

# Kyoto City from the top of 大文字山







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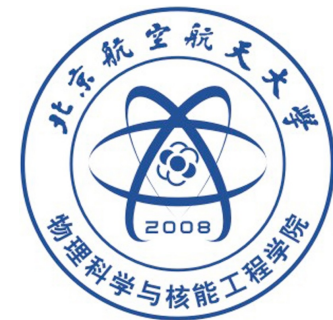
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**Towards a relativistic formulation of nucleon-nucleon  
interactions  
in chiral perturbation theory**

arXiv:1611.08475

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**Peter Ring, Technical University of Munich**

**YITP/2016.11.28**



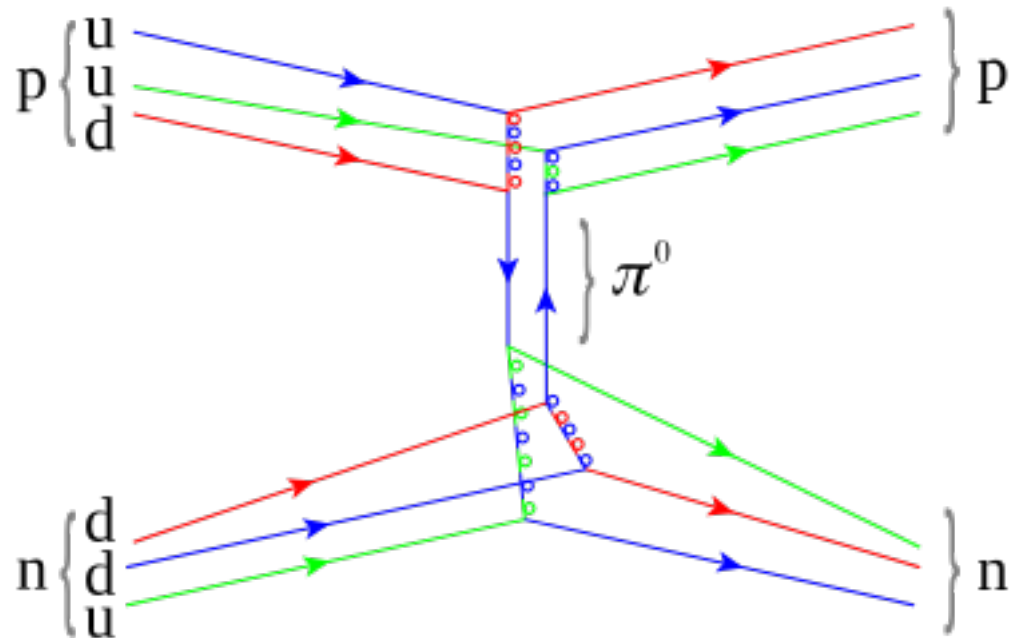
# The Nobel Prize in Physics 1949

Hideki Yukawa



"for his prediction of the existence of mesons on the basis of theoretical work on nuclear forces".

Yukawa Institute for Theoretical Physics (former Research Institute for Fundamental Physics) goes **back to 1949** when Hideki Yukawa of Kyoto University



**“On the interaction of elementary particles,” PTP17,48**

The paper was written in 1934 while he was working at Osaka U.



# *On the Interaction of Elementary Particles. I.*

By Hideki YUKAWA.

(Read Nov. 17, 1934)

## § 1. Introduction

tion.

Such quanta, if they ever exist and approach the matter close enough to be absorbed, will deliver their charge and energy to the latter. If, then, the quanta with negative charge come out in excess, the matter will be charged to a negative potential.

These arguments, of course, of merely speculative character, agree with the view that the high speed positive particles in the cosmic rays are generated by the electrostatic field of the earth, which is charged to a negative potential.<sup>(9)</sup>

The massive quanta may also have some bearing on the shower produced by cosmic rays.

In conclusion the writer wishes to express his cordial thanks to Dr. Y. Nishina and Prof. S. Kikuchi for the encouragement throughout the course of the work.

Department of Physics,  
Osaka Imperial University.

(Received Nov. 30, 1934)



# Outline

## ❖ Introduction

- **Why nuclear force; Current status (of chiral forces)**
- **Why relativistic? atomic/molecular; nuclear; one-baryon sector**

## ❖ Our strategy and some preliminary results

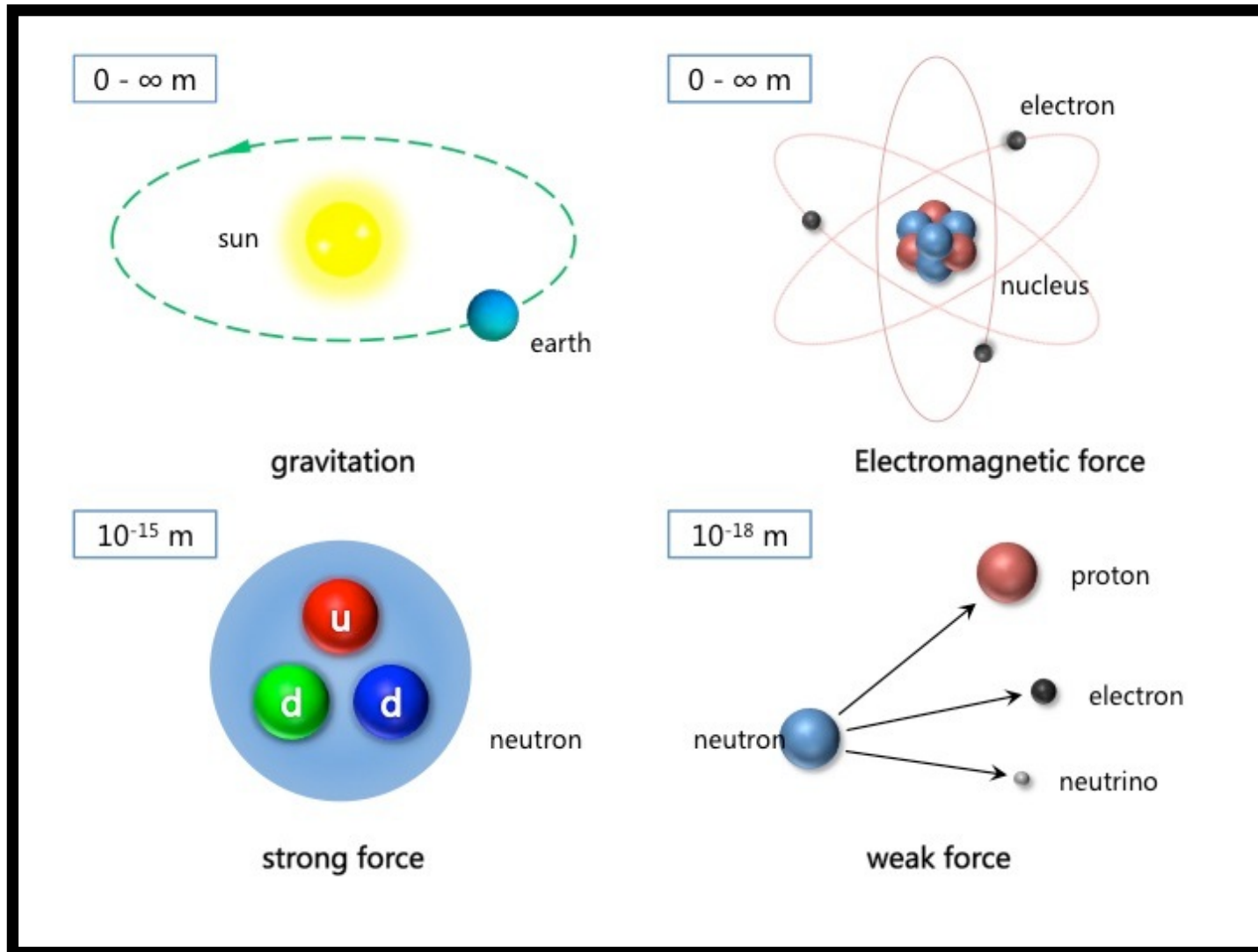
## ❖ Summary and outlook



# **Motivation: why nuclear force**



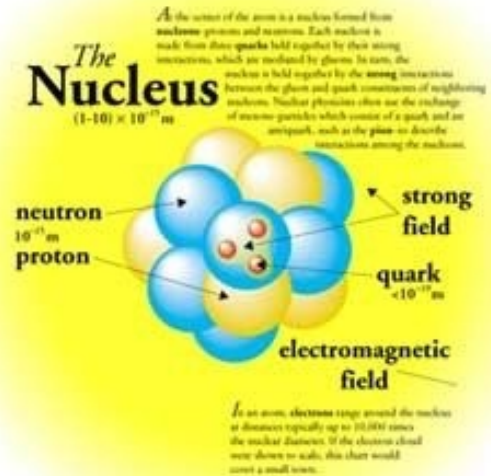
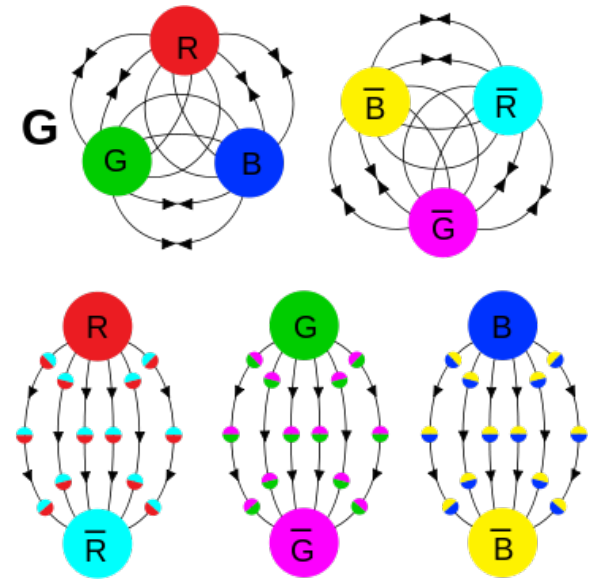
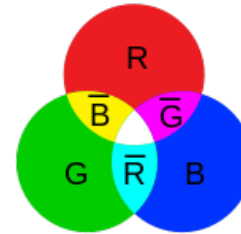
# Four (established) forces in nature





# Strong force

- **Strong force: bind quarks into hadrons**
- **Nuclear force—residual strong force: binds nucleons into nuclei**
- **Underlying theory—QCD**

