

$Z_c(3900)$ from lattice QCD

based on Y. Ikeda et al., (HAL QCD), arXiv.1602.03465(hep-lat).

Yoichi IKEDA (RCNP, Osaka Univ.)

HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

S. Aoki, D. Kawai, T. Miyamoto, K. Sasaki (YITP, Kyoto Univ.)

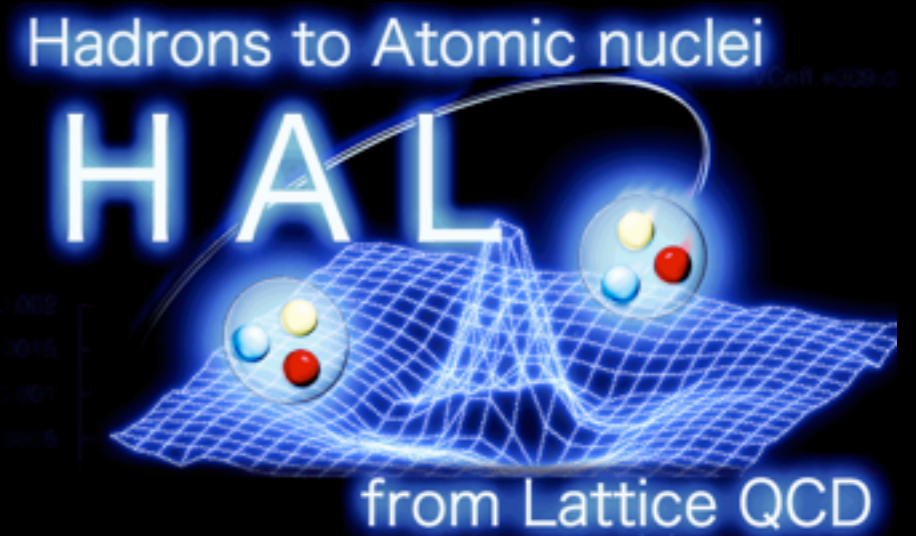
T. Doi, T. Hatsuda, T. Iritani (RIKEN)

S. Gongyo (Univ. Tours)

T. Inoue (Nihon Univ.)

Y. Ikeda, N. Ishii, K. Murano (RCNP, Osaka Univ.)

H. Nemura (Univ. Tsukuba)



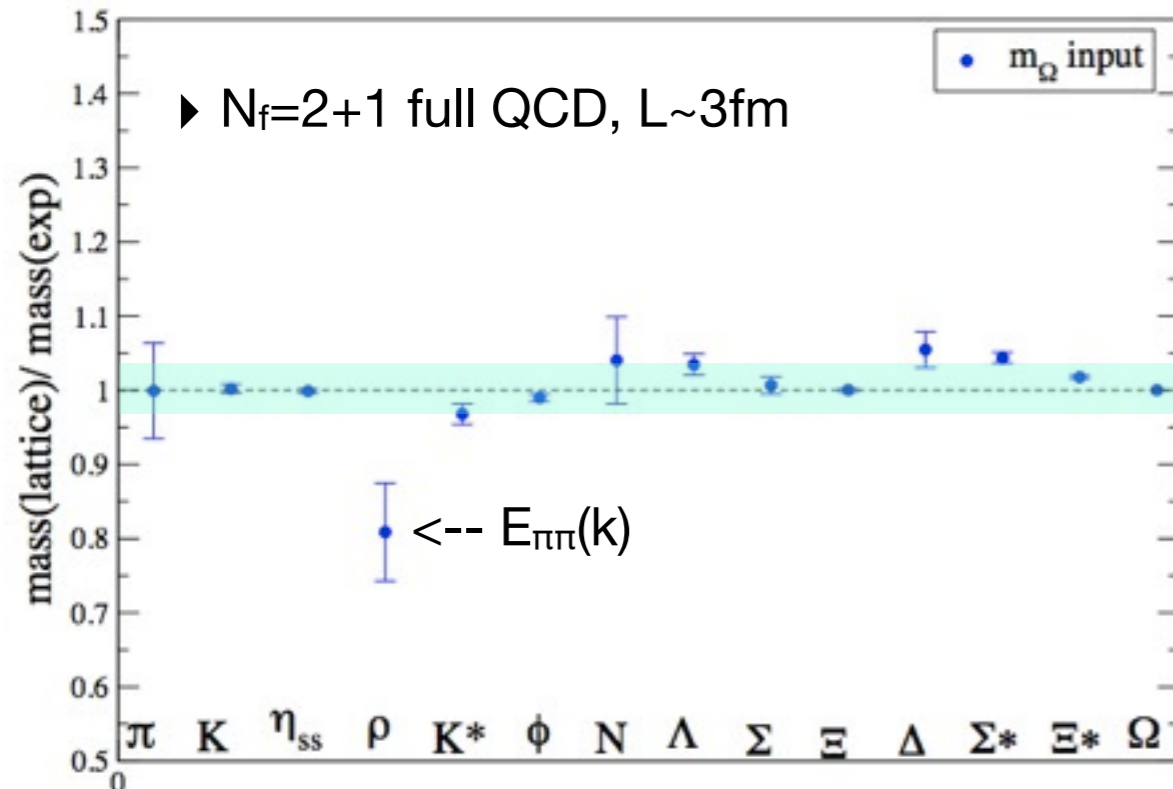
Long-term workshop in Realistic Hadron Interactions in QCD (RHIQCD2016)

@YITP, Kyoto (Dec. 2, 2016.)

Single hadron spectroscopy from LQCD

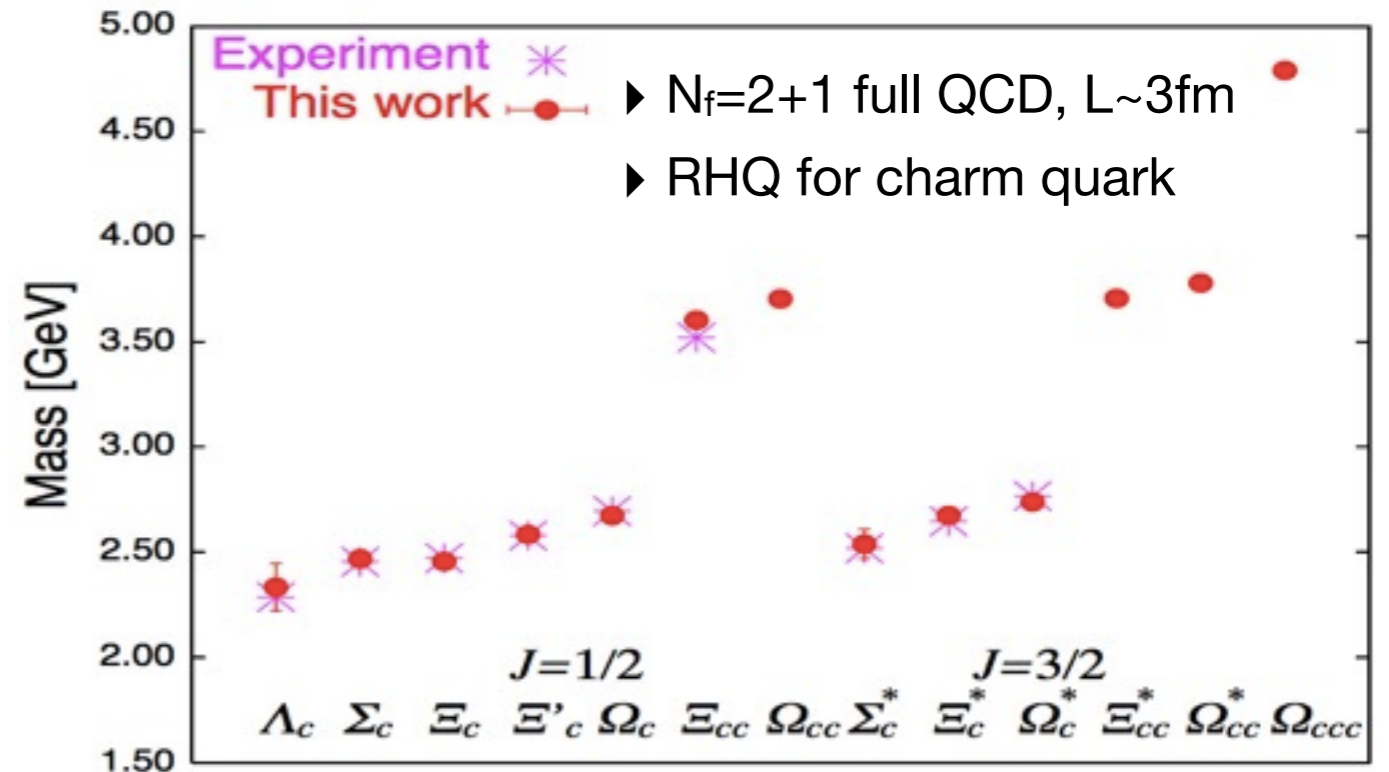
★ Low-lying (stable) hadrons on physical point (physical m_q)

light hadrons

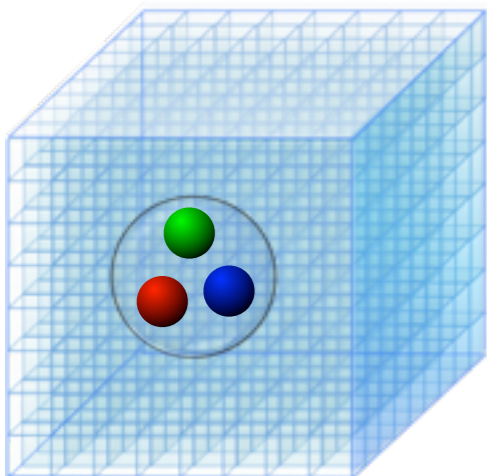


Aoki et al. (PACS-CS), PRD81 (2010).

