# Towards a relativistic formulation of nucleon-nucleon interactions

# in chiral perturbation theory


Li-Sheng Geng （耿立升）

School of Physics and Nuclear Energy Engineering,

Beihang University, Beijing, China

In collaboration with:

Xiu-Lei Ren（任修磊）, Peking University

Kai-Wen Li （李凯文）， Beihang University

Jie Meng（孟杰）, Peking University

Bing-Wei Long（龙炳蔚）, Sichuan University

Peter Ring， Technical University of Munich

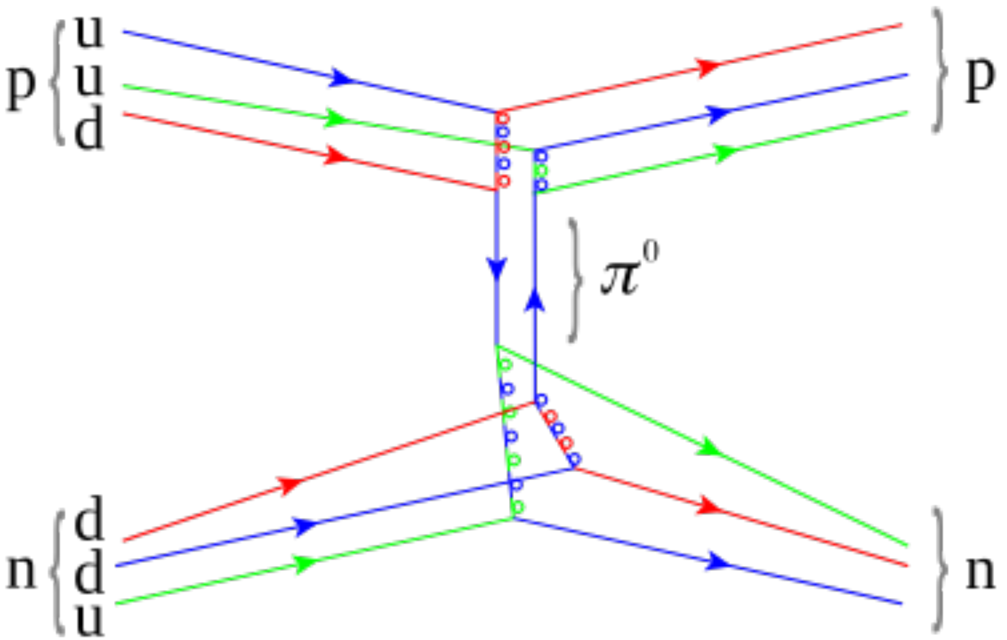

YITP/2016.11.21

# The Nobel Prize in Physics 1949

Hideki Yukawa

"for his prediction of the existence of mesons on the basis of theoretical work on nuclear forces".

Yukawa Institute for Theoretical Physics (former Research Institute for Fundamental Physics) goes **back to 1949** when Hideki Yukawa of Kyoto University




The paper was written in 1935 while he was at Osaka U.

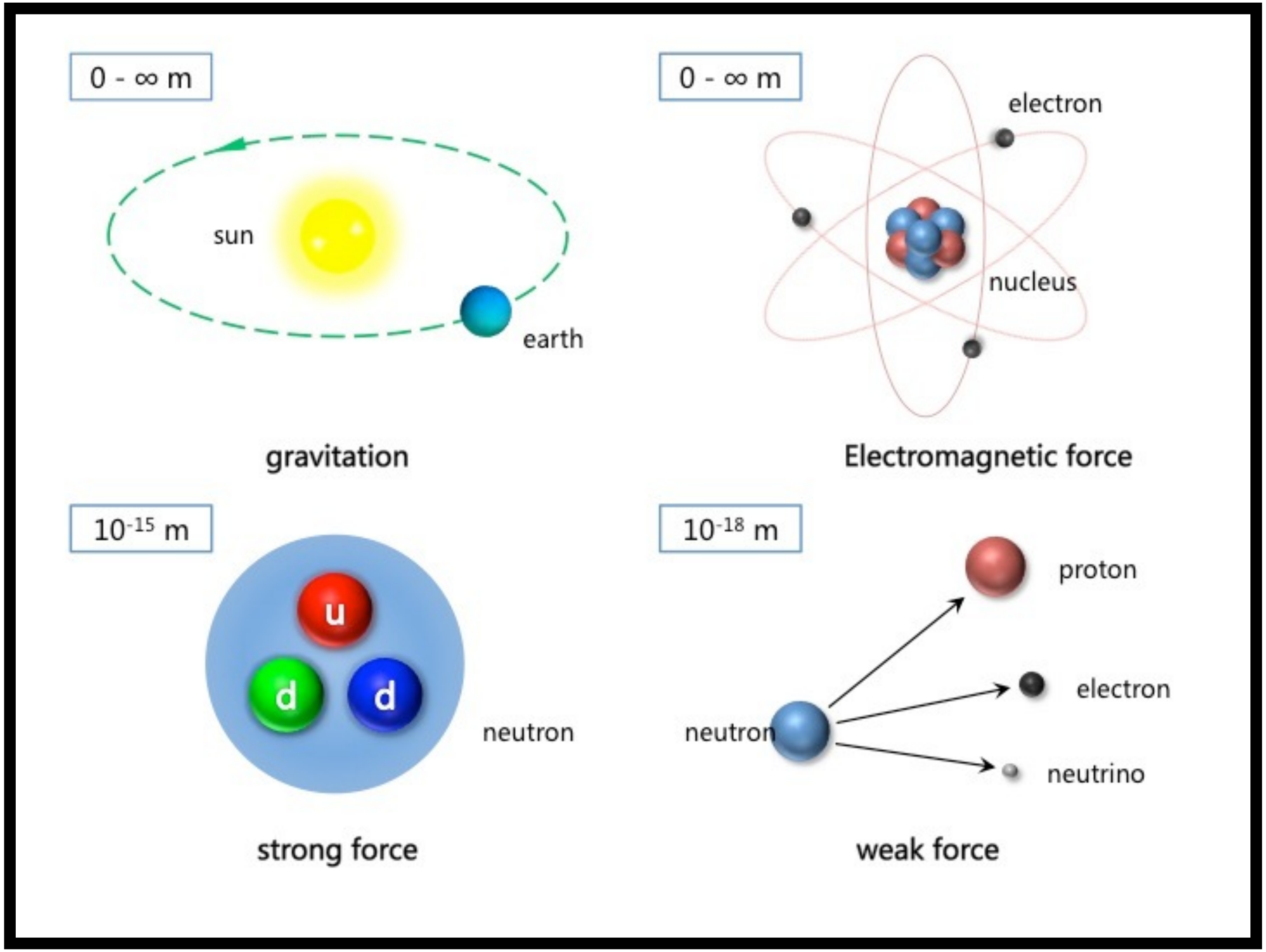
# First step in a long journey

# Outline

- ✤ (A rather lengthy) Introduction
  - Why nuclear force; Current status (of chiral forces)
  - Why relativistic?
    - atomic/molecular
    - nuclear
    - one-baryon sector
- ✤ Our strategy and some preliminary results
- ✤ Summary and outlook

# Motivation: why nuclear force

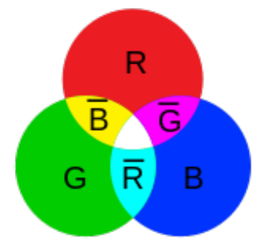
# Four (established) forces in nature

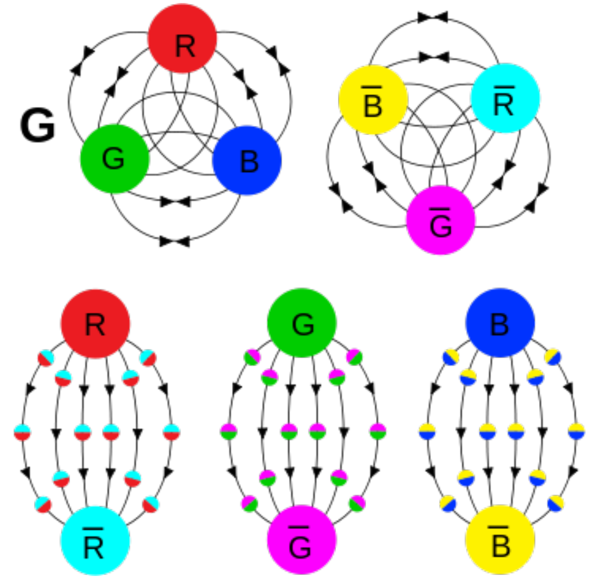



Evidence for a Protophobic Fifth Force from 8Be Nuclear Transitions,1604.07411

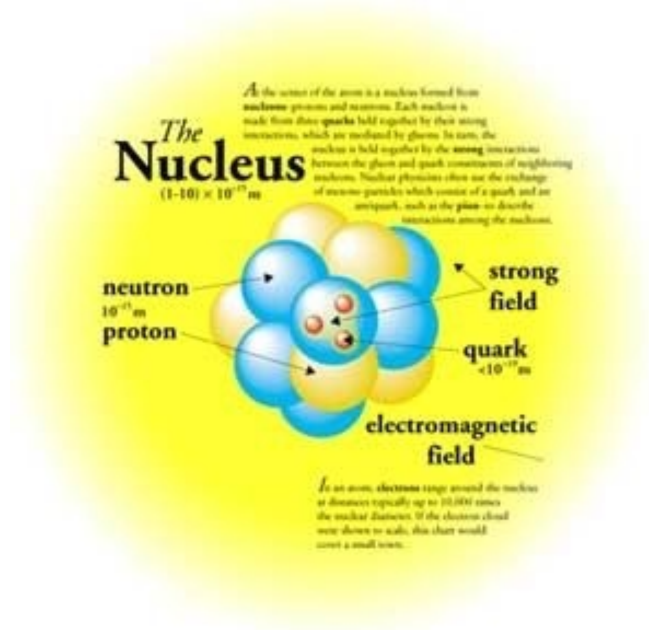
# Strong force

- **Strong force:** **bind quarks into hadrons**
- **Nuclear force—residual strong force: binds nucleons into nuclei**
- **Underlying theory—QCD**



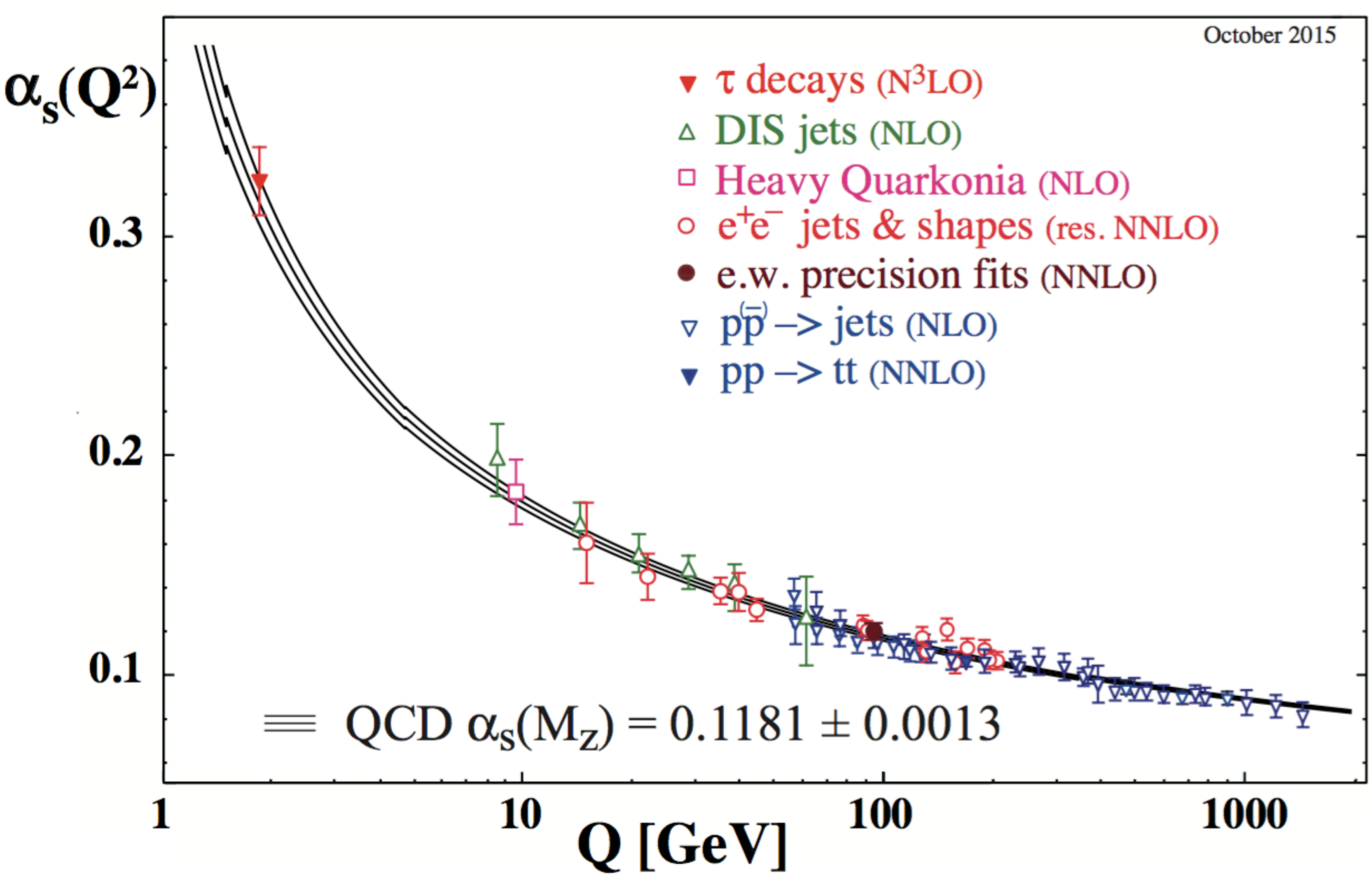




$$
\begin{aligned}
LQCD = &-\frac{1}{4}(\partial^{\mu} G_a^{\nu} - \partial^{\nu} G_a^{\mu})(\partial_{\mu} G_{\nu}^{a} - \partial_{\nu} G_{\mu}^{a}) + \sum_f \overline{q}_f^{\alpha}(i\gamma^{\mu}\partial_{\mu} - m_f) q_f^{\alpha} \\
&+ g_s G_a^{\mu} \sum_f \overline{q}_f^{\alpha} \gamma^{\mu} \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} q_f^{\beta} \\
&- \frac{g_s}{2} f^{abc}(\partial^{\mu} G_a^{\nu} - \partial^{\nu} G_a^{\mu}) G_{\mu}^{b} G_{\nu}^{c} - \frac{g_s^2}{4} f^{abc} f_{ade} G_b^{\mu} G_c^{\nu} G_{\mu}^{d} G_{\nu}^{e}
\end{aligned}
$$




