Kaon-Nucleon systems in the Skyrme model

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1. Introduction
2. Method
3. Results and discussions
4. Summary

2016/11/28 Realistic hadron interactions in QCD@YITP

1. Introduction

Introduction

Kaon nucleon systems are very attractive

- Strong attraction between the anti-kaon($\bar{\mathrm{K}}$) and the nucleon(N)

Y. Akaishi and T. Yamazaki, Phys. Rev. C **65** (2002)

- $\bar{\mathrm{K}}\mathrm{N}$ bound state = $\Lambda(1405)$
- Few body nuclear system with $\bar{\mathrm{K}}$ → under debate

$\bar{\mathrm{K}}\mathrm{N}$ interaction is important

to investigate the few body systems with $\bar{\mathrm{K}}$

Theoretical studies of $\bar{\mathrm{K}}\mathrm{N}$ interaction

- Phenomenological approach

Y. Akaishi and T. Yamazaki, Phys. Rev. **C 65** (2002) etc

- Chiral theory: based on a 4-point local interaction

T. Hyodo and W. Weise, Phys. Rev. **C 77** (2008)

K. Miyahara and T. Hyodo, Phys. Rev. C **93** (2016) etc

Investigate the KN system in the Skyrme model

where the nucleon is described as a soliton.

2. Method

The Skyrme model and our ansatz

- **Skyrme model** T.H.R. Skyrme, Nucl. Phys. **31** (1962); Proc. Roy. Soc. A **260** (1961)

- Describing the meson-baryon interaction by mesons
- Baryon emerges as a soliton of meson fields.

$$L = \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 + L_{SB} + L_{WZ}$$

F_π, e: parameter m_π: mass less, m_K: massive

- **Ansatz**

$$U = (3\times 3 \text{ matrix}) \to \sqrt{U_\pi}U_K\sqrt{U_\pi}$$

C.G. Callan and I. Klebanov, Nucl. Phys. **B 262** (1985)

C .G.Callan, K .Hornbostel and I. Klebanov, Phys. Lett. **B 202** (1988)

$$U_\pi = \begin{pmatrix} U_H & 0 \\ 0 & 1 \end{pmatrix}$$

Hedgehog soliton
(2×2 matrix)

$$\begin{cases} U_\pi \to A(t) U_\pi A^\dagger(t) \\ U_K = U_K \end{cases}$$

$A(t)$: isospin rotation matrix

$$U_K = \exp\left[i\frac{2}{F_\pi}\lambda_a K_a\right], \quad a = 3,4,5,6$$

$$\boxed{U = A(t)\sqrt{U_\pi}A^\dagger(t) U_K A(t)\sqrt{U_\pi}A^\dagger(t)}$$

$$\lambda_a K_a = \sqrt{2}\begin{pmatrix} 0_{2\times 2} & K \\ K^\dagger & 0 \end{pmatrix} \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad K^\dagger = \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix}$$

T. Ezoe. and A. Hosaka Phys. Rev. D **94**, 034022 (2016)

Obtaining Lagrangian

1. Substitute our ansatz for the Lagrangian

2. Expand U_K up to second order of the kaon field K

$$L = L_{SU(2)} + L_{KN}$$

$$\begin{aligned} L_{KN} \quad = \quad & (D_\mu K)^\dagger D^\mu K - K^\dagger a_\mu^\dagger a^\mu K - m_K^2 K^\dagger K \\ & + \frac{1}{(eF_\pi)^2}\left\{ -K^\dagger K \,\mathrm{tr}\left[\partial_\mu \tilde{U}\tilde{U}^\dagger, \partial_\nu \tilde{U}\tilde{U}^\dagger\right]^2 - 2\left(D_\mu K\right)^\dagger D_\nu K \,\mathrm{tr}\left(a^\mu a^\nu\right) \right. \\ & \qquad \left. - \frac{1}{2}\left(D_\mu K\right)^\dagger D^\mu K \,\mathrm{tr}\left(\partial_\nu \tilde{U}^\dagger \partial^\nu \tilde{U}\right) + 6\left(D_\nu K\right)^\dagger \left[a^\nu, a^\mu\right] D_\mu K \right\} \\ & + \frac{3i}{F_\pi^2} B^\mu \left[\left(D_\mu K\right)^\dagger K - K^\dagger \left(D_\mu K\right)\right] \end{aligned}$$

3. Decompose the kaon filed in partial waves

Spatial wave function

$$\begin{pmatrix} K^+ \\ K^0 \end{pmatrix} = \psi_I K\left(t, \boldsymbol{r}\right) \rightarrow \underline{\psi_I}\ \underline{K\left(\boldsymbol{r}\right)}\, e^{-iEt}$$

Isospin wave function

$$K\left(\boldsymbol{r}\right) = \sum_{l,m} C_{lm\alpha} Y_{lm}\left(\theta, \phi\right) k_l^\alpha\left(r\right)$$

$Y_{lm}(\theta, \varphi)$: Spherical harmonics

l : orbital angular momentum

m : the 3rd component of l

α : the other quantum numbers

4. Take a variation with respect to the kaon radial function

⇒Obtain the equation of motion for the kaon partial wave

3. Results and discussions

Equation of motion and potential

• Equation of motion(E.o.M)

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2 h(r)\frac{dk_l^{\alpha}(r)}{dr}\right) - E^2 f(r)k_l^{\alpha}(r) + \left(m_K^2 + V(r)\right)k_l^{\alpha}(r) = 0$$:Klein-Gordon like

$$\longrightarrow -\frac{1}{m_K+E}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dk_l^{\alpha}(r)}{dr}\right) + U(r)k_l^{\alpha}(r) = \varepsilon k_l^{\alpha}(r)$$:Schrödinger like

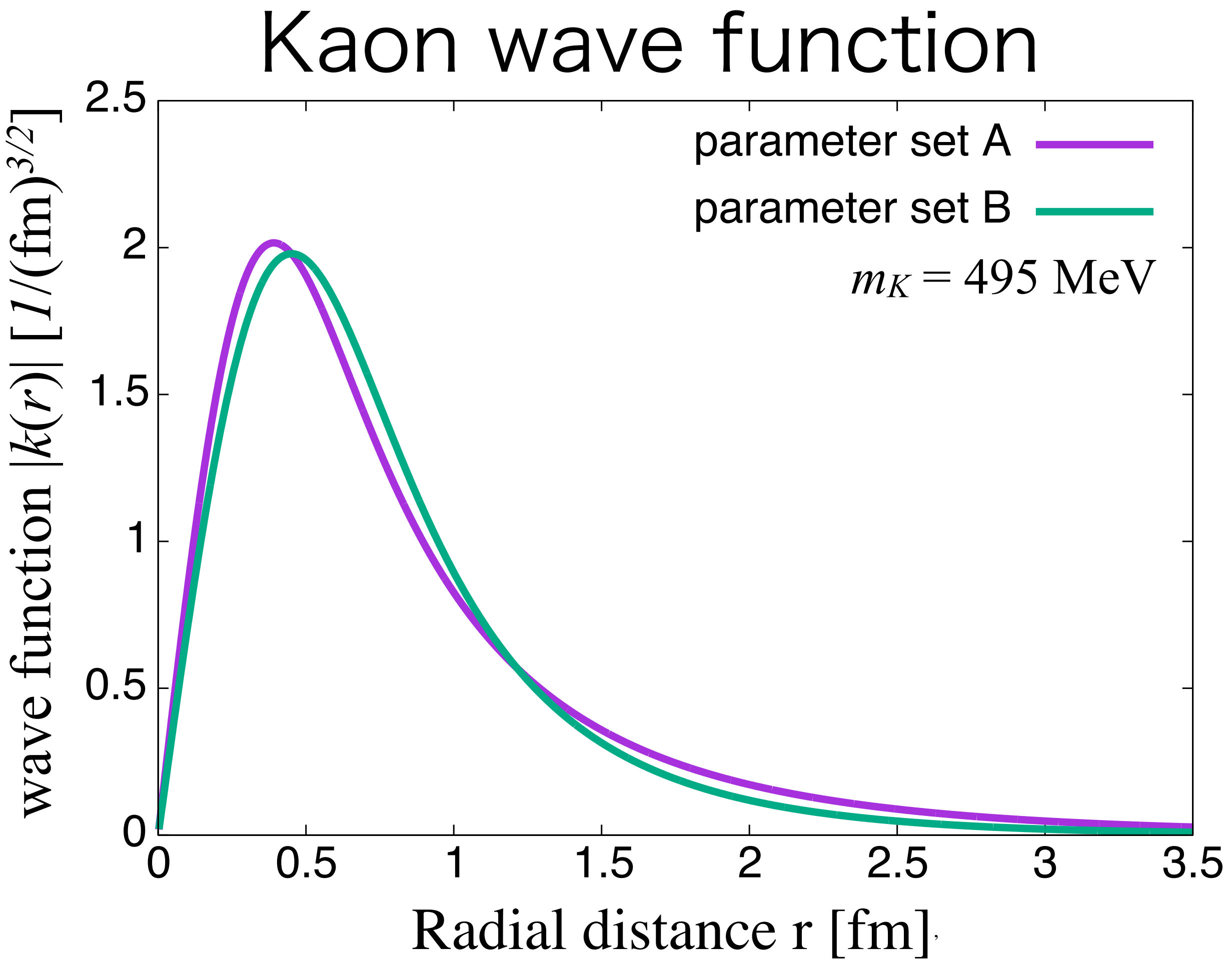
$$\boxed{\begin{aligned} U(r) \quad = & \\ - & \boxed{\frac{1}{m_K+E}}\left[\frac{h(r)-1}{r^2}\boxed{\frac{d}{dr}\left(r^2\frac{d}{dr}\right)} + \frac{dh(r)}{dr}\boxed{\frac{d}{dr}}\right] - \boxed{\frac{(f(r)-1)\,E^2}{m_K+E}} + \boxed{\frac{V(r)}{m_K+E}} \end{aligned}}$$