charm baryons



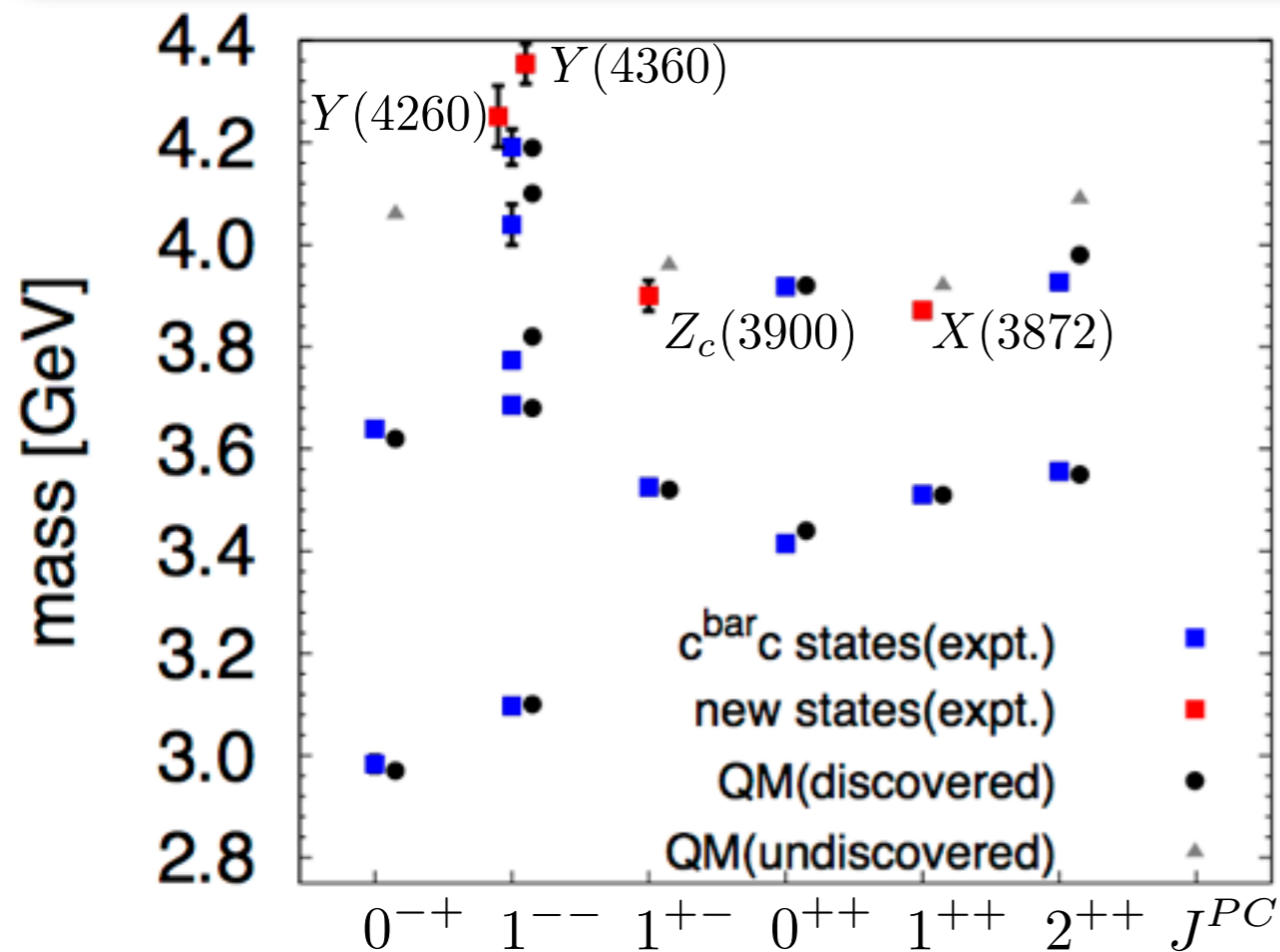
Namekawa et al. (PACS-CS), PRD84 (2011); PRD87 (2013).



- a few % accuracy already achieved for single hadrons
- LQCD now can predict undiscovered charm hadrons (Ξ_{cc} , Ξ_{cc}^* , Ω_{ccc} ,...)

➔ **Next challenge : multi-hadrons (resonances)**

Charmonium-like states



✓ Quark models well describe observed mass spectra at low energies (< 3.8 GeV)

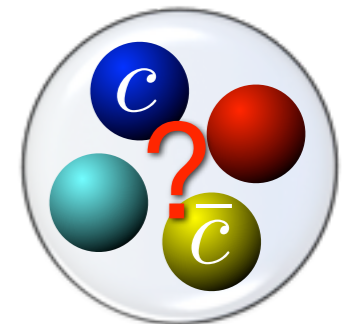
✓ Several states at high energies (> 3.8 GeV) are not discovered

Godfrey, Isgur, PRD 32 (1985).

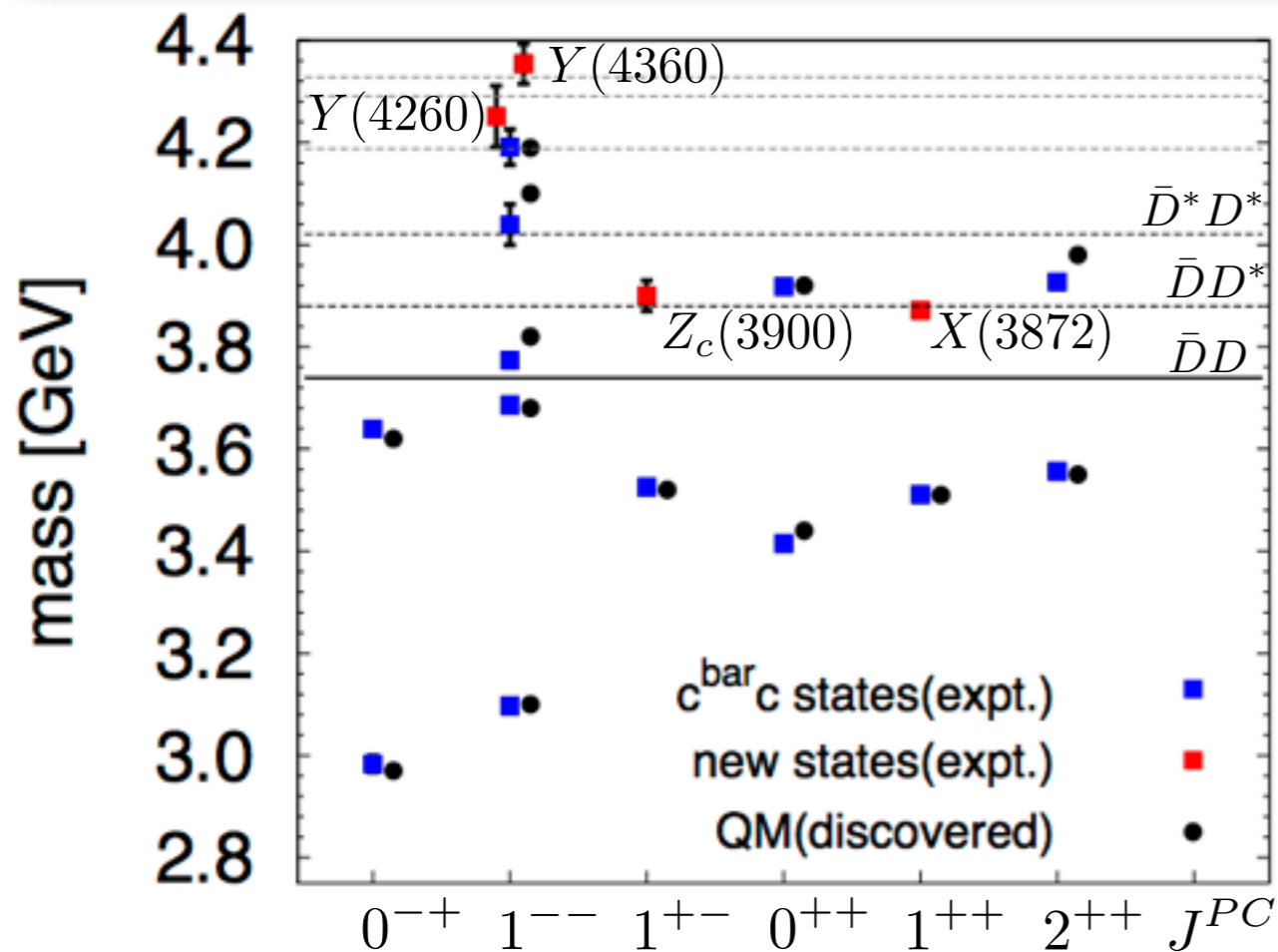
Barnes, Godfrey, Swanson, PRD 72 (2005).

📌 **NEW** (X, Y, Z) states observed in expt. which are NOT within QM spectrum

📌 Non- $c^{\bar{c}}$ structures = exotic hadrons?



Charmonium-like states



✓ Quark models well describe observed mass spectra at low energies (< 3.8 GeV)

✓ Several states at high energies (> 3.8 GeV) are not discovered

Godfrey, Isgur, PRD 32 (1985).

Barnes, Godfrey, Swanson, PRD 72 (2005).

📌 **NEW** (X, Y, Z) states observed in expt. which are NOT within QM spectrum

📌 Non- $c^{\bar{c}}$ structures = exotic hadrons?

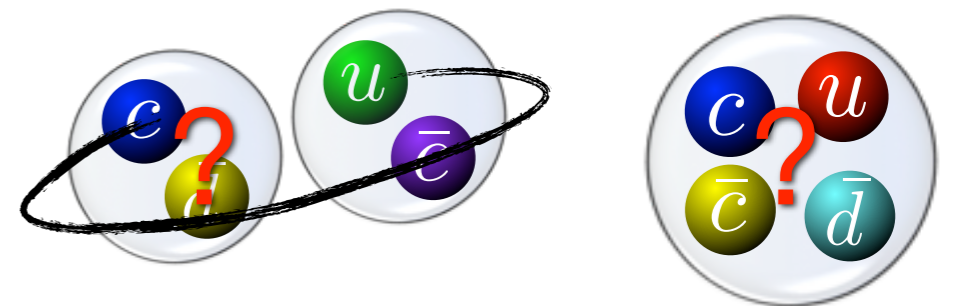
📌 All **X, Y, Z** states are found above 3.8 GeV

✓ Lowest open charm threshold ($D^{\bar{c}}D$) is 3.75 GeV

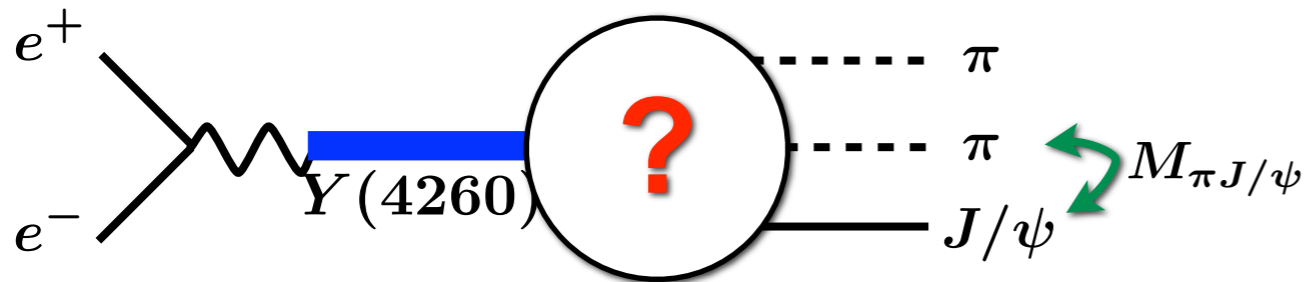
✓ All new states embedded in two-meson continuum ($D^{\bar{c}}D, D^{\bar{c}}D^*, D^{\bar{c}*}D^*, \dots$)