**2 quark masses and 1 universal coupling**

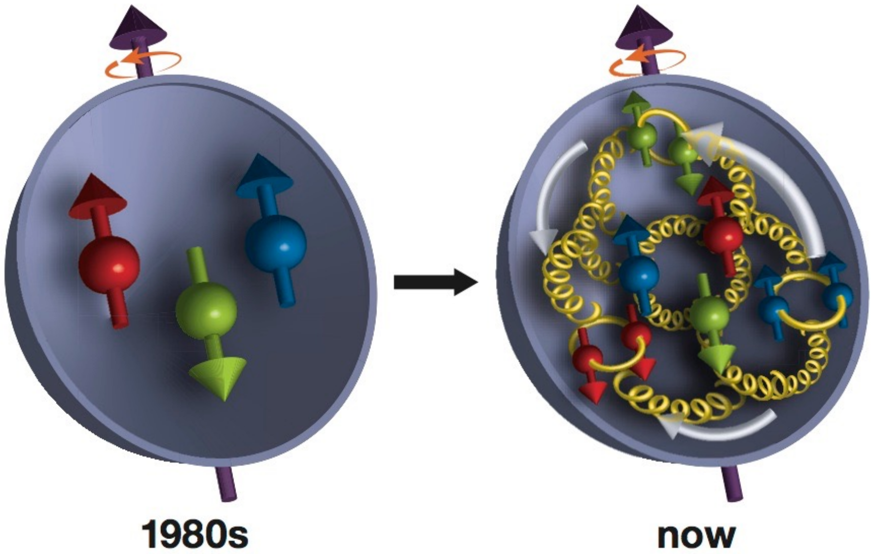
# QCD：Asymptotic freedom




PDG2015

# QCD： color confinement

- **Free quarks do not exist (color confinement), experimentally only hadrons are observed**
- **Mismatch of degrees of freedom— hadronization**




**Decomposition of the proton spin**

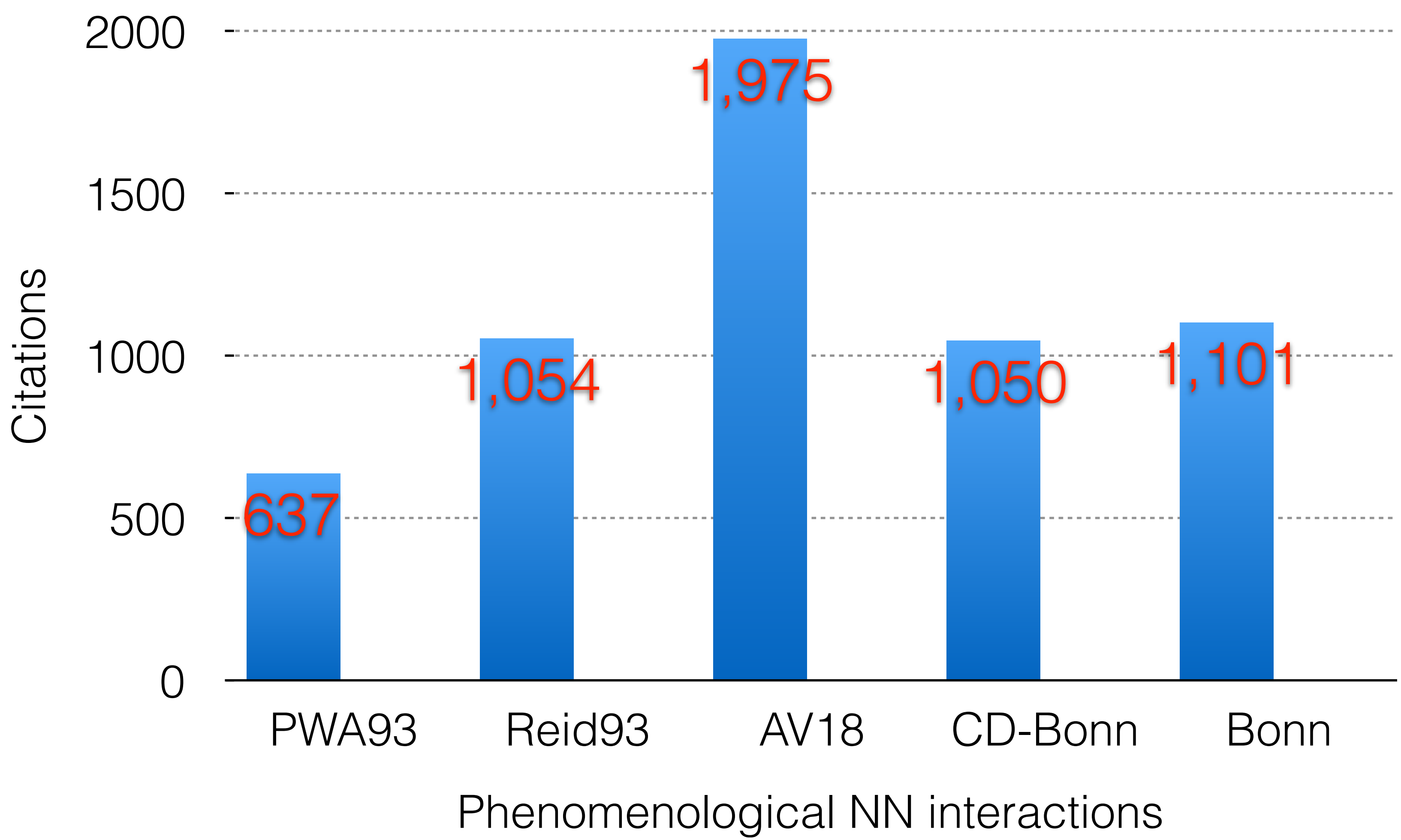
# Why construct nuclear forces?

- **Nuclear force: derivative force or residual force**
- **In this sense,  similar to intermolecular force, but because of confinement and asymptotic freedom of QCD, much richer and harder**

Fan Wang, Guang-han Wu, Li-jian Teng, J.Terrance Goldman Phys.Rev.Lett. 69 (1992) 2901-2904

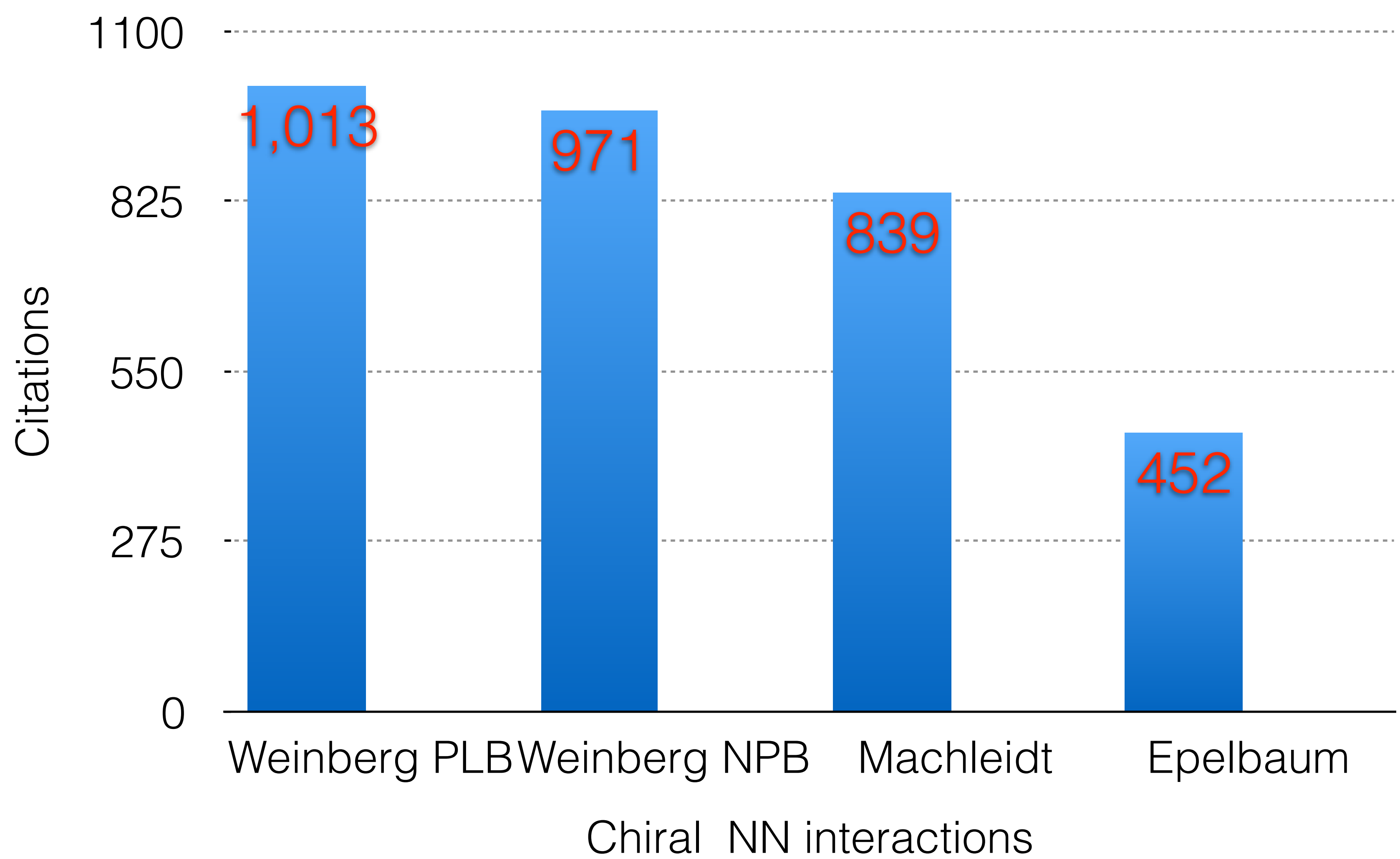
- **Constructing a nuclear force is a long-standing and interesting subject in nuclear physics; the basis of all microscopic (ab initio) nuclear structure and reaction theories**

# NN interaction—foundation of microscopic nuclear structure




as of July 8th, 2016

# NN interaction—foundation of microscopic nuclear structure




as of July 8th, 2016

# The ultimate aim:
# nuclear physics as a precision science

The Nobel Prize in Chemistry 2013

© Harvard University
Martin Karplus

Photo: © S. Fisch
Michael Levitt

Photo: Wikimedia Commons
Arieh Warshel

The Nobel Prize in Chemistry 2013 was awarded jointly to Martin Karplus, Michael Levitt and Arieh Warshel *"for the development of multiscale models for complex chemical systems"*.

for the development of multiscale models for complex chemical systems

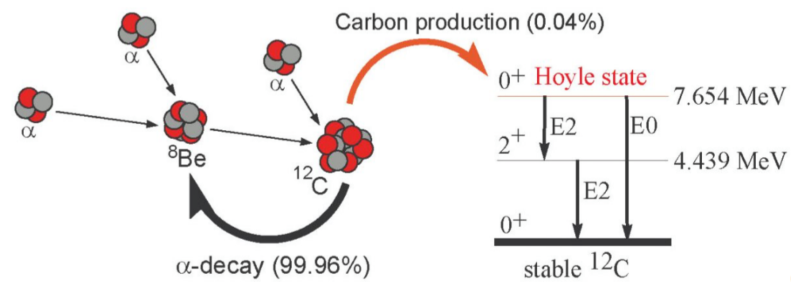
**Nuclear force+advanced numerical methods**

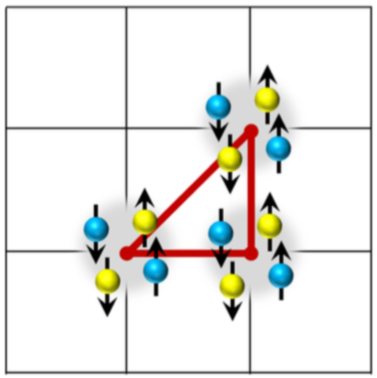
**=**

**precision nuclear physics**

# Two recent examples

## Hoyle state of Carbon



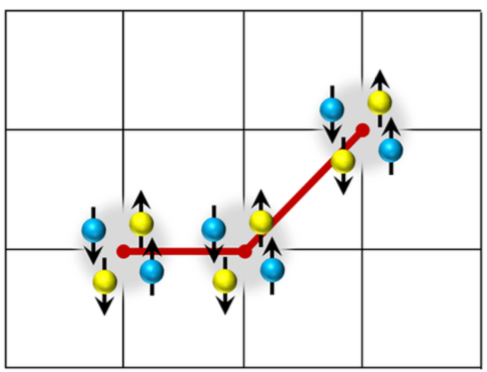


TABLE II. Lattice and experimental results for the energies of the low-lying even-parity states of $^{12}$C, in units of MeV.

| | $0_1^+$ | $2_1^+(E^+)$ | $0_2^+$ | $2_2^+(E^+)$ |
|---|---|---|---|---|
| LO | −96(2) | −94(2) | −89(2) | −88(2) |
| NLO | −77(3) | −74(3) | −72(3) | −70(3) |
| NNLO | −92(3) | −89(3) | −85(3) | −83(3) |
| Expt. | −92.16 | −87.72 | −84.51 | −82.6(1) [8,10] |
| | | | | −81.1(3) [9] |
| | | | | −82.32(6) [11] |

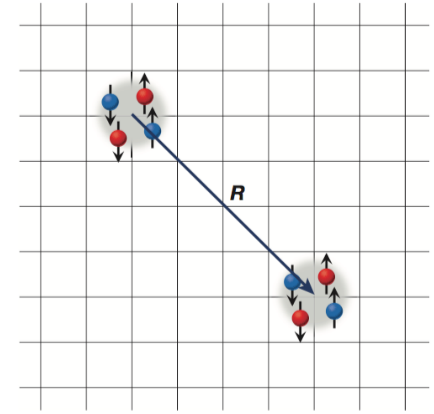
PRL **109,** 252501 (2012) PHYSICAL REVIEW LETTERS week ending 21 DECEMBER 2012

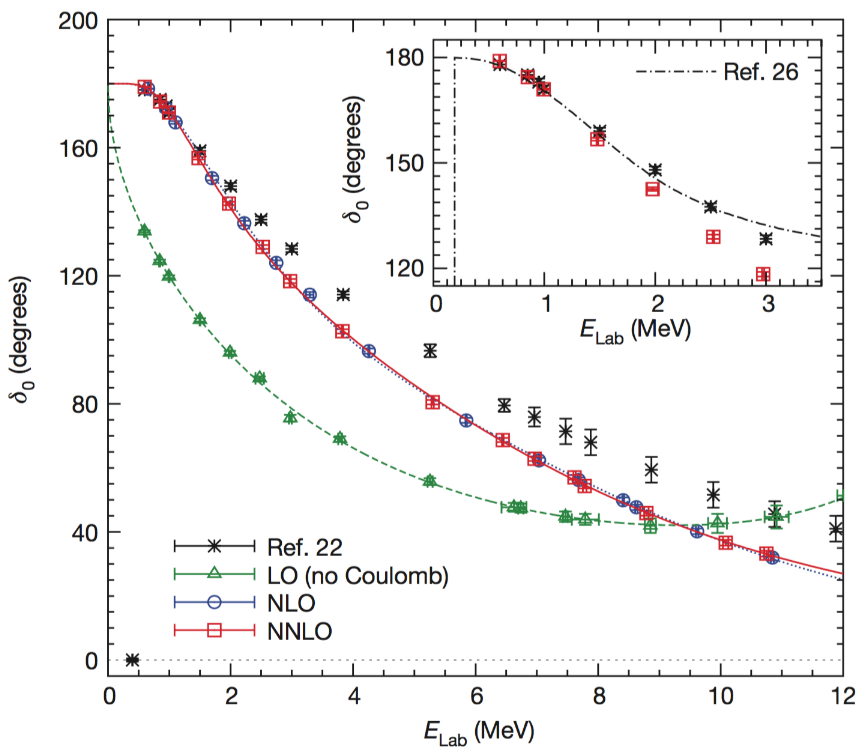
**Structure and Rotations of the Hoyle State**

