lm\alpha} Y_{lm}\left(\theta, \phi\right) k_l^{\alpha}\left(r\right)$$

$Y_{lm}(\theta, \varphi)$: Spherical harmonics

l : orbital angular momentum

m : the 3rd component of l

α : the other quantum numbers

・Take a variation with respect to the kaon radial function

⇒Obtain the equation of motion for the kaon around the nucleon

4. Results and discussions

Equation of motion and potential

• Equation of motion(E.o.M)

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2 h(r)\frac{dk_l^\alpha(r)}{dr}\right) - E^2 f(r)k_l^\alpha(r) + \left(m_K^2 + V(r)\right)k_l^\alpha(r) = 0$$:Klein-Gordon like

$$\longrightarrow -\frac{1}{m_K+E}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dk_l^\alpha(r)}{dr}\right) + U(r)k_l^\alpha(r) = \varepsilon k_l^\alpha(r)$$:Schrödinger like

$$\boxed{\begin{aligned} U(r) &= \\ &-\boxed{\frac{1}{m_K+E}}\left[\frac{h(r)-1}{r^2}\boxed{\frac{d}{dr}\left(r^2\frac{d}{dr}\right)}+\frac{dh(r)}{dr}\boxed{\frac{d}{dr}}\right]-\boxed{\frac{\left(f(r)-1\right)E^2}{m_K+E}}+\boxed{\frac{V(r)}{m_K+E}}\end{aligned}}$$

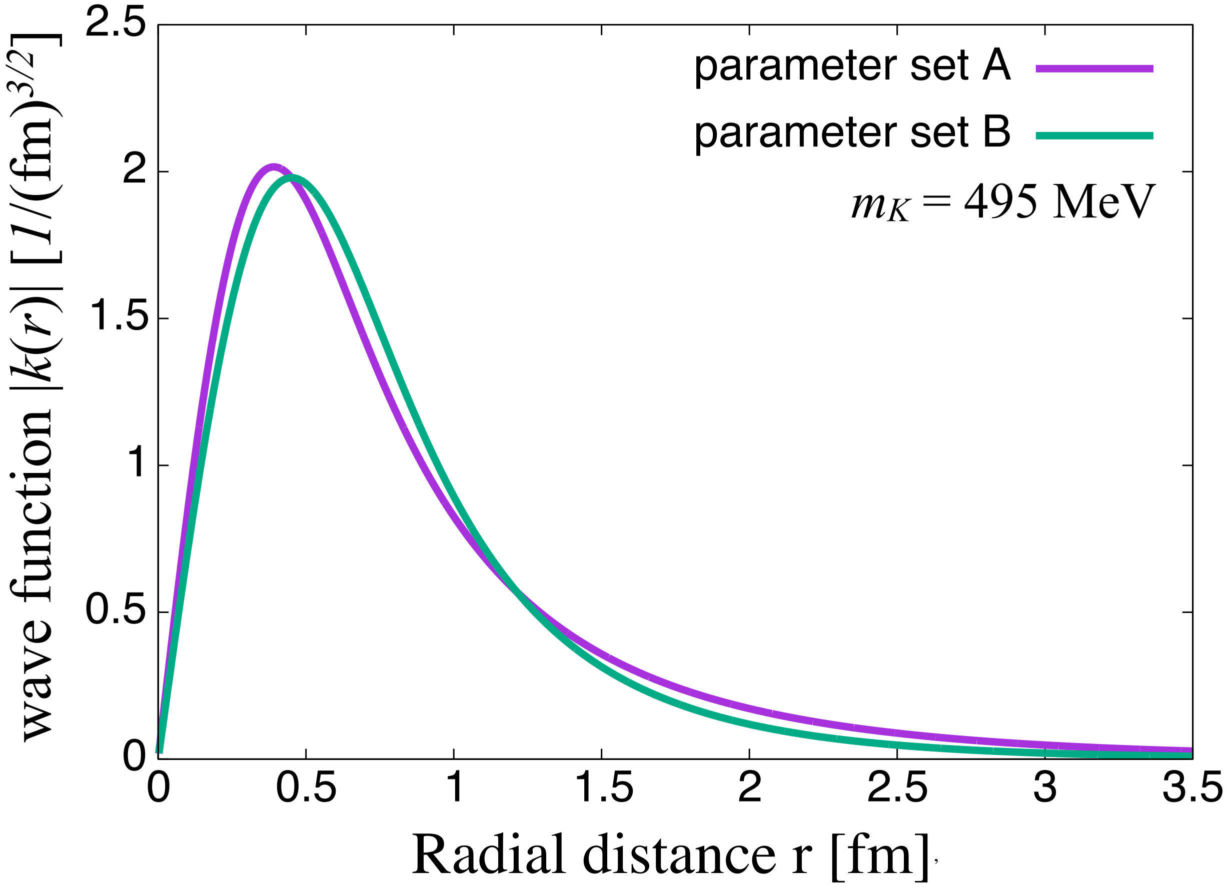
Equivalent local potential: $\boxed{\tilde{U}(r) = \frac{U(r)\,k_l^\alpha(r)}{k_l^\alpha(r)}}$

• Properties of resulting potential U

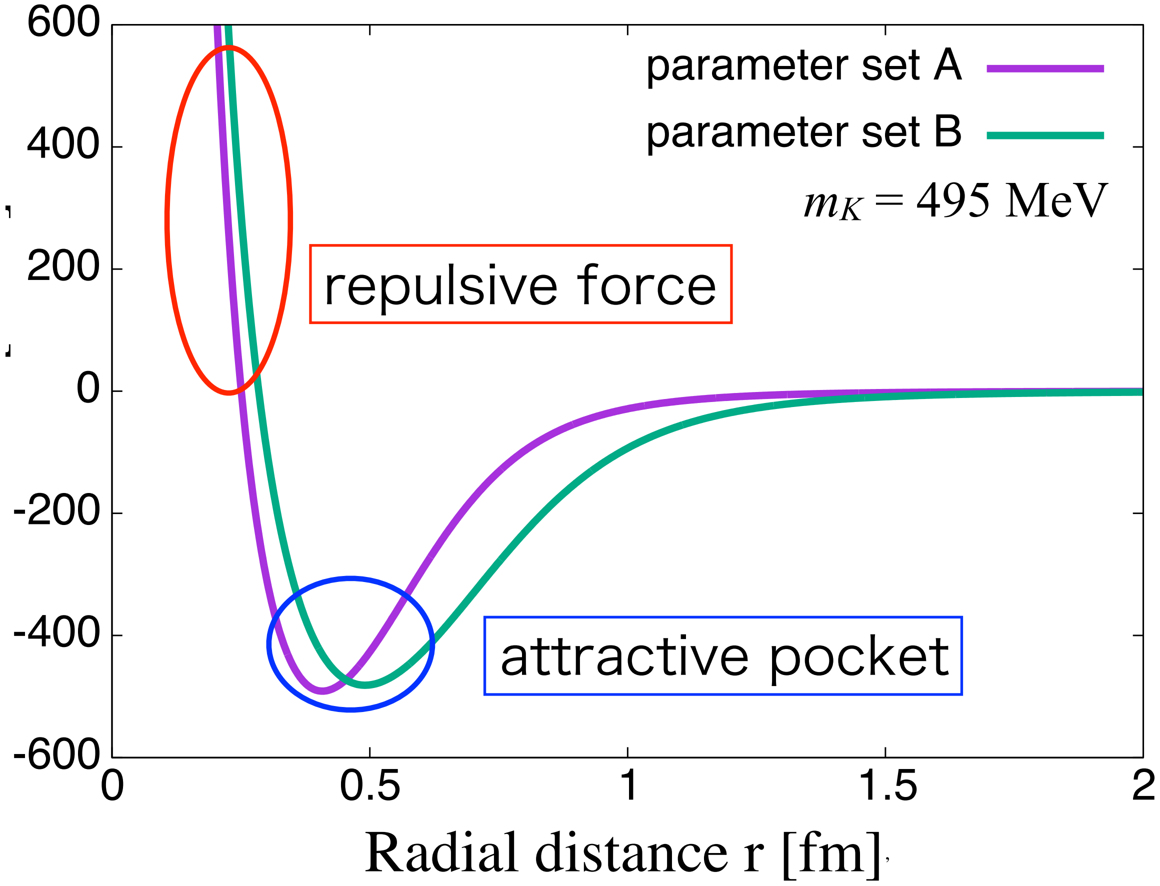
1. Nonlocal and depend on the kaon energy
2. Contain isospin dependent and independent **central forces**
   and the similar **spin-orbit(LS) forces**
3. A repulsive component is proportional to $1/r^2$ at short distances

Result 1: $\bar{\mathrm{K}}$N(I = 0, L = 0) Bound state

Kaon wave function



Potential



repulsive force

attractive pocket

• Model parameters and physical properties

	F_π [MeV]	e	B.E. [MeV]	$\langle r_N^2\rangle^{1/2}$ [fm]	$\langle r_K^2\rangle^{1/2}$ [fm]
parameter set A	186	4.82	32.9	0.46	1.18
parameter set B	129	5.45	82.9	0.59	0.99

T. Ezoe. and A. Hosaka Phys. Rev. D **94**, 034022 (2016)

$$\langle r_N^2\rangle = \int_0^\infty dr\ r^2 \rho_B(r)\,, \quad \rho_B(r) = -\frac{2}{\pi}\sin^2 F F'$$

G. S. Adkins, C. R. Nappi and E. Witten,
Nucl. Phys. B **228** (1983)

$$\langle r_K^2\rangle = \int dV\ r^2 \left[Y_{00}(\hat{r})\, k_0^0(r)\right]^2 = \int_0^\infty dr\ r^4 k^2(r) \qquad Y_{00} = \frac{1}{\sqrt{4\pi}}$$

Comparisons with the chiral theory

• Weinberg-Tomozawa intereaction

$$L_{WT} = \frac{2}{F_\pi^2}\left\{\bar{N}\boldsymbol{I}^N\gamma^\mu N\cdot\left(\partial_\mu K^\dagger \boldsymbol{I}^K K - K^\dagger \boldsymbol{I}^K \partial_\mu K\right)\right\} \propto \frac{1}{F_\pi^2}$$