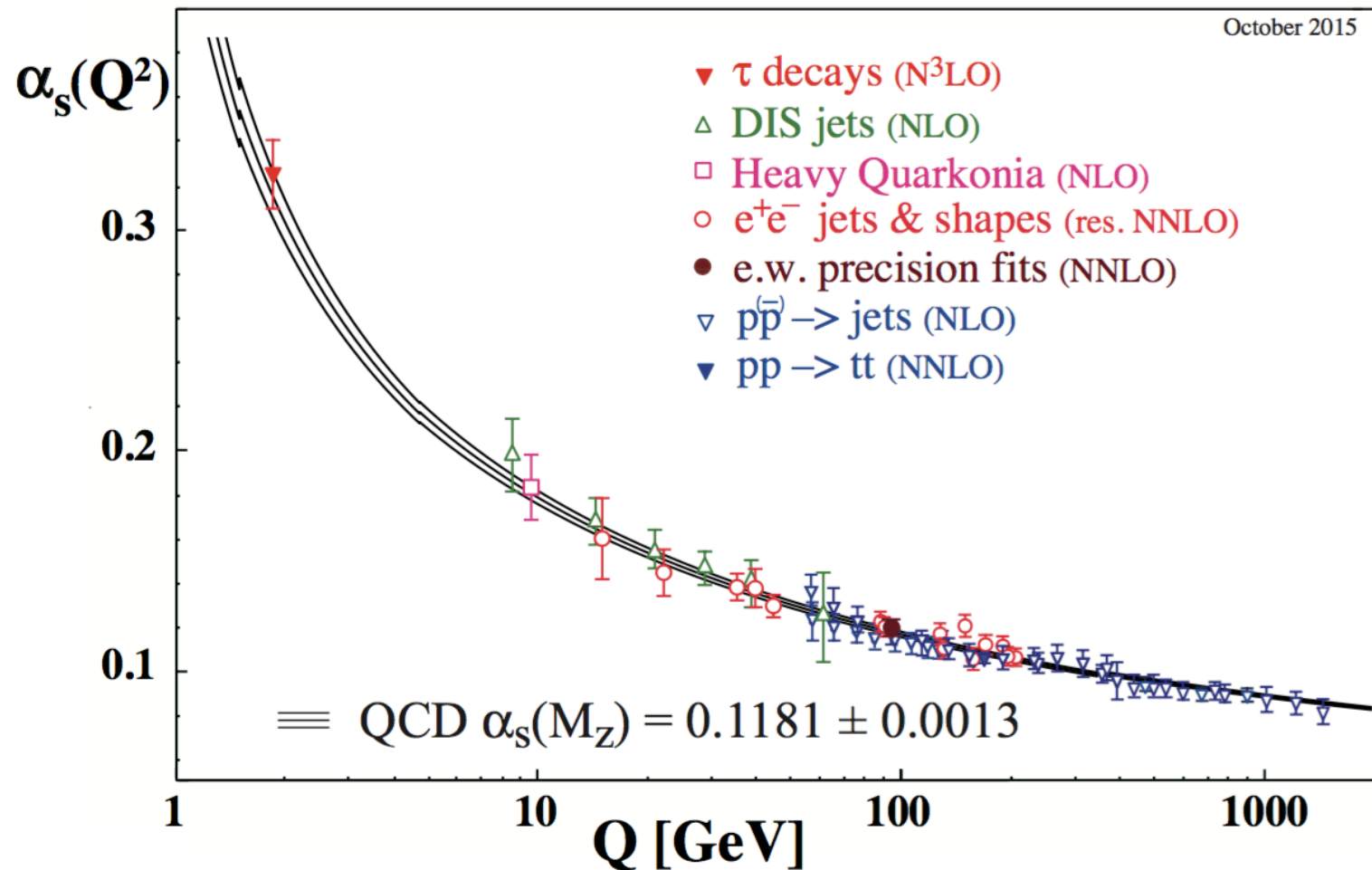


$$\begin{aligned}
 L_{QCD} = & -\frac{1}{4}(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu)(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a) + \sum_f \bar{q}_f^\alpha (i\gamma^\mu \partial_\mu - m_f) q_f^\alpha \\
 & + g_s G_a^\mu \sum_f \bar{q}_f^\alpha \gamma^\mu \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} q_f^\beta \\
 & - \frac{g_s}{2} f^{abc} (\partial^\mu G_a^\nu - \partial^\nu G_a^\mu) G_\mu^b G_\nu^c - \frac{g_s^2}{4} f^{abc} f_{ade} G_b^\mu G_c^\nu G_\mu^d G_\nu^e
 \end{aligned}$$

**2 quark masses and 1 universal coupling**

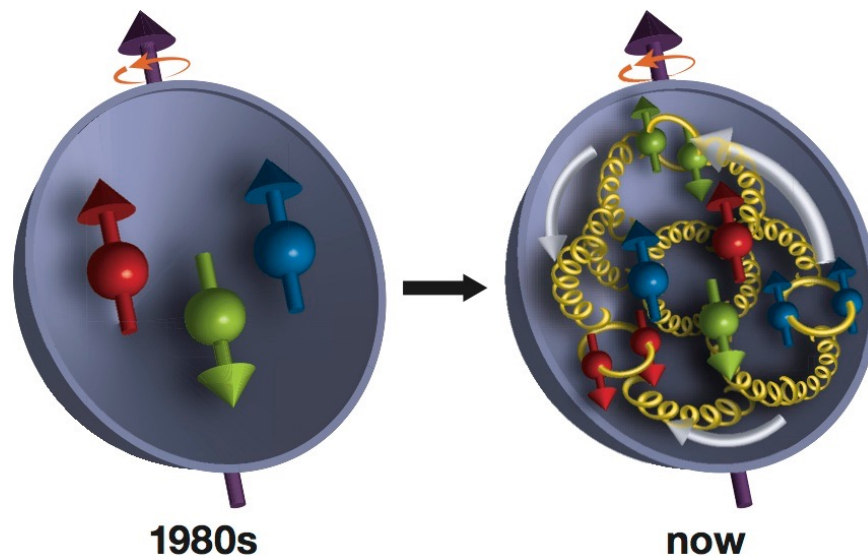


# QCD: Asymptotic freedom



# QCD: color confinement

- Free quarks do not exist (**color confinement**), experimentally only hadrons are observed
- Mismatch of degrees of freedom—**hadronization**



**Decomposition of  
the proton spin**



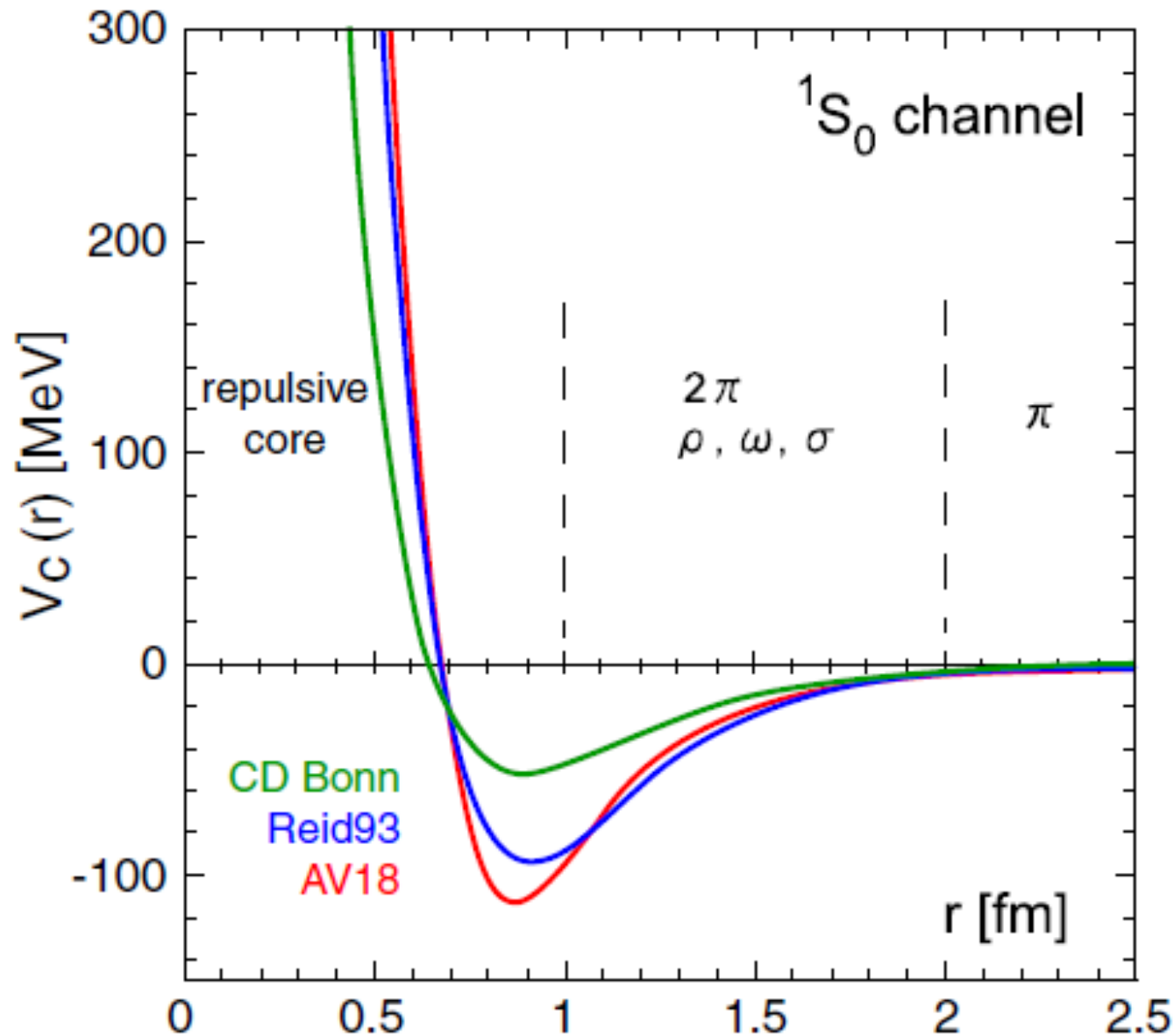
# Why construct nuclear forces?

- **Nuclear force: derivative force or residual force**
- **In this sense, similar to intermolecular force, but because of confinement and asymptotic freedom of QCD, much richer and harder**

Fan Wang, Guang-han Wu, Li-jian Teng, J.Terrance Goldman Phys.Rev.Lett. 69 (1992) 2901-2904

- **Constructing** a nuclear force is a long-standing and interesting subject in nuclear physics; **the basis** of all microscopic (ab initio) nuclear structure and reaction theories

# “High Precision” Nuclear Force



“On the interaction of elementary particles,” PTP17,48



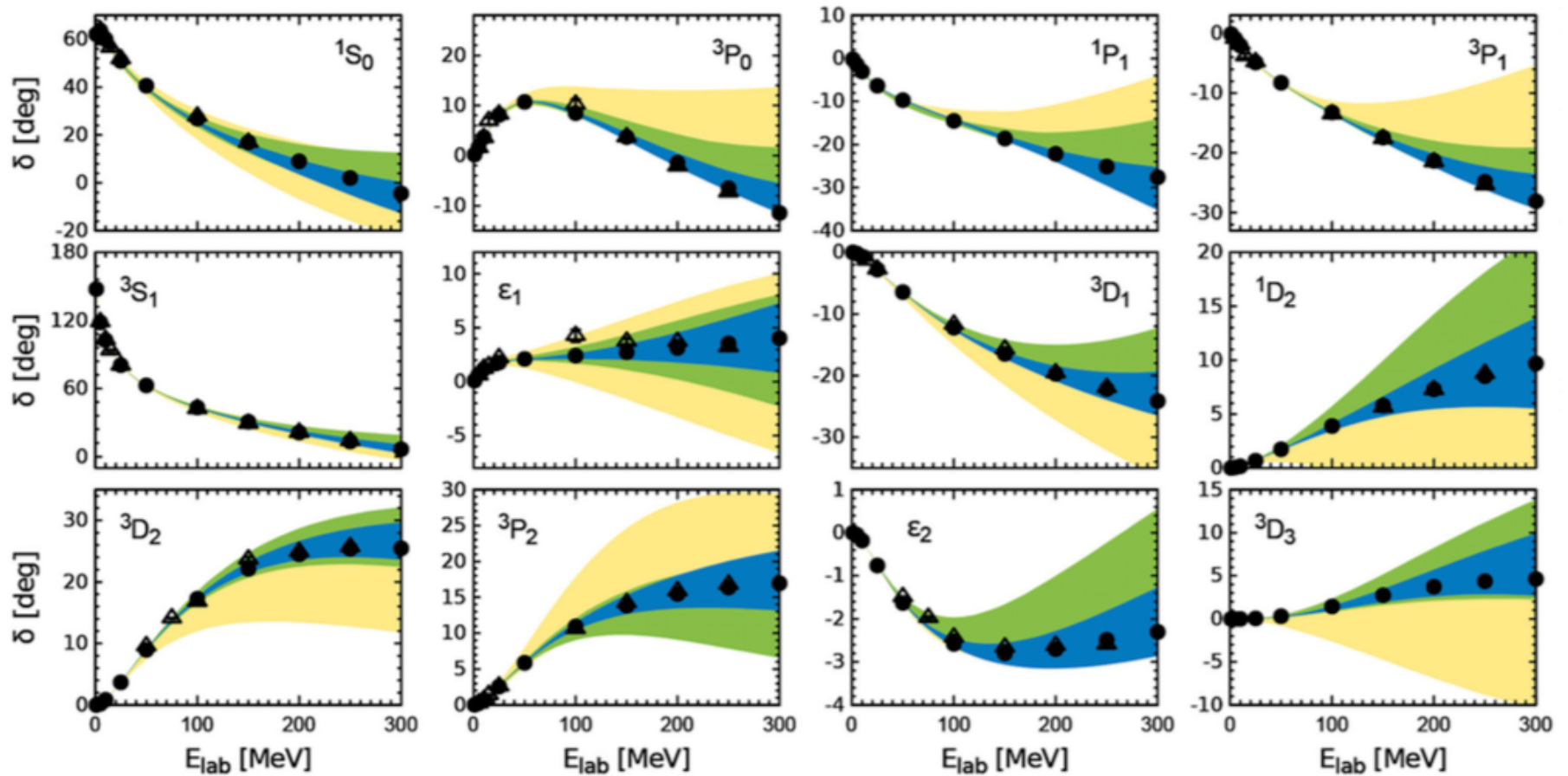
# Major milestones for NN potential development ChPT