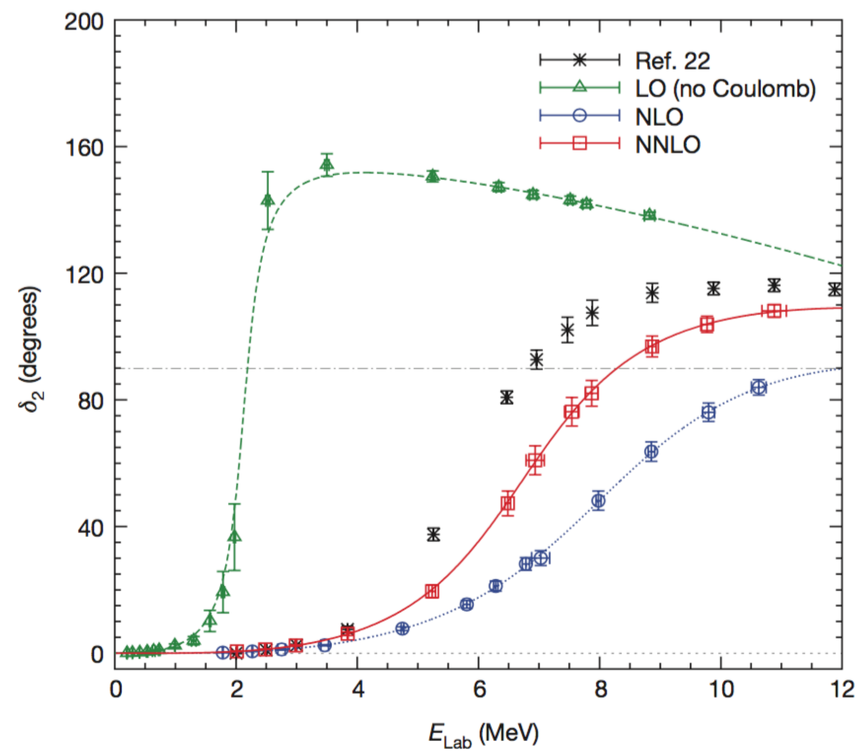
Evgeny Epelbaum,[1] Hermann Krebs,[1] Timo A. Lähde,[2] Dean Lee,[4] and Ulf-G. Meißner[5,2,3]

# Two recent examples

## alpha-alpha scattering




LETTER

doi:10.1038/nature16067

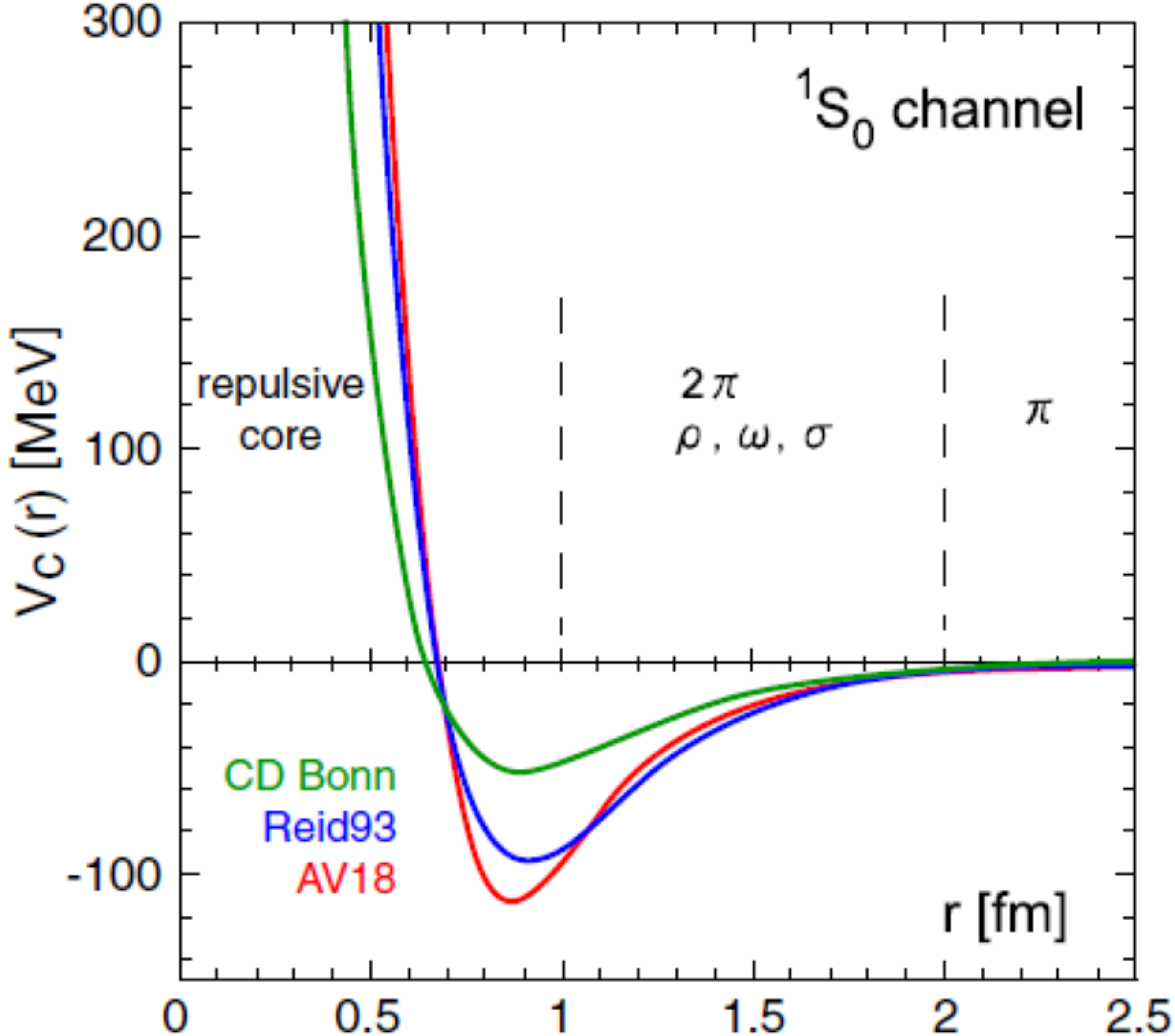
*Ab initio* alpha–alpha scattering

Serdar Elhatisari[1], Dean Lee[2], Gautam Rupak[3], Evgeny Epelbaum[4], Hermann Krebs[4], Timo A. Lähde[5], Thomas Luu[1,5] & Ulf-G. Meißner[1,5,6]

Nature 16067

# Chiral nuclear forces—current status

# “High Precision” Nuclear Force




“On the interaction of elementary particles,” PTP17,48

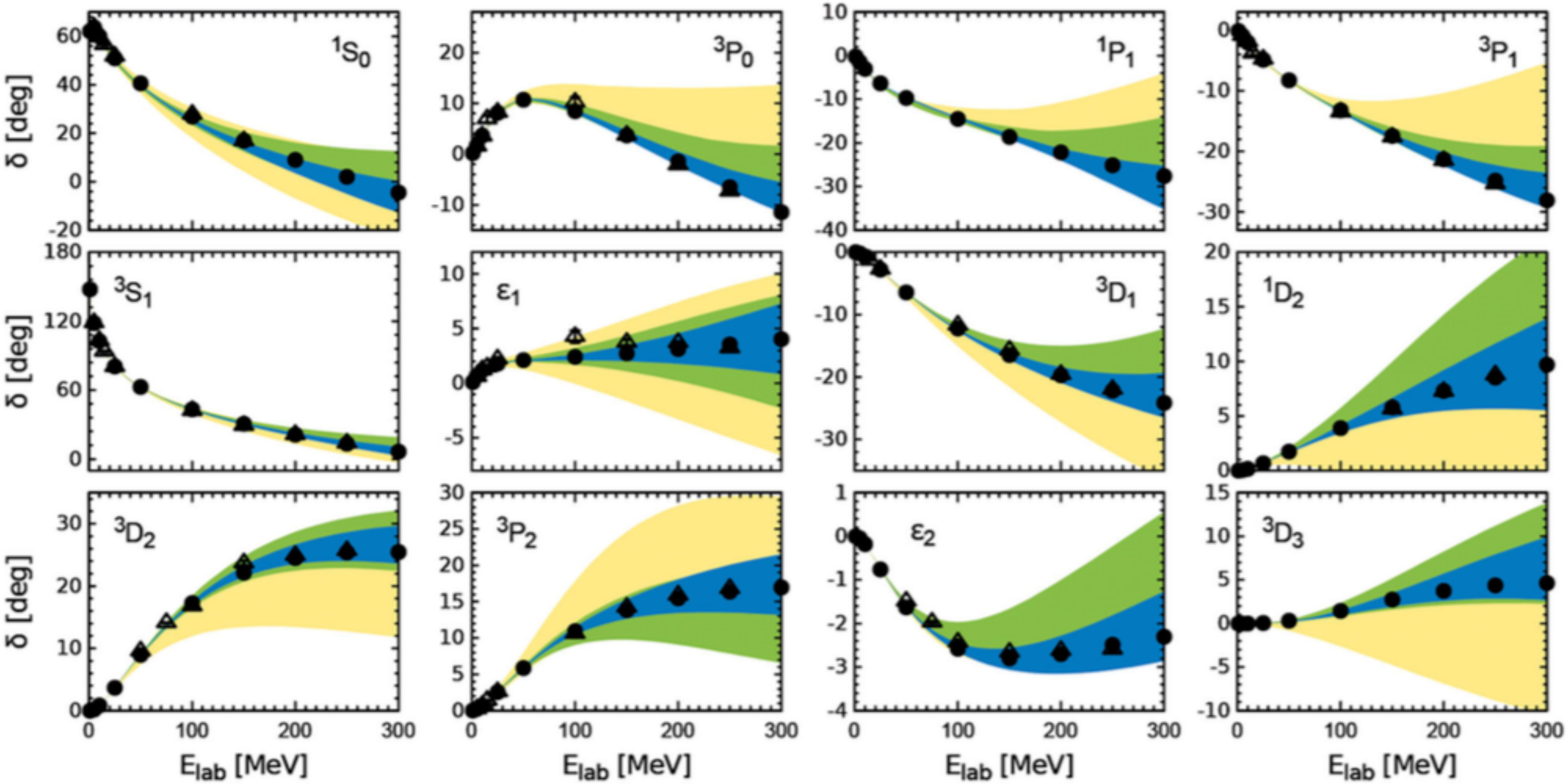
# Major milestones for NN potential development ChPT

- 1991/92: Weinberg, NN potential from ChPT
- 1994/96: Bira v. Kolck and co-workers, first ChPT based NN potential at N2LO using cutoff regularization (r-space)
- 1994-1997:
  - Robilotta and co-workers, 2-pi at N2LO
  - 1997: Kaiser et al., 2-pi at N2LO using HBChPT and DR
- 2000: Epelbaum et al. ("Bochum-Juelich" group), NN potential in momentum space at N2LO (HBChPT, DR)

High Precision Nuclear Force

- 2003:
  - Robilotta and co-workers 2-pi at N3LO in RBChPT
  - Entem & Machleidt ("Idaho" group), first NN potential at N3LO (HBChPT, DR)
- 2005: Epelbaum et al. ("Bochum-Juelich" group), NN potential at N3LO (HBChPT, SFR)
- 2015: Epelbaum et al., Entem, et al., NN potential at N4LO

# Estimate of theoretical uncertainties




- **E. Epelbaum, H. Krebs, and U.-G. Meissner, Eur. Phys. J. A (2015)51**

# PRA Editorial 2011

APS physics | Journals | Help/Feedback | Journal, vol, page, DOI, etc.

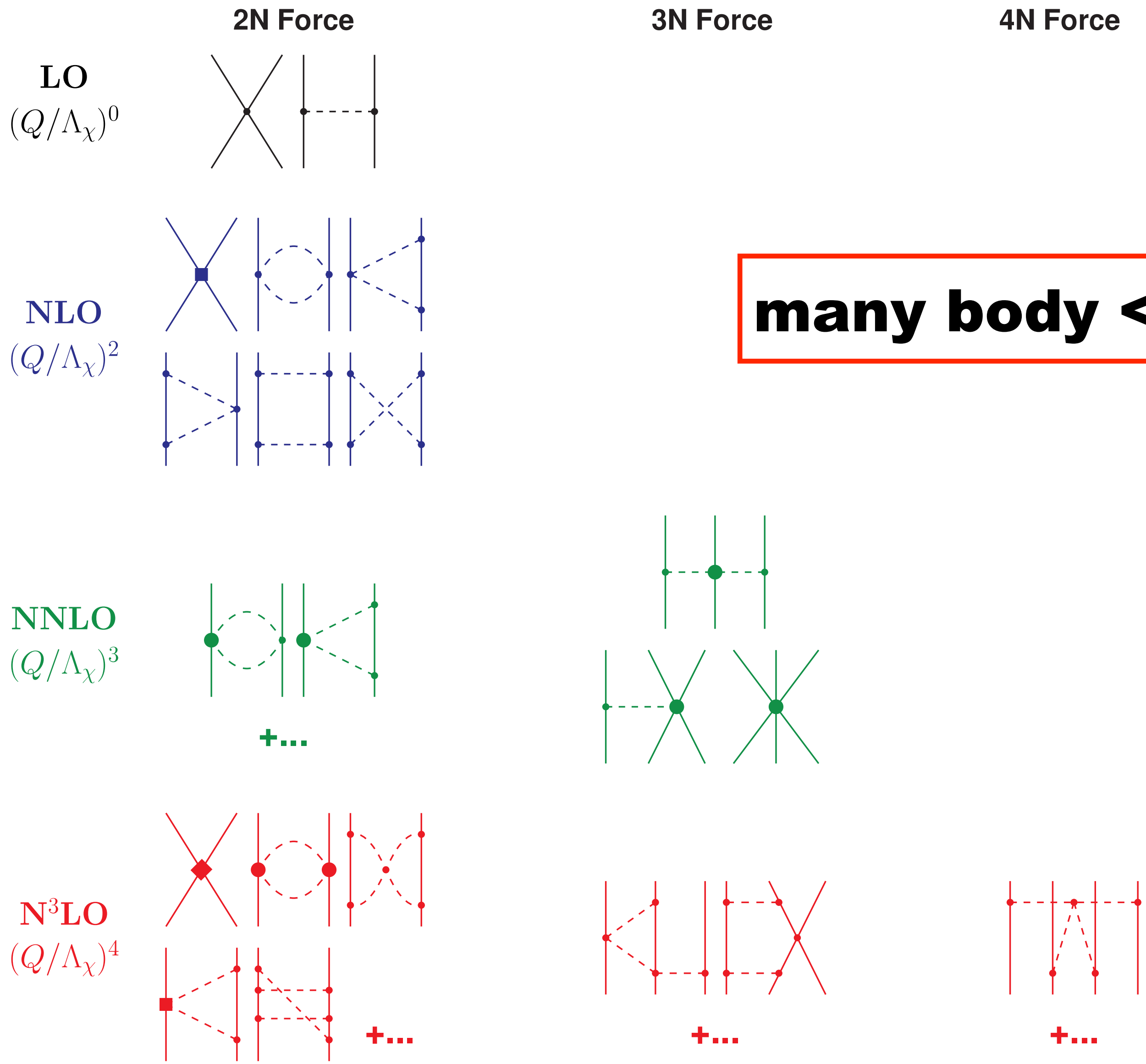
1 **If the authors claim high accuracy, or improvements on the accuracy of previous work.**
2 **If the primary motivation for the paper is to make comparisons with present or future high precision experimental measurements.**
3 **If the primary motivation is to provide interpolations or extrapolations of known experimental measurements.**

## Editorial: Uncertainty Estimates

The purpose of this Editorial is to discuss the importance of including uncertainty estimates in papers involving theoretical calculations of physical quantities.