S. Weinberg, Phys. Rev. Lett. **17** (1966)
Y. Tomozawa, Nuovo Cim. A **46** (1966)

The strength of $L_{WT} \propto 1/F_\pi^2$

→For F_π = 129 MeV and 186 MeV,

$$1/129^2 : 1/186^2 \sim \boxed{15 : 7}$$

• The interaction for K̄N($I = 0$) bound state

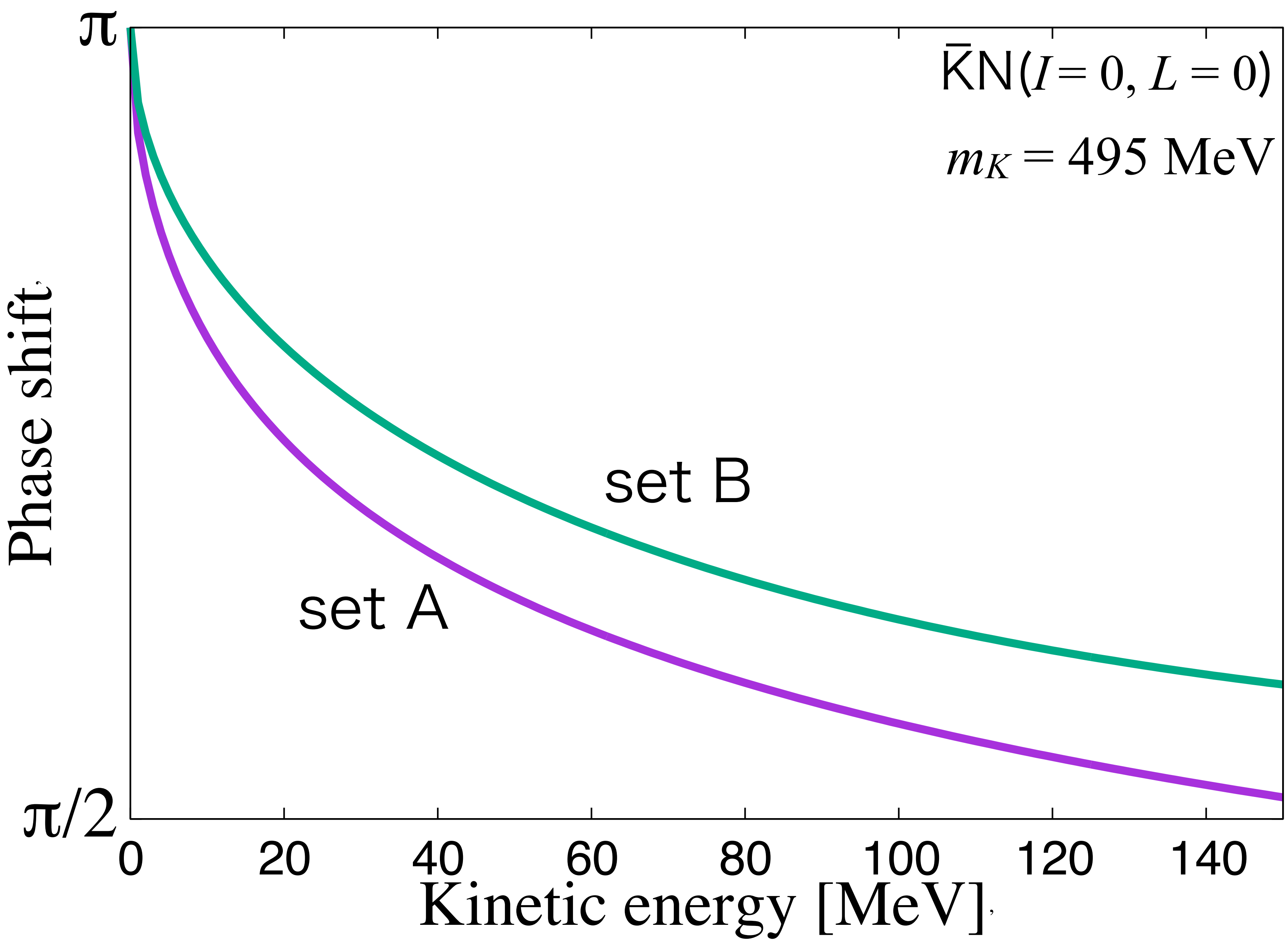
$$W \equiv 4\pi \int r^2 dr \tilde{U}\left(r\right)$$

F_π [MeV]	e	$-W \times 10^5$ [1/MeV ^{2}]
129	5.45	$1.2 \times 4\pi$
186	5.45	$0.48 \times 4\pi$

→ $\boxed{5:2}$

T. Ezoe. and A. Hosaka Phys. Rev. D **94**, 034022 (2016)

Result 2: $\bar{K}N(I=0, L=0)$ phase shift



<u>$\bar{K}N(I=0, L=0)$ binging energies</u>

	F_π [MeV]	e	B.E. [MeV]
parameter set A	186	4.82	32.9
parameter set B	129	5.45	82.9

T. Ezoe. and A. Hosaka Phys. Rev. D **94**, 034022 (2016)

Result 3: Fitting the potentials

$$\tilde{U}\left(r\right)=\tilde{U}_0^c\left(r\right)+\tilde{U}_\tau^c\left(r\right)\left(\boldsymbol{I}^K\cdot\boldsymbol{I}^N\right)+\tilde{U}_0^{LS}\left(r\right)\left(\boldsymbol{L}^K\cdot\boldsymbol{J}^N\right)+\tilde{U}_\tau^{LS}\left(r\right)\left(\boldsymbol{L}^K\cdot\boldsymbol{J}^N\right)\left(\boldsymbol{I}^K\cdot\boldsymbol{I}^N\right)$$

	Isospin	Normal term	Wess-Zumino term
Central	indep.	$u_0^c(N,r)+v_0^c(N,r)E_{kin}$ $G_{-2}\left(r\right)+G_0\left(r\right)+G_2\left(r\right)$	$u_0^c(WZ,r)+v_0^c(WZ,r)E_{kin}$ $G_0\left(r\right)+G_0\left(r\right)$
	dep.	$u_\tau^c(N,r)+v_\tau^c(N,r)E_{kin}$ $G_0\left(r\right)+G_2\left(r\right)$	— —
LS	indep.	$u_0^{LS}(N,r)+v_0^{LS}(N,r)E_{kin}$ $G_0\left(r\right)+G_0\left(r\right)$	$u_0^{LS}(WZ,r)+v_0^{LS}(WZ,r)E_{kin}$ $G_0\left(r\right)+G_0\left(r\right)$
	dep.	$u_\tau^{LS}(N,r)+v_\tau^{LS}(N,r)E_{kin}$ $G_{-2}\left(r\right)+G_{-2}\left(r\right)$	— —
Centrifugal force		$u_l\left(r\right)+v_l\left(r\right)E_{kin}$ $\propto\left(G_0\left(r\right)+G_0\left(r\right)\right)/r^2$	

$$\begin{aligned}\tilde{U}\left(r\right)&\simeq\tilde{U}\left(r\right)+\frac{\partial\tilde{U}\left(r\right)}{\partial E_{kin}}E_{kin}\\&\equiv u\left(r\right)+v\left(r\right)E_{kin}\end{aligned}$$

fit by Gaussian

$$G_{-2}\left(r\right)=C_{-2}\frac{1}{r^2/{R_{-2}}^2}\exp\left(-\frac{r^2}{{R_{-2}}^2}\right)$$

$$G_0\left(r\right)=C_0\exp\left(-\frac{r^2}{{R_0}^2}\right)$$

$$G_2\left(r\right)=C_2\frac{r^2}{{R_2}^2}\exp\left(-\frac{r^2}{{R_2}^2}\right)$$

Fitting parameters(Central terms)

parameter set A: $F_\pi = 186$ MeV, $e = 4.82$

- **$\tilde{U}_0{}^c(N, r)$**

s-wave

	$G_{-2}(r)$	$G_0(r)$	$G_2(r)$
Range [fm]	0.176	0.271	0.393
$u_0^c(N,r)$ [MeV]	2911.49	2545.87	-507.819
$v_0^c(N,r)$ [1]	-1.98786	-5.61873	-4.41952×10^{-1}

p-wave

	$G_{-2}(r)$	$G_0(r)$	$G_2(r)$
Range [fm]	0.318	0.312	0.320
$u_0^c(N,r)$ [MeV]	-2771.64	1916.04	-411.560
$v_0^c(N,r)$ [1]	2.62581	-2.87808	-1.76763

- **$\tilde{U}_\tau{}^c(N, r)$**

	$G_0(r)$	$G_2(r)$
Range [fm]	0.265	0.524
$u_\tau^c(N,r)$ [MeV]	401.337	290.964
$v_\tau^c(N,r)$ [1]	0.405391	0.293903

- **$\tilde{U}_0{}^c(WZ, r)$**

	$G_0(r)$	$G_0(r)$
Range [fm]	0.282	0.404
$u_0^c(WZ,r)$ [MeV]	-676.51	-1207.07
$v_0^c(WZ,r)$ [1]	-3.483	-0.995

Fitting parameters(LS and centrifugal terms)

parameter set A: $F_\pi = 186$ MeV, $e = 4.82$

- **$\tilde{U}_0^{LS}(N, r)$**

	$G_0(r)$	$G_0(r)$
Range [fm]	0.483	0.300
$u_0^{LS}(N,r)$ [MeV]	38.8404	63.4182
$v_0^{LS}(N,r)$ [1]	0.392332×10^{-1}	0.630481×10^{-1}

- **$\tilde{U}_\tau^{LS}(N, r)$**

	$G_{-2}(r)$	$G_{-2}(r)$
Range [fm]	0.604	0.262
$u_\tau^{LS}(N,r)$ [MeV]	-1284.46	-6954.51
$v_\tau^{LS}(N,r)$ [1]	1.29744	7.02476