✓ Channel coupling could be a key to investigate X, Y, Z states

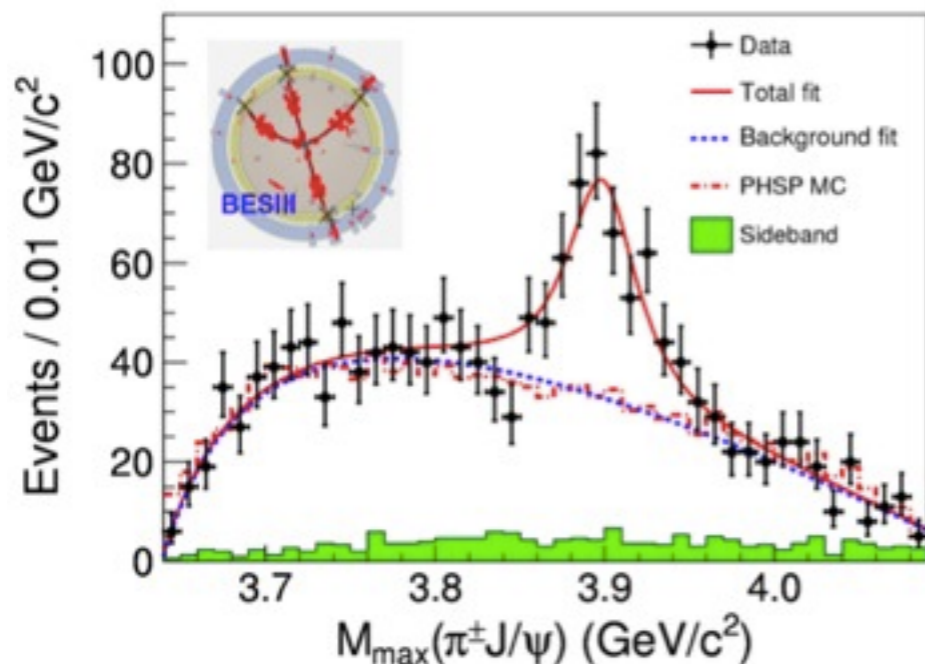
📌 **Our target : charmonium-like $Z_c(3900)$**



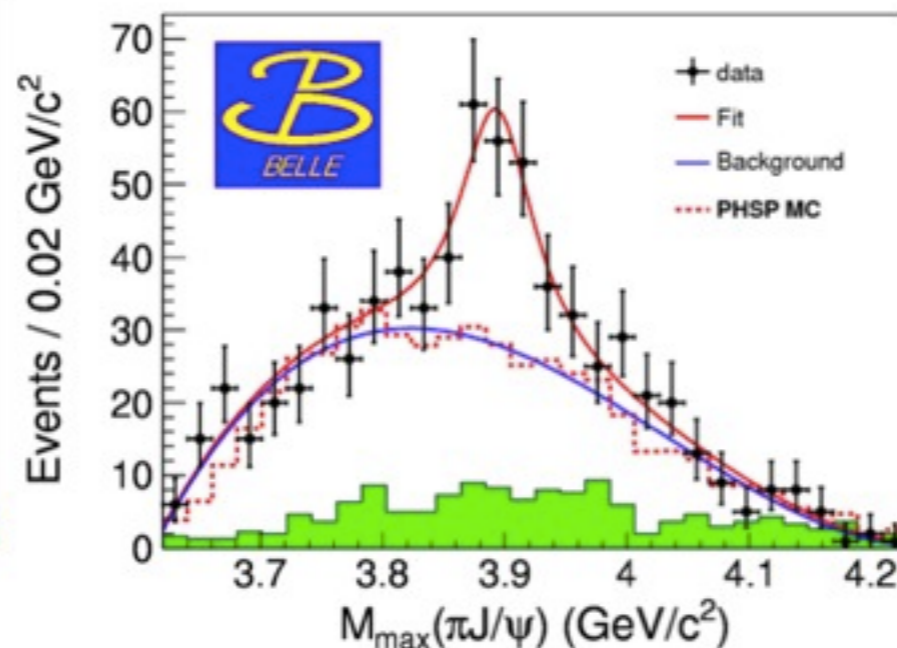
What is $Z_c(3900)$?



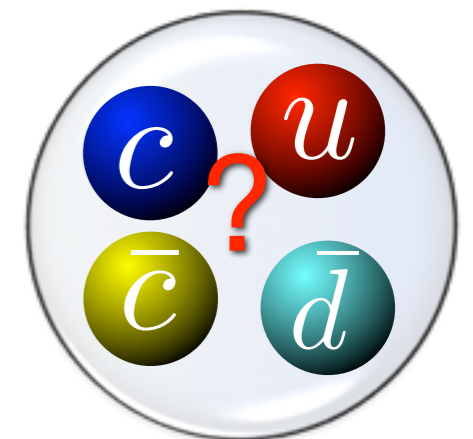
[BESIII Coll., PRL110, 252001, \(2013\).](#)



[Belle Coll., PRL110, 252002, \(2013\).](#)

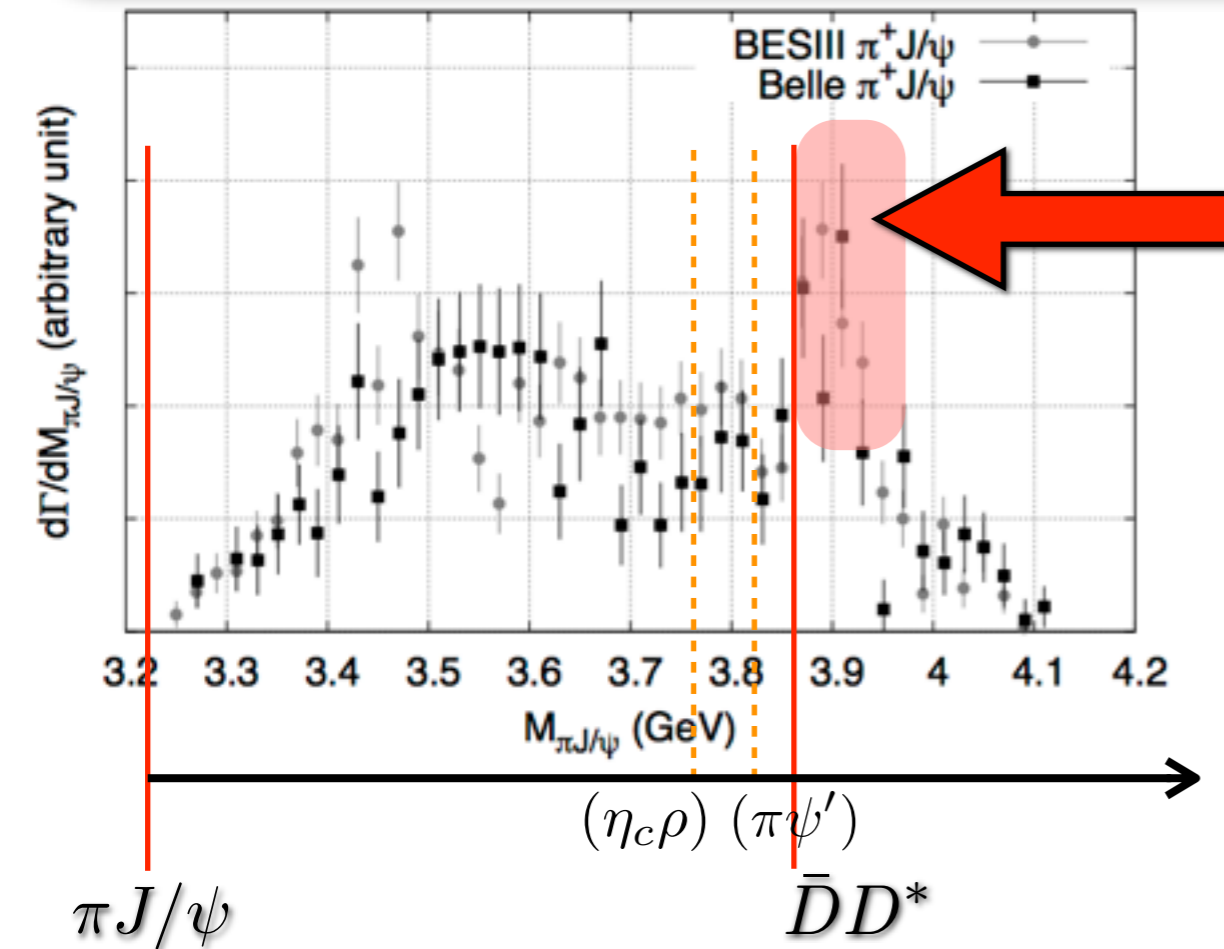


- $Z_c(3900)$ is observed in $\pi^{\pm}J/\psi$ invariant mass
- Minimal quark content : 4 quarks ($c^{\text{bar}}c u^{\text{bar}}d$)
- $M \sim 3900$, $\Gamma \sim 60$ MeV when Breit-Wigner resonance assumed
- spin-parity: $J^{PC}=1^{+-}$ by PWA



[BESIII Coll., PRL112 \(2014\), talik in MENU2016](#)

Structure of $Z_c(3900)$?



Expt. status

- Peak just above $D^{\text{bar}}D^*$ threshold found in $\pi J/\psi$ invariant mass
- $J^P=1^+$ \leftrightarrow s-wave $\pi J/\psi$ - $D^{\text{bar}}D^*$ dynamics

► Decay rate of $Z_c(3900)$

$$\frac{\Gamma(Z_c(3900) \rightarrow \bar{D}D^*)}{\Gamma(Z_c(3900) \rightarrow \pi J/\psi)} \simeq 6.2$$

BESIII Coll., PRL112 (2014).

Structure of $Z_c(3900)$?