- **1991/92: Weinberg**, NN potential from ChPT
- **1994/96: Bira v. Kolck** and co-workers, first ChPT based NN potential at N2LO using cutoff regularization (r-space)
- **1994-1997:**
  - **Robilotta** and co-workers, 2-pi at N2LO
  - **1997: Kaiser** et al., 2-pi at N2LO using HBChPT and DR
- **2000: Epelbaum** et al. (“Bochum-Juelich” group), NN potential in momentum space at N2LO (HBChPT, DR)
- **2003:**
  - **Robilotta** and co-workers 2-pi at N3LO in RBChPT
  - **Entem & Machleidt** (“Idaho” group), first NN potential at N3LO (HBChPT, DR)
- **2005: Epelbaum** et al. (“Bochum-Juelich” group), NN potential at N3LO (HBChPT, SFR)
- **2015: Epelbaum** et al., **Entem**, et al., NN potential at N4LO

High  
Precision  
Nuclear  
Force

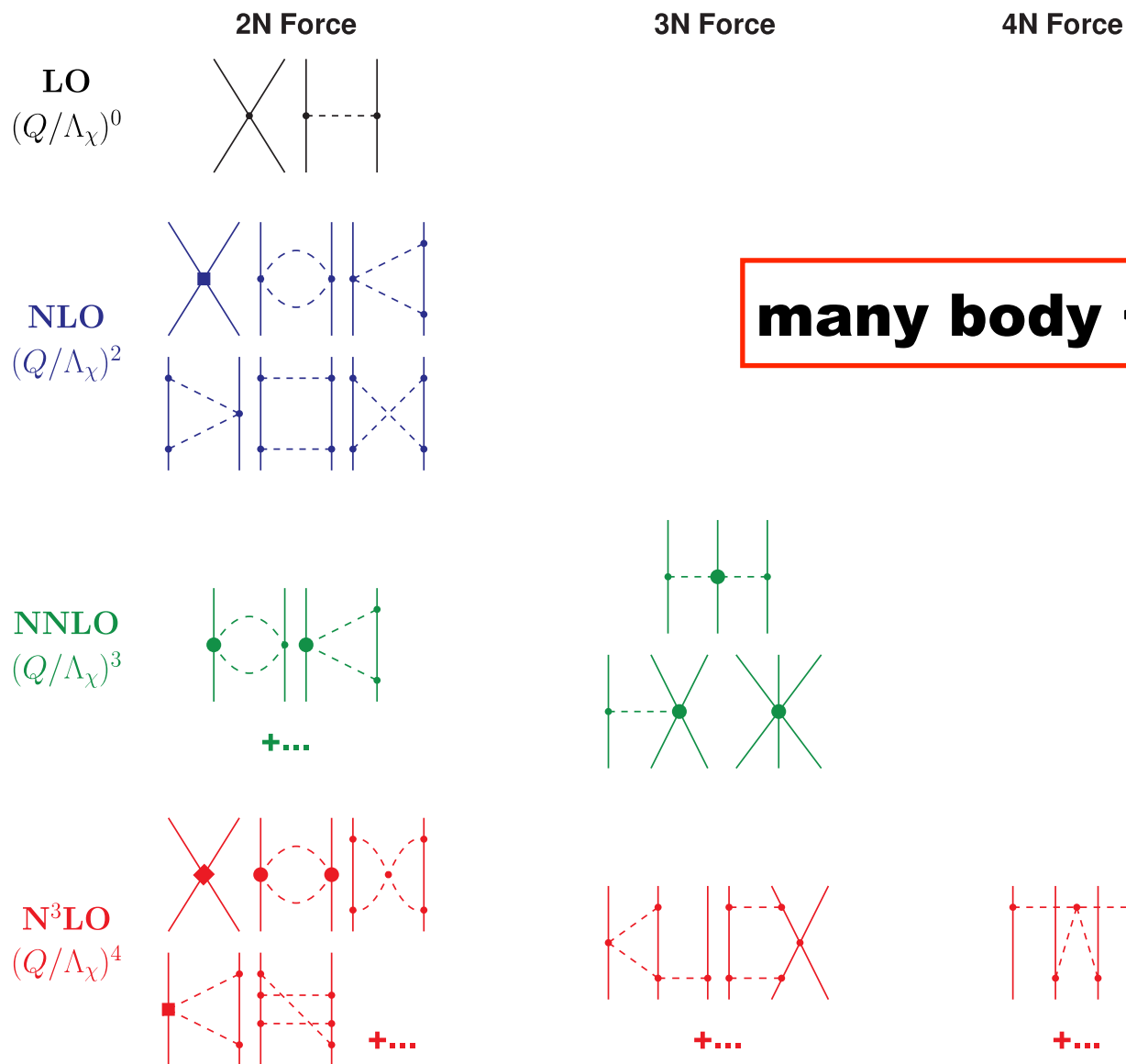
# Estimate of theoretical uncertainties



- E. Epelbaum, H. Krebs, and U.-G. Meissner, Eur. Phys. J. A (2015)51



# Hierarchy of Bare Nuclear Force in ChEFT



**many body < few body**

- E. Epelbaum, H.-W. Hammer, Ulf-G. Meissner, *Reviews of Modern Physics* 81(2009)1773
- R. Machleidt and D. R. Entem, *Physics Reports* 503(2011)1

# Nonrelativistic NF from heavy baryon (HB) ChEFT

## ● NN interaction

- up to NLO **U. van Kolck et al., PRL, PRC1992-94; N. Kaiser, NPA1997**
- up to NNLO **E. Epelbaum, et al.,NPA2000; U. van Kolck et al.,PRC1994**
- up to N<sup>3</sup>LO **R. Machleidt et al., PRC2003; E. Epelbaum et al., NPA2005**
- up to **N<sup>4</sup>LO** **E. Epelbaum et al., PRL2015, D.R. Entem, et al., PRC2015**
- dominant N<sup>5</sup>LO terms** **D.R. Entem, et al., PRC2015**

## ● 3N interaction

- up to NNLO **U. van Kolck, PRC1994**
- up to N<sup>3</sup>LO **S. Ishikwas, et al, PRC2007; V. Bernard et al, PRC2007;**
- up to N<sup>4</sup>LO **H. Krebs, et al., PRC2012-13**

## ● 4N interaction

- up to N<sup>3</sup>LO **E. Epelbaum, PLB 2006, EPJA 2007**



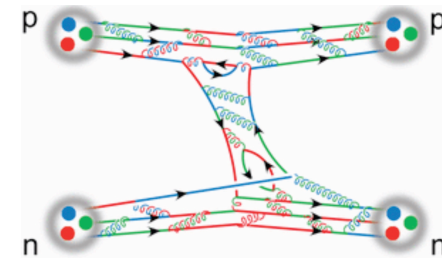
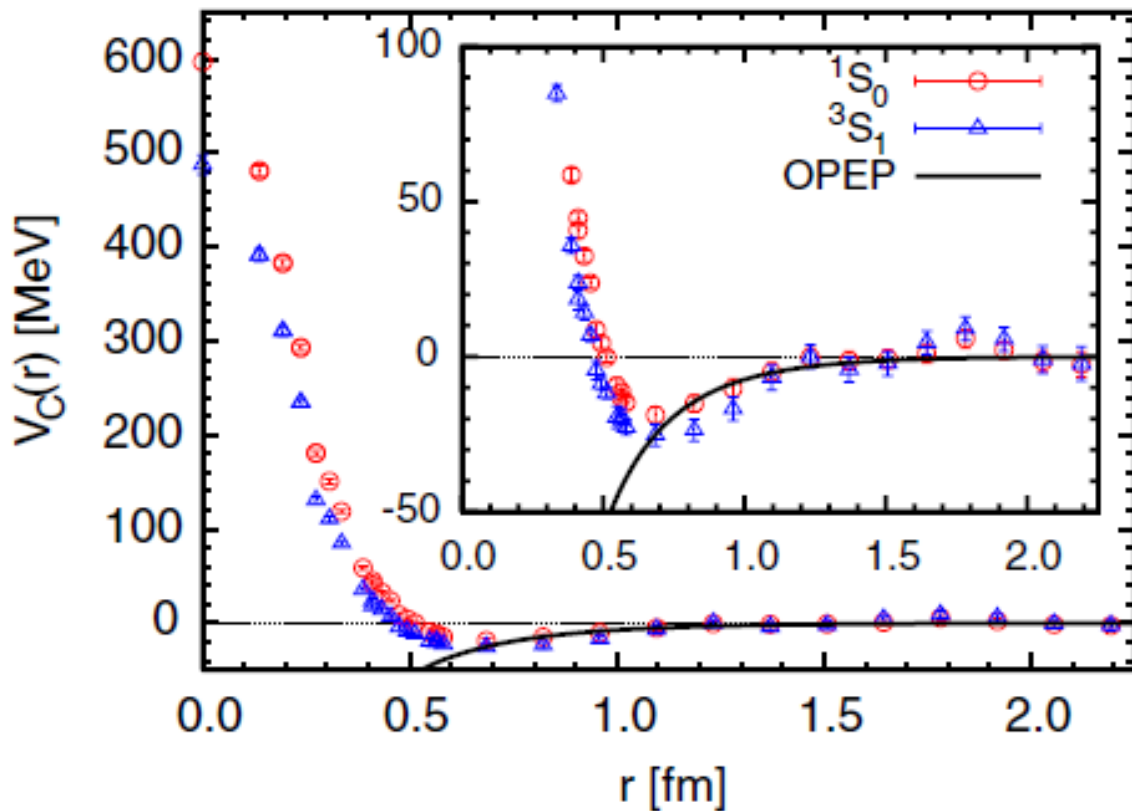
# Number of parameters in Modern Nuclear Forces

ChEFT [5]									
	PWA93 [1]	Reid93 [2]	AV18 [3]	CD- Bonn [4]	LO	NLO	NNLO	N3LO	N4LO
No. of LECs	35	50	40	38	2	9	9	24	24
$\chi^2$ / datum	1.07	1.03	1.09	1.02	480	63	21	0.7	0.3

**caution about definition of  $\chi^2$**

- [1] V.G.J. Stoks et al., PRC48, 792(1993)—Inspire cited **637** times
- [2] V.G.J. Stoks et al., PRC49, 2950(1994)—Inspire cited **1054** times
- [3] Robert B. Wiringa et al, PRC51, 38(1995)—Inspire cited **1975** times
- [4] R. Machleidt, PRC63,024001(2001)—Inspire cited **1050** times
- [5] PRL 115,122301(2015)—Inspire cited 58

# Nuclear Force from Quark-Gluon dofs



- **First qualitative nuclear force from first principles**
- **$m_\pi=461$  MeV**
- **Quenched**