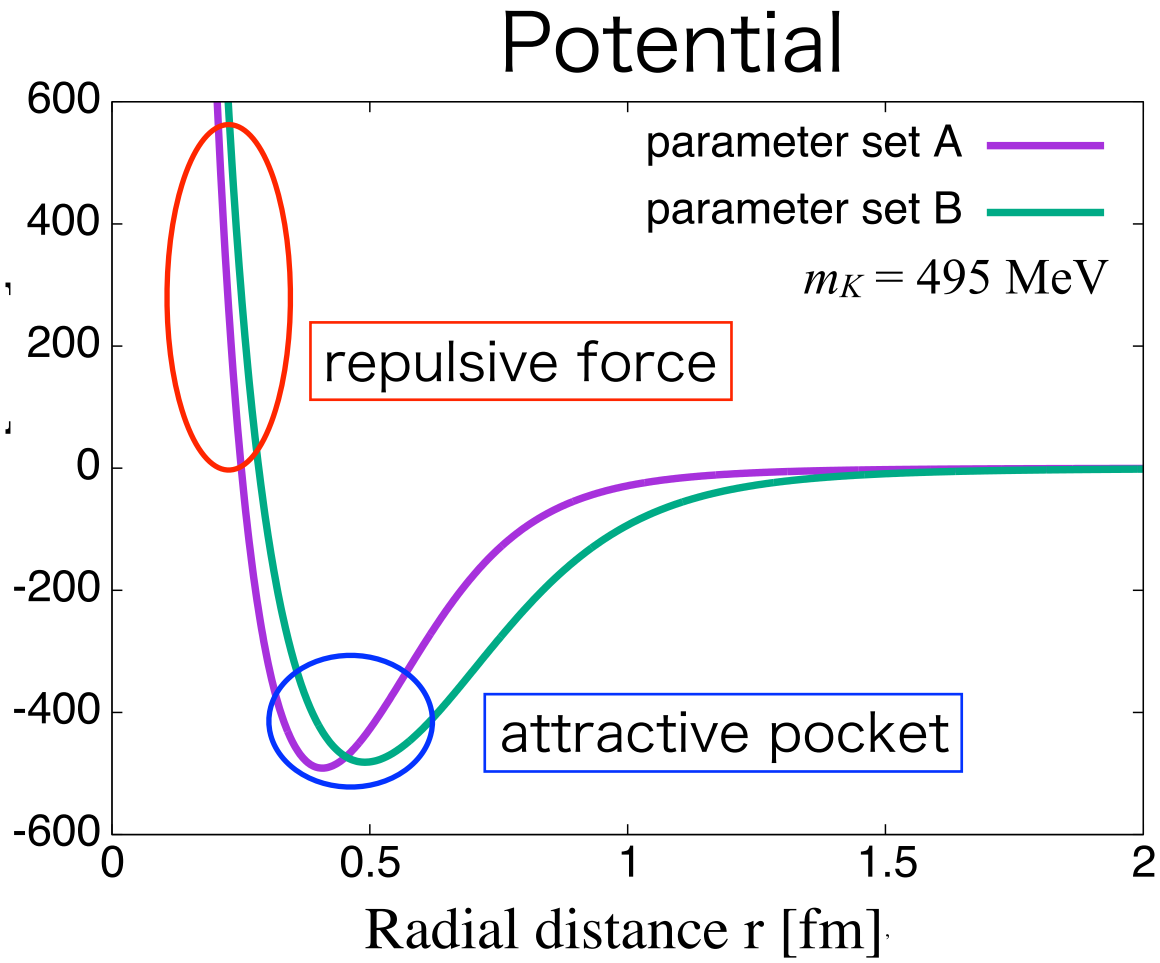
Equivalent local potential: $\boxed{\tilde{U}(r) = \frac{U(r)\,k_l^{\alpha}(r)}{k_l^{\alpha}(r)}}$

• Properties of resulting potential U

1. Nonlocal and depend on the kaon energy
2. Contain isospin dependent and independent **central forces**
   and the similar **spin-orbit(LS) forces**
3. A repulsive component is proportional to $1/r^2$ at short distances

Result 1: $\bar{\mathrm{K}}\mathrm{N}(J^P = 1/2^-, I = 0)$) Bound state


Kaon wave function


Potential

- **Model parameters and physical properties**

	F_π [MeV]	e	B.E. [MeV]	$\langle r_N^2 \rangle^{1/2}$ [fm]	$\langle r_K^2 \rangle^{1/2}$ [fm]
parameter set A	186	4.82	32.9	0.46	1.18
parameter set B	129	5.45	82.9	0.59	0.99

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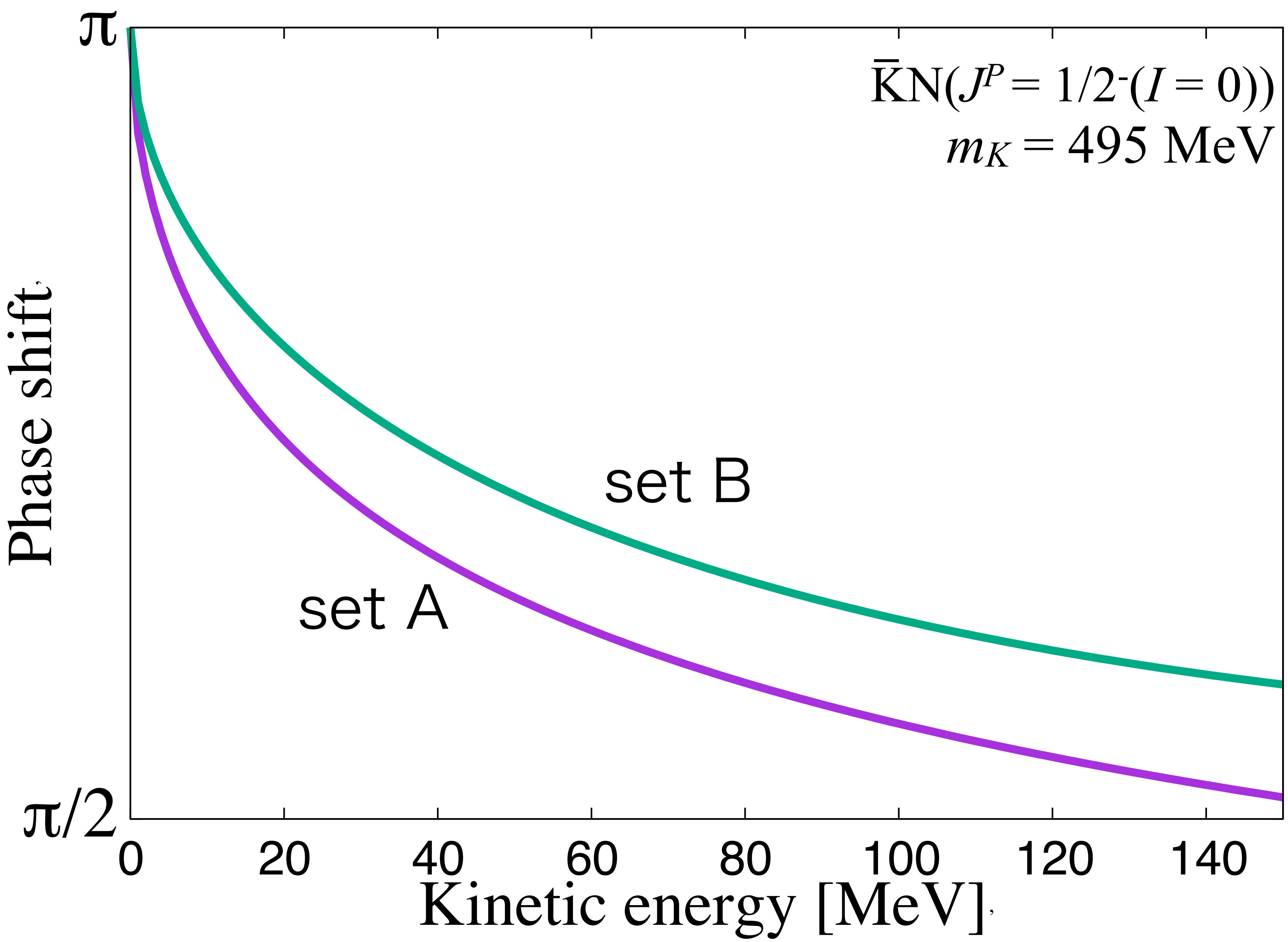
$$\langle r_N^2 \rangle = \int_0^\infty dr\ r^2 \rho_B(r), \quad \rho_B(r) = -\frac{2}{\pi} \sin^2 F F'$$