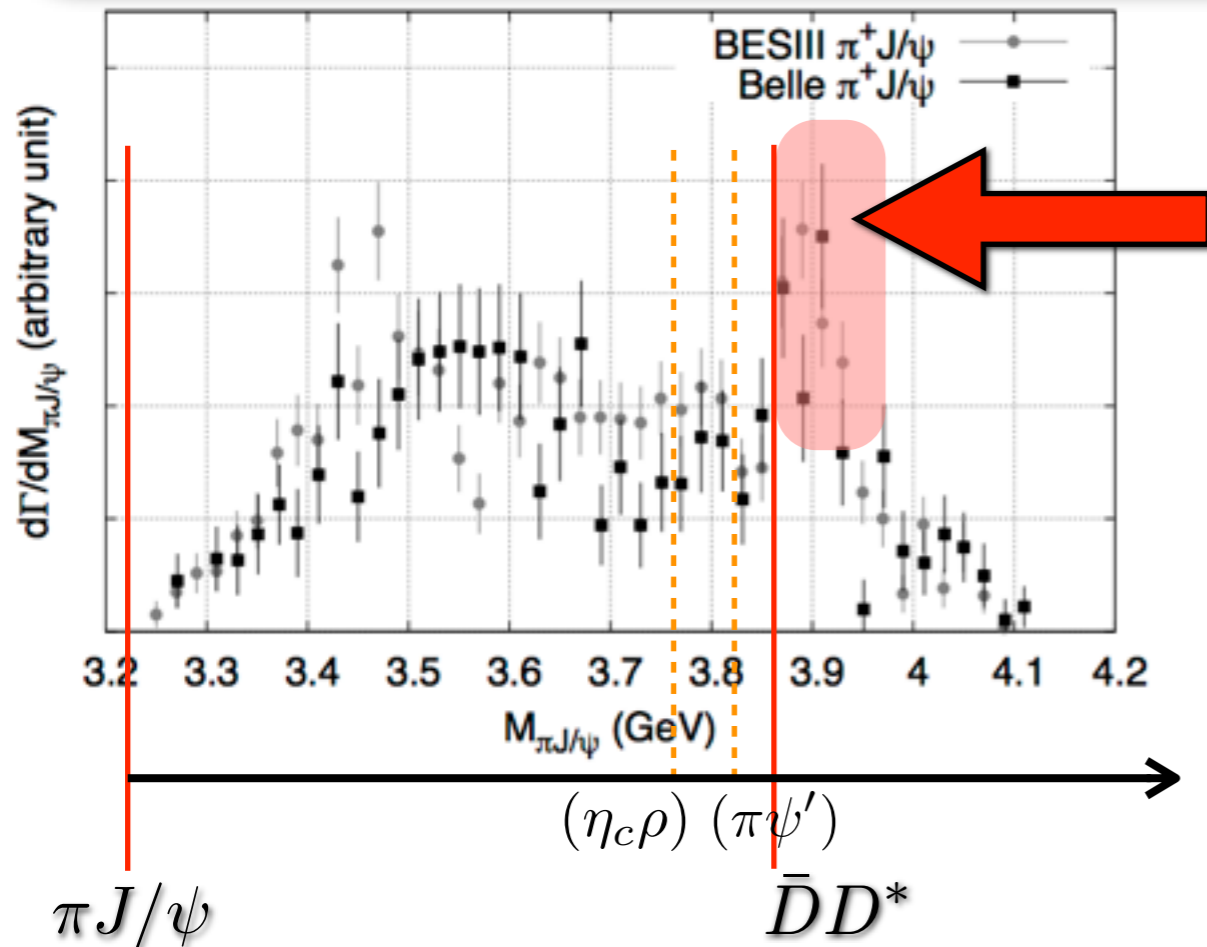
Expt. status

- Peak just above $D^{\text{bar}}D^*$ threshold found in $\pi J/\psi$ invariant mass
- $J^P=1^+$ \leftrightarrow s-wave $\pi J/\psi$ - $D^{\text{bar}}D^*$ dynamics

► Decay rate of $Z_c(3900)$

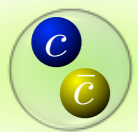
$$\frac{\Gamma(Z_c(3900) \rightarrow \bar{D}D^*)}{\Gamma(Z_c(3900) \rightarrow \pi J/\psi)} \simeq 6.2$$

BESIII Coll., PRL112 (2014).



★ Structure of $Z_c(3900)$ from models

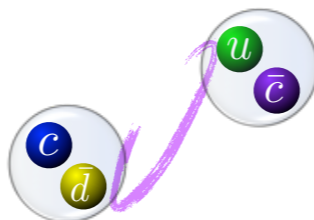
- Tetraquark? **Maiani et al. (2013).**
- J/ψ + π cloud, $D^{\text{bar}}D^*$ molecule?



**Voloshin (2008),
Nieves et al. (2011),
+ many others**

- Threshold kinematical effect?

Chen et al. (2013), Swanson (2015).



Structure of $Z_c(3900)$?

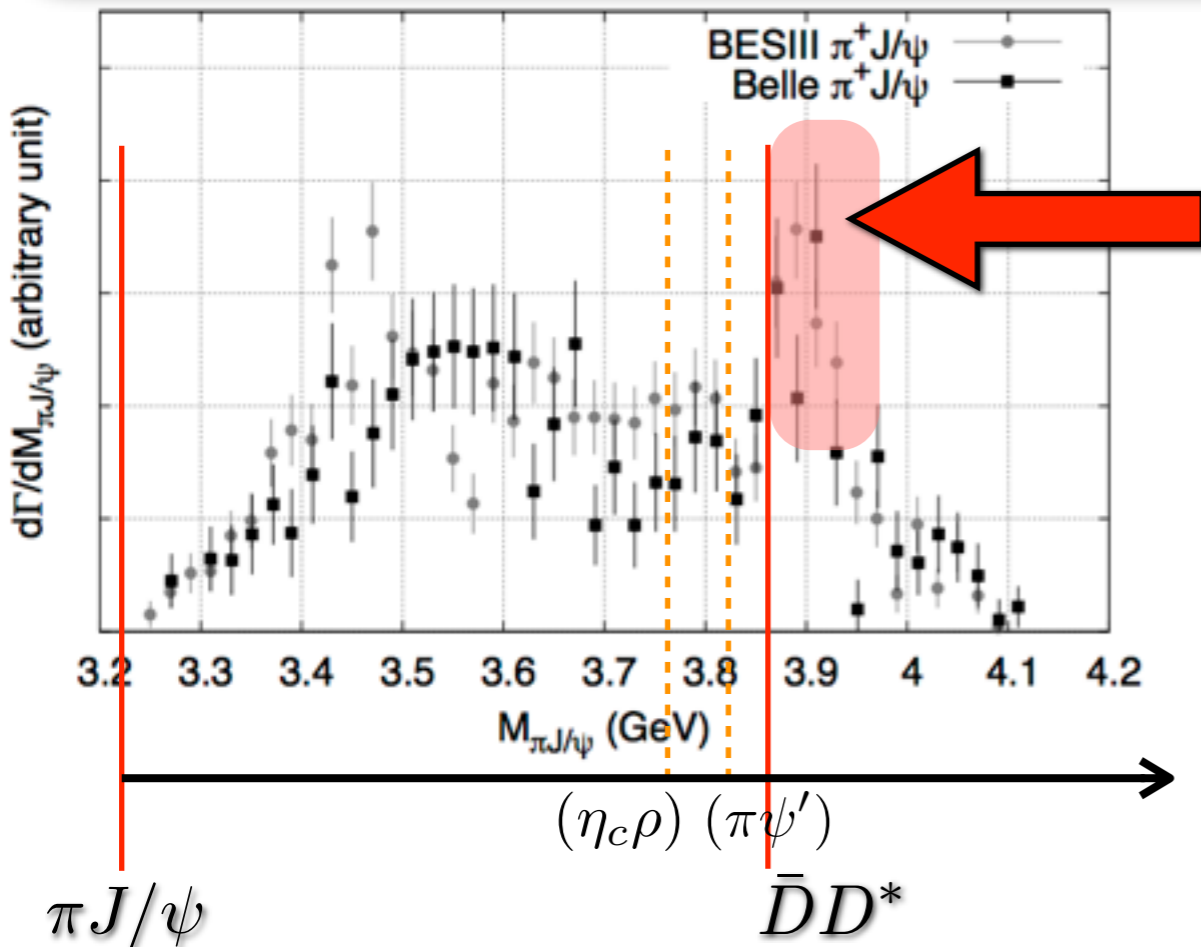
Expt. status

- Peak just above $D^{\text{bar}}D^*$ threshold found in $\pi J/\psi$ invariant mass
- $J^P=1^+$ \leftrightarrow s-wave $\pi J/\psi - D^{\text{bar}}D^*$ dynamics

► Decay rate of $Z_c(3900)$

$$\frac{\Gamma(Z_c(3900) \rightarrow \bar{D}D^*)}{\Gamma(Z_c(3900) \rightarrow \pi J/\psi)} \simeq 6.2$$

BESIII Coll., PRL112 (2014).



★ Structure of $Z_c(3900)$ from models

➡ **poor information on interactions**

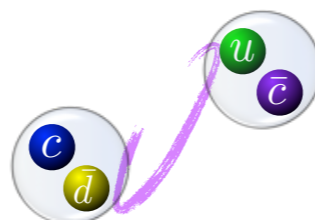
- Tetraquark? **Maiani et al. (2013).**
- $J/\psi + \pi$ cloud, $D^{\text{bar}}D^*$ molecule?



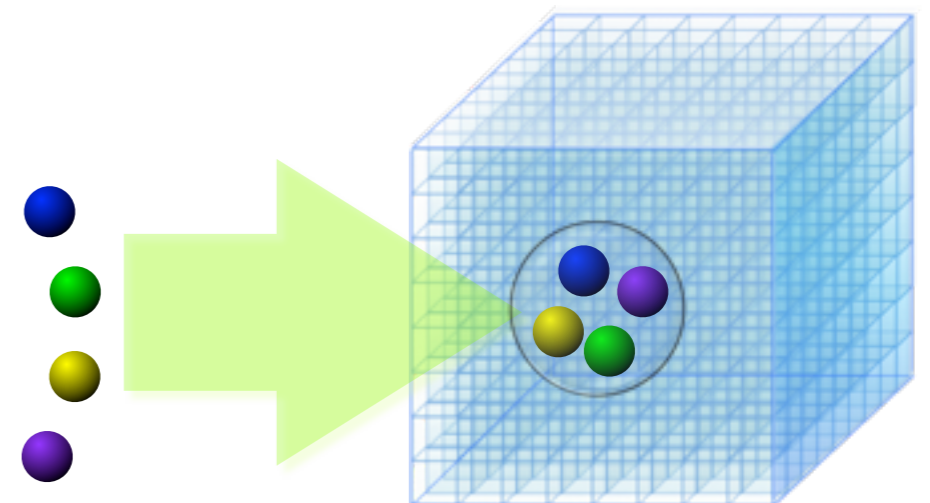
**Voloshin (2008),
Nieves et al. (2011),
+ many others**

- Threshold kinematical effect?