N. Ishii et al., PRL99,022001(2007)

**Talks from Takumi Doi, Sinya Aoki, Kenji Sasaki**

# The ultimate aim: nuclear physics as a precision science

## The Nobel Prize in Chemistry 2013



© Harvard University  
Martin Karplus



Photo: © S. Fisch  
Michael Levitt



Photo: Wikimedia  
Commons  
Arieh Warshel

The Nobel Prize in Chemistry 2013 was awarded jointly to Martin Karplus, Michael Levitt and Arieh Warshel *"for the development of multiscale models for complex chemical systems"*.

for the development  
of multiscale  
models for complex  
chemical systems

**Nuclear force+advanced numerical methods**

**=**

**precision nuclear physics**



# Two recent examples

## Hoyle state of Carbon

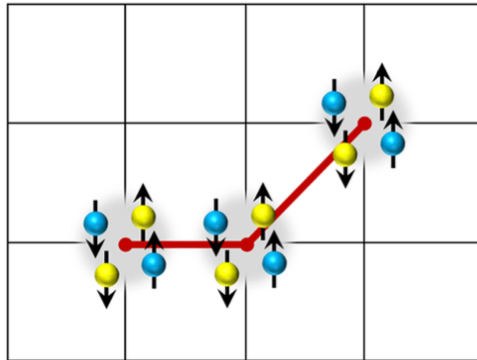
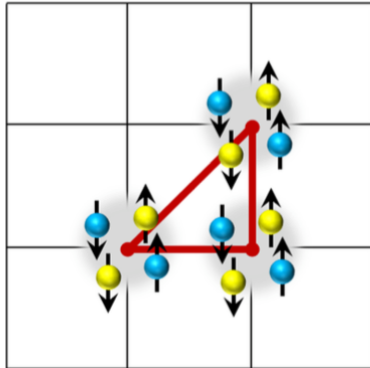
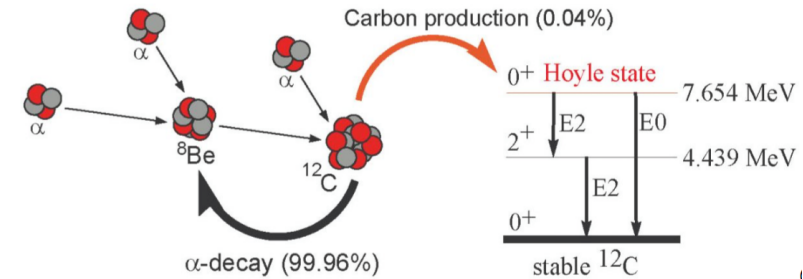


TABLE II. Lattice and experimental results for the energies of the low-lying even-parity states of  $^{12}\text{C}$ , in units of MeV.

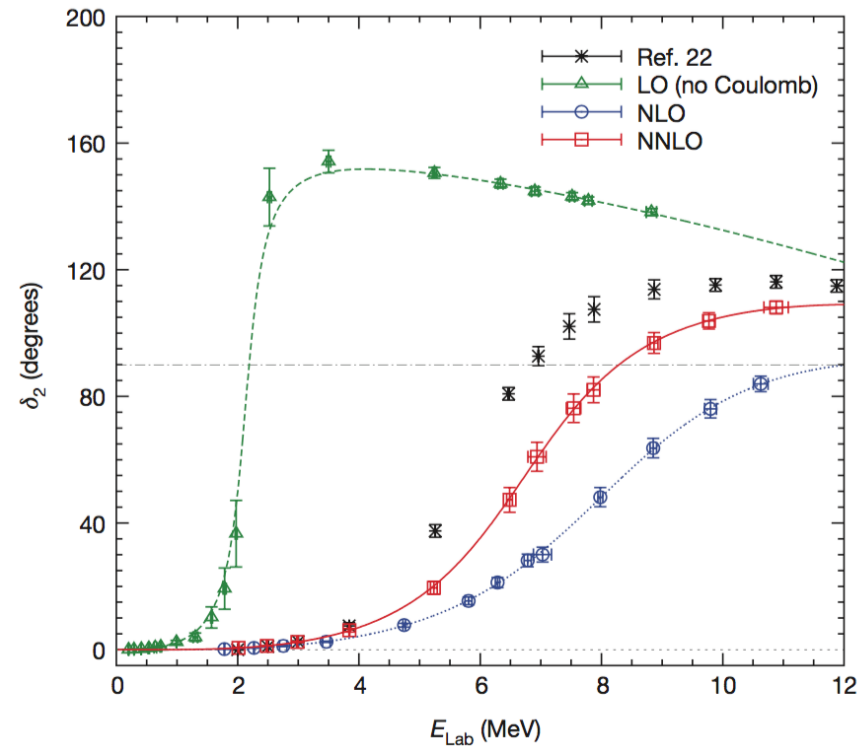
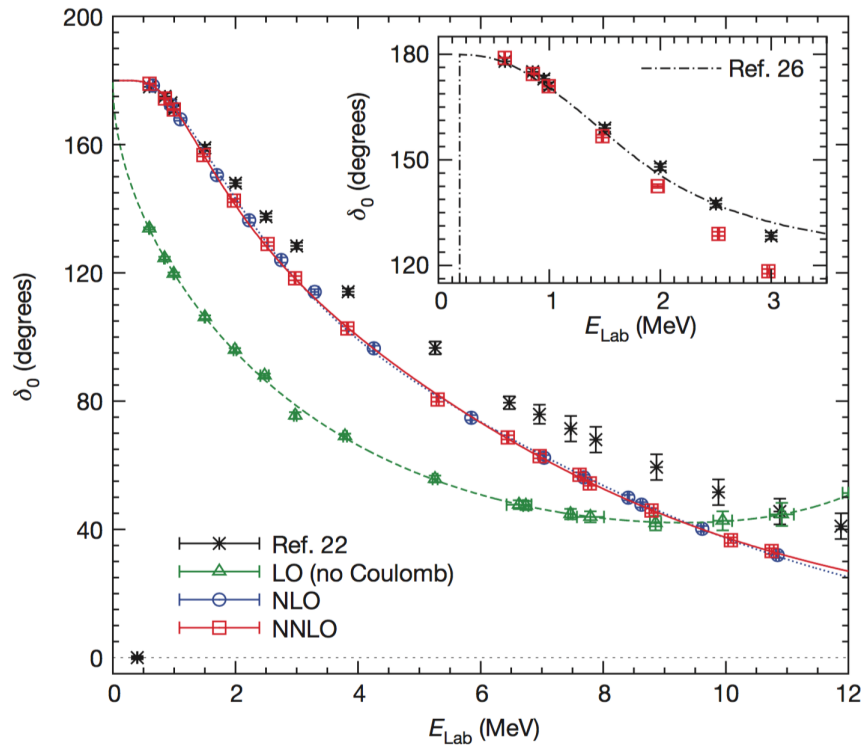
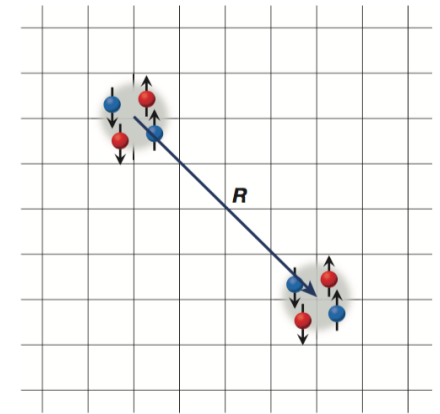
	$0_1^+$	$2_1^+(E^+)$	$0_2^+$	$2_2^+(E^+)$
LO	-96(2)	-94(2)	-89(2)	-88(2)
NLO	-77(3)	-74(3)	-72(3)	-70(3)
NNLO	-92(3)	-89(3)	-85(3)	-83(3)
Expt.	-92.16	-87.72	-84.51	-82.6(1) [8,10] -81.1(3) [9] -82.32(6) [11]



### Structure and Rotations of the Hoyle State

Evgeny Epelbaum,<sup>1</sup> Hermann Krebs,<sup>1</sup> Timo A. Lähde,<sup>2</sup> Dean Lee,<sup>4</sup> and Ulf-G. Meißner<sup>5,2,3</sup>

# Two recent examples alpha-alpha scattering



LETTER

doi:10.1038/nature16067

## *Ab initio* alpha-alpha scattering

Serdar Elhatisari<sup>1</sup>, Dean Lee<sup>2</sup>, Gautam Rupak<sup>3</sup>, Evgeny Epelbaum<sup>4</sup>, Hermann Krebs<sup>4</sup>, Timo A. Lähde<sup>5</sup>, Thomas Luu<sup>1,5</sup> & Ulf-G. Meißner<sup>1,5,6</sup>

Nature 16067

# Limitations of Current ChPT NN forces

- **Not “renormalization group invariant”**

- Sensitive to the UV cutoff, not (nonperturbatively) renormalizable
- Diverse opinion on this issue ( B.W. Long et al.)

- **Based on HBChPT**

- Slow convergence as in the one-baryon sector?

D. R. Entem, N. Kaiser, R. Machleidt, and Y. Nosyk, Phys. Rev. C92, 064001 (2015).

- Cannot be used directly in covariant calculations.

- **A relativistic nuclear force based on the EOMS BChPT more relativistic nuclear studies?**