G. S. Adkins, C. R. Nappi and E. Witten,
Nucl. Phys. B **228** (1983)

$$\langle r_K^2 \rangle = \int dV\ r^2 \left[Y_{00}(\hat{r})\, k_0^0(r) \right]^2 = \int_0^\infty dr\ r^4 k^2(r) \qquad Y_{00} = \frac{1}{\sqrt{4\pi}}$$

Result 2: Phase shift ($\bar{K}N(J^P = 1/2^-, I = 0)$)



Binging energies

	F_π [MeV]	e	B.E. [MeV]
parameter set A	186	4.82	32.9
parameter set B	129	5.45	82.9

T. Ezoe. and A. Hosaka Phys. Rev. D **94**, 034022 (2016)

Result 3: Fitting the potentials

$$\tilde{U}\left(r\right)=\tilde{U}_0^c\left(r\right)+\tilde{U}_\tau^c\left(r\right)\left(\boldsymbol{I}^K\cdot\boldsymbol{I}^N\right)+\tilde{U}_0^{LS}\left(r\right)\left(\boldsymbol{L}^K\cdot\boldsymbol{J}^N\right)+\tilde{U}_\tau^{LS}\left(r\right)\left(\boldsymbol{L}^K\cdot\boldsymbol{J}^N\right)\left(\boldsymbol{I}^K\cdot\boldsymbol{I}^N\right)$$

	Isospin	Normal term	Wess-Zumino term
Central	indep.	$u_0^c(N,r)+v_0^c(N,r)E_{kin}$ $G_{-2}\left(r\right)+G_0\left(r\right)+G_2\left(r\right)$	$u_0^c(WZ,r)+v_0^c(WZ,r)E_{kin}$ $G_0\left(r\right)+G_0\left(r\right)$
	dep.	$u_\tau^c(N,r)+v_\tau^c(N,r)E_{kin}$ $G_0\left(r\right)+G_2\left(r\right)$	— —
LS	indep.	$u_0^{LS}(N,r)+v_0^{LS}(N,r)E_{kin}$ $G_0\left(r\right)+G_0\left(r\right)$	$u_0^{LS}(WZ,r)+v_0^{LS}(WZ,r)E_{kin}$ $G_0\left(r\right)+G_0\left(r\right)$
	dep.	$u_\tau^{LS}(N,r)+v_\tau^{LS}(N,r)E_{kin}$ $G_{-2}\left(r\right)+G_{-2}\left(r\right)$	— —
Centrifugal force		$u_l\left(r\right)+v_l\left(r\right)E_{kin}$ $\propto\left(G_0\left(r\right)+G_0\left(r\right)\right)/r^2$	

$$\begin{aligned}\tilde{U}\left(r\right) &\simeq \tilde{U}\left(r\right)+\frac{\partial\tilde{U}\left(r\right)}{\partial E_{kin}}E_{kin}\\ &\equiv u\left(r\right)+v\left(r\right)E_{kin}\end{aligned}$$

fit by Gaussian

$$G_{-2}\left(r\right)=C_{-2}\frac{1}{r^2/{R_{-2}}^2}\exp\left(-\frac{r^2}{{R_{-2}}^2}\right)$$

$$G_0\left(r\right)=C_0\exp\left(-\frac{r^2}{{R_0}^2}\right)$$

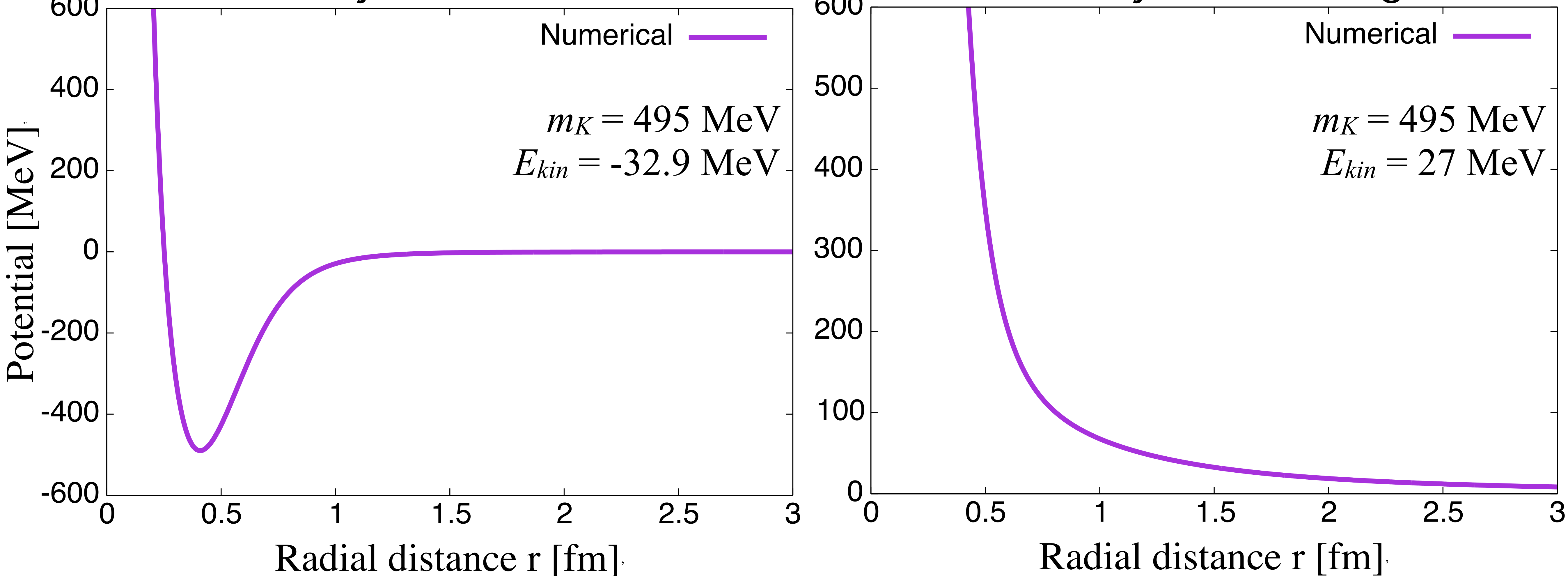
$$G_2\left(r\right)=C_2\frac{r^2}{{R_2}^2}\exp\left(-\frac{r^2}{{R_2}^2}\right)$$

Comparison

Parameter set A: $F_\pi = 186$ MeV, $e = 4.82$

$\bar{\mathrm{K}}\mathrm{N}$ potential for $J^P = 1/2^-$, $I = 0$ obtained by the bound state

$\bar{\mathrm{K}}\mathrm{N}$ potential for $J^P = 3/2^+$, $I = 0$ obtained by a scattering state



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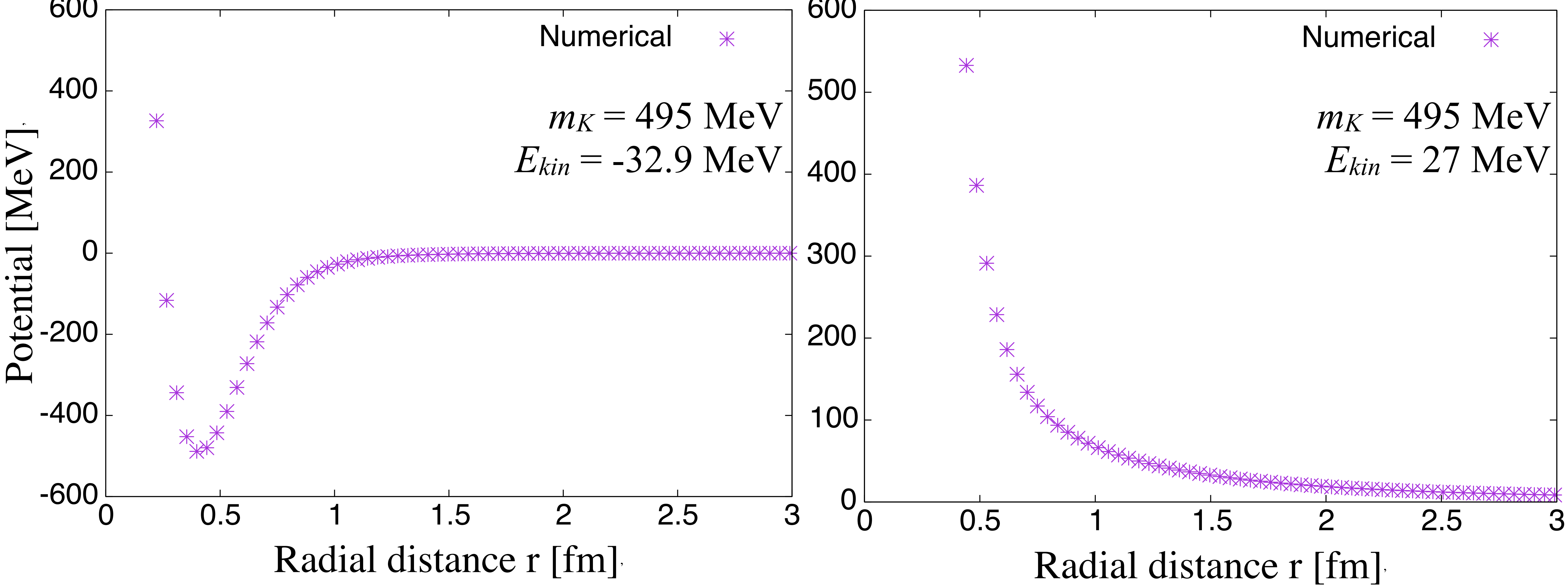