Chen et al. (2013), Swanson (2015).



★ LQCD simulations for $Z_c(3900)$



Contents

📌 Brief introduction to $Z_c(3900)$

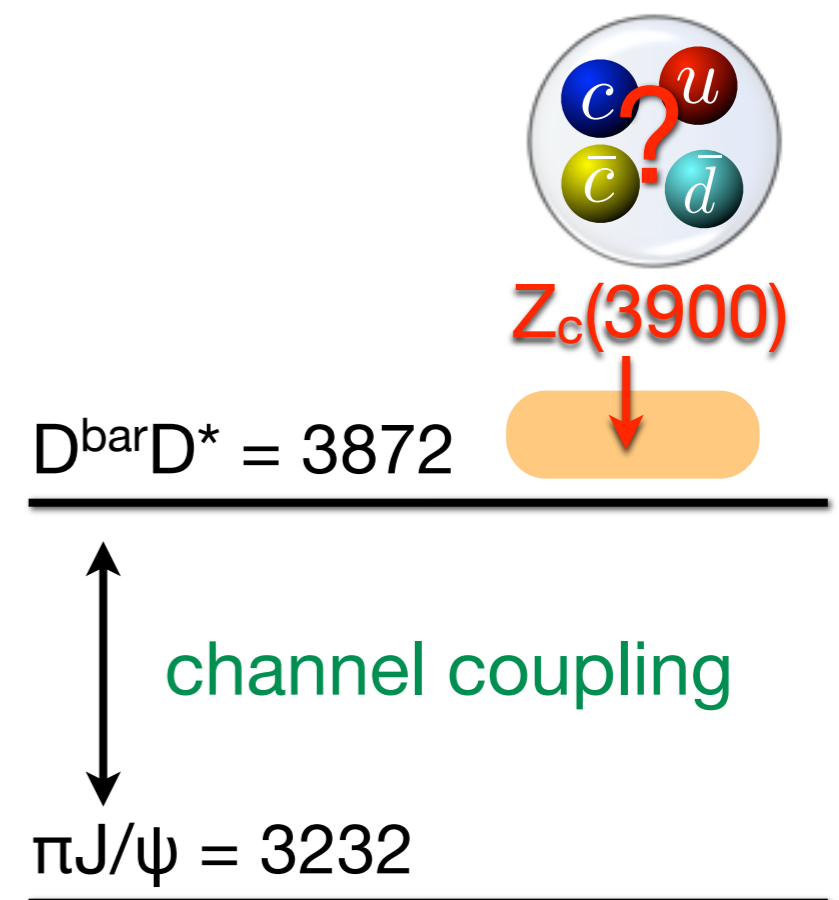
📌 How to study $Z_c(3900)$ on the lattice?

📌 Coupled-channel interactions for $Z_c(3900)$ in $I^G(J^{PC})=1^+(1^{+-})$

📌 Structure of $Z_c(3900)$

📌 Comparison with experimental data

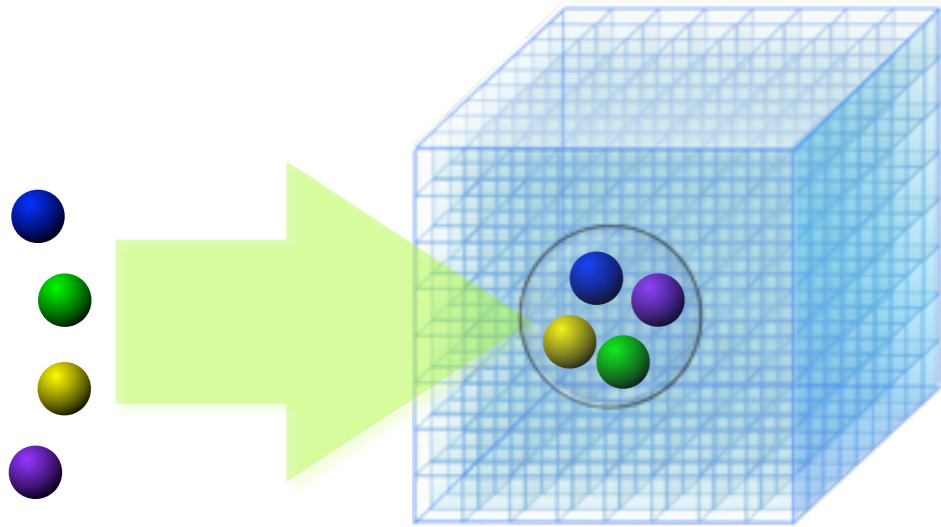
📌 Summary



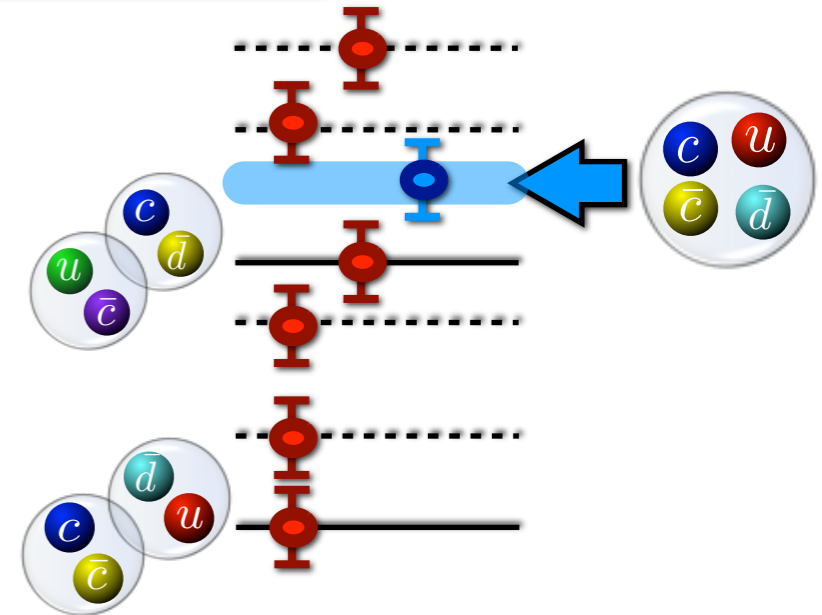
How to study $Z_c(3900)$ on the lattice?

◆ Conventional approach: LQCD spectrum

➔ identify all relevant $W_n(L)$ ($n=0,1,2,3,\dots$)



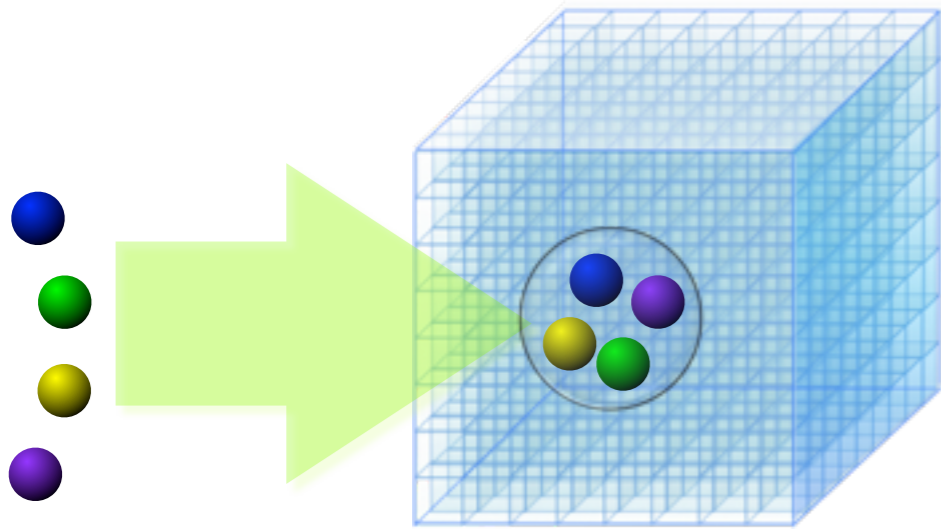
$$\langle 0 | \Phi[c\bar{c}u\bar{d}] (\tau) | W_n \rangle = e^{-W_n \tau}$$



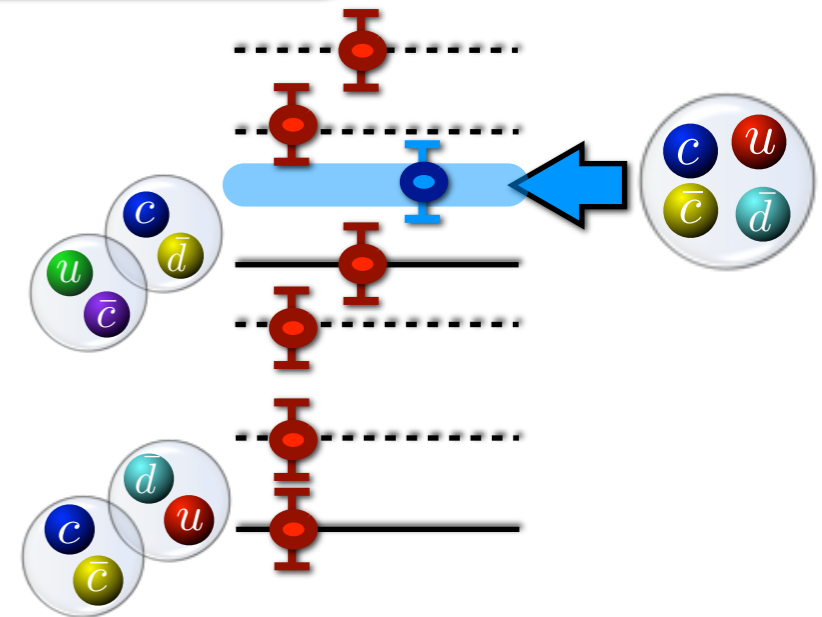
How to study $Z_c(3900)$ on the lattice?