- **$\tilde{U}_0^{LS}(WZ, r)$**

	$G_0(r)$	$G_0(r)$
Range [fm]	0.377	0.243
$u_0^{LS}(WZ,r)$ [MeV]	-363.915	-287.034
$v_0^{LS}(WZ,r)$ [1]	0.367587	0.289936

- **$\tilde{U}_l(r)$**

	$G_0(r)$	$G_0(r)$
Range [fm]	0.431	0.748
$u_l(r)$ [MeV]	62867.86	7583.59
$v_l(r)$ [1]	63.5029	7.66019

$$\tilde{U}_l(r) = \frac{l(l+1)}{2m_K r^2}\left[G_0(r) + G_0(r)\right]$$

l: Kaon angular momentum

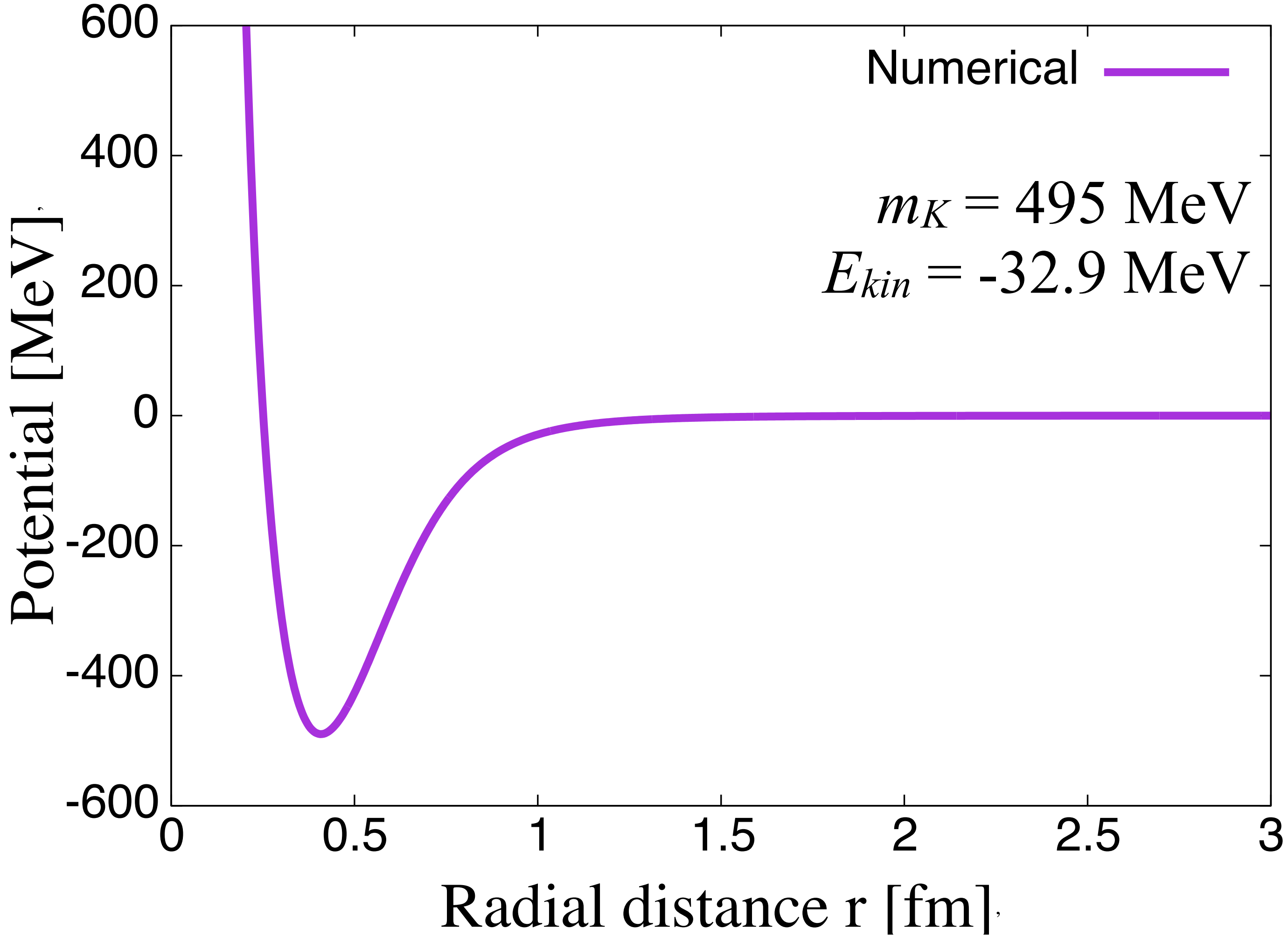
Comparing 1

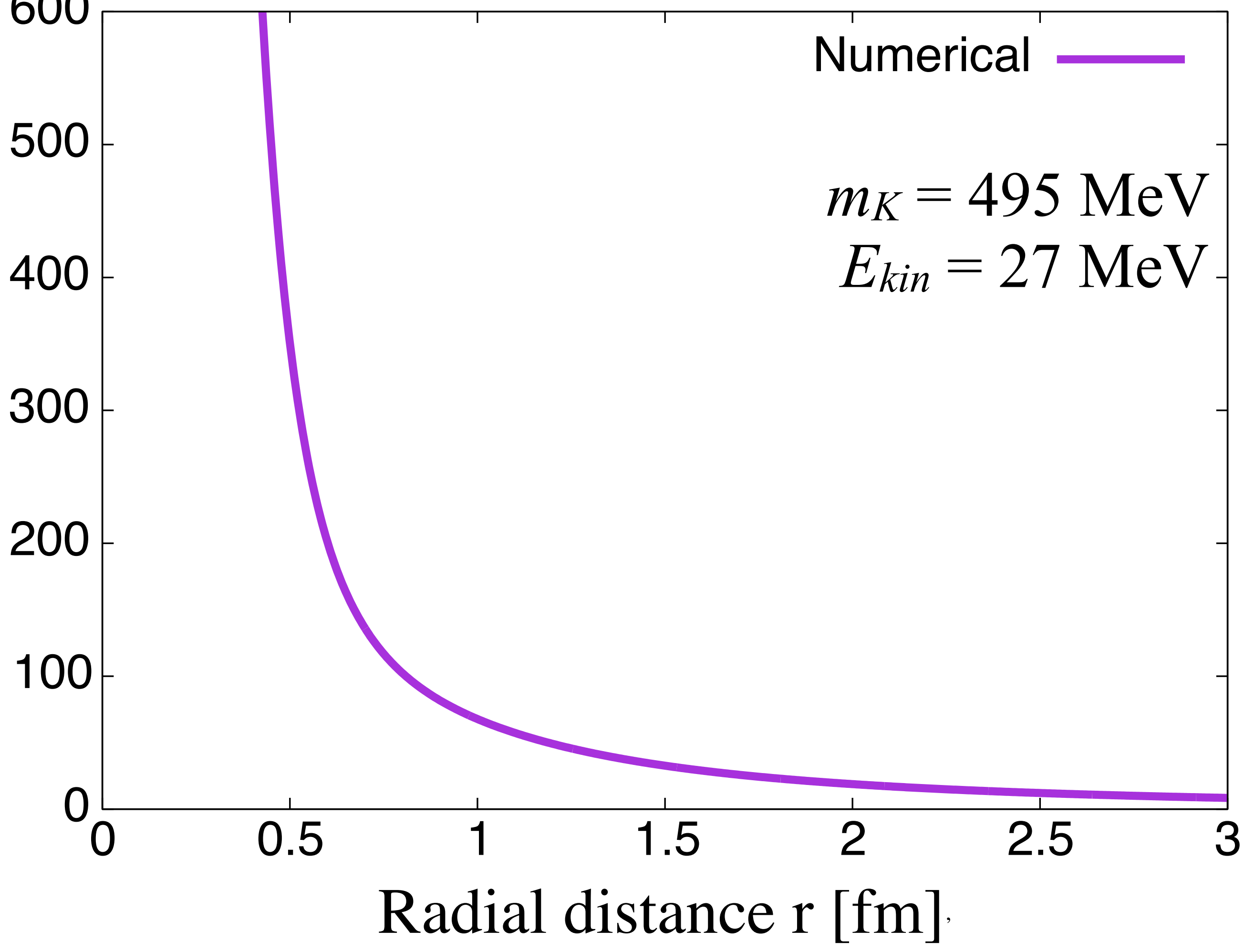
parameter set A: $F_\pi = 186\ \mathrm{MeV},\ e = 4.82$