Comparison

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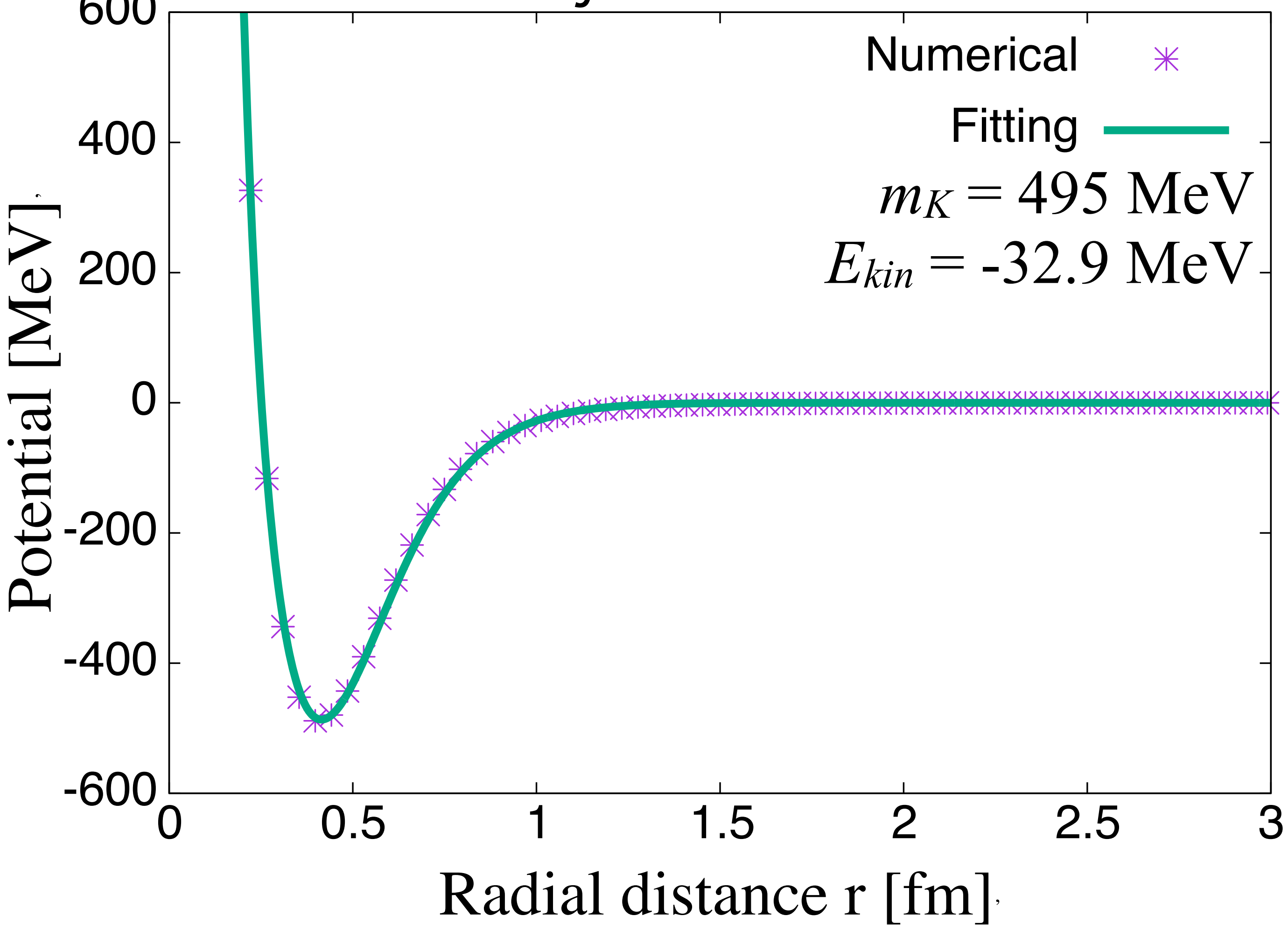
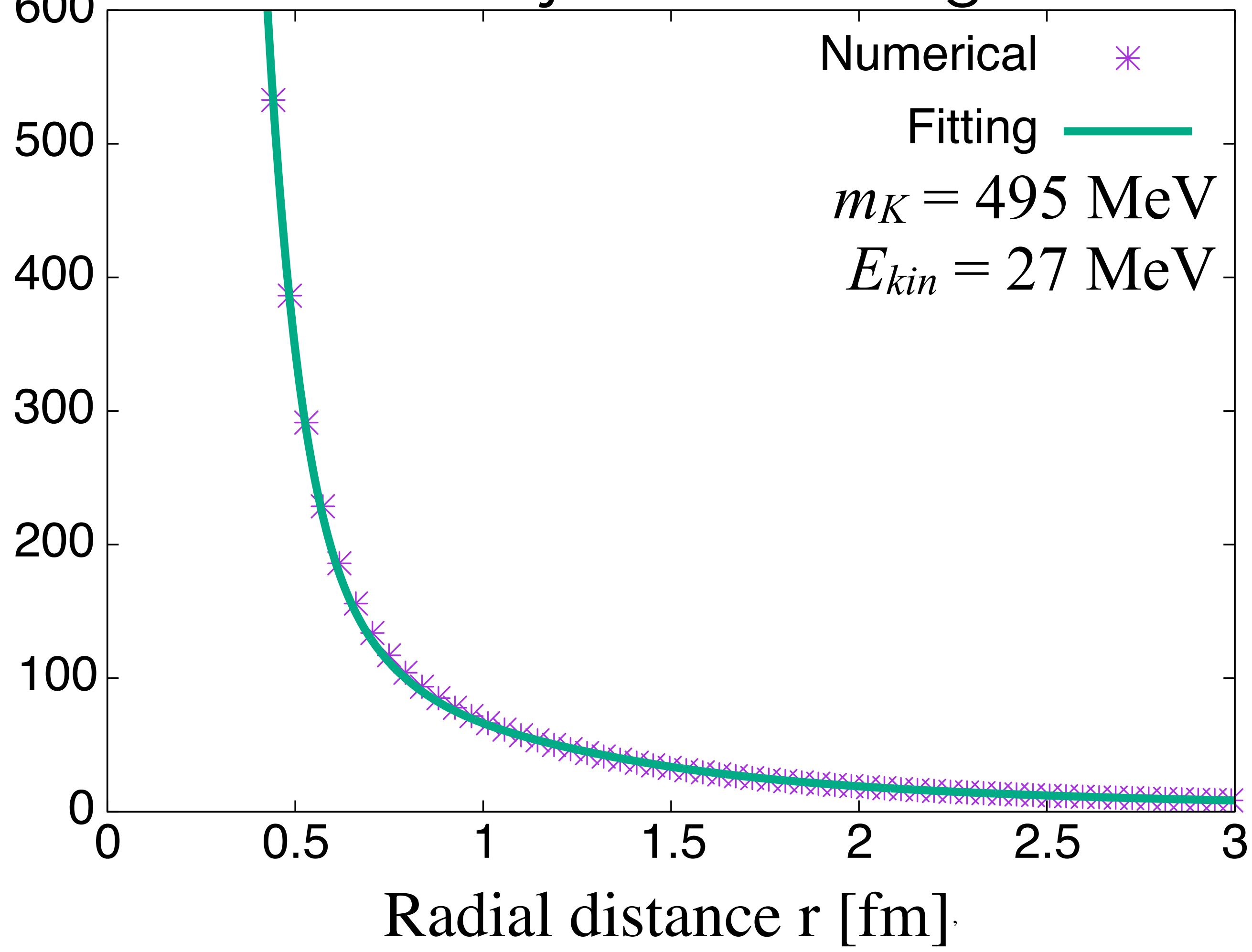
$\bar{K}N$ potential for $J^P = 1/2^-$, $I = 0$ obtained by the bound state



$\bar{K}N$ potential for $J^P = 3/2^+$, $I = 0$ obtained by a scattering state



4. Summary

Summaries

Investigate the kaon-nucleon systems
by a modified bound state approach in the Skyrme model

・Results

1. Properties of the obtained potential
   a. nonlocal and depends on the kaon energy
   b. contain **central and LS terms**
      with and without isospin dependence
   c. **repulsion proportional to $1/r^2$** for small r
2. $\bar{K}N(I=0)$ bound states exist with B.E. of order ten MeV
3. Phases as functions of energy reflect
   the property of the bound state
4. Fit the potential by a simple form of the Gaussian type

・Future works

1. The $\pi\Sigma$ system
2. The properties of Λ(1405)
3. few body nuclear system with kaon

Thank you for
your attention

back-up

The Skyrme model

The Skyrme model 1

T.H.R. Skyrme, Nucl. Phys. **31** (1962);
Proc. Roy. Soc. A **260** (1961)