◆ Conventional approach: LQCD spectrum

➔ identify all relevant $W_n(L)$ ($n=0,1,2,3,\dots$)



$$\langle 0 | \Phi [c\bar{c}u\bar{d}] (\tau) | W_n \rangle = e^{-W_n \tau}$$



✓ No positive evidence for $Z_c(3900)$ in $J^{PC}=1^{+-}$

S. Prelovsek et al., PLB 727, 172 (2013).

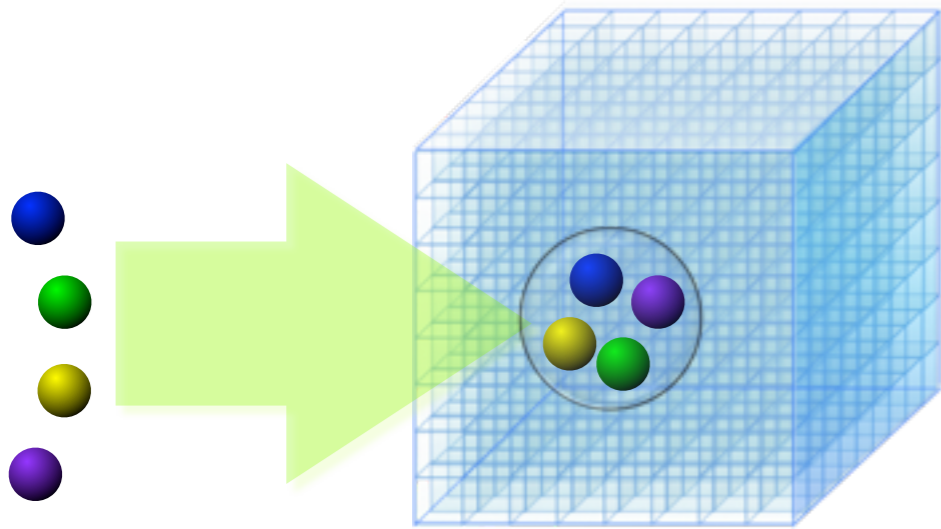
S.-H. Lee et al., PoS Lattice2014 (2014).

S. Prelovsek et al., PRD91, 014504 (2015).

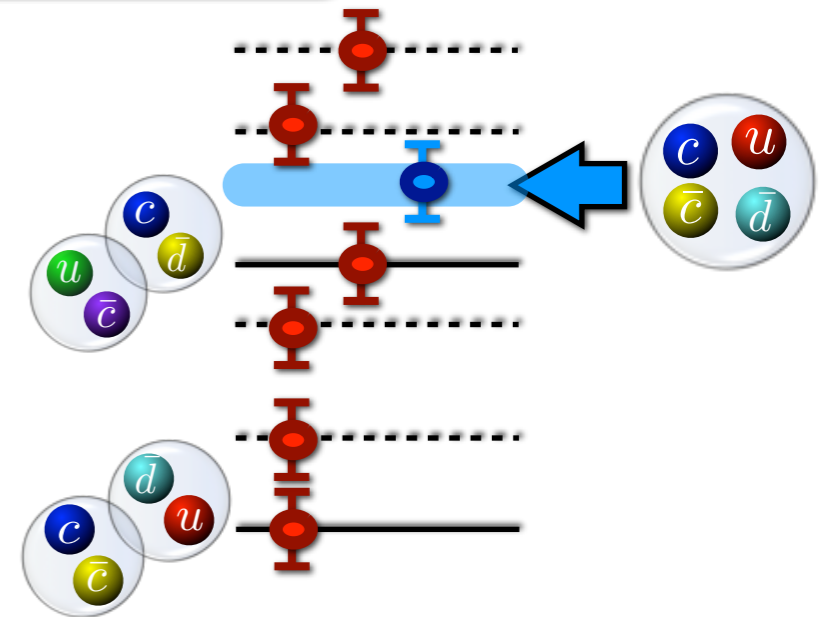
How to study $Z_c(3900)$ on the lattice?

◆ Conventional approach: LQCD spectrum

➔ identify all relevant $W_n(L)$ ($n=0,1,2,3,\dots$)



$$\langle 0 | \Phi[c\bar{c}ud\bar{d}](\tau) | W_n \rangle = e^{-W_n \tau}$$



✓ No positive evidence for $Z_c(3900)$ in $J^{PC}=1^{+-}$

[S. Prelovsek et al., PLB 727, 172 \(2013\).](#)

[S.-H. Lee et al., PoS Lattice2014 \(2014\).](#)

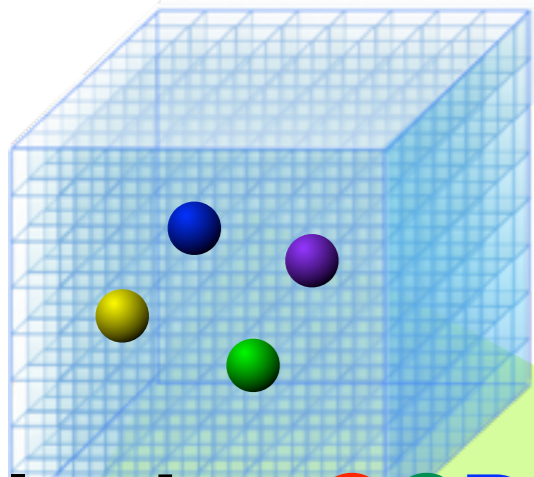
[S. Prelovsek et al., PRD91, 014504 \(2015\).](#)

★ Why is the peak observed in expt.?

- broad resonance? threshold effect?

➔ To understand expt. signals for exotics from QCD is very challenging

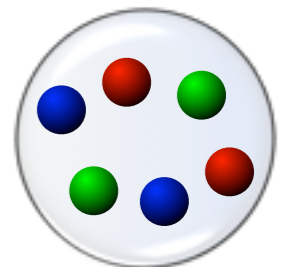
Why is resonance study so hard?



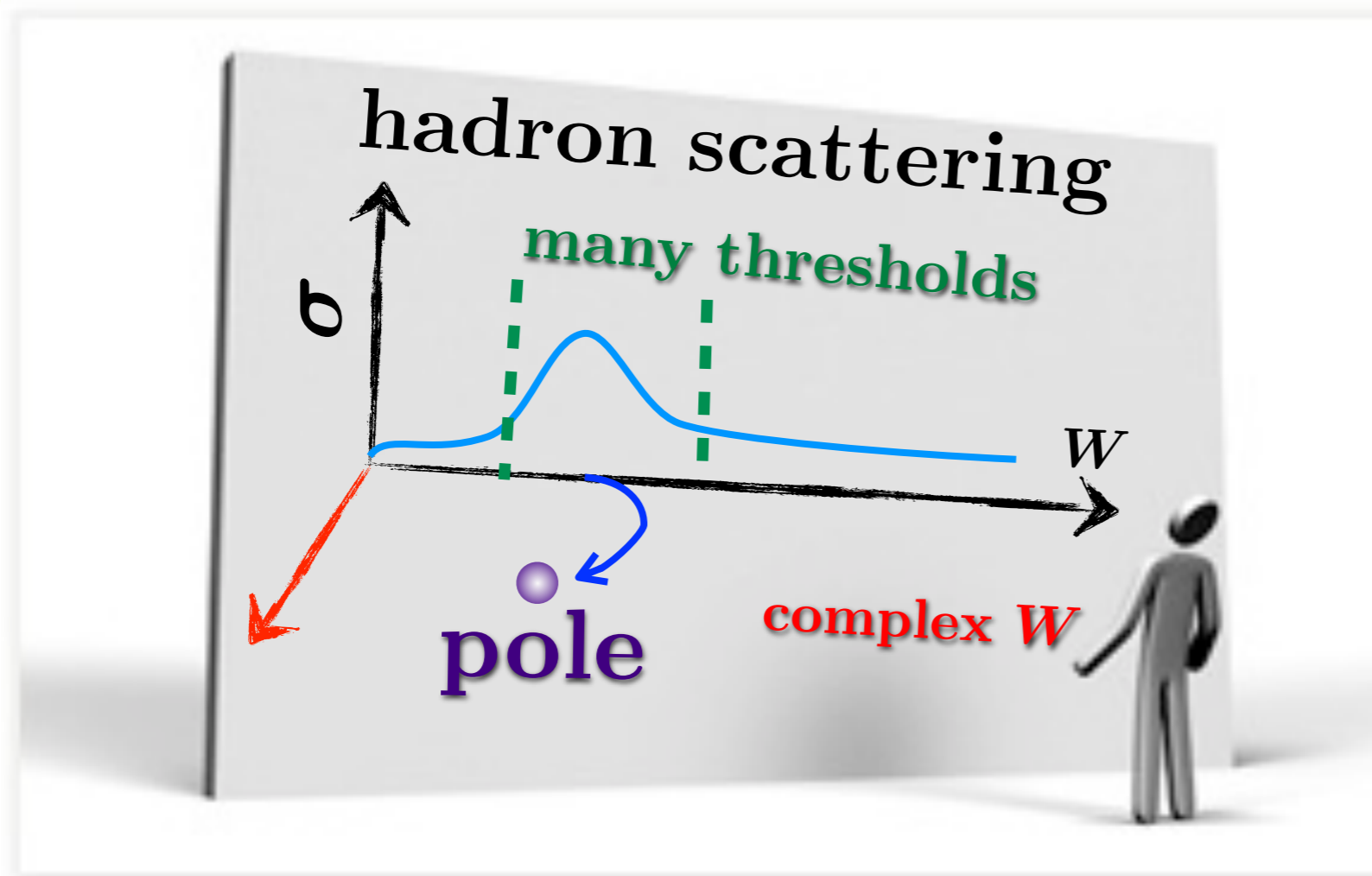
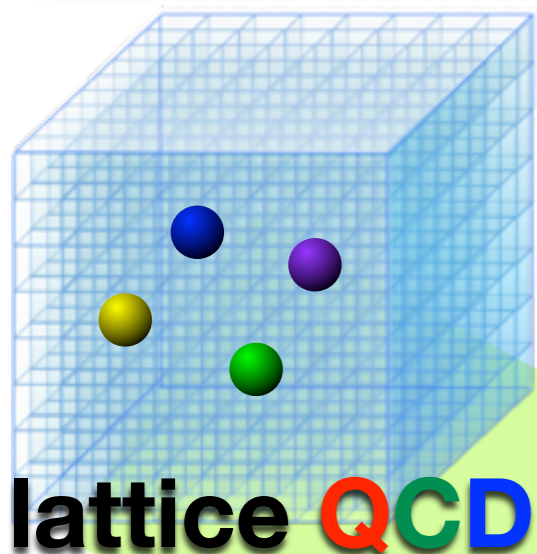
lattice QCD