# **Motivation: why relativistic**

# Importance of Relativity not so much recognized

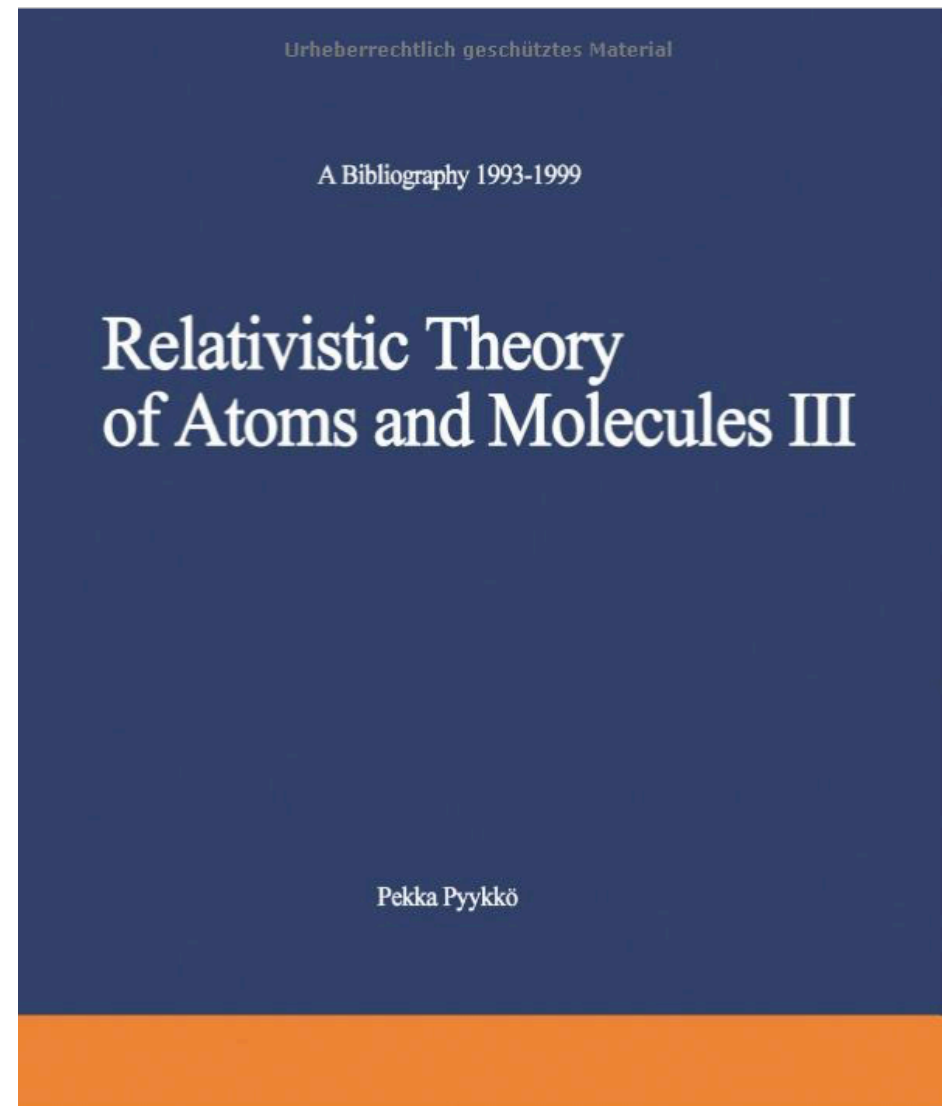
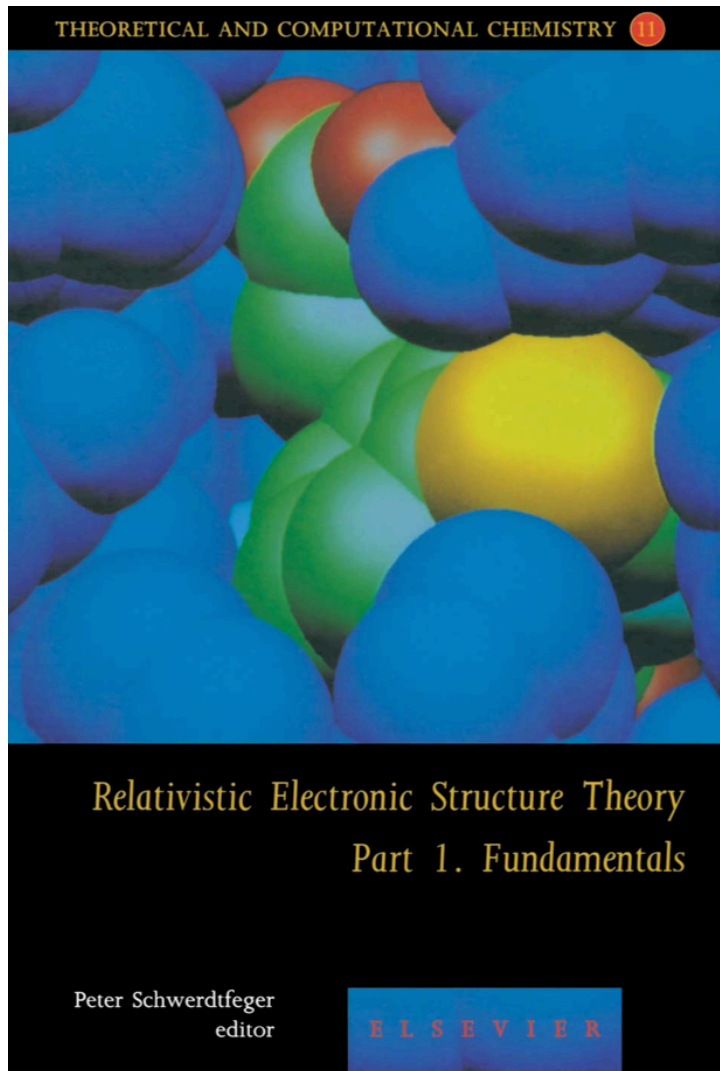
- **Two pillars of modern physics:**
  - ✓ **Quantum mechanics**
  - ✓ **Special (General) relativity, not**

S.L.Glashow, 1988, Interactions, Wamer Books, New York

*Modern elementary-particle physics is founded upon the two pillars of quantum mechanics and relativity. ....Thus it is that a satisfactory description of the atom can be obtained without Einstein's revolutionary theory.*

# Facts speak louder than words

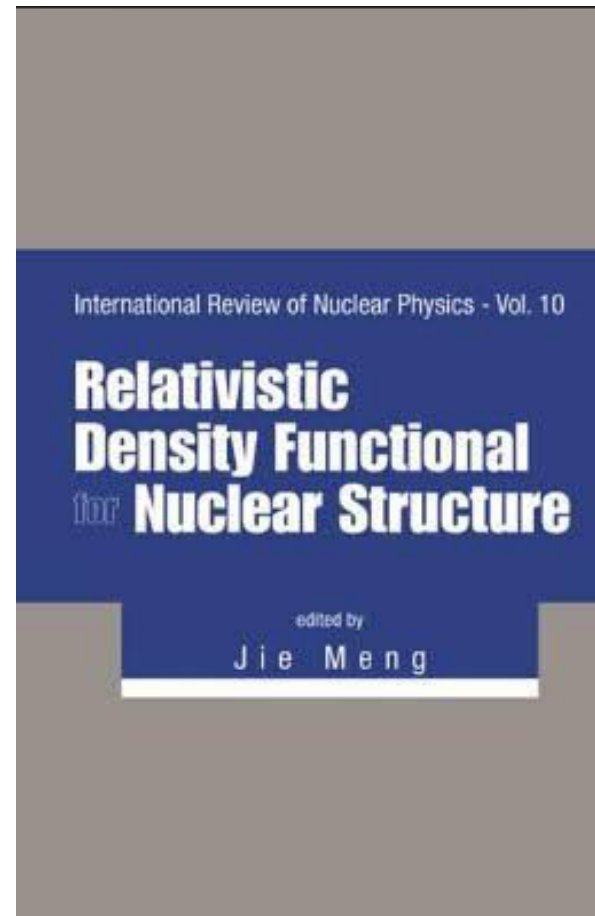
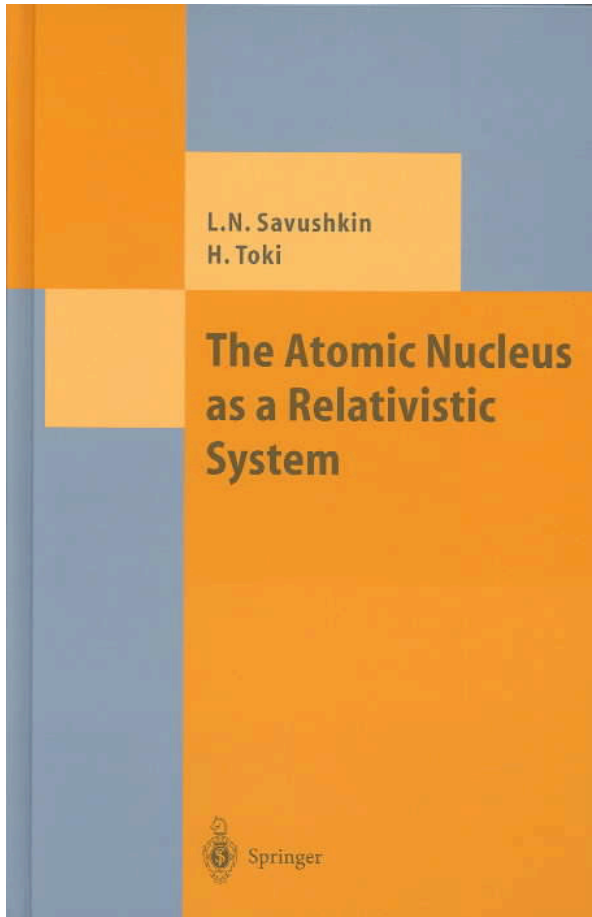
## Atomic/Molecular systems





# Facts speak louder than words

## Nuclear systems



# Facts speak louder than words

## One-Baryon-Sector

- **Heavy baryon (HB) ChPT**
  - non-relativistic
  - breaks analyticity of loop amplitudes
  - converges slowly (particularly in three-flavor sector)
  - strict PC and simple nonanalytical results
- **Infrared BChPT**
  - breaks analyticity of loop amplitudes
  - converges slowly (particularly in three-flavor sector)
  - analytical terms the same as HBChPT
- **Extended-on-mass-shell (EOMS) BChPT**
  - satisfies all symmetry and analyticity constraints
  - converges relatively faster--an appealing feature

# Some successful applications of covariant BChPT (in the three-flavor sector)

## ❖ **Magnetic moments**

PRL101:222002,2008; PLB676:63,2009; PRD80:034027,2009

## ❖ **Masses and sigma terms**

PRD82:074504,2010; PRD84:074024,2011; JHEP12:073,2012;  
PRD 87:074001,2013; PRD89:054034,2014 ; EPJC74:2754,2014 ;  
PRD91:051502,2015

## ❖ **Vector form factors (couplings)**

PRD79:094022,2009; PRD89:113007,2014

## ❖ **Axial form factors (couplings)**

PRD78:014011,2008; PRD90:054502,2014



# **Towards a relativistic nuclear force**

# Our strategy

- We construct the kernel potentials from the **covariant chiral Lagrangians**

$$\begin{aligned}\mathcal{L}_{NN}^{(0)} = & -\frac{1}{2} [C_S(\bar{\Psi}\Psi)(\bar{\Psi}\Psi) + C_A(\bar{\Psi}\gamma_5\Psi)(\bar{\Psi}\gamma_5\Psi) \\ & + C_V(\bar{\Psi}\gamma_\mu\Psi)(\bar{\Psi}\gamma^\mu\Psi) + C_{AV}(\bar{\Psi}\gamma_\mu\gamma_5\Psi)(\bar{\Psi}\gamma^\mu\gamma_5\Psi) \\ & + C_T(\bar{\Psi}\sigma_{\mu\nu}\Psi)(\bar{\Psi}\sigma^{\mu\nu}\Psi)],\end{aligned}\quad \text{5 LECs}$$

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f_\pi^2}{4} \text{Tr} [\partial_\mu U \partial^\mu U^\dagger + (U + U^\dagger) m_\pi^2],$$

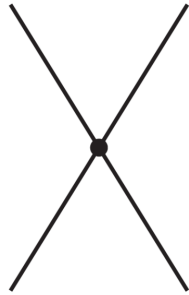
$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi} \left[ i\not{D} - M_N + \frac{g_A}{2} \gamma^\mu \gamma_5 u_\mu \right] \Psi,$$

- We retain the full form of Dirac spinors

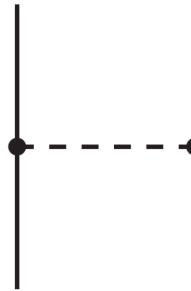
$$u(\vec{p}, s) = N_p \begin{pmatrix} 1 \\ \frac{\vec{\sigma} \cdot \vec{p}}{\epsilon_p} \end{pmatrix} \chi_s, \quad N_p = \sqrt{\frac{\epsilon_p}{2M_N}}, \quad E_p = \sqrt{M_N^2 + \vec{p}^2},$$

# NN force at leading order

- **Feynman diagrams at LO**



Contact Potential (CTP)



One-Pion Exchange Potential (OPEP)

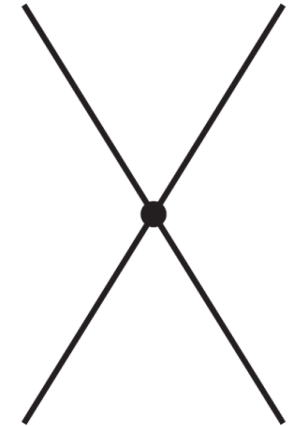
- **“Covariant power counting”**

$$n_{\chi} = 4L - 2N_{\pi} - N_n + \sum_k kV_k,$$

**Expansion parameters:**

pseudoscalar meson masses or small  
three-momenta of nucleons

# NN force at leading order



**Explicitly covariant form**  $u_i(\vec{p}, s) = \sqrt{\frac{E_N + M_N}{2M_N}} \begin{pmatrix} 1 \\ \frac{\vec{\sigma}_1 \cdot \vec{p}}{\epsilon_p} \end{pmatrix} \chi_{s,i}$

$$V_{\text{CTP}} = C_S(\bar{u}_4 u_2)(\bar{u}_3 u_1) + C_A(\bar{u}_4 \gamma_5 u_2)(\bar{u}_3 \gamma_5 u_1) \\ + C_V(\bar{u}_4 \gamma_\mu u_2)(\bar{u}_3 \gamma^\mu u_1) + C_{AV}(\bar{u}_4 \gamma_\mu \gamma_5 u_2)(\bar{u}_3 \gamma^\mu \gamma_5 u_1) \\ + C_T(\bar{u}_4 \sigma_{\mu\nu} u_2)(\bar{u}_3 \sigma_{\mu\nu} u_1).$$