1 Development of new theoretical techniques or formalisms.
2 Development of approximation methods, where the comparison with experiment, or other theory, itself provides an assessment of the error in the method of calculation.
3 Explanation of previously unexplained phenomena, where a semiquantitative agreement with experiment is already significant.
4 Proposals for new experimental arrangements or configurations, such as optical lattices.
5 **Quantitative comparisons with experiment for the purpose of (a) verifying that all significant physical effects have been taken into account, and/or (b) interpolating or extrapolating known experimental data.**
6 **Provision of benchmark results intended as reference data or standards of comparison with other less accurate methods.**

# Hierarchy of Nuclear Force in ChEFT




**many body < few body**

- **E. Epelbaum, H.-W. Hammer, Ulf-G. Meissner, Reviews of Modern Physics 81(2009)1773**
- **R. Machleidt and D. R. Entem, Physics Reports 503(2011)1**

# Nonrelativistic NF from heavy baryon (HB) ChEFT

- **NN interaction**
  - up to NLO U. van Kolck et al., PRL, PRC1992-94; N. Kaiser, NPA1997
  - up to NNLO E. Epelbaum, et al.,NPA2000; U. van Kolck et al.,PRC1994
  - up to $N^3LO$ R. Machleidt et al., PRC2003; E. Epelbaum et al., NPA2005
  - up to $N^4LO$ E. Epelbaum et al., PRL2015, D.R. Entem, et al., PRC2015
  - dominant $N^5LO$ terms D.R. Entem, et al., PRC2015
- **3N interaction**
  - up to NNLO U. van Kolck, PRC1994
  - up to $N^3LO$ S. Ishikwas, et al, PRC2007; V. Bernard et al, PRC2007;
  - up to $N^4LO$ H. Krebs, et al., PRC2012-13
- **4N interaction**
  - up to $N^3LO$ E. Epelbaum, PLB 2006, EPJA 2007

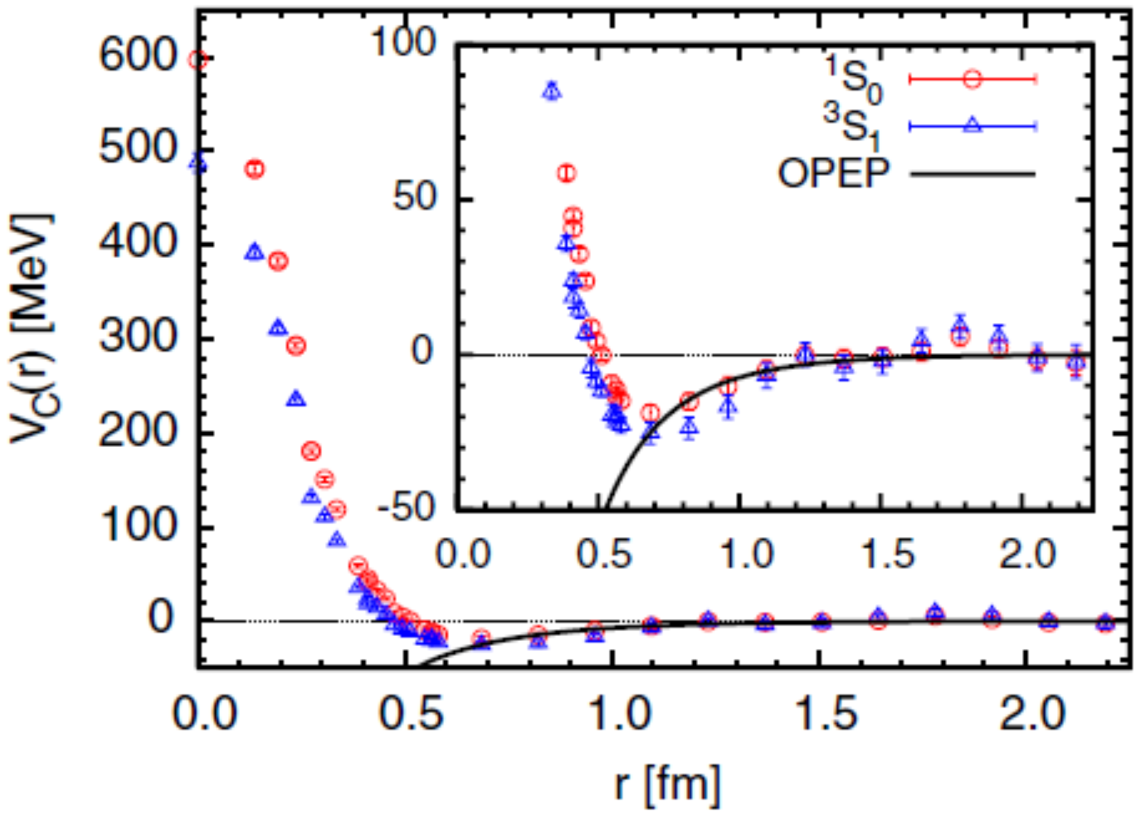
# Number of parameters in Modern Nuclear Forces

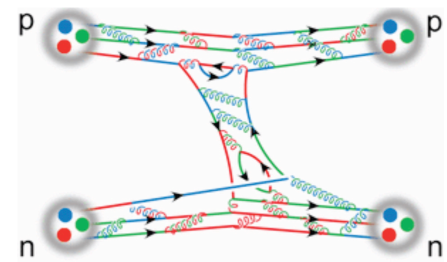
| | | | | | ChEFT [5] | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | PWA93 [1] | Reid93 [2] | AV18 [3] | CD-Bonn [4] | LO | NLO | NNLO | N3LO | N4LO |
| No. of LECs | 35 | 50 | 40 | 38 | 2 | 9 | 9 | 24 | 24 |
| $\chi^2$/ datum | 1.07 | 1.03 | 1.09 | 1.02 | 480 | 63 | 21 | 0.7 | 0.3 |

caution about definition of $\chi^2$

[1] V.G.J. Stocks et al., PRC48, 792(1993)—Inspire cited 637 times
[2] V.G.J. Stocks et al., PRC49, 2950(1994)—Inspire cited 1054 times
[3] Robert B. Wiringa et al, PRC51, 38(1995)—Inspire cited 1975 times
[4] R. Machleidt, PRC63,024001(2001)—Inspire cited 1050 times
[5] PRL 115,122301(2015)—Inspire cited 58

# Nuclear Force from Quark-Gluon dofs




- **First qualitative nuclear force from first principles**
- **$m_\pi$=461 MeV**
- **Quenched**

N. Ishii et al., PRL99,022001(2007)

# LQCD-predicted n$\Sigma^-$ phase shift

LO ChPT better?

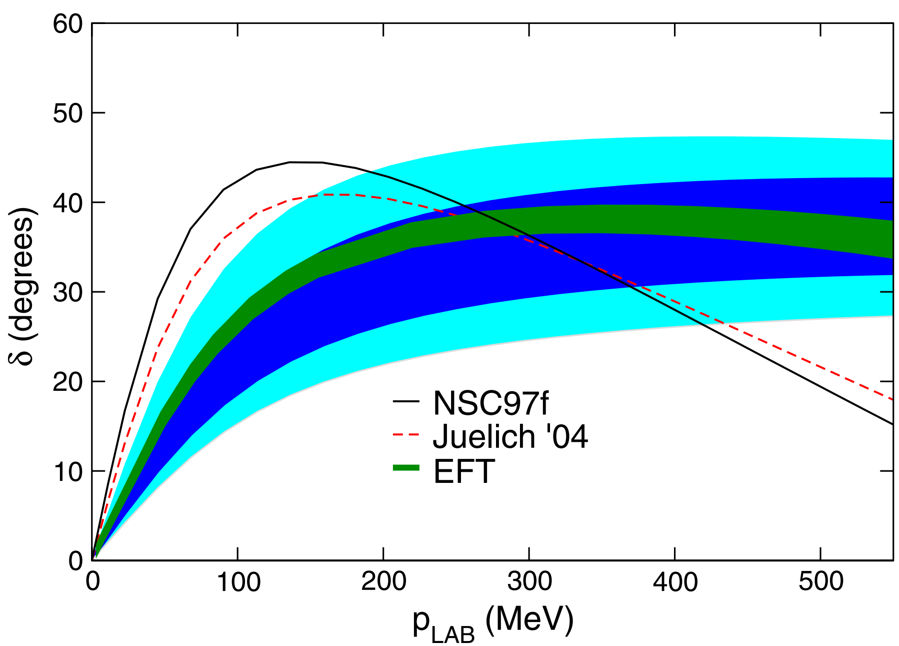



FIG. 1 (color online). LQCD-predicted $^1S_0$ $n\Sigma^-$ phase shift versus laboratory momentum at the physical pion mass (very dark and light blue bands), compared with other determinations, as discussed in the text.

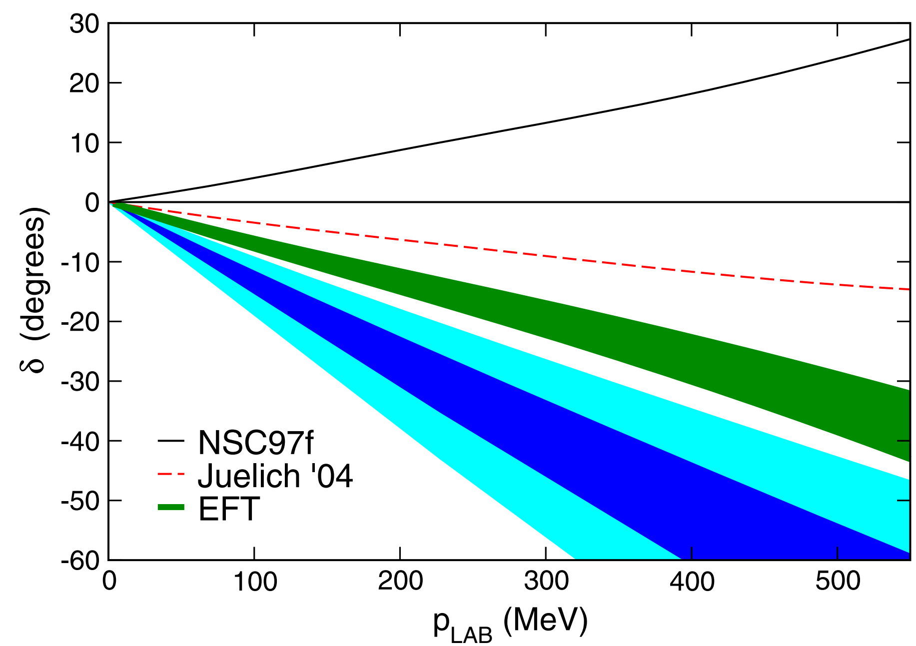



FIG. 2 (color online). LQCD-predicted $^3S_1$ $n\Sigma^-$ phase shift versus laboratory momentum at the physical pion mass (very dark and light blue bands), compared with other determinations, as discussed in the text.

NPLQCD, PRL109(2012)172001

# Limitations of Current ChPT NN forces

- **Not “renormalization group invariant”**
  - Sensitive to the UV cutoff, not (nonperturbatively) renormalizable
  - Diverse opinion on this issue (many discussions)
- **Based on HBChPT**
  - Slow convergence as in the one-baryon sector?
  - Cannot be used directly in covariant calculations.
- **A relativistic nuclear force based on the EOMS BChPT?**

# Motivation: why relativistic

# Importance of Relativity not so much recognized

- **Two pillars of modern physics:**
  - ✓ **Quantum mechanics**
  - ✓ **Special (General) relativity, not**

*Modern elementary-particle physics is founded upon the two pillars of quantum mechanics and relativity. I have made little mention of relativity so far because, while the atom is very much a quantum system, it is not very relativistic at all. Relativity becomes important only when velocities become comparable to the speed of light. Electrons in atoms move rather slowly, at a mere one percent of light speed. Thus it is that a satisfactory description of the atom can be obtained without Einstein's revolutionary theory.*

S.L.Glashow, 1988, Interactions, Wamer Books, New York

**Facts speak louder than words**

# Atomic/Molecular systems

# Relativistic corrections in heavy Atoms

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

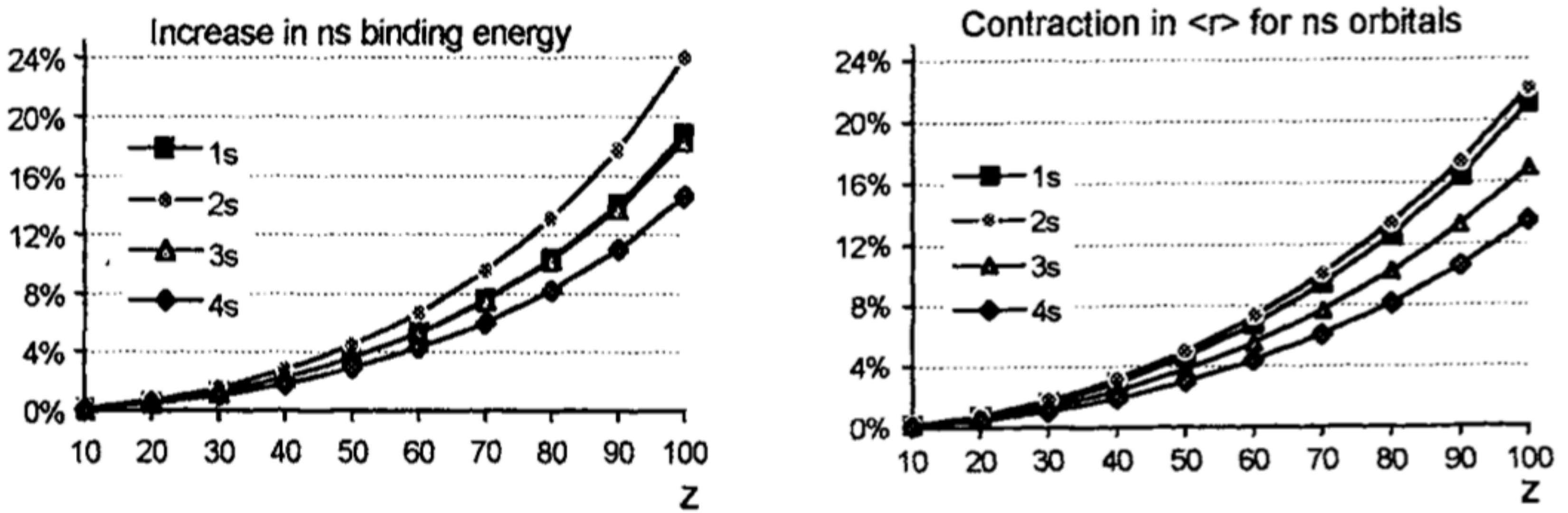



Fig. 2. Relativistic corrections for hydrogenic s orbitals (as percentage of the non relativistic value).

# QED effects

Table 1
Contributions to 1s electron binding energy in $U^{+91}$

| Contribution | Value (in eV) |
| --- | --- |
| Point nucleus Dirac equation | -132 279.61 |
| Finite nucleus correction | 198.82 |
| Self Energy | 355.05 |
| Vacuum Polarization | -88.60 |
| Higher order QED corrections | -0.64 |
| Nuclear recoil and nuclear polarization | 0.28 |
| Reduced mass correction | -0.30 |
| Lamb shift (i.e. sum of all above corrections) | 464.61 |
| 1s binding energy | -131 815.00 |

468±13 eV.