- Describe the interaction between mesons and baryons by mesons
- Baryon emerges as a soliton of meson fields.

$$\phi = \frac{1}{\sqrt{2}}\lambda_a\phi_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$U = \exp\left[i\frac{2}{F_\pi}\lambda_a\phi_a\right]$$

λ_a: Gell-Mann matrices (a = 1, 2, … ,8)

The Skyrme model 1

T.H.R. Skyrme, Nucl. Phys. **31** (1962);
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- Describe the interaction between mesons and baryons by mesons
- Baryon emerges as a soliton of meson fields.

$$\phi = \frac{1}{\sqrt{2}}\lambda_a \phi_a = \left(\begin{array}{cc|c} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ \cline{1-2} K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{array}\right)$$

$$U = \exp\left[i\frac{2}{F_\pi}\lambda_a\phi_a\right]$$

λ_a: Gell-Mann matrices (a = 1, 2, … ,8)

- **For SU(2)**

$$L = \underbrace{\frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)}_{\text{kinetic term}} + \underbrace{\frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2}_{\text{the Skyrme term}}$$

F_π, e: parameters

The Skyrme model 2

・Hedgehog ansatz

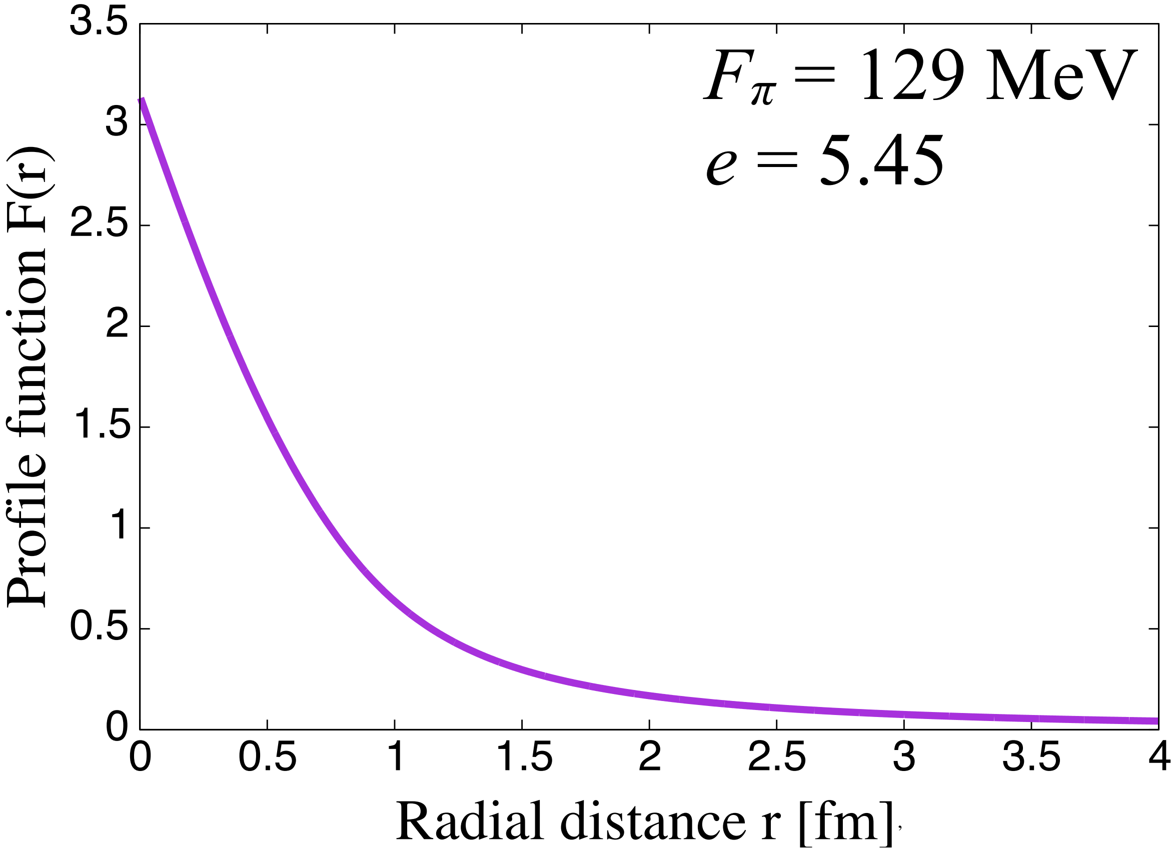
π has three degrees of freedom(π^0, π^+, π^-)

・two of these: the angles of the radial vector, θ, φ

・the rest: a function depending on r

⇒a special configuration called the hedgehog ansatz

Hedgehog ansatz: $U_H = \exp\left[i\boldsymbol{\tau}\cdot\hat{r}\boxed{F(r)}\right]$



minimize the mass of the soliton

with B.C.: $F(\infty) = 0,\ F(0) = \pi$

G. S. Adkins, C. R. Nappi and E. Witten,
Nucl. Phys. B **228** (1983)

The Skyrme model 3

・Quantization

The hedgehog solution is a classical field configuration

→without spin or isospin

→become a physical state by quantization

$$U_H(\boldsymbol{x}) \to U_H(t, \boldsymbol{x}) = A(t) \exp\left[i\tau_a R_{ab}(t) \hat{r}_b F(r)\right] A^\dagger(t)$$

$A(t)$: 2×2 isospin rotation matrix

$R_{ab}(t)$: 3×3 spatial rotation matrix

Baryon with $I = J$ are generated due to the symmetry which the hedgehog ansatz has

・Quantized Hamiltonian

$$H = M_{sol} + \boxed{\frac{J(J+1)}{2\Lambda}}$$

the rotation energy

M_{sol}: soliton mass

J: spin or isospin value

Λ: moment of inertia

Our method and results
(For Bound states)

Method

SU(3) symmetry is broken → $m_u = m_d = 0, m_s \neq 0$

Callan-Klebanov approach (CK approach)

- Introduce the kaon as fluctuations around the hedgehog soliton
- Form a bound state of the kaon and the hedgehog soliton
- rotate the system to generate hyperons
- Follow the $1/N_c$ counting rule
-

C.G. Callan and I. Klebanov, Nucl. Phys. **B 262** (1985)

C .G.Callan, K .Hornbostel and I. Klebanov, Phys. Lett. **B 202** (1988)

Our approach

- Rotate the hedgehog soliton to generate the nucleon
- Introduce the kaon as fluctuations around the nucleon
- describe kaon-nucleon systems
- Violate the $1/N_c$ counting rule
-

Method

SU(3) symmetry is broken → $m_u = m_d = 0,\ m_s \neq 0$

Callan-Klebanov approach (CK approach)

- Introduce the kaon as fluctuations around the hedgehog soliton
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- Projection after variation, The strong coupling

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Our approach

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T. Ezoe. and A. Hosaka Phys. Rev. D **94**, 034022 (2016)

Lagrangian and ansatz

- **Extension to the SU(3) Skyrme model**

$$
\begin{aligned}
L \quad = \quad & \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 \\
& + L_{SB} + L_{WZ}
\end{aligned}
$$

- **Ansatz**

$$
U = \begin{cases} A\left(t\right)\sqrt{U_\pi}U_K\sqrt{U_\pi}A^\dagger\left(t\right) & \text{: Callan-Klebanov ansatz} \\ A\left(t\right)\sqrt{U_\pi}A^\dagger\left(t\right)U_K A\left(t\right)\sqrt{U_\pi}A^\dagger\left(t\right) & \textbf{: Our ansatz} \end{cases}
$$

$$
U_\pi = \begin{pmatrix} U_H & 0 \\ 0 & 1 \end{pmatrix}
$$

Hedgehog ansatz
(2×2 matrix)

$$
U_K = \exp\left[i\frac{2}{F_\pi}\lambda_a K_a\right], \quad a = 3, 4, 5, 6
$$

$$
\lambda_a K_a = \sqrt{2}\begin{pmatrix} 0_{2\times 2} & K \\ K^\dagger & 0 \end{pmatrix} \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad K^\dagger = \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix}
$$

Lagrangian and ansatz

- **Extension to the SU(3) Skyrme model**

$$\begin{aligned} L \quad = \quad & \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 \\ & + L_{SB} + L_{WZ} \end{aligned}$$

- **Ansatz**

the kaon around the hedgehog soliton

$$U = \begin{cases} A\left(t\right)\boxed{\sqrt{U_\pi}U_K\sqrt{U_\pi}}A^\dagger\left(t\right) & \text{: Callan-Klebanov ansatz} \\ \boxed{A\left(t\right)\sqrt{U_\pi}A^\dagger\left(t\right)}U_K\boxed{A\left(t\right)\sqrt{U_\pi}A^\dagger\left(t\right)} & \text{: } \textbf{Our ansatz} \end{cases}$$

the kaon **around the rotating hedgehog soliton**

Derivation 1

・Substitute our ansatz for the Lagrangian

Ansatz

$$U = A(t)\sqrt{U_\pi}A^\dagger(t)U_K A(t)\sqrt{U_\pi}A^\dagger(t)$$

$$U_K = \exp\left[i\frac{2}{F_\pi}\lambda_a K_a\right], \quad a = 3,4,5,6 \qquad U_\pi = \begin{pmatrix} U_H & 0 \\ 0 & 1 \end{pmatrix}$$

$$\lambda_a K_a = \sqrt{2}\begin{pmatrix} 0_{2\times 2} & K \\ K^\dagger & 0 \end{pmatrix} \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad K^\dagger = \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix}$$