## Expressed in terms of pauli matrices

$$V_{\text{CTP}} = \sum_{i=S,A,V,AV,T} C_i \left[ V_C^i(E_N) + V_\sigma^i(E_N) \sigma_1 \cdot \sigma_2 + V_{SO}^i(E_N) \frac{i}{2} (\sigma_1 + \sigma_2) \cdot (\mathbf{k} \times \mathbf{q}) \right. \\ \left. + V_{\sigma q}^i(E_N) \sigma_1 \cdot \mathbf{q} \sigma_2 \cdot \mathbf{q} + V_{\sigma k}^i(E_N) \sigma_1 \cdot \mathbf{k} \sigma_2 \cdot \mathbf{k} \right. \\ \left. + V_{\sigma L}^i(E_N) \sigma_1 \cdot (\mathbf{q} \times \mathbf{k}) \sigma_2 \cdot (\mathbf{q} \times \mathbf{k}) \right].$$

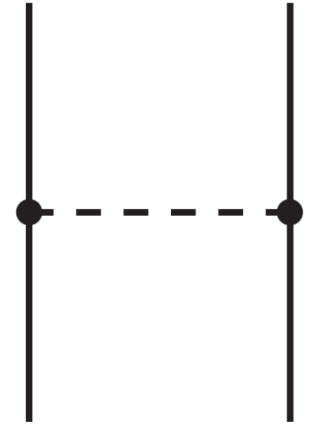
- all allowed **six** spin operators,
- potential energy dependent

## Non-relativistic (static) limit

$$V_{\text{CTP}}^{\text{NonRel.}} = -(C_S + C_V) + (C_{AV} - 2C_T) \sigma_1 \cdot \sigma_2 + \mathcal{O}\left(\frac{1}{M_N}\right).$$



# NN force at leading order



Explicitly covariant form

$$u_i(\vec{p}, s) = \sqrt{\frac{E_N + M_N}{2M_N}} \begin{pmatrix} 1 \\ \frac{\vec{\sigma}_1 \cdot \vec{p}}{\epsilon_p} \end{pmatrix} \chi_{s,i}$$

$$V_{\text{OPEP}} = \xi_{N_1 N_2 \rightarrow N_3 N_4} \frac{g_A^2}{4f_\pi^2} \frac{(\bar{u}_4 \gamma^\mu \gamma_5 q_\mu u_2)(\bar{u}_3 \gamma^\nu \gamma_5 q_\nu u_1)}{\mathbf{q}^2 + m_\pi^2}.$$

Expressed in terms of pauli matrices and NR wfs

$$V_{\text{OPEP}} = \frac{g_A^2}{4f_\pi^2} \frac{1}{\mathbf{q}^2 + m_\pi^2 + i\epsilon} [V_{\sigma q}(E_N) \boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{q} \\ + V_C(E_N) + U_\sigma \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + V_{SO}(E_N) \frac{i}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{k} \times \mathbf{q}) \\ + V_{\sigma k}(E_N) \boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k} + V_{\sigma L}(E_N) \boldsymbol{\sigma}_1 \cdot (\mathbf{q} \times \mathbf{k}) \boldsymbol{\sigma}_2 \cdot (\mathbf{q} \times \mathbf{k})]$$

Non-relativistic (static) limit

$$V_{\text{OPEP}}^{\text{NonRel.}} = -\frac{g_A^2}{4f_\pi^2} \tau_1 \cdot \tau_2 \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{q}}{\mathbf{q}^2 + m_\pi^2 + i\epsilon} + \mathcal{O}\left(\frac{1}{M_N}\right).$$

# A hint at a more efficient formulation

$$V_{1S0} = 4\pi \left[ C_{1S0} + (C_{1S0} + \hat{C}_{1S0}) \left( \frac{\vec{p}^2 + \vec{p}'^2}{4M_N^2} + \dots \right) \right] \\ - \frac{3\pi g_A^2}{f_\pi^2} \int_{-1}^1 \frac{dz}{\vec{q}^2 + m_\pi^2} \left[ \vec{q}^2 - \left( \frac{(\vec{p}^2 - \vec{p}'^2)^2}{4M_N^2} + \dots \right) \right] \quad (18)$$

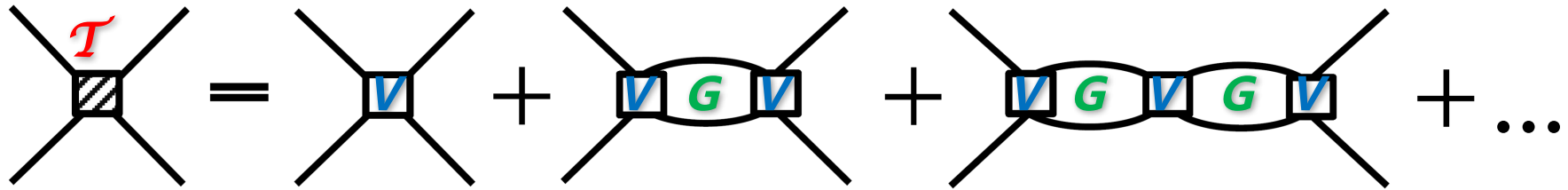
$$C_{1S0} = (C_S + C_V + 3C_{AV} - 6C_T),$$

$$\hat{C}_{1S0} = (3C_V + C_A + C_{AV} + 6C_T),$$

**A large contribution of the correction terms is essential to describe the 1S0 phase shift**

# The nuclear force is non-perturbative

## Non-perturbative summation of the tree-level potential



## 3D reduction of the Bethe-Salpeter equation (Kadyshevsky)

$$T(p', p) = V(p', p) + \int_0^{+\infty} \frac{k^2 dk}{(2\pi)^3} V(p', k) \frac{2\pi M_N^2}{(k^2 + M_N^2)(\sqrt{p^2 + M_N^2} - \sqrt{k^2 + M_N^2} + i\epsilon)} T(k, p).$$

With the implicit mass “on-shell” approximation of the potential.

$$E_p = \sqrt{M_N^2 + \vec{p}^2},$$

# NN force at leading order

- **5 LECs to fit the np phase shifts of Nijmegen 93**

- **7** partial waves:  $J=0, 1$   $^1S_0, ^3P_0, ^1P_1, ^3P_1, ^3D_1, ^3S_1, \epsilon_1$
- **42** data points: 6 data points for each partial wave  
( $E_{\text{lab}} = 1, 5, 10, 25, 50, 100$  MeV)

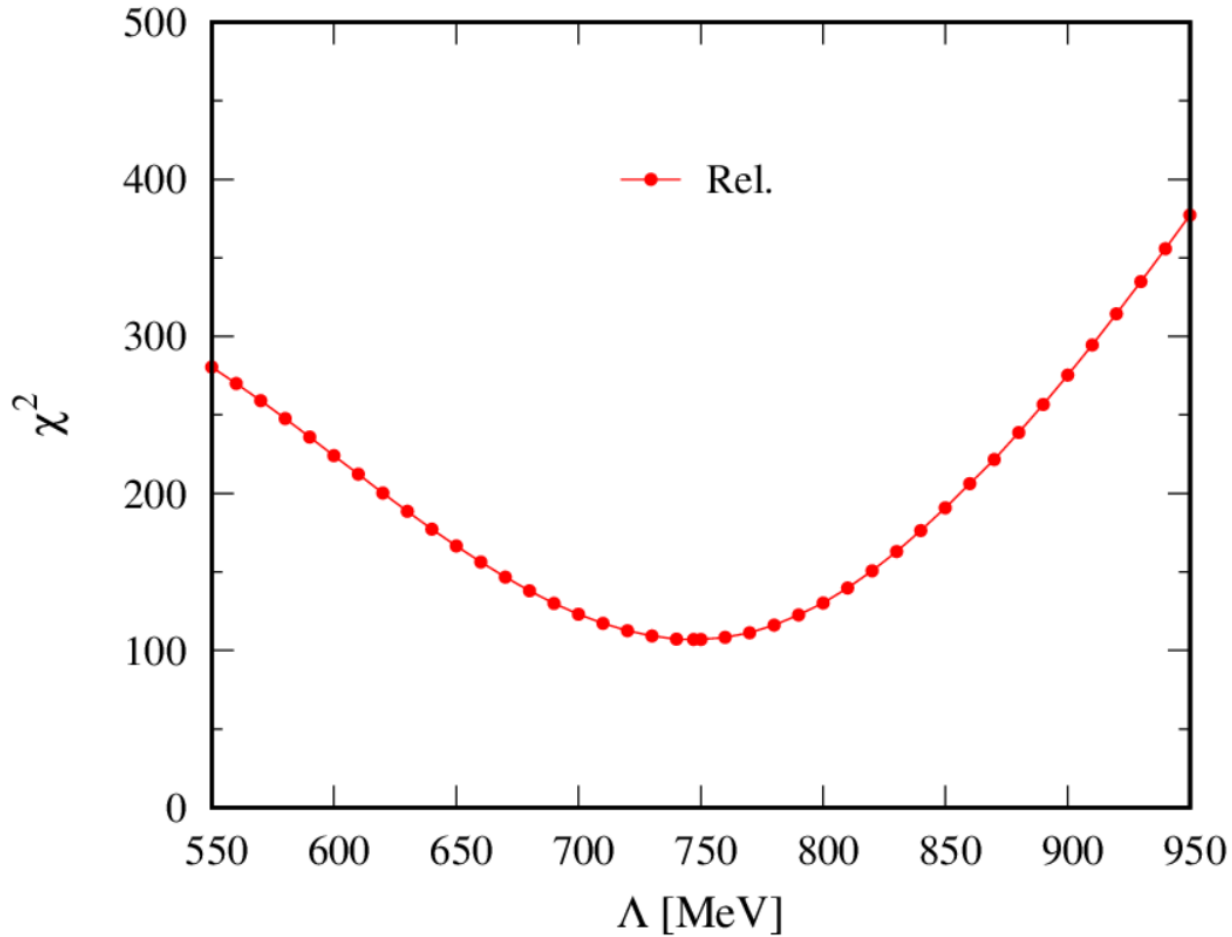
$$\tilde{\chi}^2 = \sum_i \left( \delta_i^{\text{Theory}} - \delta_i^{\text{Nij93}} \right)^2.$$

- **Cutoff renormalization in solving the scattering eq.**

$$V(p', p) \rightarrow V(p', p) \mathbf{f}(p', p). \quad \mathbf{f}(p', p) = \exp[-(p'/\Lambda)^{2n} - (p/\Lambda)^{2n}].$$