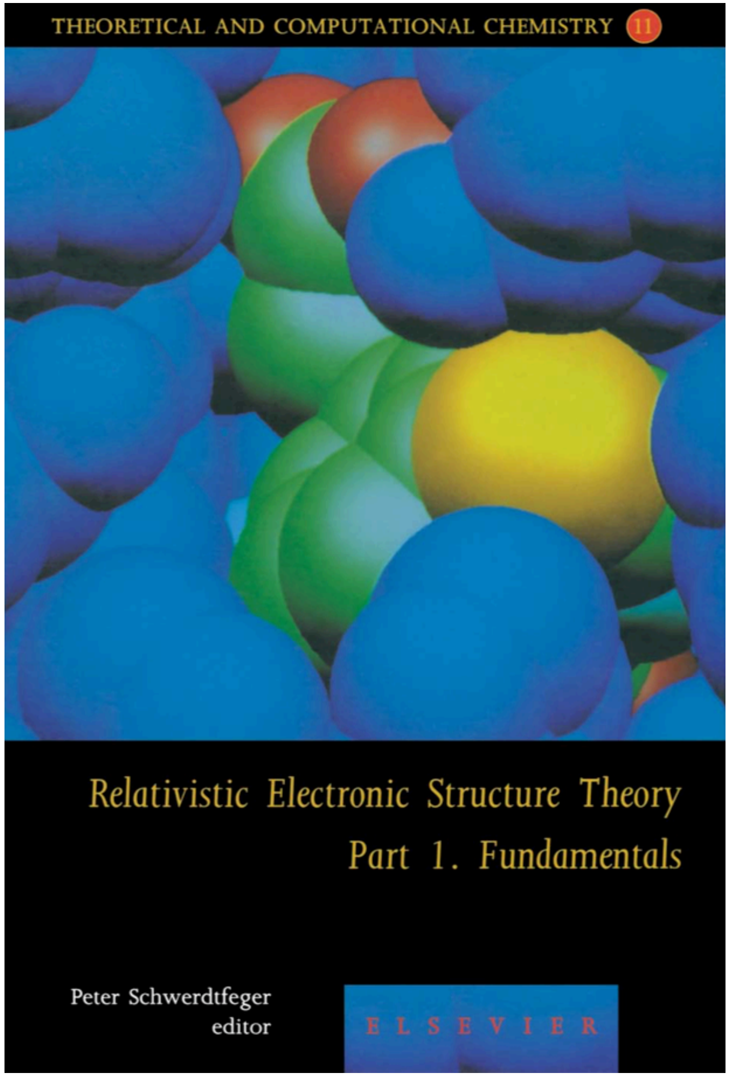
# Molecular systems studied in the Dirac-Fock one center approximation

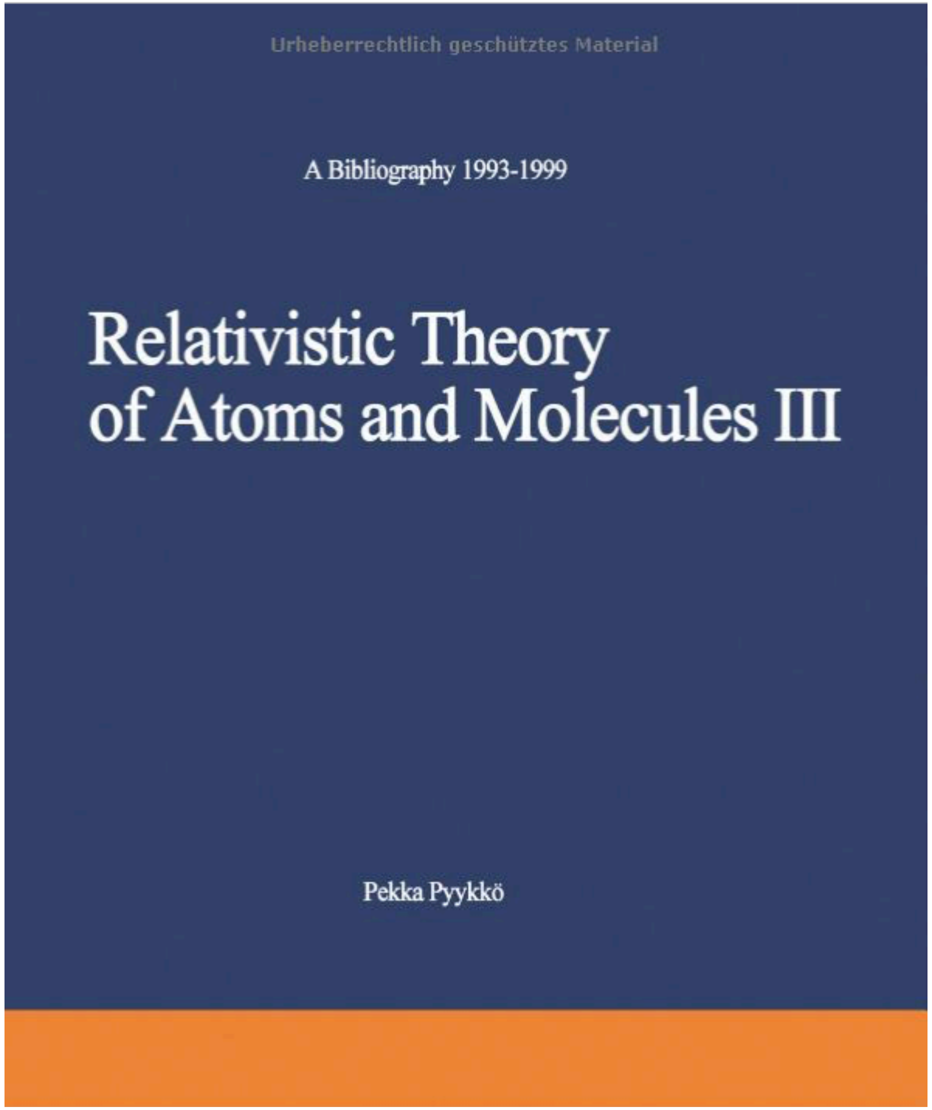
Table 2
Hydrides studied by the one-centre Dirac-Fock method

| Molecules | Main impact of relativistic corrections |
|---|---|
| $CH_4$ to $PbH_4$ | Bond length contraction and increase of force constants |
| CuH, AgH and AuH | Increase of the dissociation energy. Explanation of the difference between Ag and Au. |
| BH to TlH | Decrease of the dissociation energy for TlH and monovalency of TlH partially due to 6p spin-orbit splitting. Transition from LS to jj coupling in bonding orbitals. |
| $TiH_4$ to $(104)H_4$ | Small bond length expansion for $TiH_4$ and $ZrH_4$. |
| $CeH_4$ , $ThiH_4$ ; $UH_6$ , $CrH_6$ to $(106)H_6$ | 5d orbital of W moves to bonding region and W-H bonds strengthened. Further evidence for 5f participation in U-H bonds. Contraction in actinide series found of the order of 30 pm. |
| $MH^+$ and $MH_2$ with M = Be to Ra, Zn to Hg, Yb and No | Increasingly strong d contributions to the bonds from Ca to Ra. Ra-H bonds longer than Ba-H ones. Yb-H and No-H bonds are about the same. Explanation of the linear two coordination of Hg. |
| $^1\Sigma$ states of ScH to AcH, TmH, LuH and LrH | Trends in group 3. Lu-H and Lr-H bond lengths comparable. |

Performed before 1980

# Two nice books



# Nuclear Systems

# Recent Progresses in Nuclear Structure Physics 2016 (NSP2016)

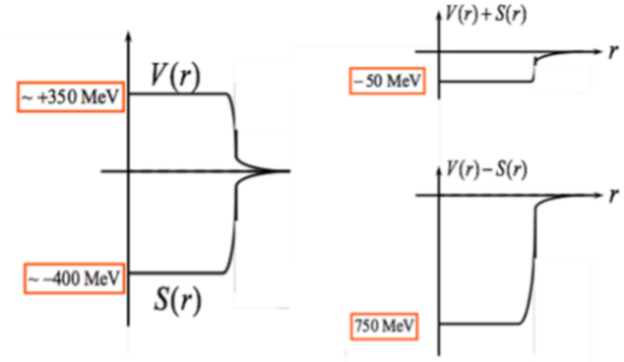
2016/12/05 --- 2016/12/23

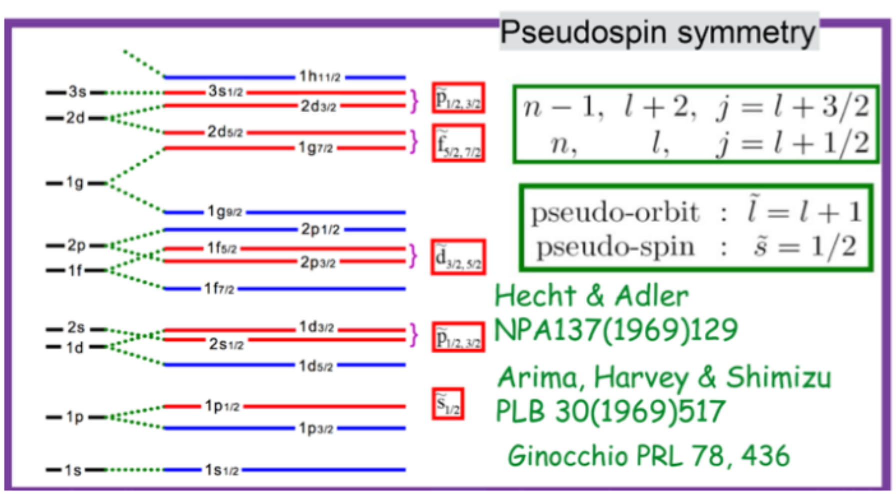
京都大学 基礎物理学研究所 研究棟・会議室 K102

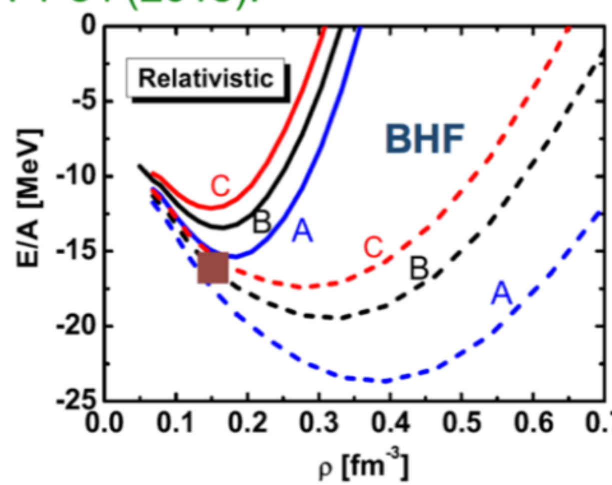
## Why Covariant?

P. Ring Physica Scripta, T150, 014035 (2012)

- ✓ Spin-orbit automatically included
- ✓ Lorentz covariance restricts parameters
- ✓ Pseudo-spin Symmetry
- ✓ Connection to QCD: big V/S ~ ±400 MeV
- ✓ Consistent treatment of time-odd fields
- ✓ Relativistic saturation mechanism
- ✓ … Liang, Meng, Zhou, Physics Reports **570** : 1-84 (2015).



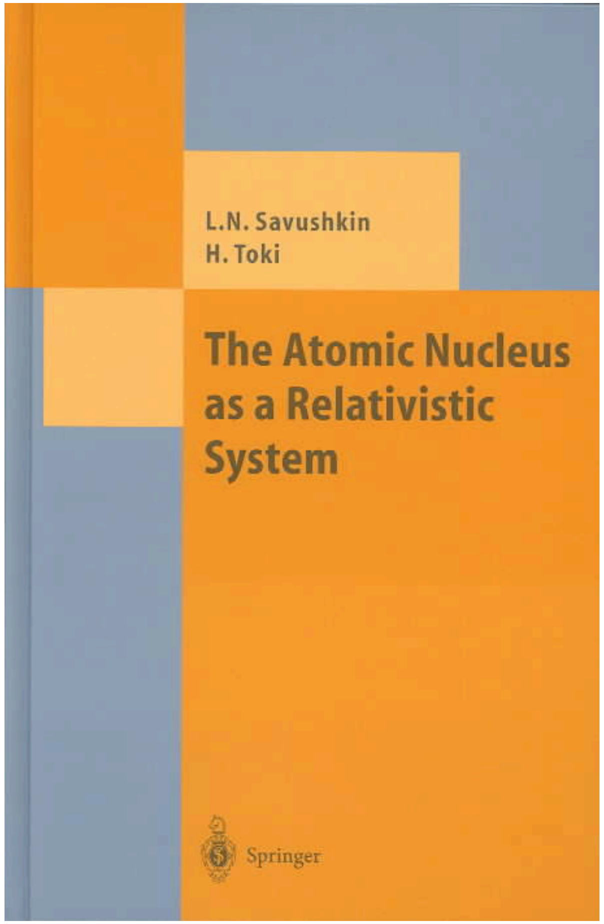



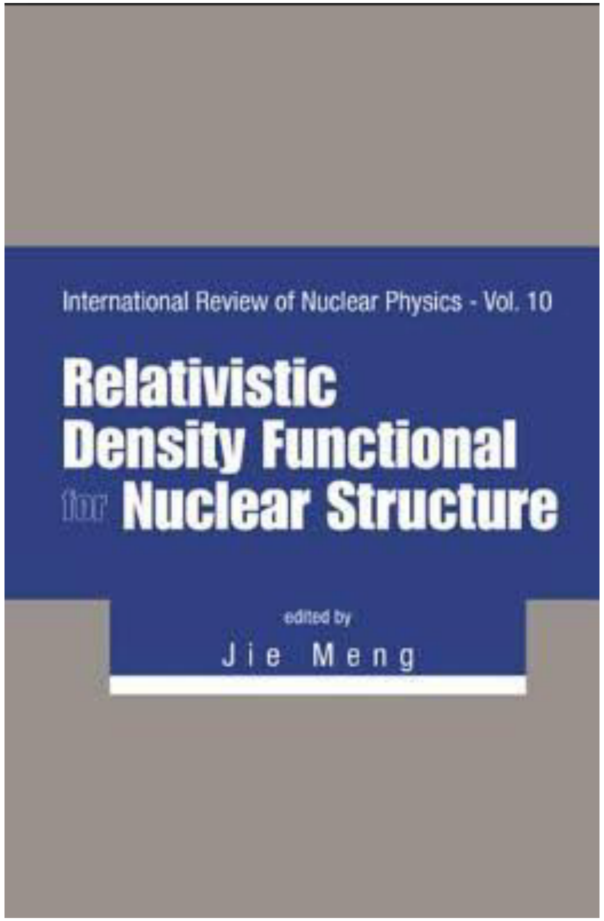




Brockmann & Machleidt, PRC42, 1965 (1990)