$\bar{\mathrm{K}}\mathrm{N}\ (I = 0, L = 0)$ bound state

$\bar{\mathrm{K}}\mathrm{N}\ (I = 0, L = 1,\ J = 3/2)$

scattering state



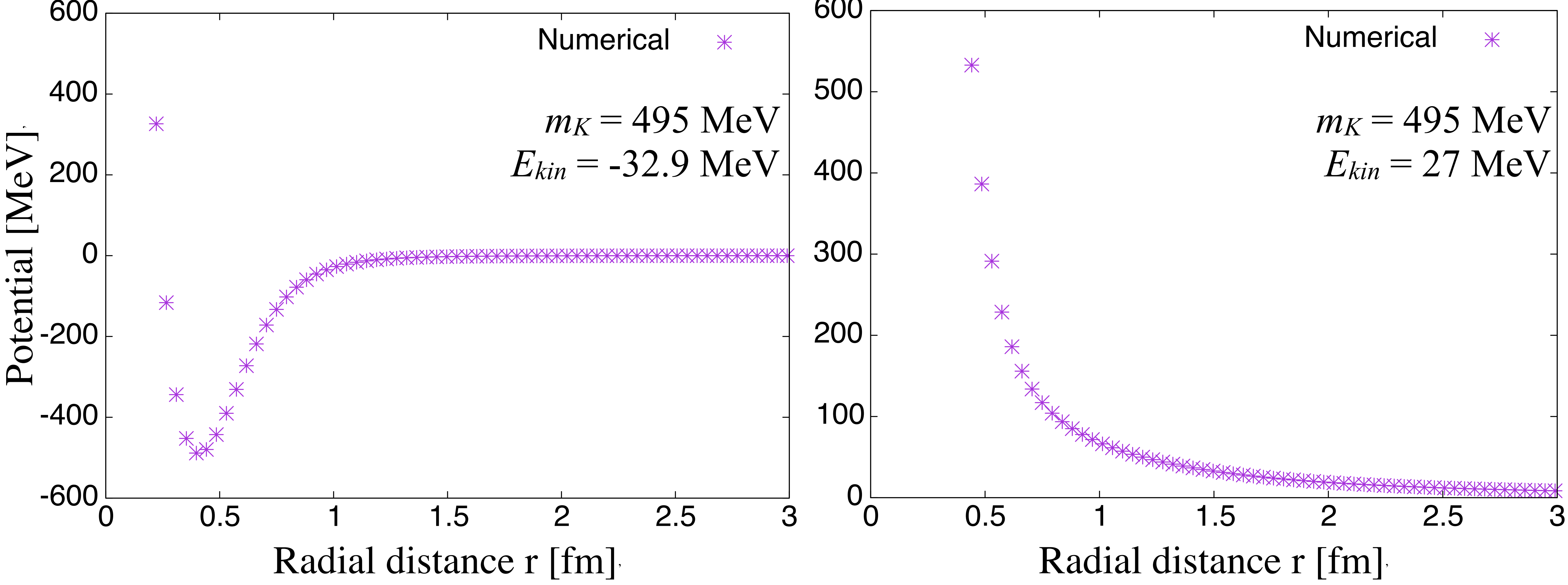


Comparing 1

parameter set A: $F_\pi = 186$ MeV, $e = 4.82$

K̄N $(I = 0, L = 0)$ bound state

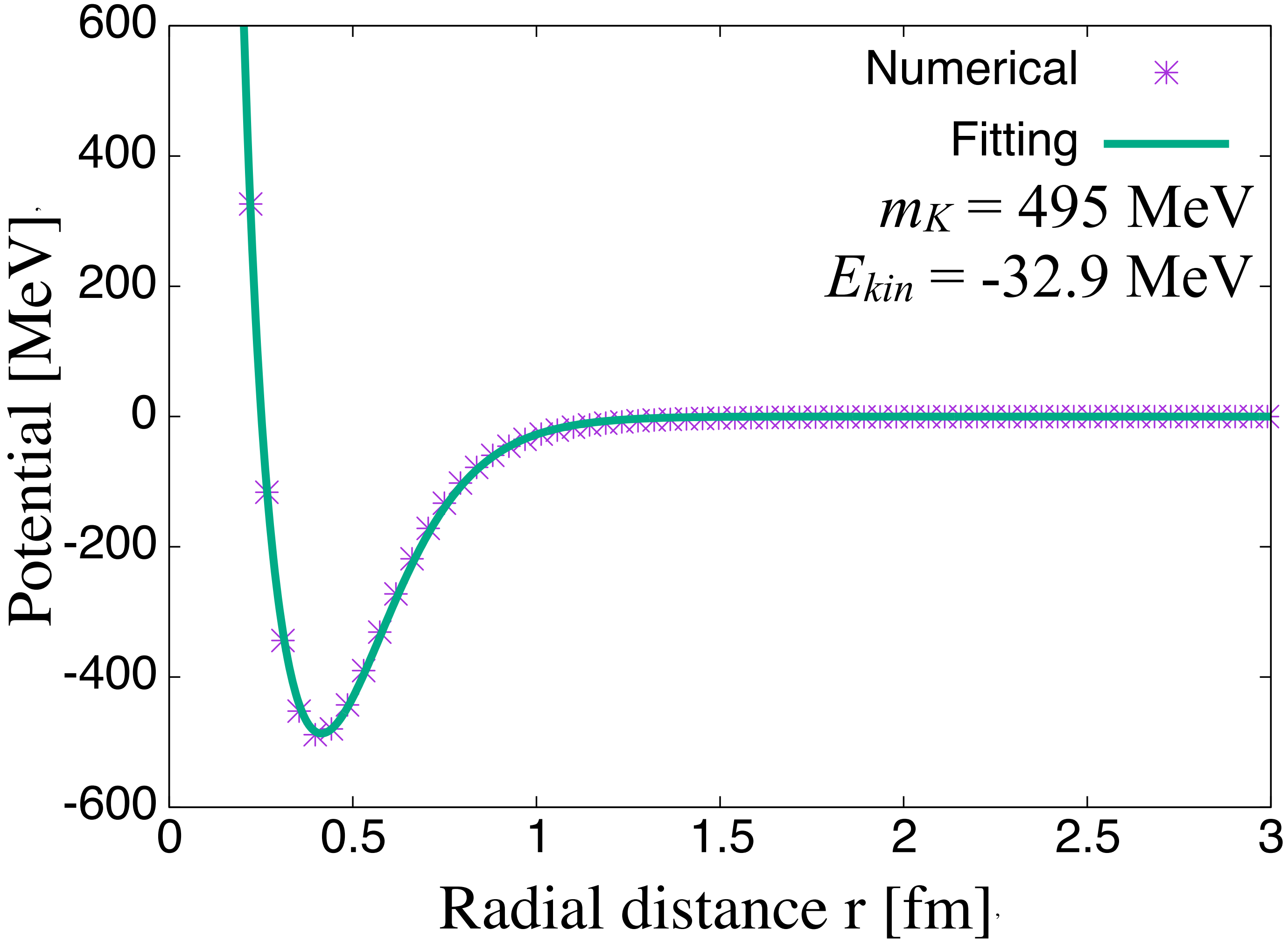
K̄N $(I = 0, L = 1,\ J = 3/2)$ scattering state



Comparing 1

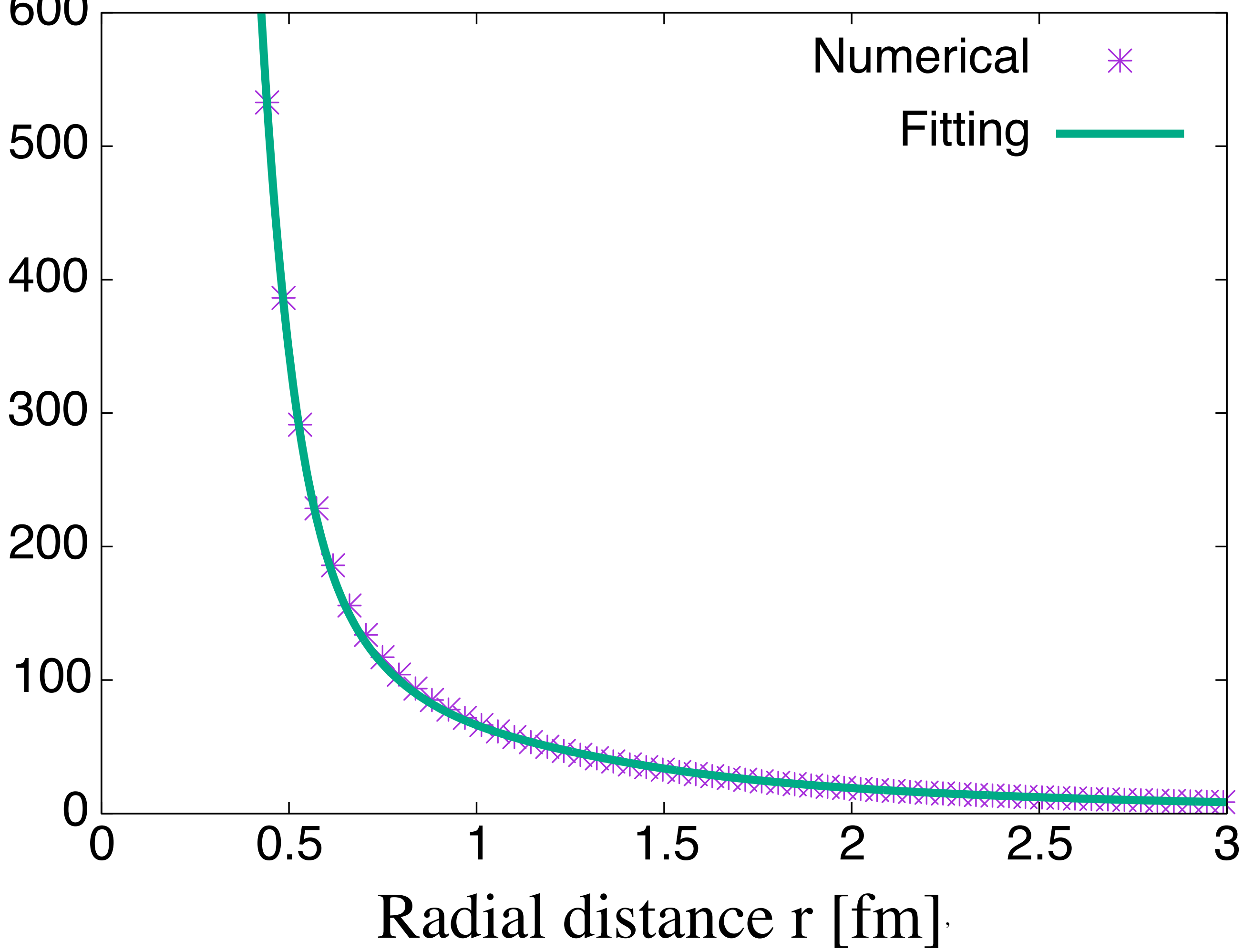
parameter set A: $F_\pi = 186$ MeV, $e = 4.82$