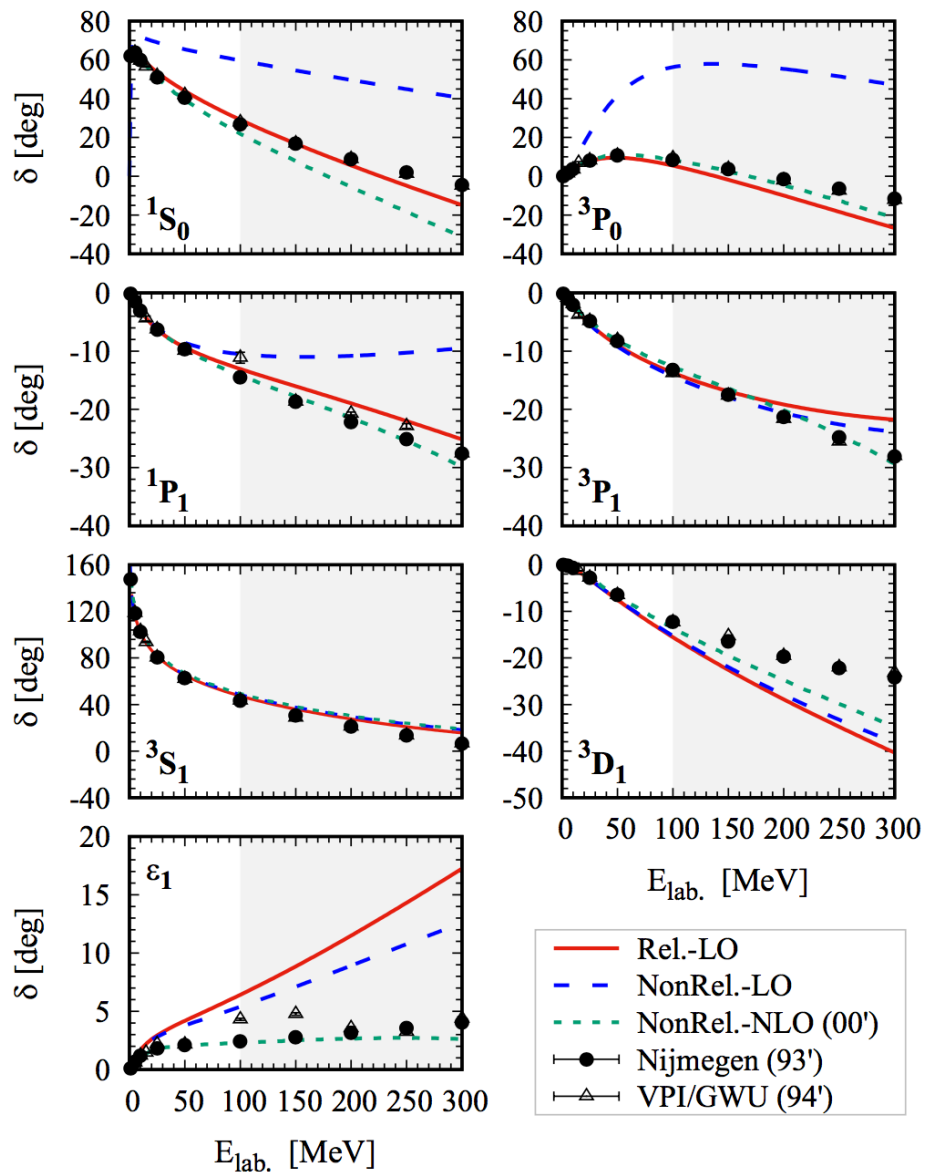
# Best fit



LECs	Values [ $10^4 \text{ GeV}^{-2}$ ]
$C_S$	0.1339
$C_A$	-0.05477
$C_V$	-0.2673
$C_{AV}$	-0.2454
$C_T$	-0.06310

$\Lambda=747$  MeV, the minimum of fit- $\chi^2=106.90$ ,  $\chi^2/\text{d.o.f.} = 2.89$

# A closer look at the partial waves



- **Improved description of  $^1S_0$  and  $^3P_0$  phase shifts**
- **Quantitatively similar with the nonrelativistic case for J=1 partial waves**

# Relativistic vs. non-relativistic

## Very promising

	Relativistic Chiral NF	Non-relativistic Chiral NF	
Chiral order	LO	LO	NLO*
No. of LECs	5	2	9
$\chi^2/\text{d.o.f.}$	<b>2.9</b>	<b>147.9</b>	<b>2.5</b>

**A more efficient description is achieved**

# BbS vs. Kadeshevsky scattering equation

BbS(Blankenbecler-Sugar)

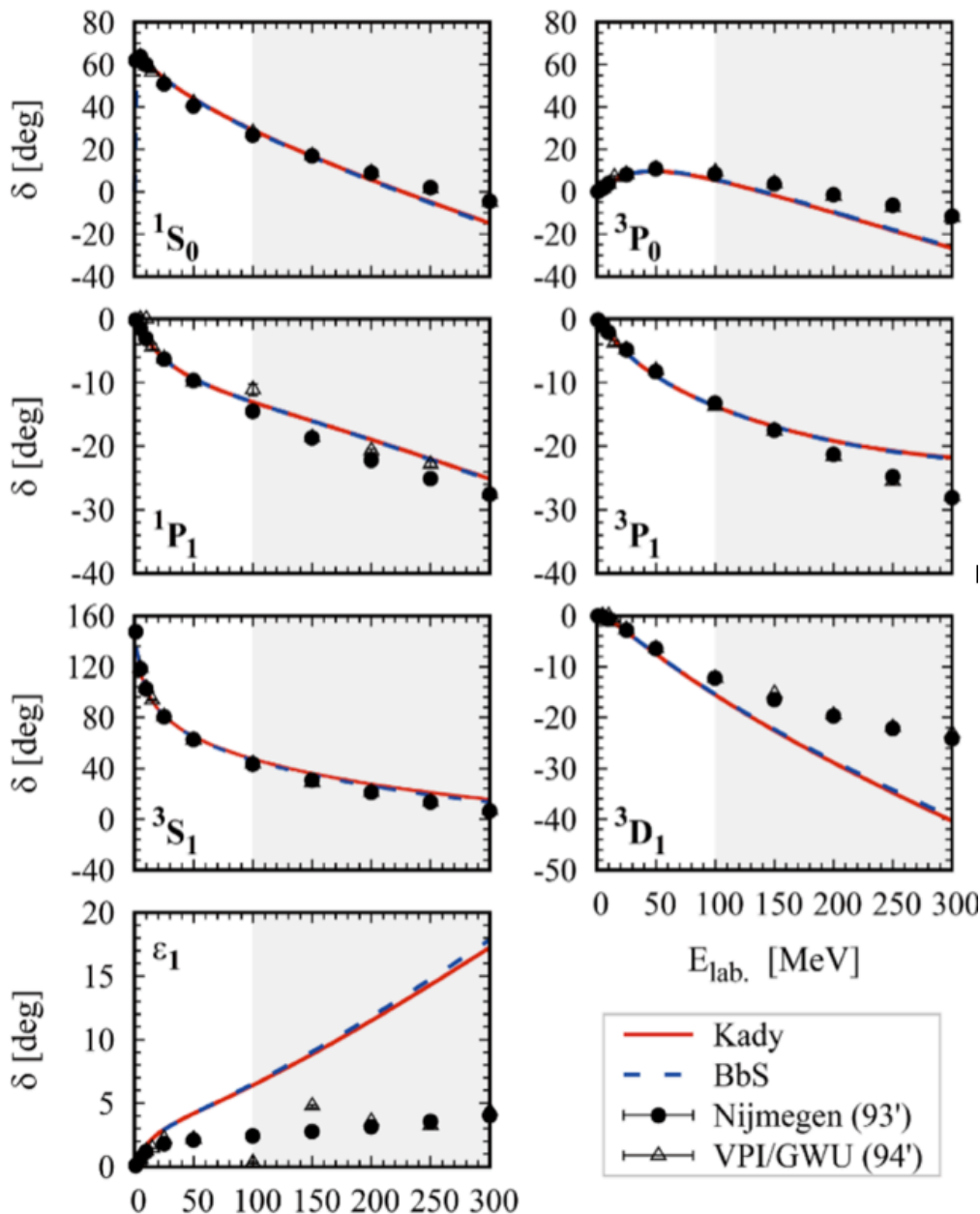
- Replace the scattering function from the **Kadyshevsky** eq. to the **Blankenbecler-Sugar** eq.

$$T(p', p) = V(p', p) + \int_0^{+\infty} \frac{dk}{(2\pi)^3} V(p', k) \times \frac{1}{M_N^2 \sqrt{k^2 + M_N^2(p^2 - k^2) + i\epsilon}} T(k, p).$$

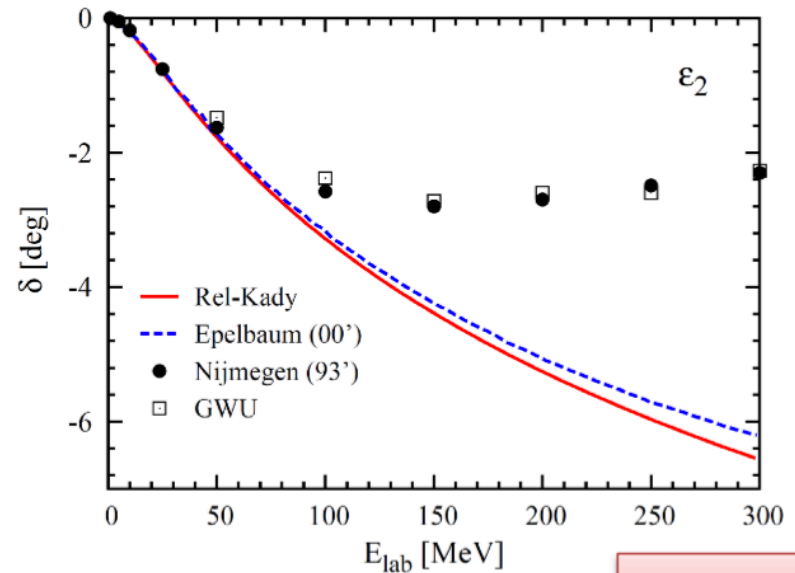
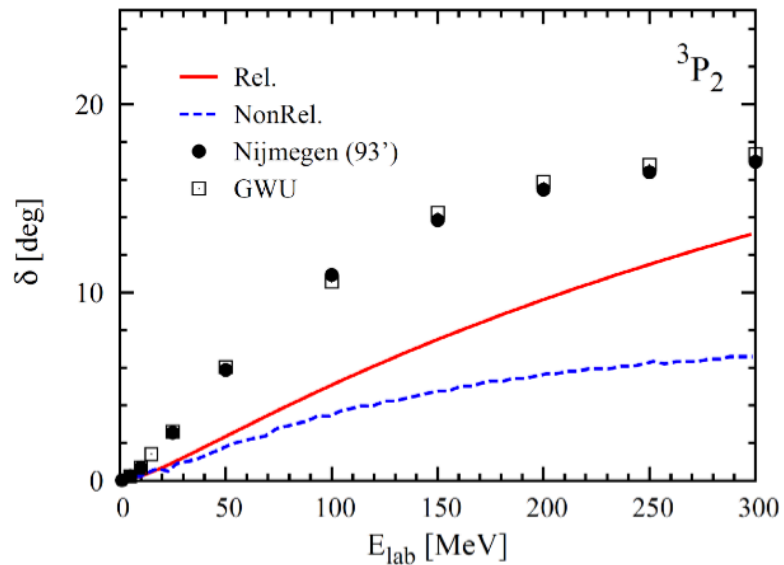
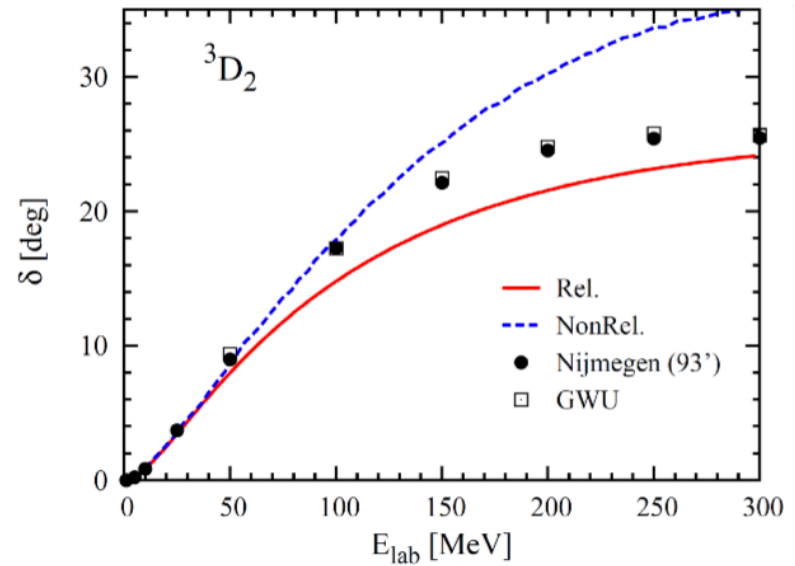
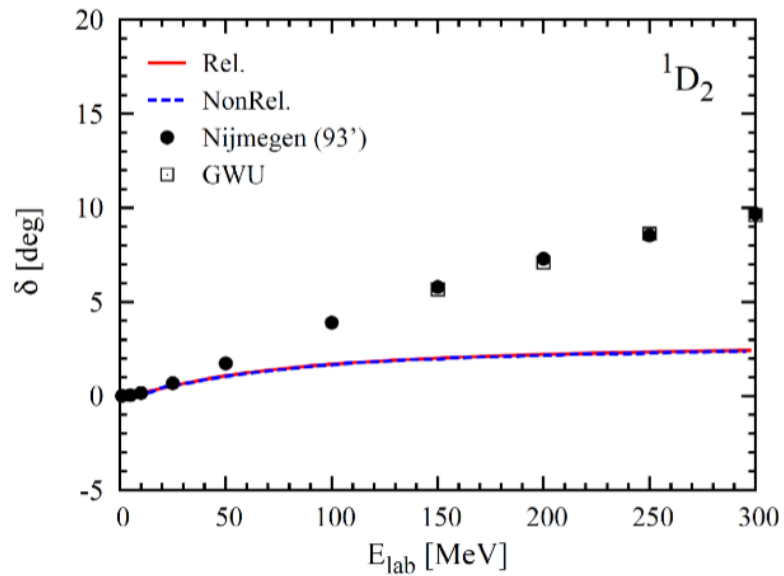
*R.Blankenbecler, Phys.Rev.(1966)*