**from Jie Meng's talk**

# Two nice books



# One-Baryon(Nucleon) Sector

# Chiral Perturbation Theory (ChPT) in essence

- **Maps quark (u, d, s) dof's to those of the asymptotic states, hadrons**

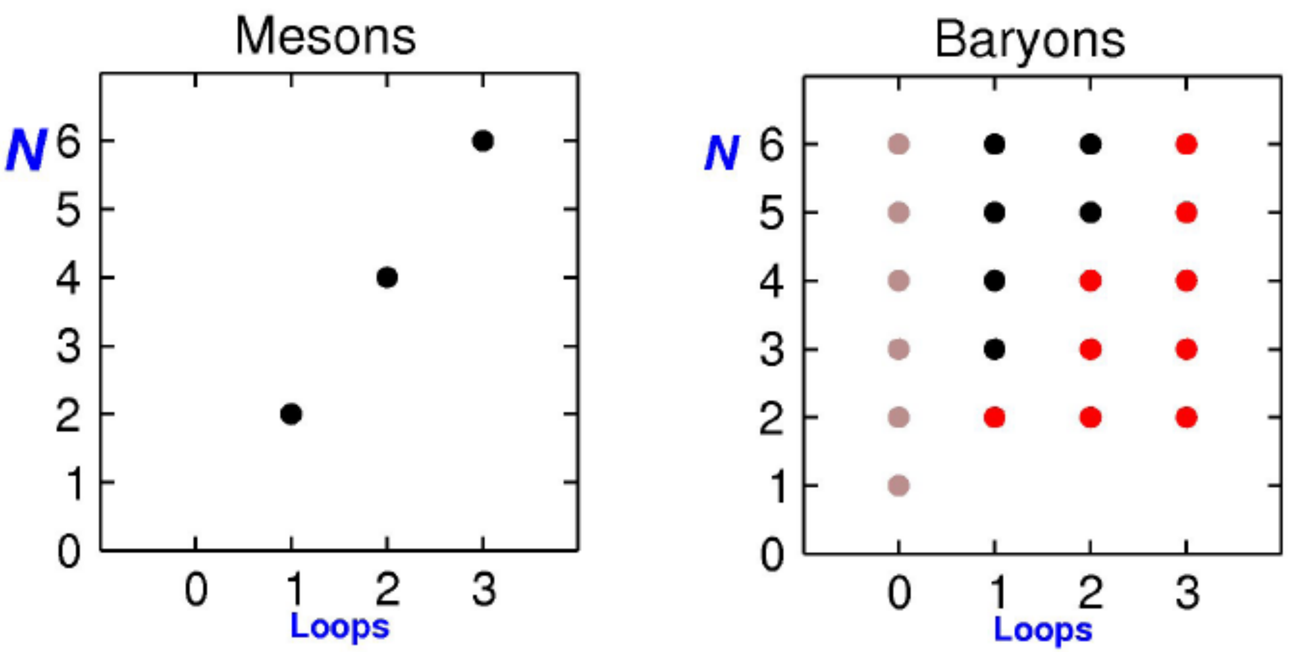
$$\boxed{\mathcal{L}_{\rm QCD}[q,\bar{q};G] \to \mathcal{L}_{\rm ChPT}[U,\partial U,\ldots,\mathcal{M},N]}$$

- $U$ parameterizes the Nambu-Goldstone bosons
- $\partial U$ vanishes at $E=\vec{p}=0$ (Nambu-Goldstone theorem)
- $M$ parameterizes the explicit symmetry breaking
- $N$ denotes interactions with matter fields
- Exact mapping via chiral Ward identities

- **ChPT exploits the symmetry of the QCD Lagrangian and its ground state; in practice, one solves in a perturbative manner the constraints imposed by chiral symmetry and unitarity by expanding the Green functions in powers of the external momenta and of the quark masses. (J. Gasser, 2003)**

# Power-counting-breaking (PCB) in the one-baryon sector

- ChPT very successful in the study of Nanbu-Goldstone boson self-interactions. (at least in SU(2))
- In the one-baryon sector, things become problematic because of the nonzero (large) baryon mass in the chiral limit, which leads to the fact that high-order loops contribute to lower-order results, i.e., a systematic power counting is lost!



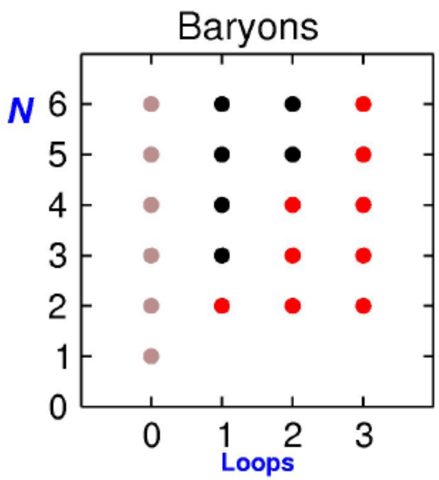

***red dots** denote possible PCB terms (pion-nucleon scattering)*

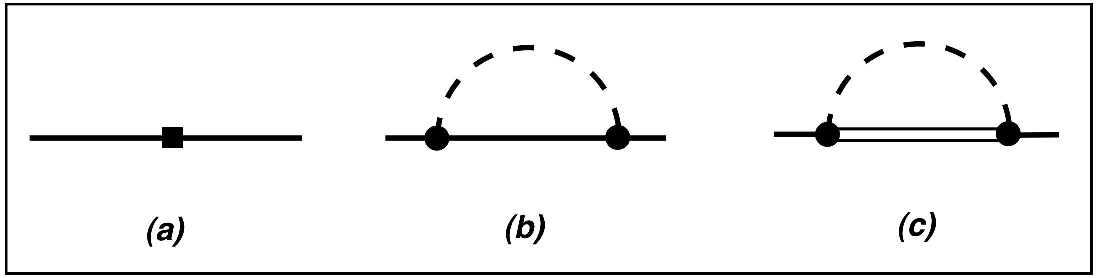
*J. Gasser et al., NPB 307, 779(1988)*

$$\boxed{\text{Chiral order} = 4L - 2N_M - N_B + \sum_k kV_k}$$

$\boxed{\text{Chiral order} = 4L - 2N_M - N_B + \sum_k kV_k}$

# Nucleon mass up to $O(p^3)$




order of the loop $= 1+1+4-1-2 = 3$

**Naively (no PCB)** $$M_N = M_0 + bm_\pi^2 + \text{loop}$$
$$\text{loop}(= cm_\pi^3 + \cdots)$$

**However** $$\boxed{\text{loop} = aM_0^3 + b' M_0 m_\pi^2 + cm_\pi^3 + \cdots}$$

**No need to calculate, simply recall that $M_0 \sim O(p^0)$**

# Power-counting-restoration methods

- **Heavy Baryon ChPT**: baryons are treated "semi-relativistically" by a simultaneous expansion in terms of external momenta and $1/M_N$ (*Jenkins et al., 1993*). It converges slowly for certain observables!

- **Relativistic baryon ChPT**: removing power counting breaking terms but retaining higher-order relativistic corrections, thus, keeping relativity.
  - Infrared baryon ChPT (*T. Becher and H. Leutwyler, 1999*)
  - Fully relativistic baryon ChPT–Extended On-Mass-Shell (EOMS) scheme (*J. Gegelia et al., 1999; T. Fuchs et al.,2003*)

- IR scheme separates the full integral into the Infrared and Regular parts:

$$H = \text{Infrared}$$

# Extended-on-Mass-Shell (EOMS)

- **"Drop" the PCB terms**

$$\boxed{\text{tree} = M_0 + bm_\pi^2} \quad + \quad \boxed{\text{loop} = aM_0^3 + b'M_0 m_\pi^2 + cm_\pi^3 + \cdots}$$

$$\Downarrow \; a = 0; b' = 0$$

$$\boxed{M_N = M_0 + b\ m_\pi^2 + cm_\pi^3 + \cdots \quad (\mathcal{O}(p^3))}$$

- **Equivalent to redefinition of the LECs**

$$\boxed{\text{tree} = M_0 + bm_\pi^2} \quad + \quad \boxed{\text{loop} = aM_0^3 + b'M_0 m_\pi^2 + cm_\pi^3 + \cdots}$$

**ChPT contains all possible terms allowed by symmetries, therefore whatever analytical terms come out from a loop amplitude, they must have a corresponding LEC**

# HB vs. Infrared vs. EOMS

- Heavy baryon (HB) ChPT
  - non-relativistic
  - breaks analyticity of loop amplitudes
  - converges slowly (particularly in three-flavor sector)
  - strict PC and simple nonanalytical results
- Infrared BChPT
  - breaks analyticity of loop amplitudes
  - converges slowly (particularly in three-flavor sector)
  - analytical terms the same as HBChPT
- Extended-on-mass-shell (EOMS) BChPT
  - satisfies all symmetry and analyticity constraints
  - converges relatively faster--an appealing feature

# Some successful applications of covariant BChPT (in the three-flavor sector)

- **Magnetic moments**

  PRL101:222002,2008; PLB676:63,2009; PRD80:034027,2009

- **Masses and sigma terms**

  PRD82:074504,2010; PRD84:074024,2011; JHEP12:073,2012; PRD 87:074001,2013; PRD89:054034,2014 ; EPJC74:2754,2014 ; PRD91:051502,2015

- **Vector form factors (couplings)**

  PRD79:094022,2009； PRD89:113007,2014

- **Axial form factors (couplings)**

  PRD78:014011,2008； PRD90:054502,2014

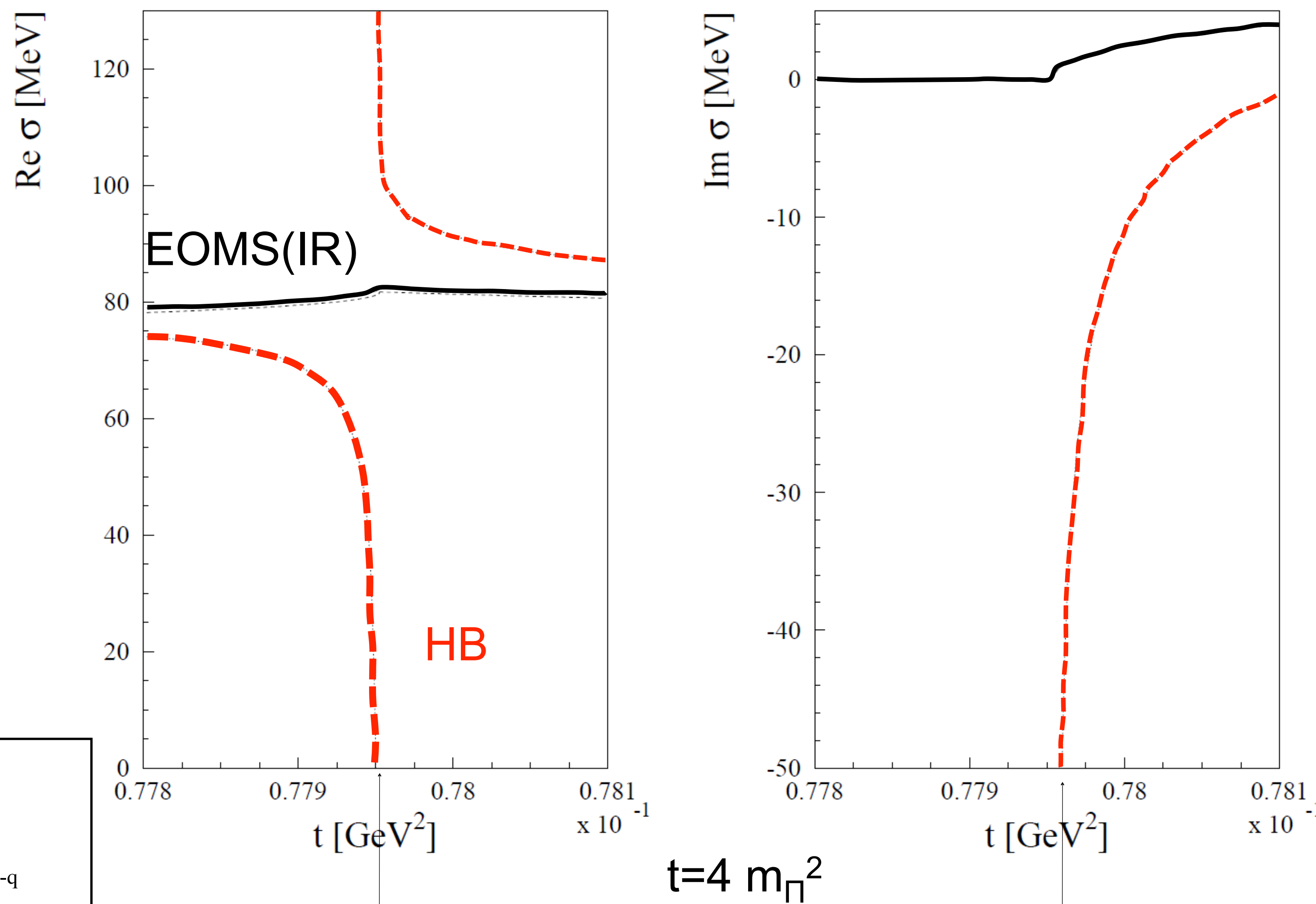
Recent developments in SU(3) covariant baryon chiral perturbation theory
Li-sheng Geng, Front.Phys.(Beijing) 8 (2013) 328-348

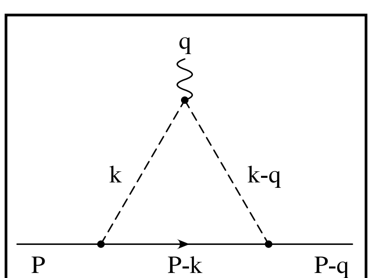
# The nucleon scalar form factor at $q^3$

t

$$\boxed{\langle p(p',s')|\mathcal{H}_{\rm sb}(0)|p(p,s)\rangle = \bar{u}(p',s')u(p,s)\sigma(t), \quad t=(p'-p)^2}$$

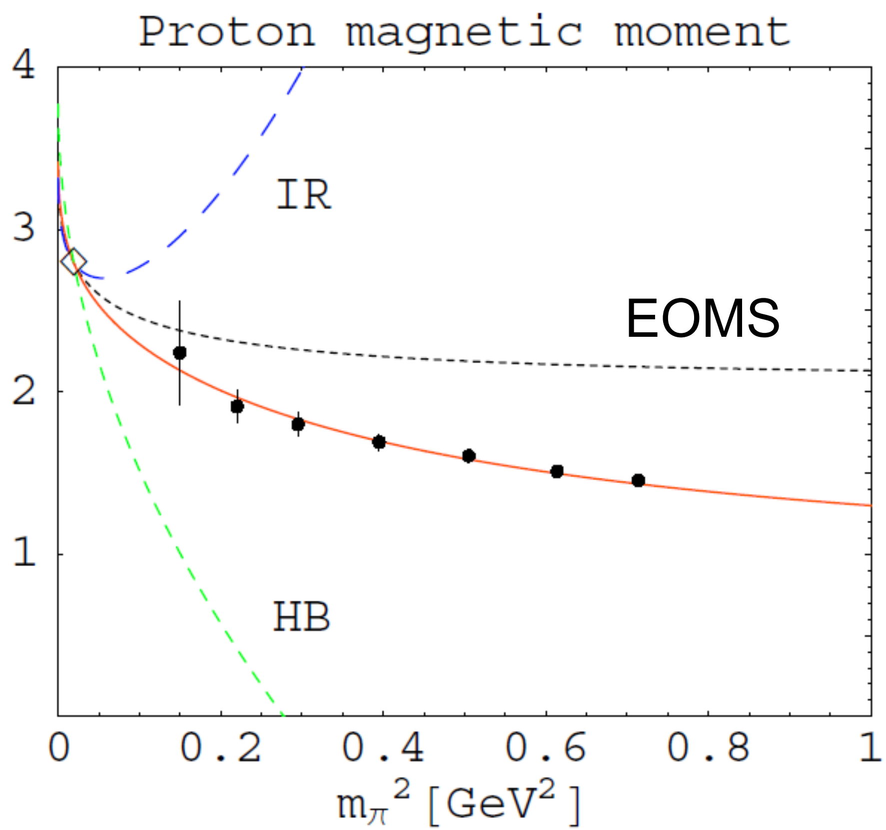
$$\mathcal{H}_{\rm sb} = \hat{m}(\bar{u}u + \bar{d}d)$$