$\bar{K}N$ $(I = 0, L = 0)$ bound state



$\bar{K}N$ $(I = 0, L = 1,\ J = 3/2)$

scattering state



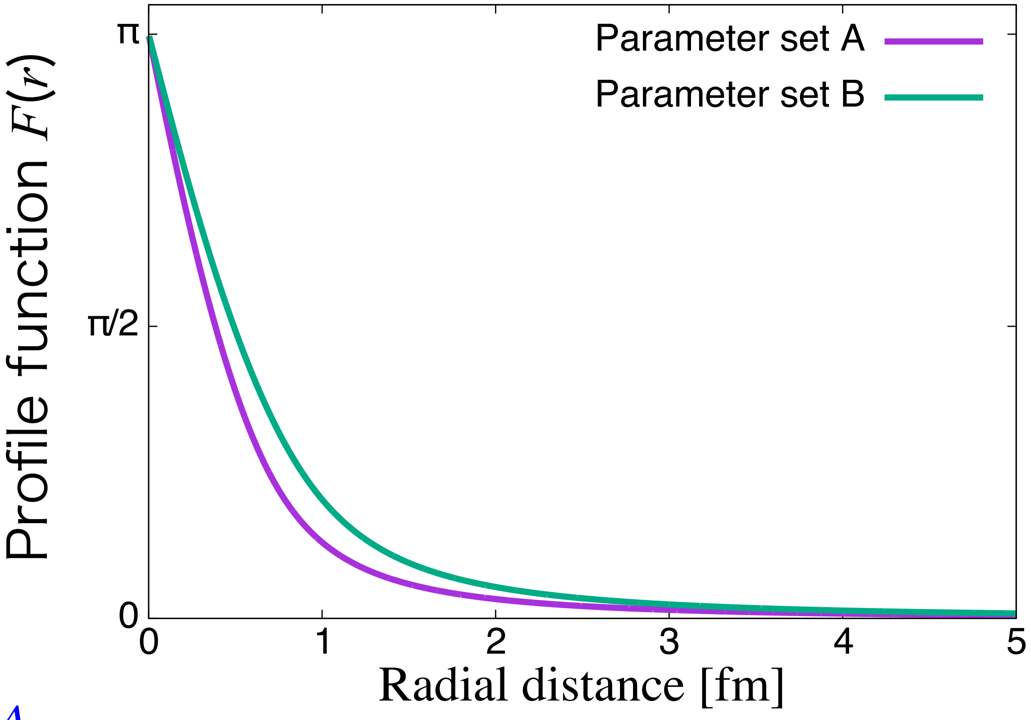
Scaling rule

Scaling rule for the profile function

$$\boxed{y = eF_\pi r}$$

y: radial distance [1]

r: radial distance [fm]



Set A: $F_\pi = 186$ MeV, $e = 4.82$

Set B: $F_\pi = 129$ MeV, $e = 5.45$

Scaling rule for the fitting functionals

$$G_0(r) = C_0 \exp\left(-\frac{r^2}{{R_0}^2}\right)$$

$$G_2(r) = C_2 \frac{r^2}{{R_2}^2} \exp\left(-\frac{r^2}{{R_2}^2}\right)$$

$$G_4(r) = C_4 \frac{r^4}{{R_4}^4} \exp\left(-\frac{r^2}{{R_4}^2}\right)$$

$$R^B = \frac{\alpha^A}{\alpha^B} R^A$$

$$C_0^B = C_0^A$$

$$C_2^B = \left(\frac{\alpha^B}{\alpha^A}\right)^2 C_2^A$$

$$C_4^B = \left(\frac{\alpha^B}{\alpha^A}\right)^4 C_4^A$$

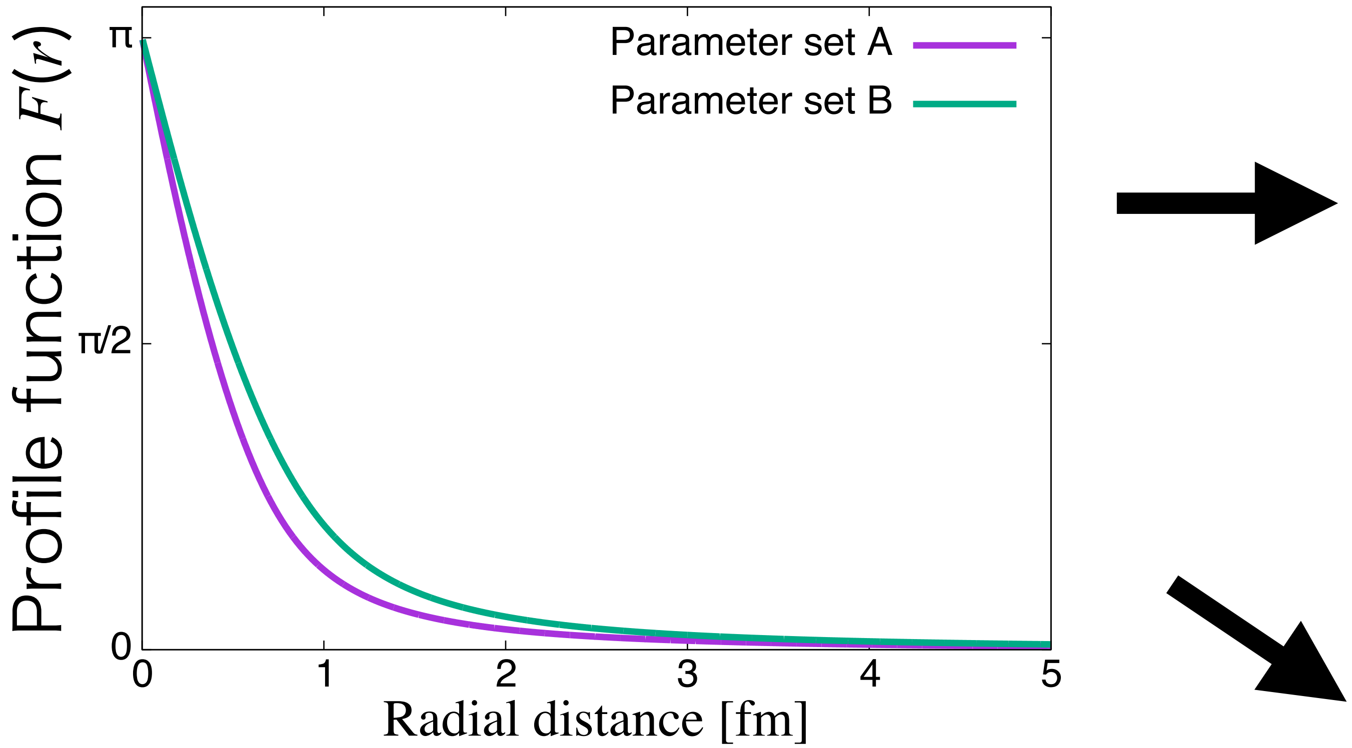
$$\boldsymbol{\alpha \equiv eF_\pi} \rightarrow \alpha^A = 186 \times 4.82, \quad \alpha^B = 129 \times 5.45$$

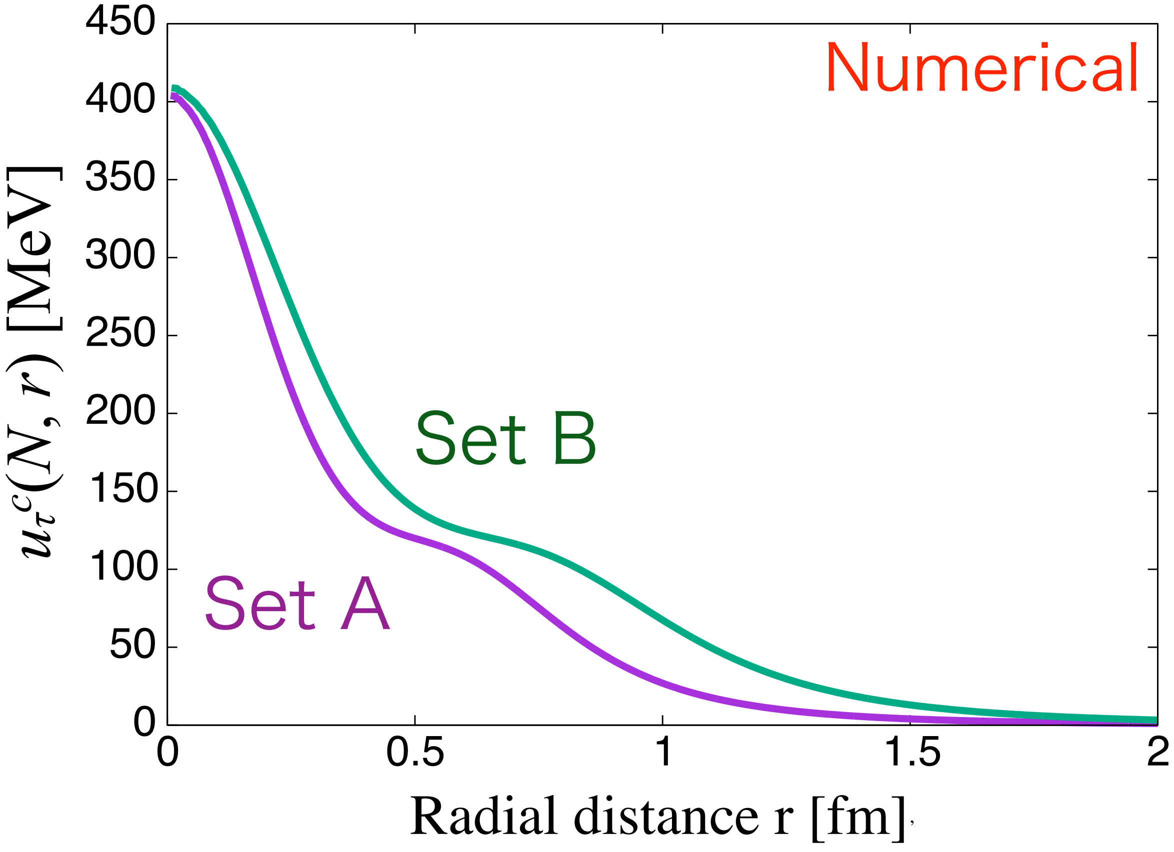
Scaling the fitting parameters from set A to set B