Lagrangian

$$\begin{aligned} L \quad = \quad & \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 \\ & + L_{SB} + L_{WZ} \end{aligned}$$

・Expand U_K up to second order of the kaon field K

Obtaining Lagrangian

$$L = L_{SU(2)} + L_{KN}$$

$$L_{SU(2)} = \frac{1}{16} F_\pi{}^2 \mathrm{tr}\left[\partial_\mu \tilde{U}^\dagger \partial^\mu \tilde{U}\right] + \frac{1}{32e^2}\mathrm{tr}\left[\partial_\mu \tilde{U} \tilde{U}^\dagger , \partial_\nu \tilde{U} \tilde{U}^\dagger\right]^2$$

$$\begin{aligned} L_{KN} \quad = \quad & \left(D_\mu K\right)^\dagger D^\mu K - K^\dagger a_\mu^\dagger a^\mu K - m_K^2 K^\dagger K \\ & + \frac{1}{(eF_\pi)^2}\left\{ -K^\dagger K \mathrm{tr}\left[\partial_\mu \tilde{U} \tilde{U}^\dagger , \partial_\nu \tilde{U} \tilde{U}^\dagger\right]^2 - 2\left(D_\mu K\right)^\dagger D_\nu K \mathrm{tr}\left(a^\mu a^\nu\right) \right. \\ & \qquad \left. - \frac{1}{2}\left(D_\mu K\right)^\dagger D^\mu K \mathrm{tr}\left(\partial_\nu \tilde{U}^\dagger \partial^\nu \tilde{U}\right) + 6\left(D_\nu K\right)^\dagger \left[a^\nu , a^\mu\right] D_\mu K \right\} \\ & + \frac{3i}{F_\pi^2} B^\mu \left[\left(D_\mu K\right)^\dagger K - K^\dagger \left(D_\mu K\right)\right] \end{aligned}$$

$$\tilde{U} = A(t) U_H A^\dagger(t), \quad \tilde{\xi} = A(t)\sqrt{U_H} A^\dagger(t) \qquad\qquad D_\mu K = \partial_\mu K + v_\mu K$$

$$v_\mu = \frac{1}{2}\left(\tilde{\xi}^\dagger \partial_\mu \tilde{\xi} + \tilde{\xi} \partial_\mu \tilde{\xi}^\dagger\right)$$

$$a_\mu = \frac{1}{2}\left(\tilde{\xi}^\dagger \partial_\mu \tilde{\xi} - \tilde{\xi} \partial_\mu \tilde{\xi}^\dagger\right)$$

$$B^\mu = -\frac{\varepsilon^{\mu\nu\alpha\beta}}{24\pi^2} \mathrm{tr}\left[\left(U_H^\dagger \partial_\nu U_H\right)\left(U_H^\dagger \partial_\alpha U_H\right)\left(U_H^\dagger \partial_\beta U_H\right)\right]$$

G. S. Adkins, C. R. Nappi and E. Witten,
Nucl. Phys. B **228** (1983)

Derivation 2

・Decompose the kaon filed

Spatial wave function

$$\begin{pmatrix} K^+ \\ K^0 \end{pmatrix} = \psi_I K\left(t, \boldsymbol{r}\right) \rightarrow \underline{\psi_I}\ \underline{K\left(\boldsymbol{r}\right)}\, e^{-iEt}$$

Isospin wave function

・Expand the $K(r)$ by the spherical harmonics

$$K\left(\boldsymbol{r}\right) = \sum_{l,m} C_{lm\alpha} Y_{lm}\left(\theta, \phi\right) k_l^{\alpha}\left(r\right)$$

$Y_{lm}(\theta, \varphi)$: Spherical harmonics

l : orbital angular momentum

m : the 3rd component of l

α : the other quantum numbers

・Take a variation with respect to the kaon radial function

⇒Obtain the equation of motion for the kaon around the nucleon

Interaction term

$$V(r) = V_{nor}(r) + V_{WZ}(r)$$

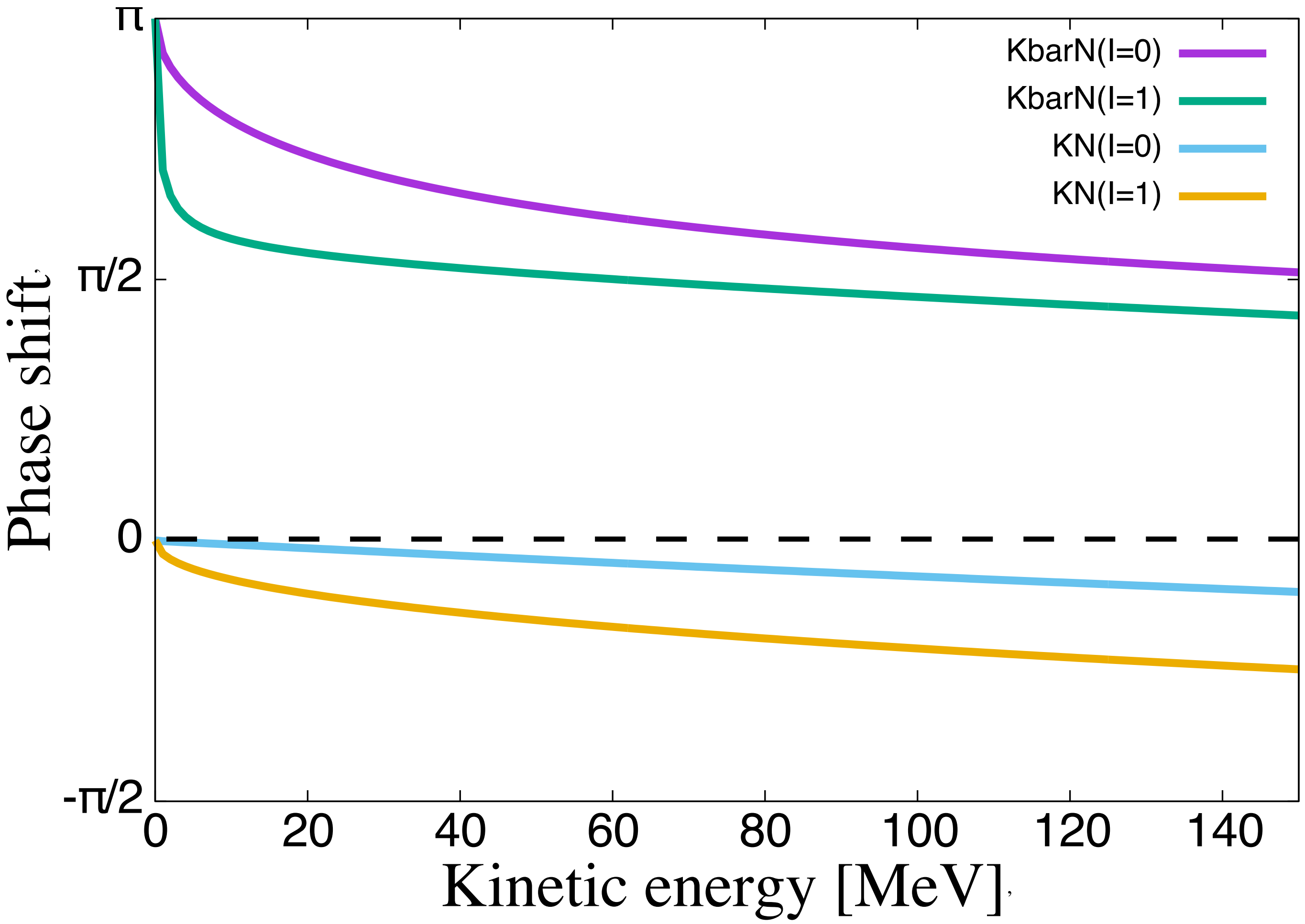
$$\begin{aligned}
V_{nor}(r) \quad = \quad & -\frac{1}{4}\left(2\frac{\sin^2 F}{r^2} + F'^2\right) + 2\frac{s^4}{r^2} - \frac{1}{(eF_\pi)^2}\left[2\frac{\sin^2 F}{r^2}\left(\frac{\sin^2 F}{r^2} + 2F'^2\right) - 2\frac{s^4}{r^2}\left(F'^2 + \frac{\sin^2 F}{r^2}\right)\right] \\
& + \frac{1}{(eF_\pi)^2}\frac{6}{r^2}\left[\frac{s^4 \sin^2 F}{r^2} + \frac{d}{dr}\left\{s^2 \sin F F'\right\}\right] \\
& + \frac{2E}{\Lambda}s^2\left[1 + \frac{1}{(eF_\pi)^2}\left(F'^2 + \frac{5}{r^2}\sin^2 F\right)\right] + \frac{8E}{3\Lambda}s^2 \boxed{I_{KN}} + \frac{1}{(eF_\pi)^2}\frac{8Es^2}{3\Lambda}\left[F'^2 + \frac{4}{r^2}\sin^2 F\right]\boxed{I_{KN}} \\
& + \frac{1}{r^2}\frac{d}{dr}\left[r^2\left(\frac{4}{(eF_\pi)^2}\frac{EF'\sin F}{\Lambda}\boxed{I_{KN}} + \frac{3}{(eF_\pi)^2}\frac{EF'\sin F}{\Lambda}\right)\right] \\
& + \left[1 + \frac{1}{(eF_\pi)^2}\left(\frac{\sin^2 F}{r^2} + F'^2\right)\right]\frac{l\,(l+1)}{r^2} - \left[1 + \frac{1}{(eF_\pi)^2}\left(4\frac{\sin^2 F}{r^2} + F'^2\right)\right]\frac{16s^2}{3r^2}\boxed{J_{KN}}\boxed{I_{KN}} \\
& + \frac{1}{(eF_\pi)^2}\frac{2E\sin^2 F}{\Lambda r^2}\boxed{J_{KN}} - \frac{1}{(eF_\pi)^2}\frac{8}{r^2}\frac{d}{dr}\left(\sin F F'\right)\boxed{J_{KN}}\boxed{I_{KN}}
\end{aligned}$$