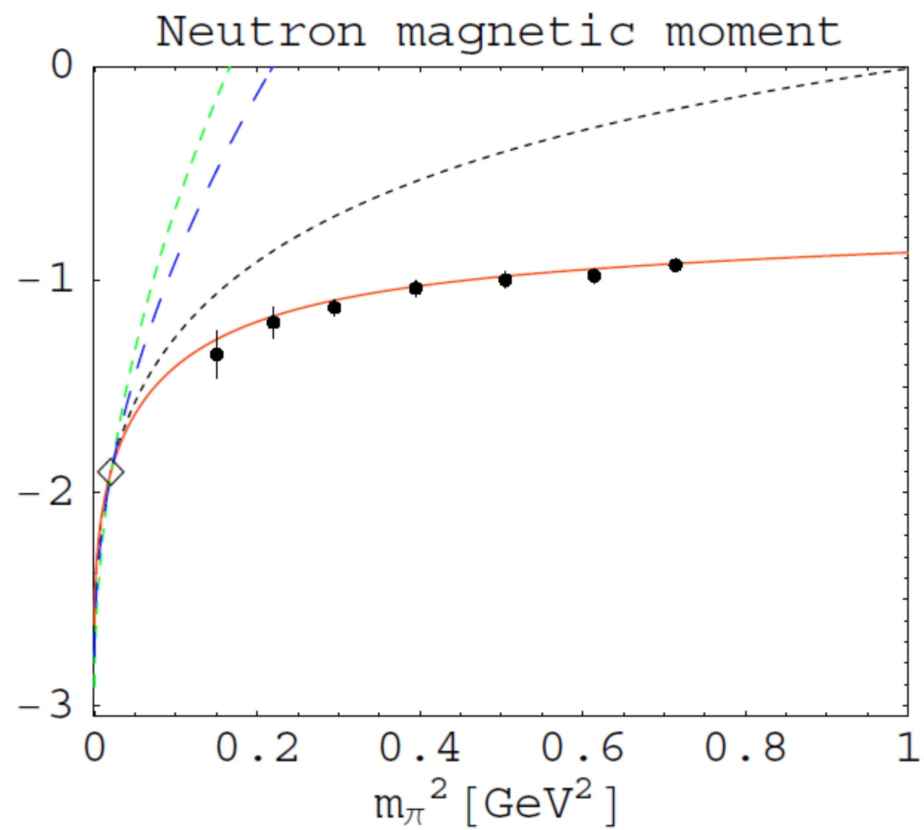



S. Scherer, Prog.Part.Nucl.Phys.64:1-60,2010

# Proton and neutron magnetic moments: chiral extrapolation




V. Pascalutsa et al., Phys.Lett.B600:239-247,2004.

# Octet baryon magnetic moments at NLO BChPT

$$\chi^2 = \sum(\mu_{th} - \mu_{exp})^2$$

| | | $p$ | $n$ | $\Lambda$ | $\Sigma^-$ | $\Sigma^0$ | $\Sigma^+$ | $\Xi^-$ | $\Xi^0$ | $\Lambda\Sigma^0$ | $\chi^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| LO | C-G | 2.56 | -1.60 | -0.80 | -0.97 | 0.80 | 2.56 | -0.97 | -1.60 | 1.38 | **0.46** |
| NLO | HB | 3.01 | -2.62 | -0.42 | -1.35 | 0.42 | 2.18 | -0.52 | -0.70 | 1.68 | **1.01** |
| | IR | 2.08 | -2.74 | -0.64 | -1.13 | 0.64 | 2.41 | -1.17 | -1.45 | 1.89 | **1.83** |
| | EOMS | 2.58 | -2.10 | -0.66 | -1.10 | 0.66 | 2.43 | -0.95 | -1.27 | 1.58 | **0.18** |
| | Exp. | 2.79 | -1.91 | -0.61 | -1.16 | — | 2.46 | -0.65 | -1.25 | 1.61 | — |

- Contribution of the chiral series [LO(1+NLO/LO)]:

$$\mu_p = 3.47(1-0.257), \quad \mu_n = -2.55(1-0.175), \quad \mu_\Lambda = -1.27(1-0.482),$$

$$\mu_{\Sigma^-} = -0.93(1+0.187), \quad \mu_{\Sigma^+} = 3.47(1-0.300), \quad \mu_{\Sigma^0} = 1.27(1-0.482),$$

$$\mu_{\Xi^-} = -0.93(1+0.025), \quad \mu_{\Xi^0} = -2.55(1-0.501), \quad \mu_{\Lambda\Sigma^0} = 2.21(1-0.284).$$

LSG, J. Martin Camalich , L. Alvarez-Ruso, M.J. Vicente Vacas, Phys.Rev.Lett. 101:222002,2008

# Towards a relativistic nuclear force

# Our strategy

- **We construct the kernel potentials from the covariant chiral Lagrangians**

$$
\begin{aligned}
\mathcal{L}_{NN}^{(0)} &= -\frac{1}{2}\left[C_S(\bar\Psi\Psi)(\bar\Psi\Psi) + C_A(\bar\Psi\gamma_5\Psi)(\bar\Psi\gamma_5\Psi)\right.\\
&+ C_V(\bar\Psi\gamma_\mu\Psi)(\bar\Psi\gamma^\mu\Psi) + C_{AV}(\bar\Psi\gamma_\mu\gamma_5\Psi)(\bar\Psi\gamma^\mu\gamma_5\Psi)\\
&+ \left. C_T(\bar\Psi\sigma_{\mu\nu}\Psi)(\bar\Psi\sigma^{\mu\nu}\Psi)\right],
\end{aligned}
$$

5 LECs

$$
\mathcal{L}_{\pi\pi}^{(2)} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger + (U+U^\dagger)m_\pi^2\right],
$$

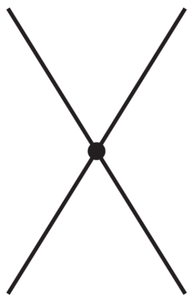
$$
\mathcal{L}_{\pi N}^{(1)} = \bar\Psi\left[i\not{D} - M_N + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right]\Psi,
$$

- **We retain the full form of Dirac spinors**

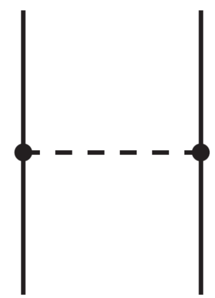
$$
u(\vec p, s) = N_p \begin{pmatrix} 1 \\ \frac{\vec\sigma\cdot\vec p}{\epsilon_p} \end{pmatrix}\chi_s, \quad N_p = \sqrt{\frac{\epsilon_p}{2M_N}}, \qquad E_p = \sqrt{M_N^2 + \vec p^{\,2}},
$$

# NN force at leading order

- **Feynman diagrams at LO**



Contact Potential (CTP)



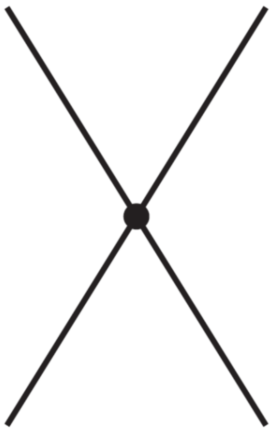
One-Pion Exchange Potential (OPEP)

- **"Covariant power counting"**

$$n_\chi = 4L - 2N_\pi - N_n + \sum_k kV_k,$$

Expansion parameters:
pseudscalar meson masses or small three-momenta of nucleons

# NN force at leading order



## Explicitly covariant form

$$\boxed{u_i(\vec{p},s) = \sqrt{\frac{E_N+M_N}{2M_N}}\begin{pmatrix} 1 \\ \frac{\vec{\sigma}_1\cdot\vec{p}}{\epsilon_p}\end{pmatrix}\chi_{s,i}}$$

$$\begin{aligned}
V_{\rm CTP} \;=\; & C_S(\bar{u}_4 u_2)(\bar{u}_3 u_1) + C_A(\bar{u}_4\gamma_5 u_2)(\bar{u}_3\gamma_5 u_1) \\
& + C_V(\bar{u}_4\gamma_\mu u_2)(\bar{u}_3\gamma^\mu u_1) + C_{AV}(\bar{u}_4\gamma_\mu\gamma_5 u_2)(\bar{u}_3\gamma^\mu\gamma_5 u_1) \\
& + C_T(\bar{u}_4\sigma_{\mu\nu} u_2)(\bar{u}_3\sigma_{\mu\nu} u_1).
\end{aligned}$$

## Expressed in terms of pauli matrices

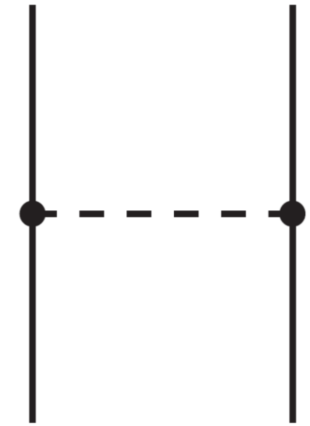
$$\begin{aligned}
V_{\rm CTP} \;=\; & \sum_{i=S,A,V,AV,T} C_i\Big[V_C^i(E_N) + V_\sigma^i(E_N)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + V_{SO}^i(E_N)\frac{\boldsymbol{i}}{\boldsymbol{2}}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\boldsymbol{k}\times\boldsymbol{q}) \\
& + V_{\sigma q}^i(E_N)\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\boldsymbol{\sigma}_2\cdot\boldsymbol{q} + V_{\sigma k}^i(E_N)\boldsymbol{\sigma}_1\cdot\boldsymbol{k}\boldsymbol{\sigma}_2\cdot\boldsymbol{k} \\
& + V_{\sigma L}^i(E_N)\boldsymbol{\sigma}_1\cdot(\boldsymbol{q}\times\boldsymbol{k})\boldsymbol{\sigma}_2\cdot(\boldsymbol{q}\times\boldsymbol{k})\Big].
\end{aligned}$$

all allowed spin operators

## Non-relativistic (static) limit

$$V_{\rm CTP}^{\rm NonRel.} = -(C_S+C_V) + (C_{AV}-2C_T)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + \mathcal{O}\Big(\frac{1}{M_N}\Big).$$

# NN force at leading order



Explicitly covariant form $\boxed{u_i(\vec{p},s) = \sqrt{\frac{E_N+M_N}{2M_N}}\begin{pmatrix} 1 \\ \frac{\vec{\sigma}_1\cdot\vec{p}}{\epsilon_p}\end{pmatrix}\chi_{s,i}}$

$$V_{\mathrm{OPEP}} = \xi_{N_1N_2\to N_3N_4}\frac{g_A^2}{4f_\pi^2}\frac{(\bar{u}_4\gamma^\mu\gamma_5 q_\mu u_2)(\bar{u}_3\gamma^\nu\gamma_5 q_\nu u_1)}{\boldsymbol{q}^2+m_\pi^2}.$$

Expressed in terms of pauli matrices and NR wfs

$$\begin{aligned} V_{\mathrm{OPEP}} \;=\; & \frac{g_A^2}{4f_\pi^2}\frac{1}{\boldsymbol{q}^2+m_\pi^2+i\epsilon}\left[V_{\sigma q}(E_N)\boldsymbol{\sigma_1}\cdot\boldsymbol{q}\boldsymbol{\sigma_2}\cdot\boldsymbol{q}\right. \\ & + V_C(E_N)+U_\sigma\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+V_{SO}(E_N)\frac{i}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\boldsymbol{k}\times\boldsymbol{q}) \\ & \left.+V_{\sigma k}(E_N)\boldsymbol{\sigma}_1\cdot\boldsymbol{k}\boldsymbol{\sigma}_2\cdot\boldsymbol{k}+V_{\sigma L}(E_N)\boldsymbol{\sigma}_1\cdot(\boldsymbol{q}\times\boldsymbol{k})\boldsymbol{\sigma}_2\cdot(\boldsymbol{q}\times\boldsymbol{k})\right] \end{aligned}$$

Non-relativistic (static) limit

$$V_{\mathrm{OPEP}}^{\mathrm{NonRel.}} = -\frac{g_A^2}{4f_\pi^2}\boxed{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}{\boldsymbol{q}^2+m_\pi^2+i\epsilon}+\mathcal{O}(\frac{1}{M_N}).$$

# A hint at a more efficient formulation

$$V_{1S0} = 4\pi\left[C_{1S0} + (C_{1S0} + \hat{C}_{1S0})\underline{\left(\frac{\vec{p}^2 + \vec{p}'^2}{4M_N^2} + \cdots\right)}\right]$$

$$- \frac{3\pi g_A^2}{f_\pi^2}\int_{-1}^{1}\frac{dz}{\vec{q}^2 + m_\pi^2}\left[\vec{q}^2 - \underline{\left(\frac{(\vec{p}^2 - \vec{p}'^2)^2}{4M_N^2} + \cdots\right)}\right] \quad (1.8)$$

$$C_{1S0} = (C_S + C_V + 3C_{AV} - 6C_T),$$
$$\hat{C}_{1S0} = (3C_V + C_A + C_{AV} + 6C_T),$$

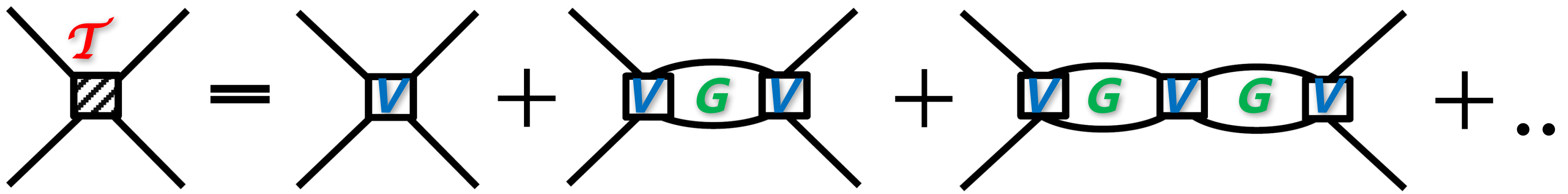
**A large contribution of the correction terms is essential to describe the 1S0 phase shift**

J. Soto and J. Tarrus, Phys. Rev. C78, 024003 (2008).

B. Long, Phys. Rev. C88, 014002 (2013).

# The nuclear force is non-perturbative

**Non-perturbative summation of the tree-level potential**




**3D reduction of the Bethe-Salpeter equation (Kadyshevsky)**

$$T(p',p) = V(p',p) + \int_0^{+\infty} \frac{k^2 dk}{(2\pi)^3} V(p',k) \frac{2\pi M_N^2}{(k^2+M_N^2)(\sqrt{p^2+M_N^2}-\sqrt{k^2+M_N^2}+i\epsilon)} T(k,p).$$

With the implicit mass "on-shell" approximation of the potential.