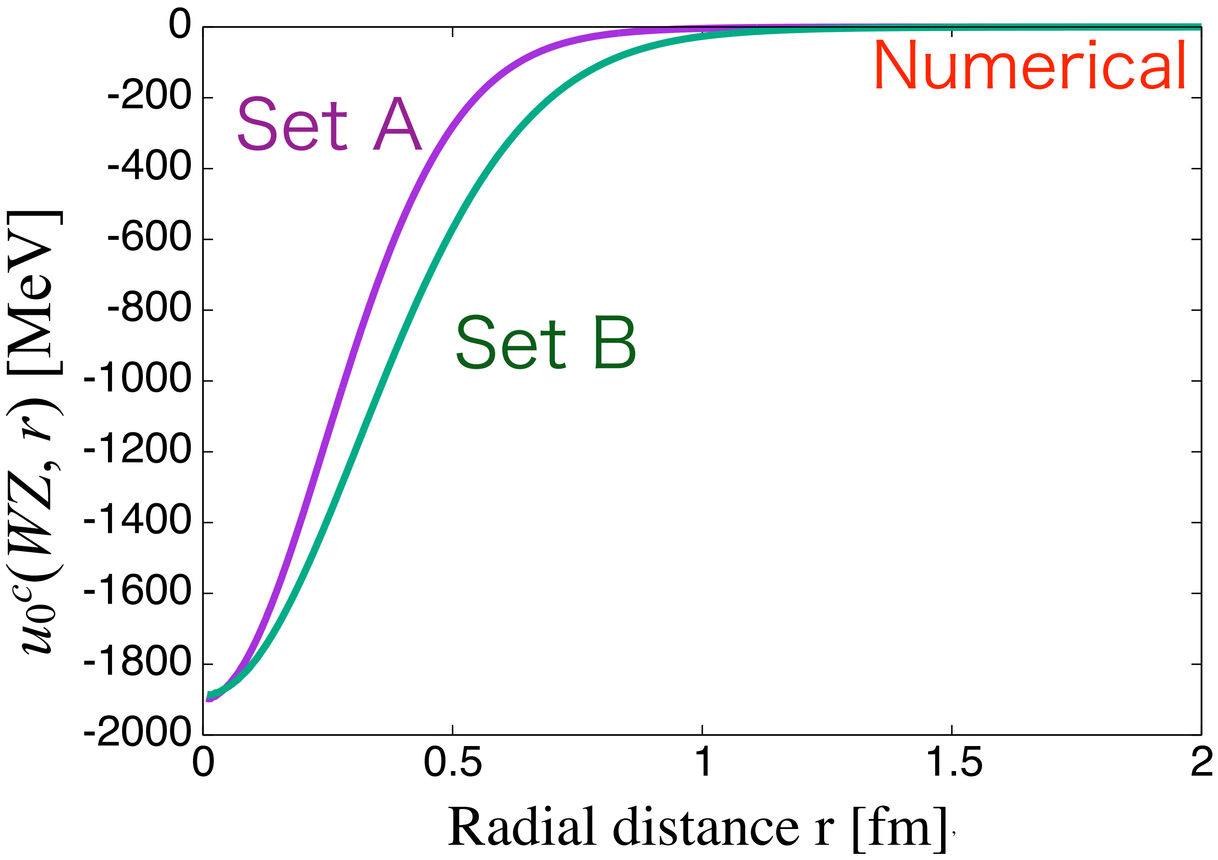
with the scaling rules

Scaling rule for convergent terms

Scaling rule for the profile function







Convergent contributions:

$\widetilde{U}_\tau{}^c(N, r)$, $\widetilde{U}_0{}^{LS}(N, r)$, $\widetilde{U}_0{}^c(WZ, r)$, $\widetilde{U}_0{}^{LS}(WZ, r)$

$$R^B = \frac{\alpha^A}{\alpha^B} R^A$$: Range

$$C_0^B = C_0^A$$: Strength for $G_0(r)$

$$C_2^B = \left(\frac{\alpha^B}{\alpha^A}\right)^2 C_2^A$$: Strength for $G_2(r)$

Scaling rule for divergent terms

$1/r^2$ repulsion does not depend on the scaling for $F(r)$

→Come from $\partial_i(\exp[i\tau_a r_a F(r)])$ and $\partial_i K(r)$

Divergent contributions: $\tilde{U}_0^c(N, r)$, $\tilde{U}_\tau^{LS}(N, r)$, $\tilde{U}_l(r)$

→Make them convergent by multiplying r^2 and divided by r^2 after scaling

	Isospin	Normal term
Central	indep.	$r^2 \times (u_0^c(N,r) + v_0^c(N,r)E_{kin})$ $G_0(r) + G_2(r) + G_4(r)$
LS	dep.	$r^2 \times \left(u_\tau^{LS}(N,r) + v_\tau^{LS}(N,r)E_{kin}\right)$ $G_0(r) + G_0(r)$
Centrifugal force		$r^2 \times (u_l(r) + v_l(r)E_{kin})$ $G_0(r) + G_0(r)$

$$G_0(r) = C_0 \exp\left(-\frac{r^2}{{R_0}^2}\right)$$

$$G_2(r) = C_2 \frac{r^2}{{R_2}^2} \exp\left(-\frac{r^2}{{R_2}^2}\right)$$

$$G_4(r) = C_4 \frac{r^4}{{R_4}^4} \exp\left(-\frac{r^2}{{R_4}^2}\right)$$

$$R^B = \frac{\alpha^A}{\alpha^B} R^A$$

$$C_0^B = C_0^A$$

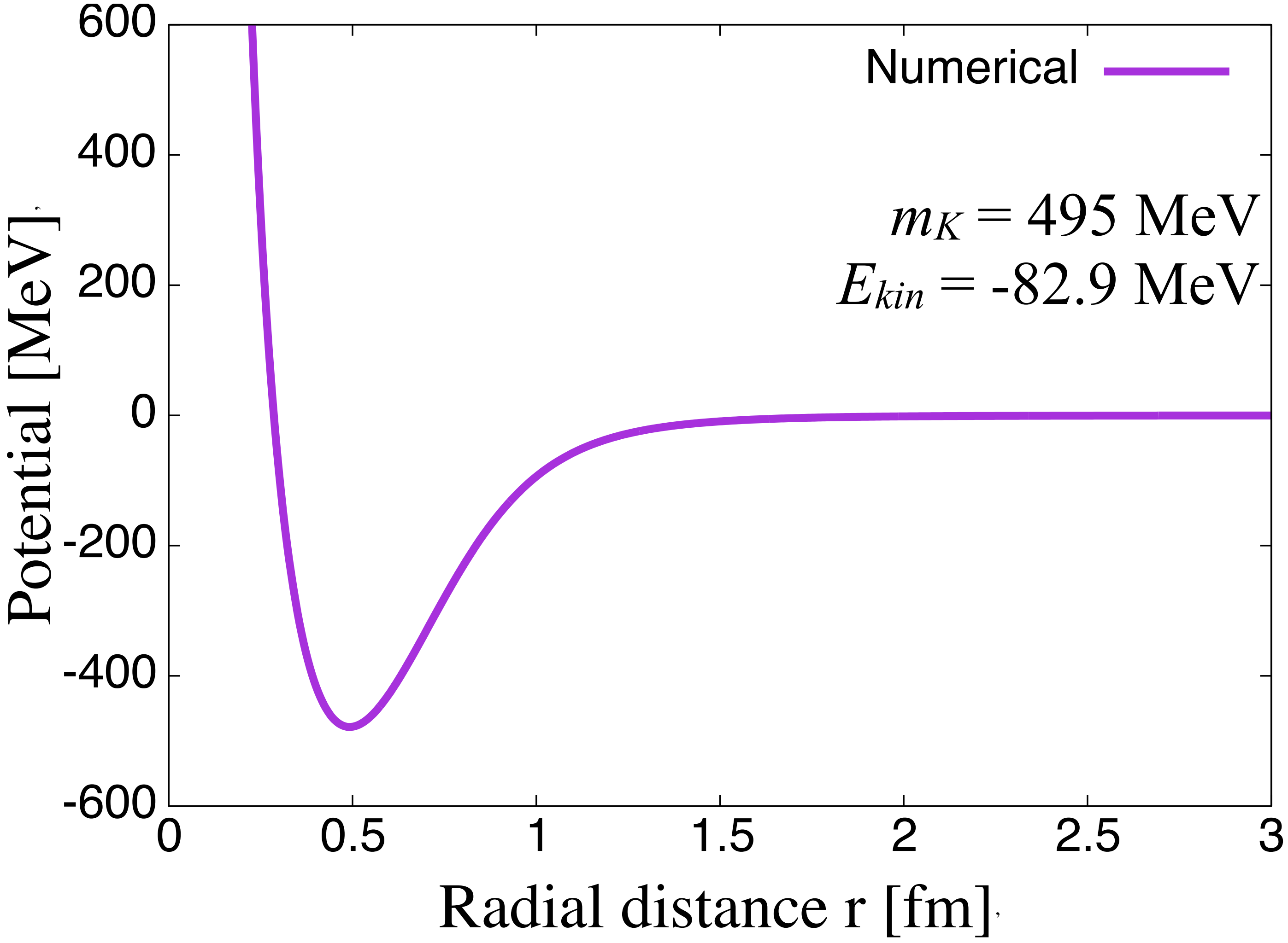
$$C_2^B = \left(\frac{\alpha^B}{\alpha^A}\right)^2 C_2^A$$