$$V_{WZ}(r) = \frac{3E}{(\pi F_\pi)^2}\frac{\sin^2 F}{r^2}F' - \frac{3}{(\pi F_\pi)^2}\frac{\sin^2 F s^2}{\Lambda r^2}F' + \frac{3}{(\pi F_\pi)^2}\frac{\sin^2 F}{\Lambda r^2}F'\boxed{J_{KN}}$$

$$s = \sin(F/2) \quad \boxed{I_{KN} = \boldsymbol{I}^K \cdot \boldsymbol{I}^N}, \quad \boxed{J_{KN} = \boldsymbol{L}^K \cdot \boldsymbol{J}^N}$$

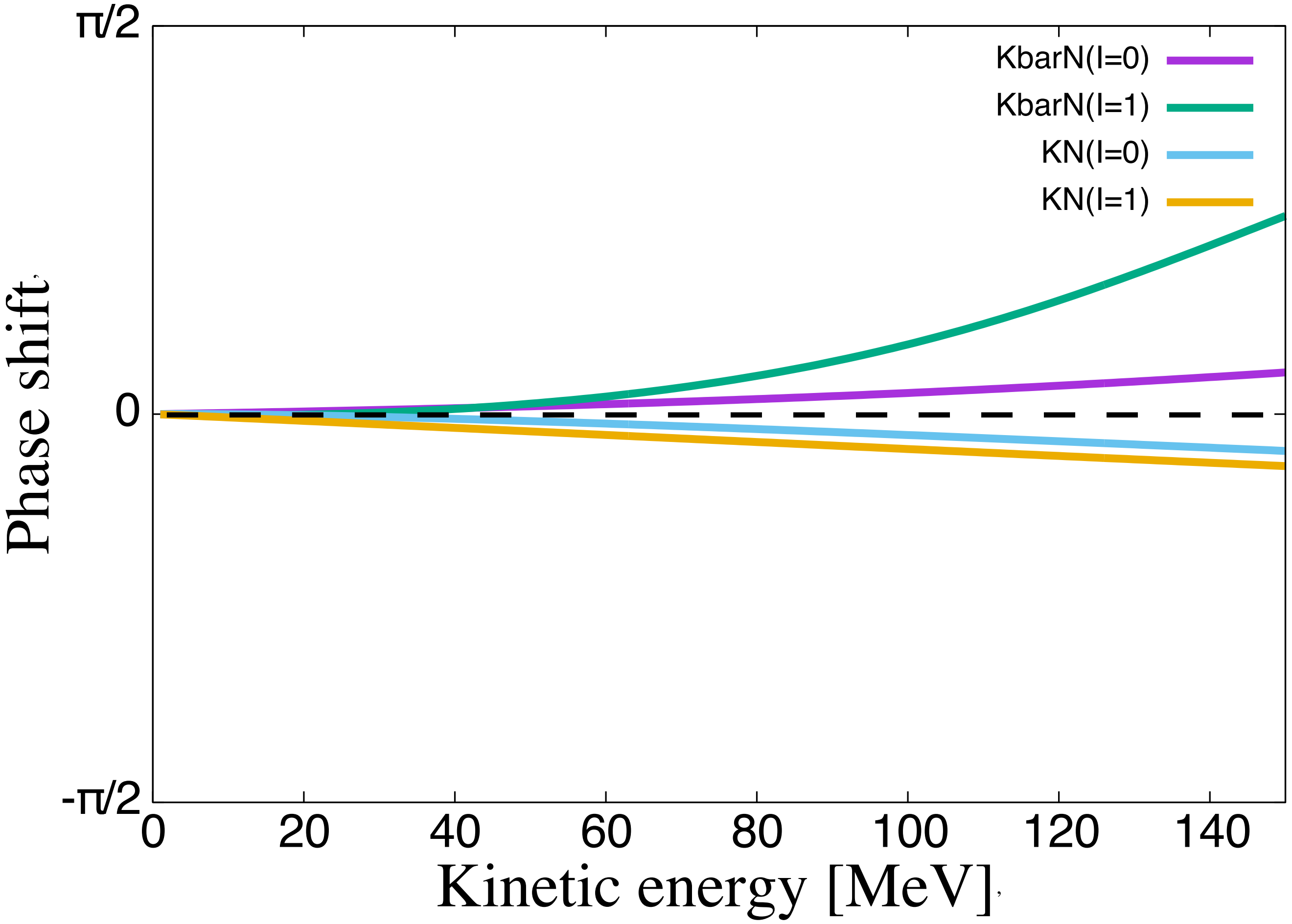
Our method and results (For scattering states)

Phase shift (parameter set A)

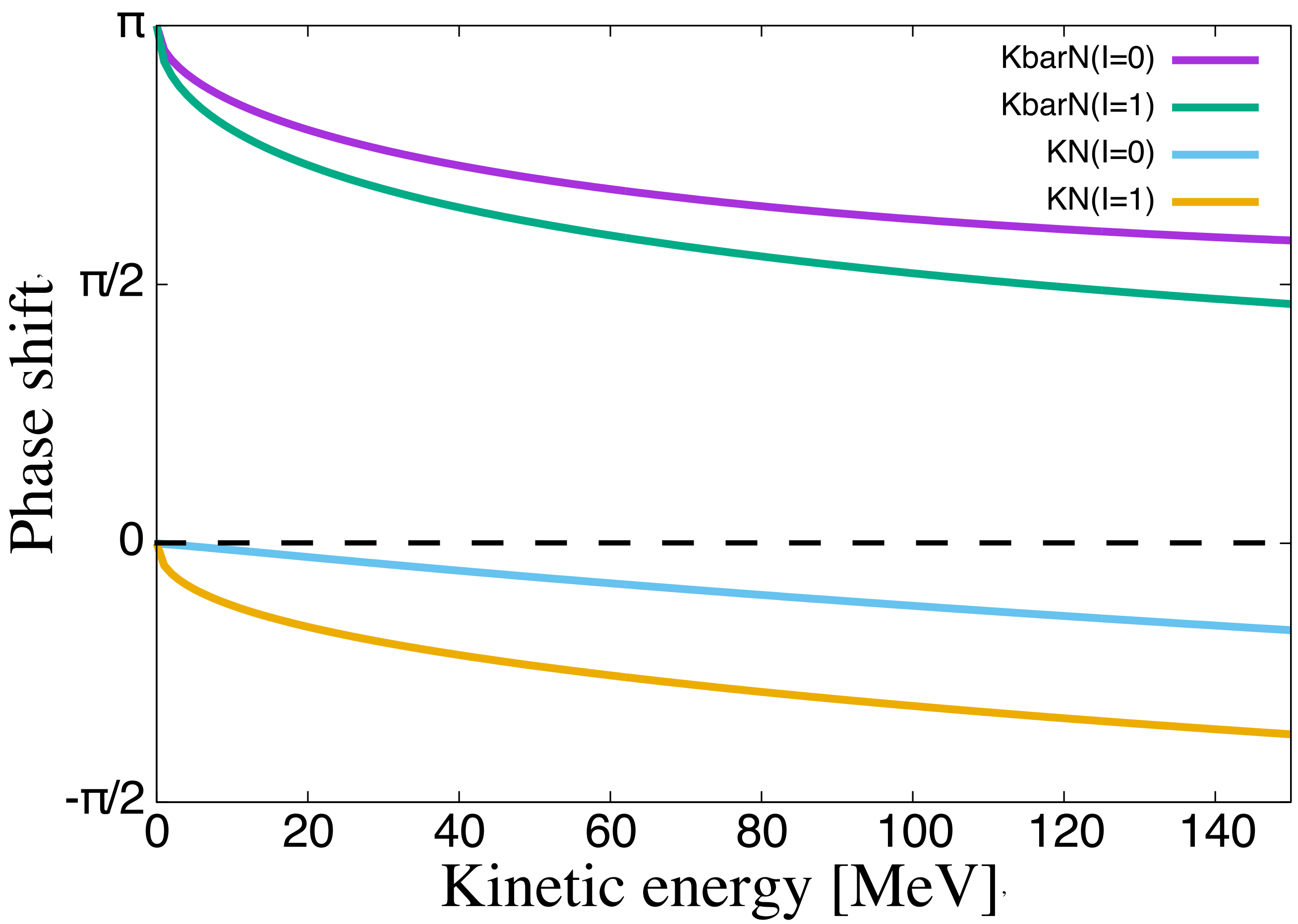


s-wave

p-wave(J = 1+1/2)