lattice spectroscopy

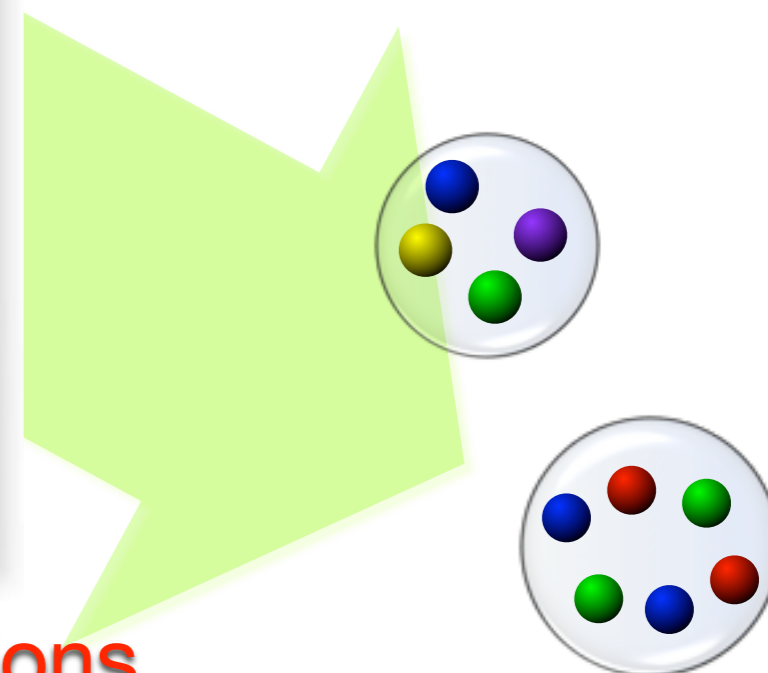
exotic hadrons
~ resonances



Why is resonance study so hard?



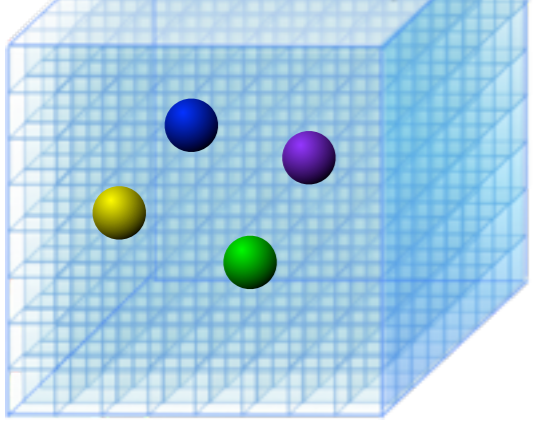
exotic hadrons
~ resonances



Key is coupled-channel hadronic interactions

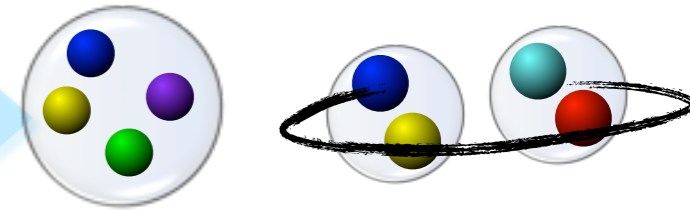
How to extract hadron interactions?

lattice QCD



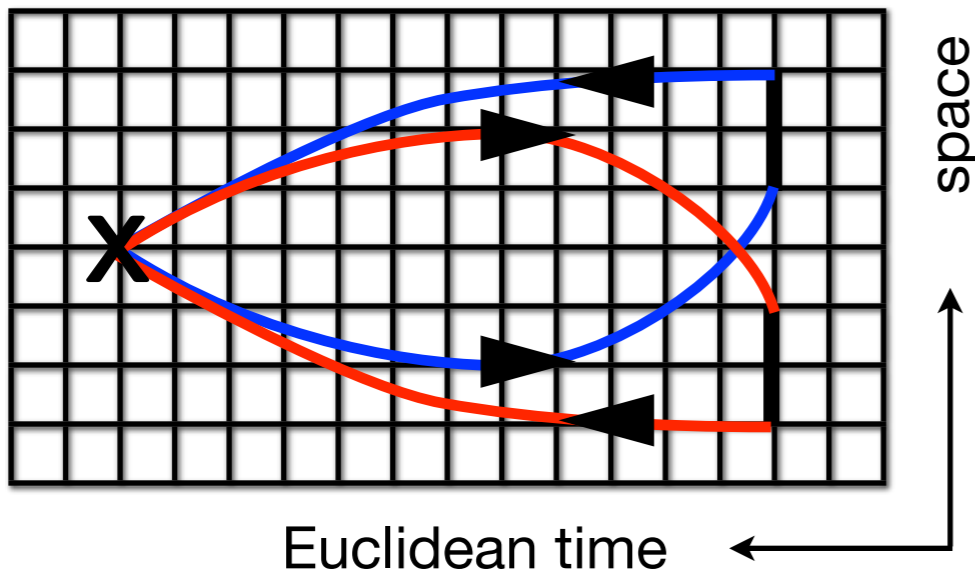
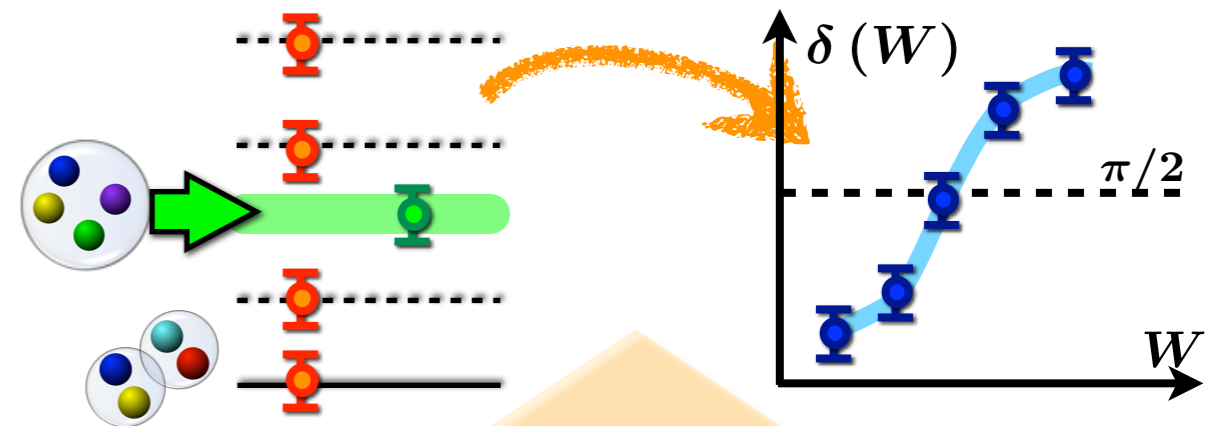
hadron interaction
faithful to S-matrix

✓ exotics, molecules



★ single channel scattering

$$\langle 0 | \Phi(\tau) \Phi^\dagger(0) | 0 \rangle = \sum_n A_n e^{-W_n \tau}$$



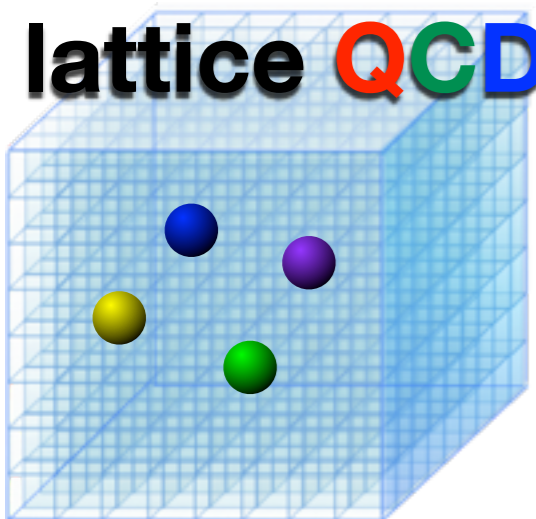
❖ Lüscher's formula Lüscher, NPB354 (1991).

► finite V spectrum --> phase shift $\delta(W_n)$

$$k_n \cot \delta(k_n) = \frac{4\pi}{L^3} \sum_{m \in \mathbb{Z}^3} \frac{1}{\vec{p}_m^2 - k_n^2}$$

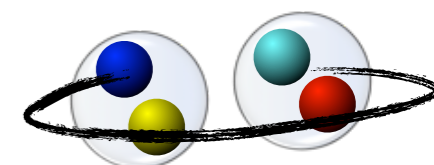
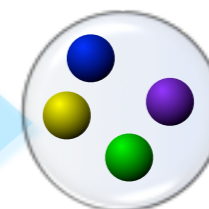
Problem in coupled-channel scattering

lattice QCD



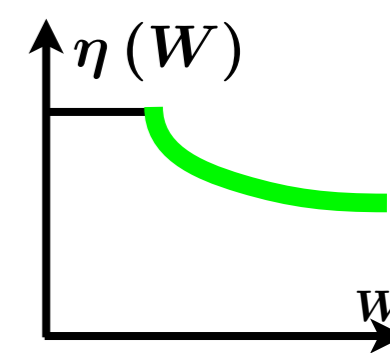
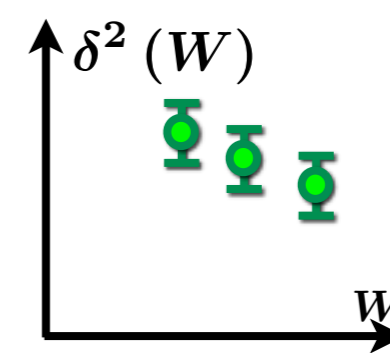
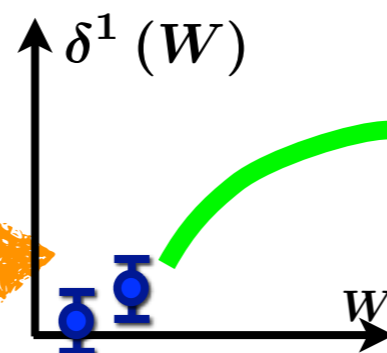
hadron interaction
faithful to S-matrix

✓ exotics, molecules



★ coupled-channel scattering

$$\langle 0 | \Phi(\tau) \Phi^\dagger(0) | 0 \rangle = \sum_n A_n e^{-W_n \tau}$$



➔ coupled-channel Lüscher's formula

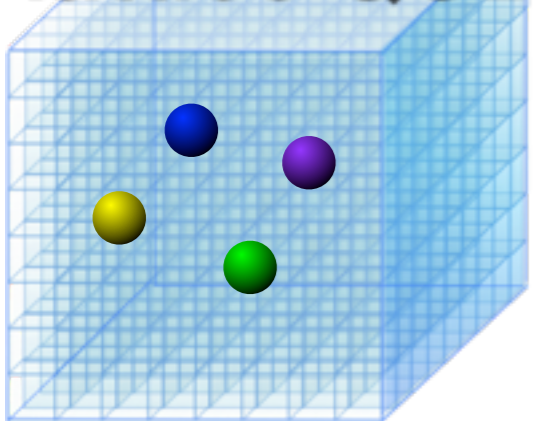
elastic region: $W \rightarrow \delta(W)$

inelastic region: $W \rightarrow \delta^1(W), \delta^2(W), \eta(W) \rightarrow$ find $W(L_1)=W(L_2)=W(L_3)$

➔ **assumptions** about interactions or K-matrices necessary...