$$C_4^B = \left(\frac{\alpha^B}{\alpha^A}\right)^4 C_4^A$$

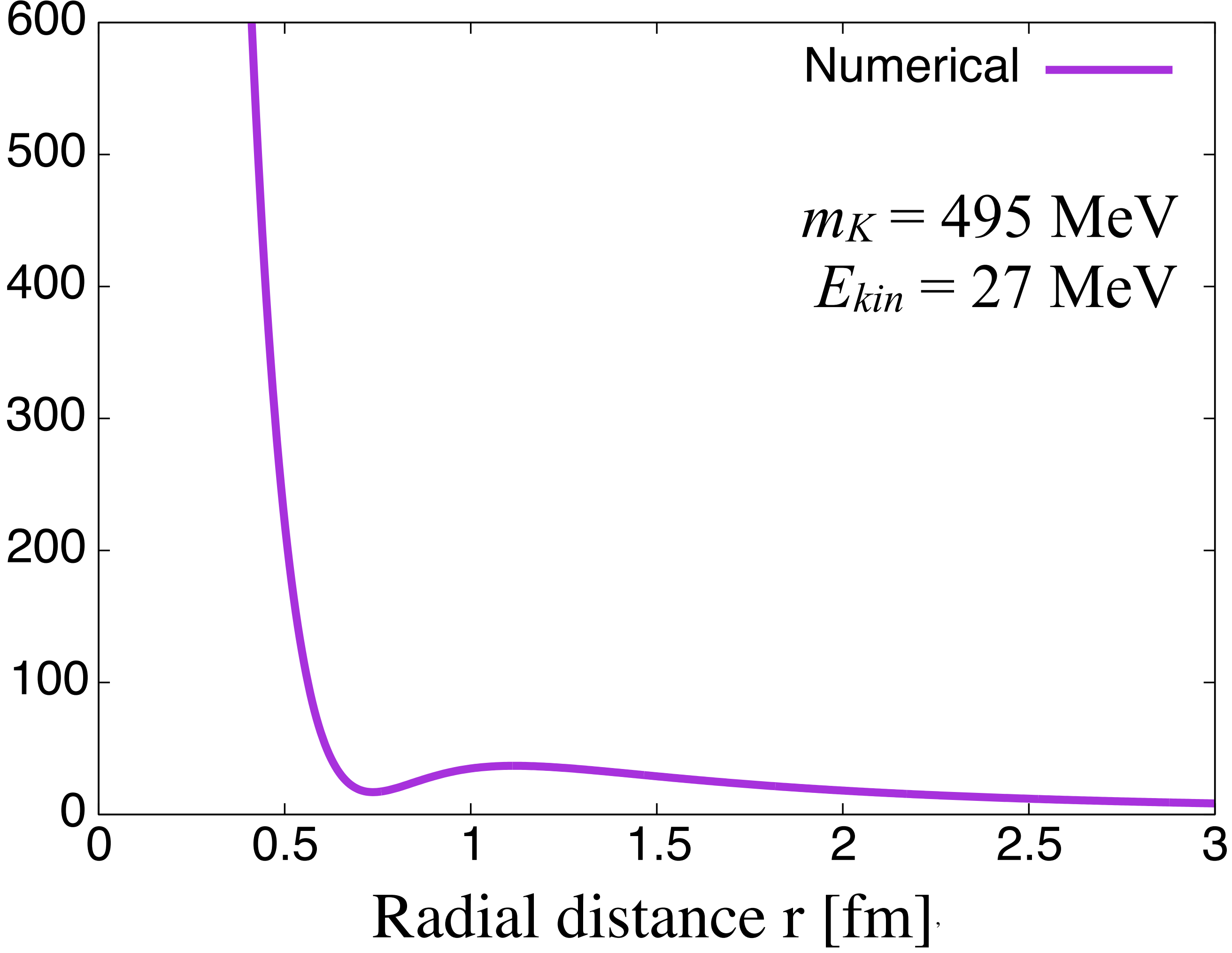
Comparing 2

parameter set B: $F_\pi = 129\ \mathrm{MeV}, e = 5.45$

$\bar{K}N$ $(I = 0, L = 0)$ bound state



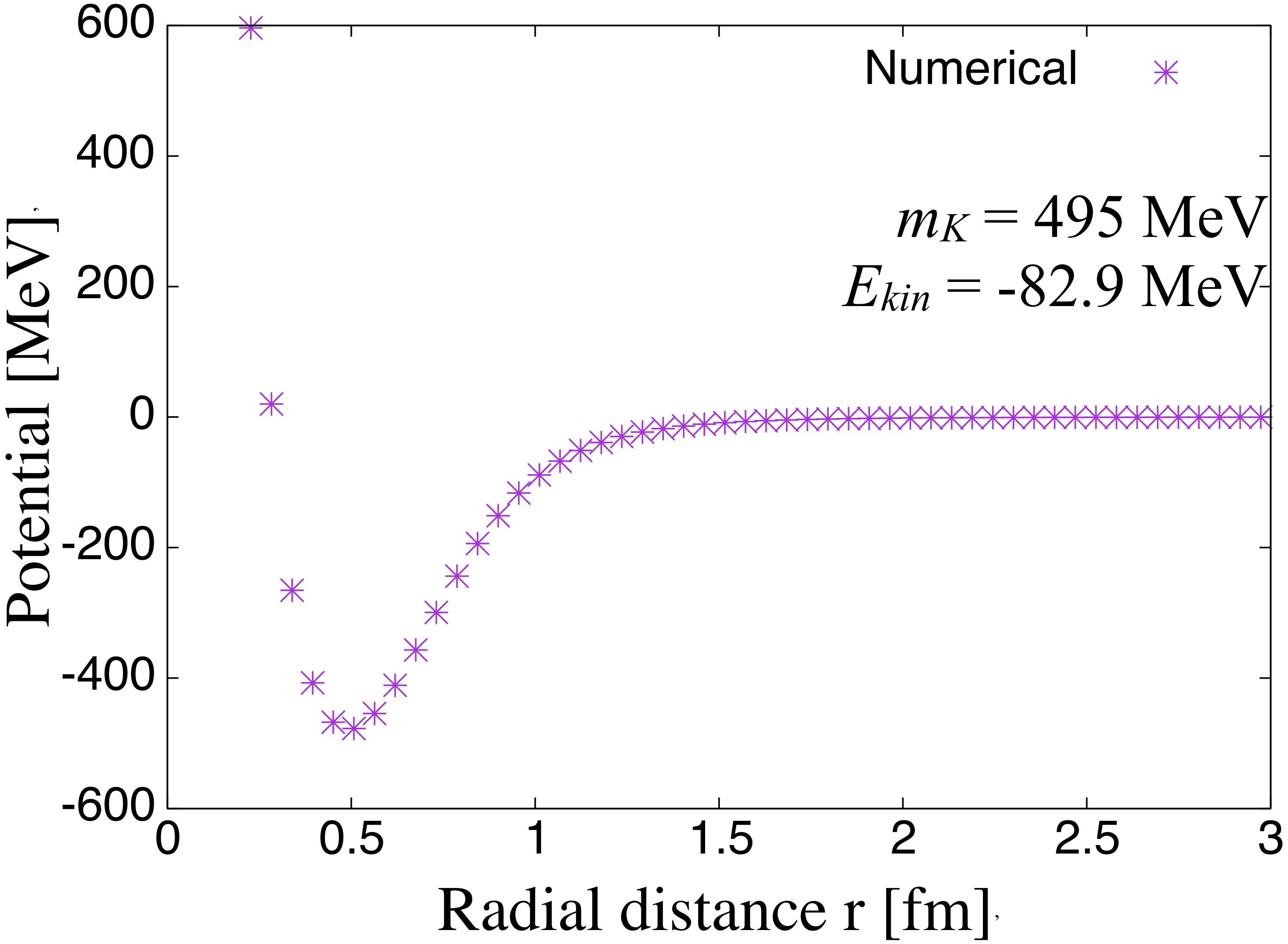
$\bar{K}N$ $(I = 0, L = 1,\ J = 3/2)$ scattering state



Comparing 2

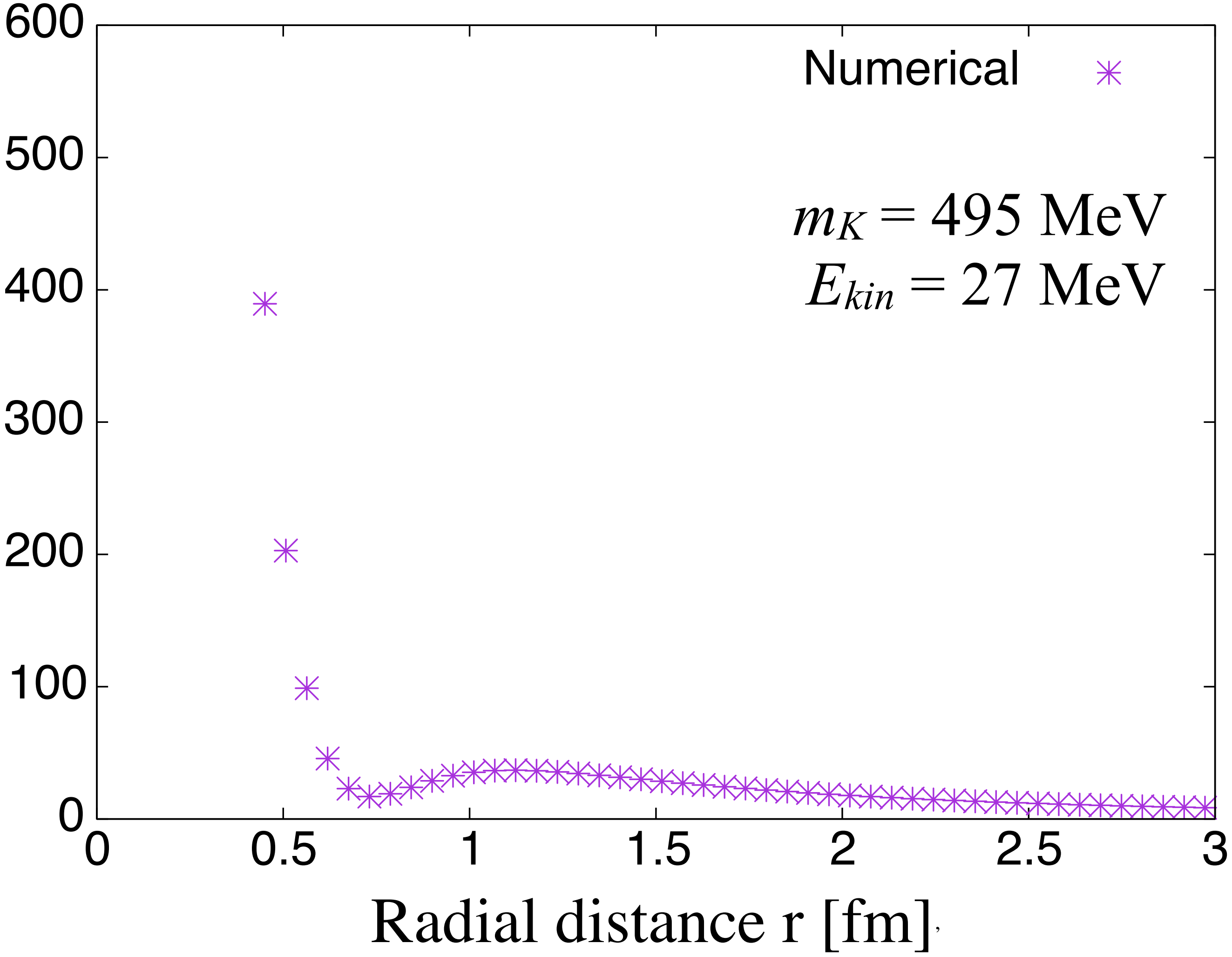
parameter set B: $F_\pi = 129$ MeV, $e = 5.45$