$$E_p = \sqrt{M_N^2 + \vec{p}^{\,2}},$$

# NN force at leading order

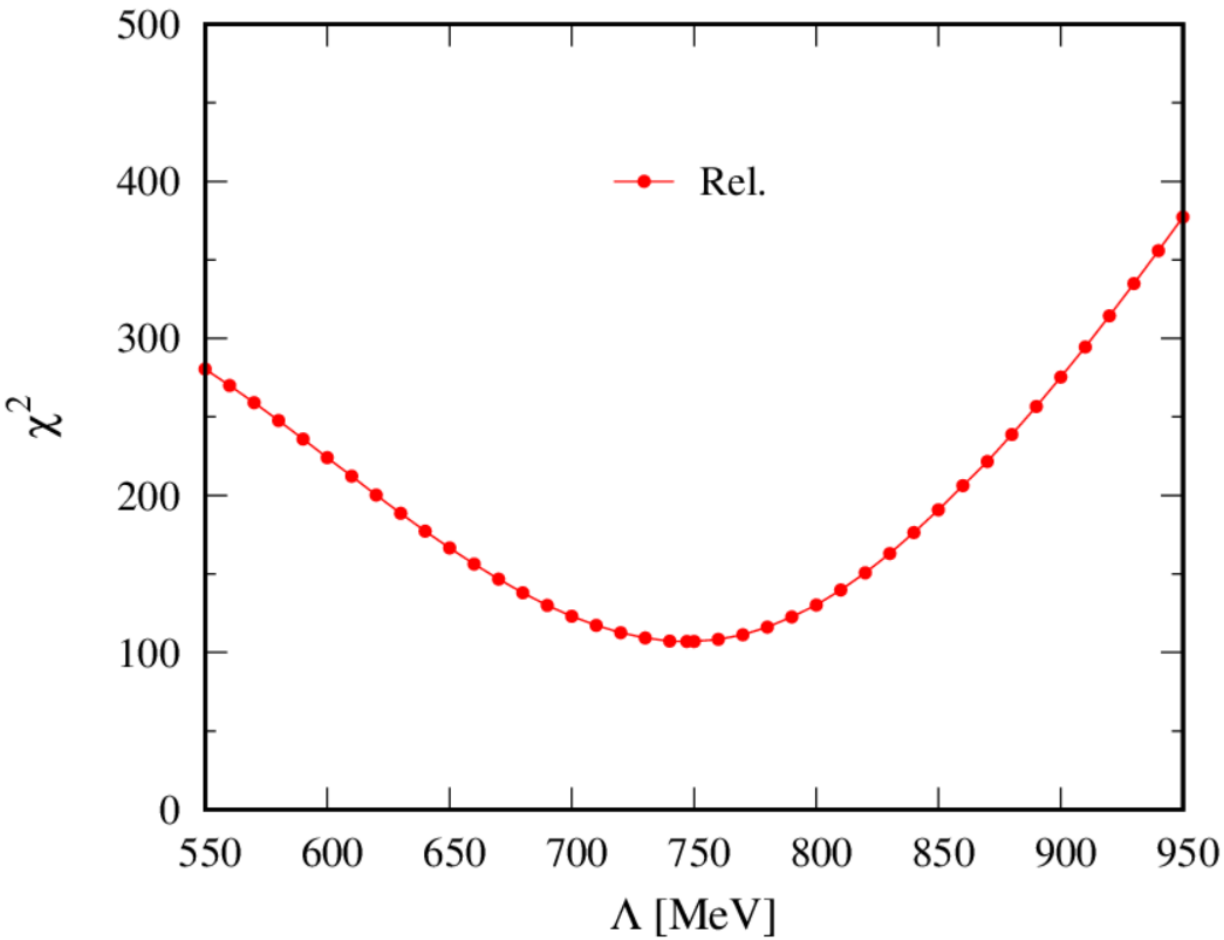
- **5 LECs to fit the np phase shifts of Nijmegen 93**
  - **7** partial waves: ***J=0, 1*** $^1S_0,\ ^3P_0,\ ^1P_1,\ ^3P_1,\ ^3D_1,\ ^3S_1,\ \epsilon_1$
  - **42** data points: 6 data points for each partial wave $(E_{\rm lab} = 1,\ 5,\ 10,\ 25,\ 50,\ 100\ {\rm MeV})$

$$\tilde{\chi}^2 = \sum_i \left(\delta_i^{\rm Theory} - \delta_i^{\rm Nij93}\right)^2 .$$

- **Cutoff renormalization in solving the scattering eq.**

$$V(p',p) \to V(p',p)\boldsymbol{f(p',p)}. \quad f(p',p) = \exp[-(p'/\Lambda)^{2n} - (p/\Lambda)^{2n}].$$

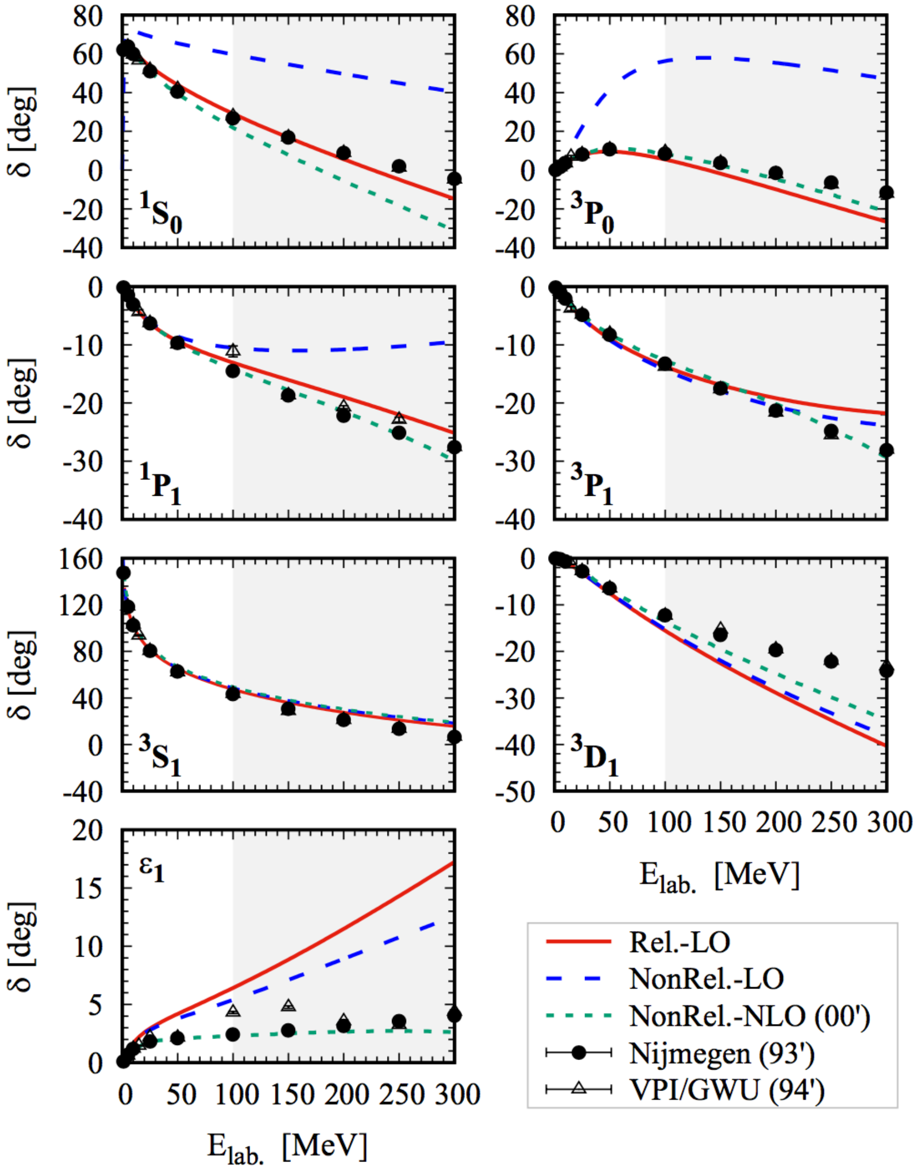
# Best fit




| LECs | Values [$10^4$ $GeV^{-2}$] |
|---|---|
| $C_S$ | 0.1339 |
| $C_A$ | -0.05477 |
| $C_V$ | -0.2673 |
| $C_{AV}$ | -0.2454 |
| $C_T$ | -0.06310 |

**$\Lambda$=747 MeV, the minimum of fit-$\chi^2$=106.90, $\chi^2$/d.o.f. = 2.89**

# A closer look at the partial waves




- **Improved description** of $^1S_0$ and $^3P_0$ phase shifts
- **Quantitatively similar** with the nonrelativistic case for J=1 partial waves

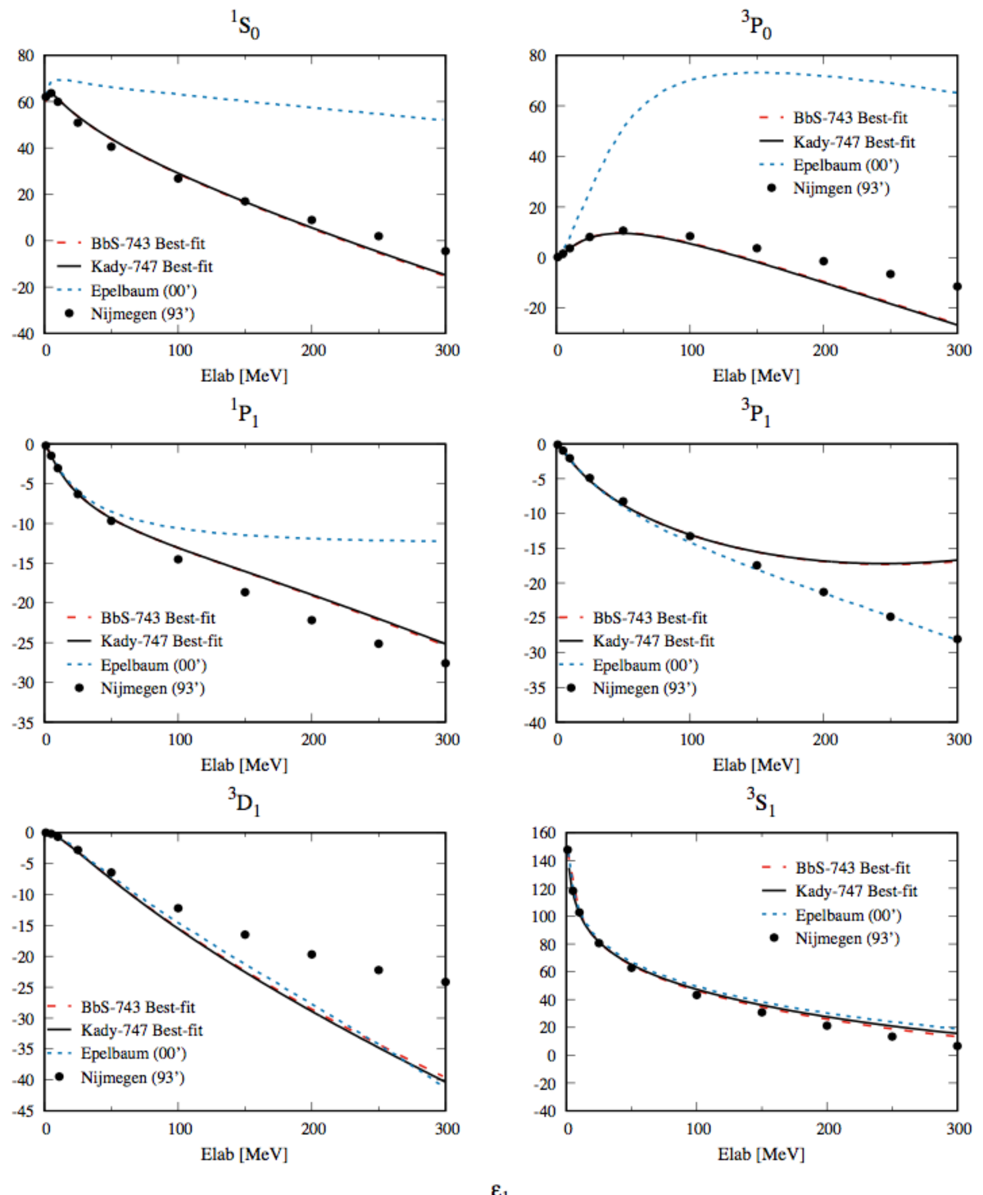
# Relativistic vs. non-relativistic
# Very promising

| | **Relativistic Chiral NF** | **Non-relativistic Chiral NF** | |
|---|---|---|---|
| Chiral order | LO | LO | NLO* |
| No. of LECs | 5 | 2 | 9 |
| **$\chi^2$/d.o.f.** | **2.9** | **147.9** | **2.5** |

**A more efficient description is achieved**

# BbS vs. Kadeshevsky scattering equation

BbS(Blankenbecler-Sugar)



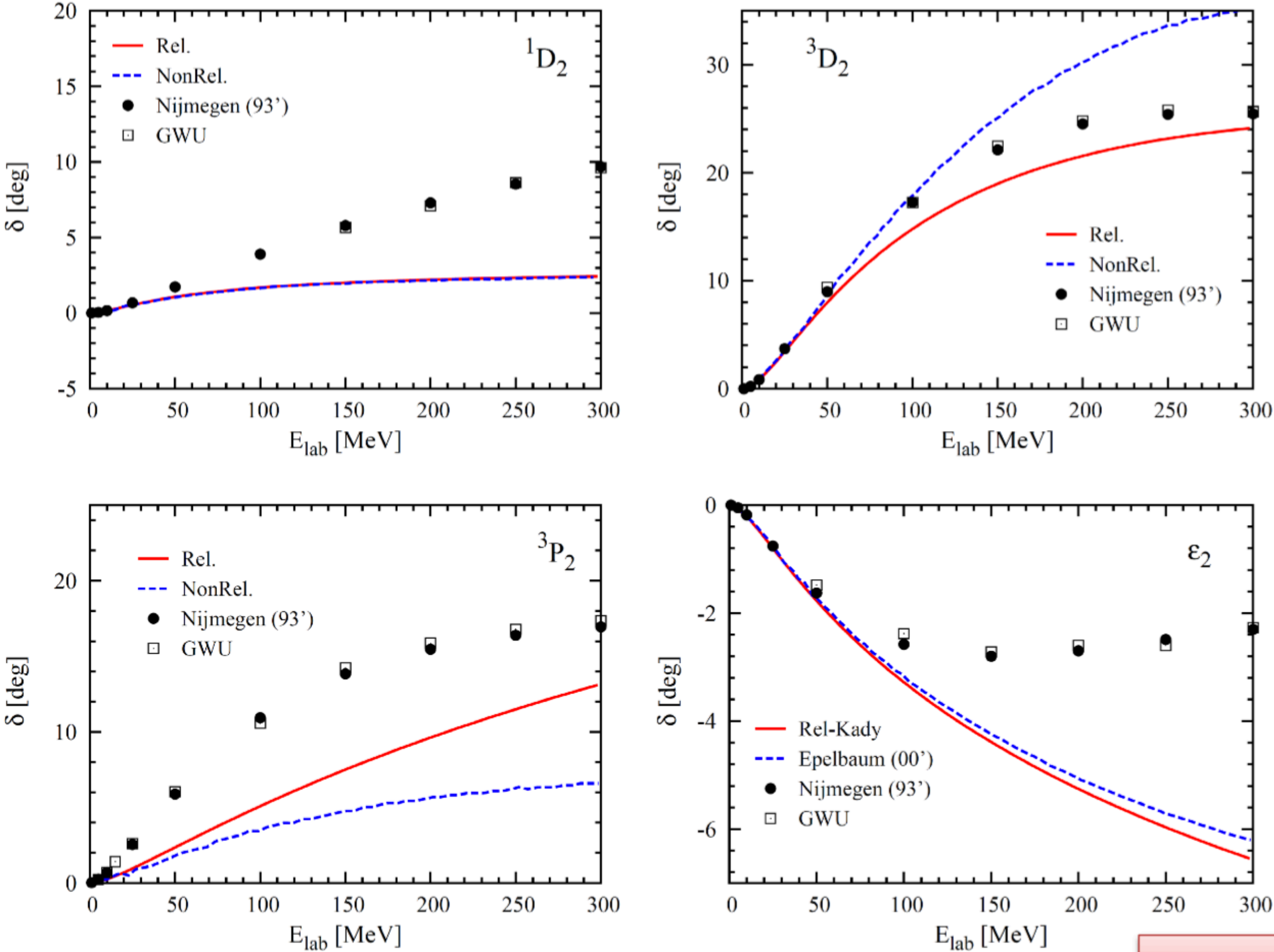

**(almost) Independent from the scattering equation**

# Summary and Outlook

- ✤ Nuclear forces based on Chiral EFT have made remarkable progress in the past decade.
- ✤ Covariant descriptions of the one-baryon and nuclear systems have been quite successful as well.
- ✤ Time is mature to develop a covariant formulation of baryon-baryon forces in chiral EFT.
- ✤ Initial (preliminary) results are very promising.
- ✤ More is coming. Remain tuned.

**Thank you very much for your attention!**

# Description of J=2 PWs phase shift




$\Lambda_F$=600 MeV

OPE prediction