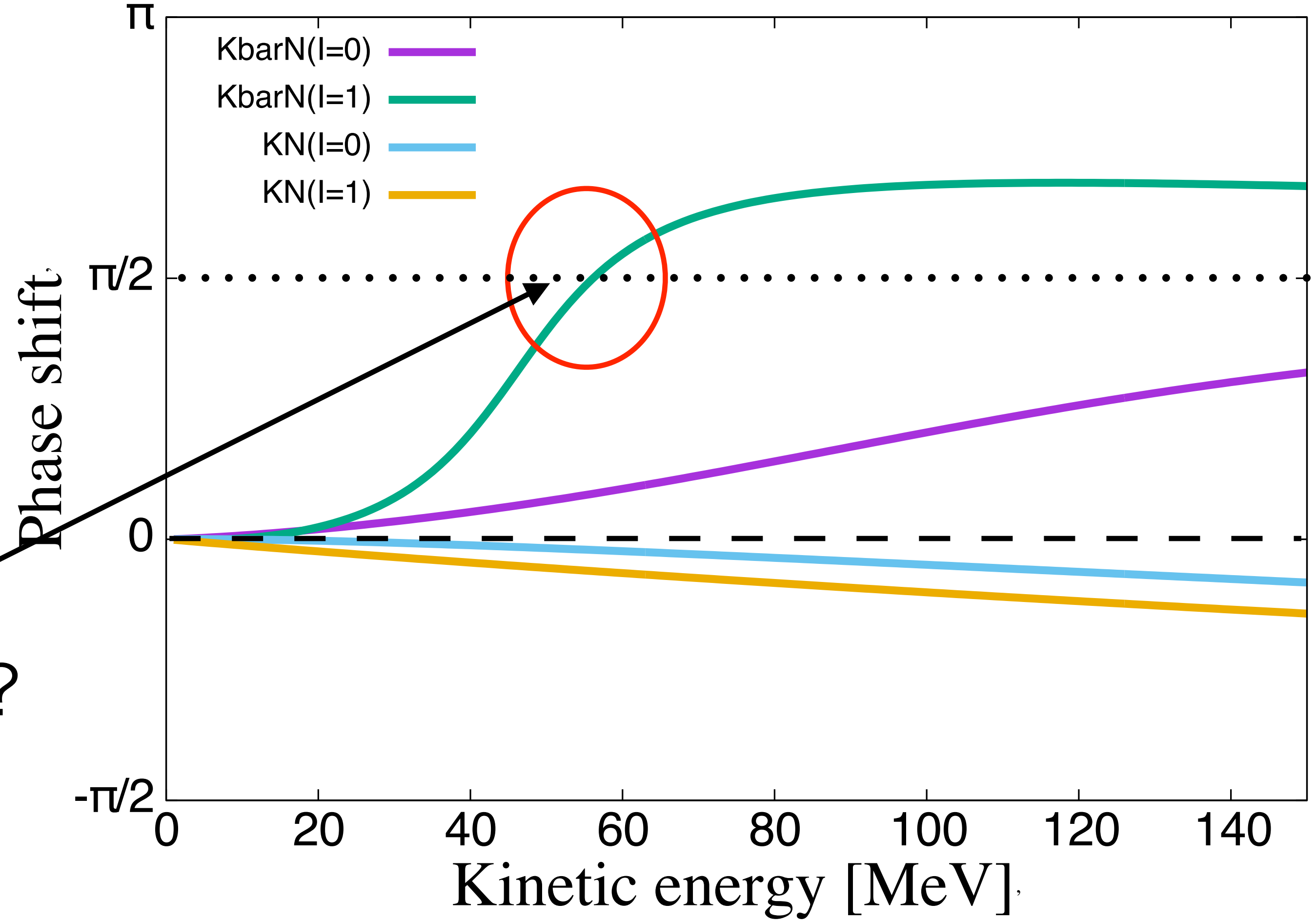


Phase shift (parameter set B)



s-wave

p-wave(J = 1+1/2)



Σ (1480) resonance?

$I(J^P) = 1(3/2^+)$
$\Gamma = 46.5$ MeV

Our method and results
(fitting potential)

Fitting parameters(s-wave)

parameter set A: $F_\pi = 186$ MeV, $e = 4.82$

• Isospin independent normal term

$$C_{-2}\frac{1}{r^2/R_{-2}{}^2}\exp\left(-\frac{r^2}{R_{-2}{}^2}\right)+C_0\exp\left(-\frac{r^2}{R_0{}^2}\right)+C_2\frac{r^2}{R_2{}^2}\exp\left(-\frac{r^2}{R_2{}^2}\right)$$

	$G_{-2}(r)$	$G_0(r)$	$G_2(r)$
Range [fm]	0.176	0.271	0.393
$u_0^c(N,r)$ [MeV]	2911.49	2545.87	-507.819
$v_0^c(N,r)$ [1]	-1.98786	-5.61873	-4.41952×10^{-1}

• Isospin dependent normal term

$$C_0\exp\left(-\frac{r^2}{R_0{}^2}\right)+C_2\frac{r^2}{R_2{}^2}\exp\left(-\frac{r^2}{R_2{}^2}\right)$$

	$G_0(r)$	$G_2(r)$
Range [fm]	0.265	0.524
$u_\tau^c(N,r)$ [MeV]	401.337	290.964
$v_\tau^c(N,r)$ [1]	0.405391	0.293903

• Wess-Zumino term

$$C_0\exp\left(-\frac{r^2}{R_0{}^2}\right)+C_0'\exp\left(-\frac{r^2}{R_0'{}^2}\right)$$

	$G_0(r)$	$G_0(r)$
Range [fm]	0.282	0.404
$u_\tau^c(WZ,r)$ [MeV]	-676.51	-1207.07
$v_\tau^c(WZ,r)$ [1]	-3.483	-0.995

Fitting parameters(Central terms)

parameter set A: $F_\pi = 186$ MeV, $e = 4.82$

- **$\tilde{U}_0{}^c(N, r)$**

s-wave

	$G_{-2}(r)$	$G_0(r)$	$G_2(r)$
Range [fm]	0.176	0.271	0.393
$u_0^c(N,r)$ [MeV]	2911.49	2545.87	-507.819
$v_0^c(N,r)$ [1]	-1.98786	-5.61873	-4.41952×10^{-1}

p-wave

	$G_{-2}(r)$	$G_0(r)$	$G_2(r)$
Range [fm]	0.318	0.312	0.320
$u_0^c(N,r)$ [MeV]	-2771.64	1916.04	-411.560
$v_0^c(N,r)$ [1]	2.62581	-2.87808	-1.76763

- **$\tilde{U}_\tau{}^c(N, r)$**

	$G_0(r)$	$G_2(r)$
Range [fm]	0.265	0.524
$u_\tau^c(N,r)$ [MeV]	401.337	290.964
$v_\tau^c(N,r)$ [1]	0.405391	0.293903

- **$\tilde{U}_0{}^c(WZ, r)$**

	$G_0(r)$	$G_0(r)$
Range [fm]	0.282	0.404
$u_0^c(WZ,r)$ [MeV]	-676.51	-1207.07
$v_0^c(WZ,r)$ [1]	-3.483	-0.995

Fitting parameters(LS and centrifugal terms)

parameter set A: $F_\pi = 186$ MeV, $e = 4.82$

- **$\tilde{U}_0^{LS}(N, r)$**

	$G_0(r)$	$G_0(r)$
Range [fm]	0.483	0.300
$u_0^{LS}(N,r)$ [MeV]	38.8404	63.4182
$v_0^{LS}(N,r)$ [1]	0.392332×10^{-1}	0.630481×10^{-1}

- **$\tilde{U}_\tau^{LS}(N, r)$**

	$G_{-2}(r)$	$G_{-2}(r)$
Range [fm]	0.604	0.262
$u_\tau^{LS}(N,r)$ [MeV]	-1284.46	-6954.51
$v_\tau^{LS}(N,r)$ [1]	1.29744	7.02476

- **$\tilde{U}_0^{LS}(WZ, r)$**

	$G_0(r)$	$G_0(r)$
Range [fm]	0.377	0.243
$u_0^{LS}(WZ,r)$ [MeV]	-363.915	-287.034
$v_0^{LS}(WZ,r)$ [1]	0.367587	0.289936

- **$\tilde{U}_l(r)$**

	$G_0(r)$	$G_0(r)$
Range [fm]	0.431	0.748
$u_l(r)$ [MeV]	62867.86	7583.59
$v_l(r)$ [1]	63.5029	7.66019

$$\tilde{U}_l(r) = \frac{l(l+1)}{2m_K r^2}\left[G_0(r) + G_0(r)\right]$$

l: Kaon angular momentum