- Best fit results:

	Kady.	BbS
Cutoff $\Lambda$ [MeV]	<b>747</b>	<b>743</b>
Fit- $\chi^2$ /d.o.f.	<b>2.9</b>	<b>2.5</b>



# Higher partial waves remain the same



$\Lambda_F = 600$  MeV

OPE prediction



# Deuteron Properties and scattering lengths

in reasonable agreement with data

## □ Deuteron binding energy

	Expt.	Kady.	BbS
$B_d$ [MeV]	2.22457	<b>1.86700</b>	<b>1.93900</b>

## □ $S$ -wave scattering length

	Expt.	Kady.	BbS
$a_{1S0}$ [fm]	-23.739	<b>-20.299</b>	<b>-20.415</b>
$a_{3S1}$ [fm]	5.420	<b>5.746</b>	<b>5.667</b>

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- ❖ More is coming. Remain tuned.

**Thank you very much  
for your attention!**

# Covariant BChPT in the NN case

- **E. Epelbaum, J. Gegelia, PLB716(2013)338**
  - LO, kernel potential consistent with HB, plus Kadeshevsky equaiton
- **E. Epelbaum, A.M. Gasparyan, J. Gegelia, Eur.Phys.J. A51 (2015), 71**
  - NLO contact terms treated non-perturbatively to solve 1S0 discrepancy
- **J. Behrendt, E. Epelbaum, J. Gegelia et al., 1606.01489**
  - LO: higher derivative terms added—equivalent to add form factors

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- D.R. Entem and R. Machleidt, “Accurate charge dependent nucleon nucleon potential at fourth order of chiral perturbation theory,” Phys.Rev. C68 (2003) 041001 —839 times
- E. Epelbaum, W. Glockle, Ulf-G. Meissner, “The Two-nucleon system at next-to-next-to-next-to-leading order ,” Nucl.Phys. A747 (2005) 362-424 —452 times

as of July 8th, 2016



# Weinberg Power Counting

❖ Potential organized by

$$V_{\text{eff}} = V_{\text{eff}}(q, g, \mu) = \sum_{\nu} q^{\nu} \mathcal{V}_{\nu}(q/\mu, g)$$

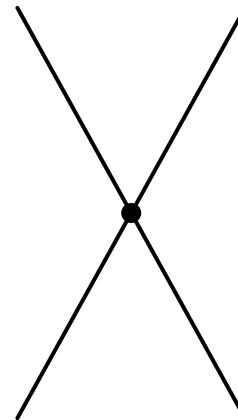
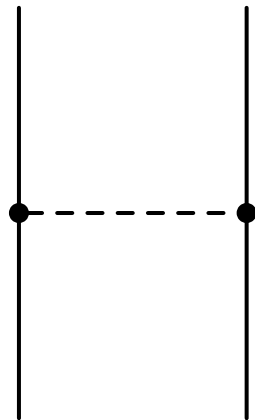
❖ Chiral power counting

$$\nu = 2 - \frac{1}{2}B + 2L + \sum_i v_i \Delta_i, \quad \Delta_i = d_i + \frac{1}{2}b_i - 2$$

- B: number of external baryons
- L: number of GB loops
- $v_i$ : number of vertices with dimension  $\Delta_i$ 
  - $d_i$ : number of derivatives or NGB masses
  - $b_i$ : number of baryon fields in the interaction  $\Delta_i$

# Leading order: $v=0$

- $B=4, L=0, \Delta_i=0$ 
  - contact:  $d_i=0; b_i=4$
  - one pion exchange:  $d_i=1, b_i=2$

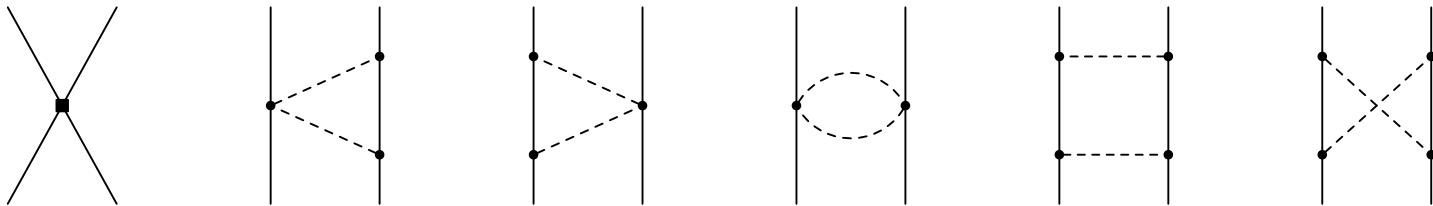


$v=1$  vanishes

- $B=4, L=0, \Delta=1$
- Parity conservation

# Next-to-leading order $v=2$

- $B=4, L=0, \Delta_i=2$  or  $2 \times 1$
- $B=4, L=1, \Delta_i=0$



# Standard Model of Particle Physics

mass →	$\approx 2.3 \text{ MeV}/c^2$	$\approx 1.275 \text{ GeV}/c^2$	$\approx 173.07 \text{ GeV}/c^2$	0	$\approx 126 \text{ GeV}/c^2$
charge →	$2/3$	$2/3$	$2/3$	0	0
spin →	$1/2$	$1/2$	$1/2$	1	0
	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b>g</b> gluon	<b>H</b> Higgs boson
<b>QUARKS</b>	$\approx 4.8 \text{ MeV}/c^2$	$\approx 95 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0	
	$-1/3$	$-1/3$	$-1/3$	0	
	$1/2$	$1/2$	$1/2$	1	
	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b><math>\gamma</math></b> photon	
	$0.511 \text{ MeV}/c^2$	$105.7 \text{ MeV}/c^2$	$1.777 \text{ GeV}/c^2$	$91.2 \text{ GeV}/c^2$	
	-1	-1	-1	0	
	$1/2$	$1/2$	$1/2$	1	
	<b>e</b> electron	<b><math>\mu</math></b> muon	<b><math>\tau</math></b> tau	<b>Z</b> Z boson	
<b>LEPTONS</b>	$< 2.2 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 15.5 \text{ MeV}/c^2$	$80.4 \text{ GeV}/c^2$	
	0	0	0	$\pm 1$	
	$1/2$	$1/2$	$1/2$	1	
	<b><math>\nu_e</math></b> electron neutrino	<b><math>\nu_\mu</math></b> muon neutrino	<b><math>\nu_\tau</math></b> tau neutrino	<b>W</b> W boson	
				<b>GAUGE BOSONS</b>	

# Number of parameters for the np potential

for the  $np$  potential

	Nijmegen PWA93	CD-Bonn “high precision”	NLO $Q^2$ (NNLO)	N <sup>3</sup> LO $Q^4$ (N <sup>4</sup> LO)	N <sup>5</sup> LO $Q^6$
$^1S_0$	3	4	2	4	6
$^3S_1$	3	4	2	4	6
$^3S_1$ - $^3D_1$	2	2	1	3	6
$^1P_1$	3	3	1	2	4
$^3P_0$	3	2	1	2	4
$^3P_1$	2	2	1	2	4
$^3P_2$	3	3	1	2	4
$^3P_2$ - $^3F_2$	2	1	0	1	3
$^1D_2$	2	3	0	1	2
$^3D_1$	2	1	0	1	2
$^3D_2$	2	2	0	1	2
$^3D_3$	1	2	0	1	2
$^3D_3$ - $^3G_3$	1	0	0	0	1
$^1F_3$	1	1	0	0	1
$^3F_2$	1	2	0	0	1
$^3F_3$	1	2	0	0	1
$^3F_4$	2	1	0	0	1
$^3F_4$ - $^3H_4$	0	0	0	0	0
$^1G_4$	1	0	0	0	0
$^3G_3$	0	1	0	0	0
$^3G_4$	0	1	0	0	0
$^3G_5$	0	1	0	0	0
<b>Total</b>	<b>35</b>	<b>38</b>	<b>9</b>	<b>24</b>	<b>50</b>

# Covariance Matrix

	$C_S$	$C_A$	$C_V$	$C_{AV}$	$C_T$
$C_S$	1.00	0.21	-0.93	-0.58	-0.39
$C_A$	0.23	1.00	-0.15	0.45	0.21
$C_V$	-0.93	-0.15	1.00	0.77	0.69
$C_{AV}$	-0.57	0.45	0.77	1.00	0.89
$C_T$	-0.39	0.21	0.69	0.89	1.00



# 核力—微观核物理的基础

## ● 唯象传统核力

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# LO Lagrangians

$$\begin{aligned}\mathcal{L}_{NN}^{(0)} &= C_S(\bar{\Psi}\Psi)(\bar{\Psi}\Psi) + C_A(\bar{\Psi}\gamma_5\Psi)(\bar{\Psi}\gamma_5\Psi) \\ &+ C_V(\bar{\Psi}\gamma_\mu\Psi)(\bar{\Psi}\gamma^\mu\Psi) + C_{AV}(\bar{\Psi}\gamma_\mu\gamma_5\Psi)(\bar{\Psi}\gamma^\mu\gamma_5\Psi) \\ &+ C_T(\bar{\Psi}\sigma_{\mu\nu}\Psi)(\bar{\Psi}\sigma^{\mu\nu}\Psi),\end{aligned}\quad (9)$$

$$\begin{aligned}\mathcal{L}_{NN} &= C_S^a \bar{\Psi}\tau^a\Psi \bar{\Psi}\tau^a\Psi + C_T^a \bar{\Psi}\tau^a\sigma_{\mu\nu}\Psi \bar{\Psi}\tau^a\sigma^{\mu\nu}\Psi \\ &+ C_{AV}^a \bar{\Psi}\tau^a\gamma_5\gamma_\mu\Psi \bar{\Psi}\tau^a\gamma_5\gamma^\mu\Psi \\ &+ C_V^a \bar{\Psi}\tau^a\gamma_\mu\Psi \bar{\Psi}\tau^a\gamma^\mu\Psi,\end{aligned}$$

indices, unless necessary, will be suppressed hereafter.) To have flavor singlets, the isospin structure of the two bilinears must be either  $1 \otimes 1$  or  $\tau^a \otimes \tau^a$ . However, the latter needs not be considered, as it can be eliminated by Fierz rearrangement.