$\bar{\text{K}}$N $(I = 0, L = 0)$ bound state



$\bar{\text{K}}$N $(I = 0, L = 1,\ J = 3/2)$

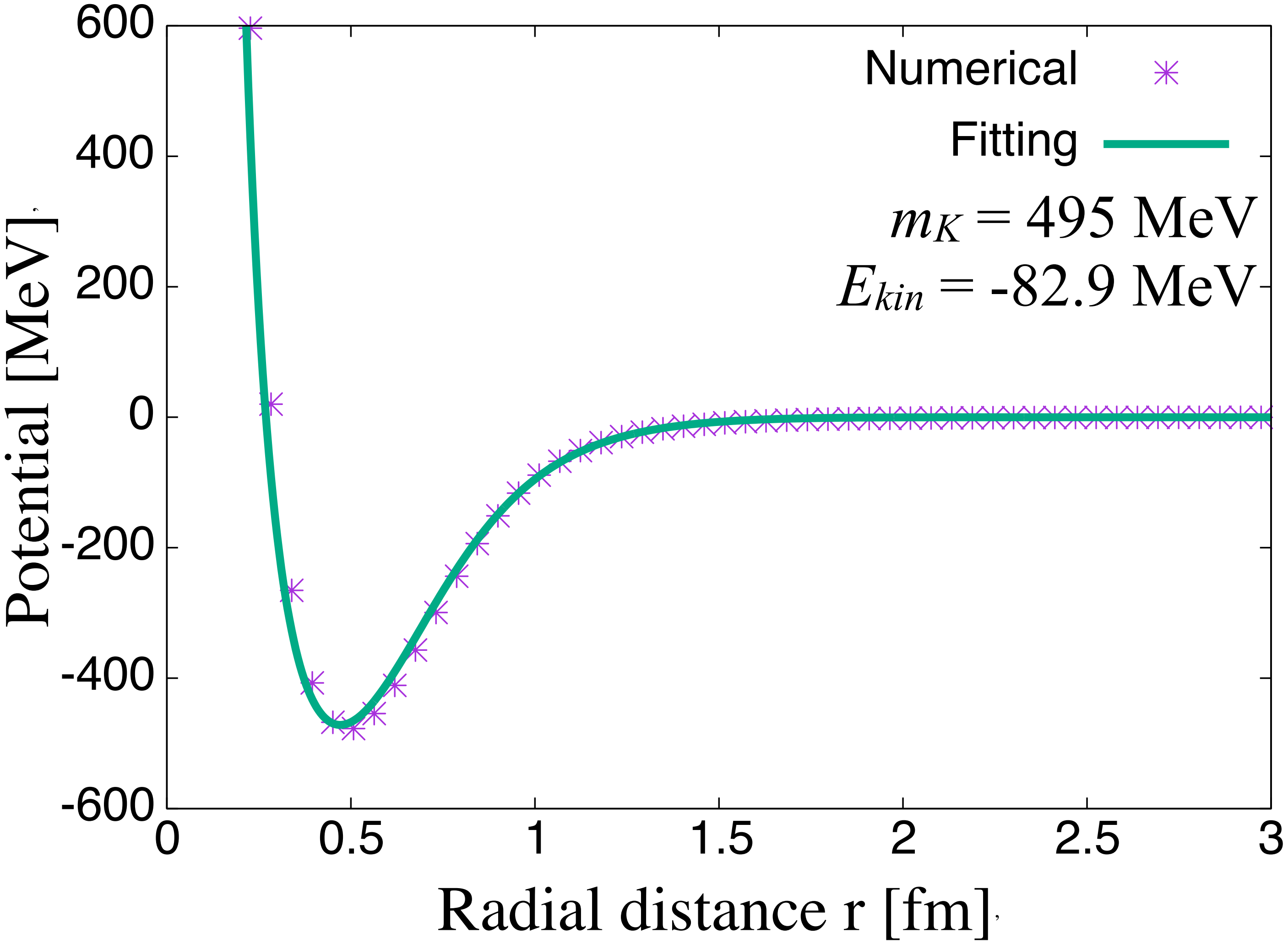
scattering state



Comparing 2

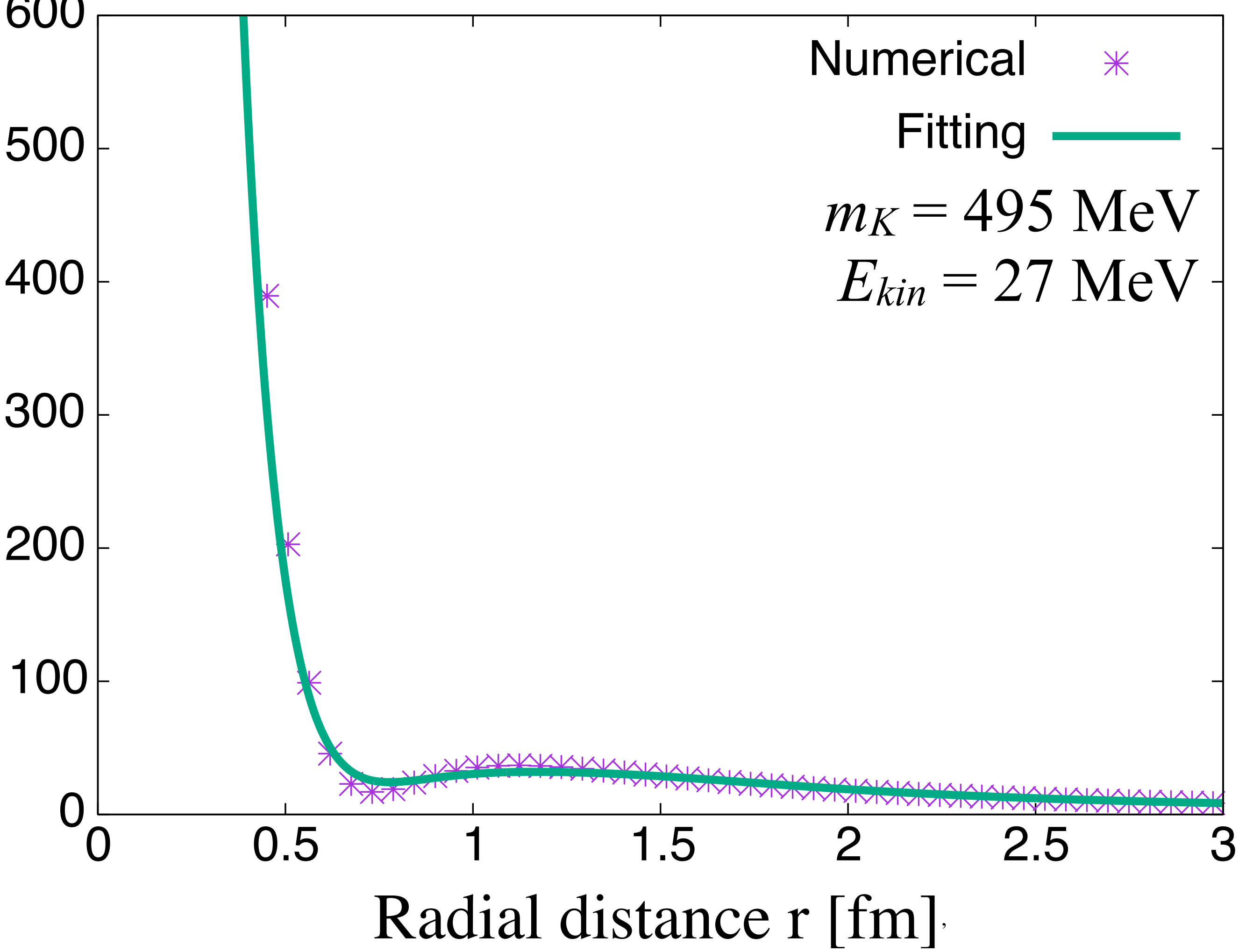
parameter set B: $F_\pi = 129$ MeV, $e = 5.45$

$\bar{\mathrm{K}}\mathrm{N}$ $(I = 0, L = 0)$ bound state



$\bar{\mathrm{K}}\mathrm{N}$ $(I = 0, L = 1,\ J = 3/2)$

scattering state



5. Summary

Summaries

Investigate the kaon-nucleon systems
by a modified bound state approach in the Skyrme model

・Results

1. Properties of the obtained potential
   - a. nonlocal and depends on the kaon energy
   - b. contain **central and LS terms**
     with and without isospin dependence
   - c. **repulsion proportional to $1/r^2$** for small r
2. $\bar{\mathrm{K}}$N$(I=0)$ bound states exist with B.E. of order ten MeV
3. Phases as functions of energy reflect
   the property of the bound state
4. Fit the potential by a simple form of the Gaussian type

・Future works

1. The $\pi\Sigma$ system
2. The properties of $\Lambda(1405)$
3. few body nuclear system with kaon

Thank you for
your attention