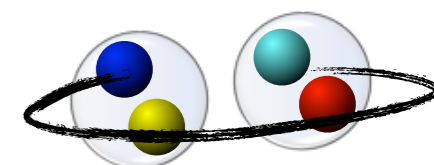
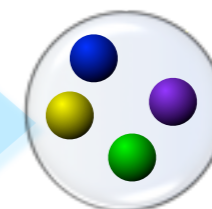
Problem in coupled-channel scattering

lattice QCD



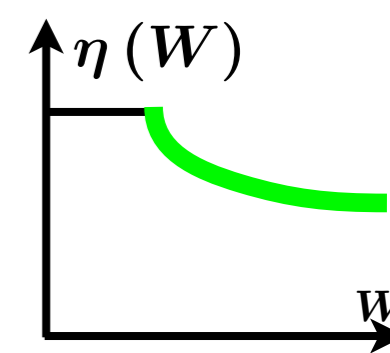
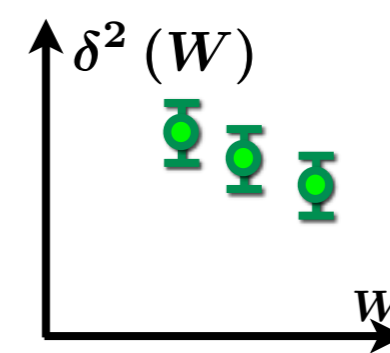
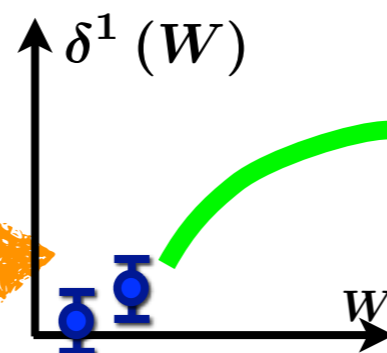
hadron interaction
faithful to S-matrix

✓ exotics, molecules



★ coupled-channel scattering

$$\langle 0 | \Phi(\tau) \Phi^\dagger(0) | 0 \rangle = \sum_n A_n e^{-W_n \tau}$$



➔ coupled-channel Lüscher's formula

elastic region: $W \rightarrow \delta(W)$

inelastic region: $W \rightarrow \delta^1(W), \delta^2(W), \eta(W) \rightarrow$ find $W(L_1)=W(L_2)=W(L_3)$

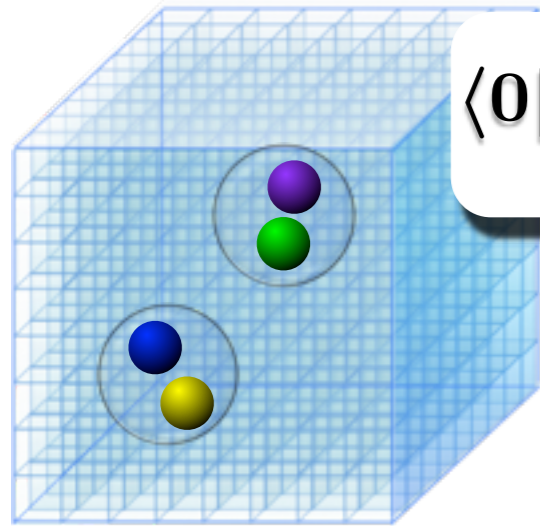
➔ **assumptions** about interactions or K-matrices necessary...

★ indicate **more information mandatory** to solve coupled-channel scatterings

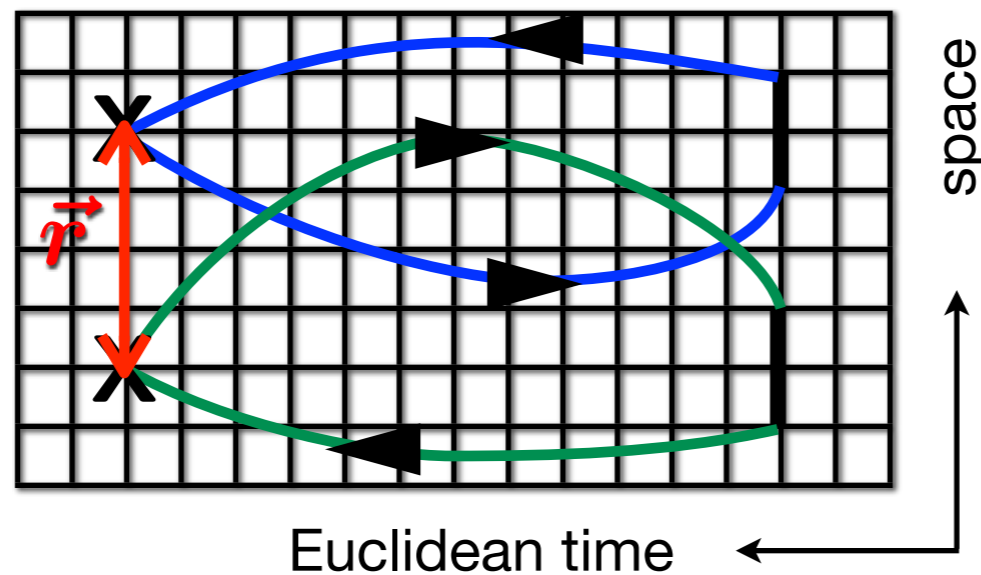
➔ What can we measure in addition to temporal correlations?

HAL QCD approach “potential” as representation of S-matrix

- ◆ HAL QCD approach: extract **energy-independent** interaction kernel
 - ➔ measure **spatial** correlation as well as temporal correlation



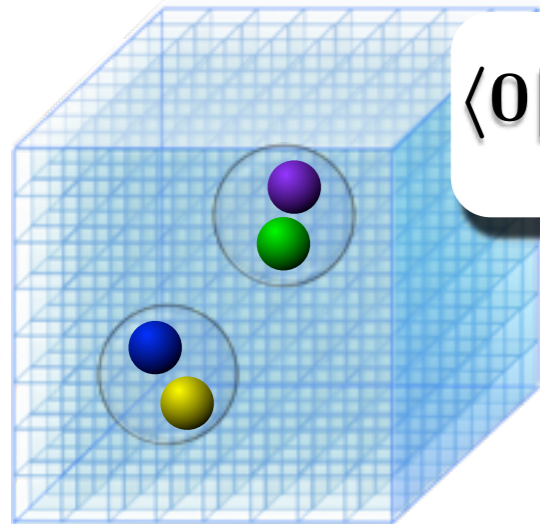
$$\langle 0 | \phi_1(\vec{x} + \vec{r}, \tau) \phi_2(\vec{x}, \tau) \Phi^\dagger(0) | 0 \rangle = \sqrt{Z_1 Z_2} \sum_n A_n \psi_n(\vec{r}) e^{-W_n \tau}$$



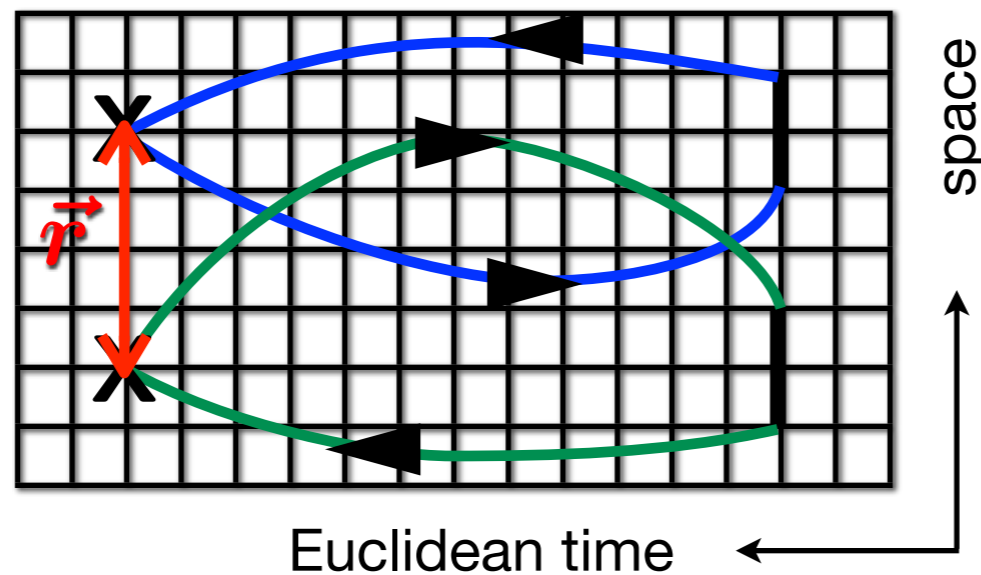
Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al, (HAL QCD), PLB712, 437(2012).

HAL QCD approach “potential” as representation of S-matrix

- ◆ HAL QCD approach: extract **energy-independent** interaction kernel
 - ➔ measure **spatial** correlation as well as temporal correlation



$$\langle 0 | \phi_1(\vec{x} + \vec{r}, \tau) \phi_2(\vec{x}, \tau) \Phi^\dagger(0) | 0 \rangle = \sqrt{Z_1 Z_2} \sum_n A_n \psi_n(\vec{r}) e^{-W_n \tau}$$



Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al, (HAL QCD), PLB712, 437(2012).

★ Nambu-Bethe-Salpeter wave functions: $\psi_n(r)$

- ▶ NBS wave functions outside interactions --> **Helmholtz equation**

$$\left(\nabla^2 + \vec{k}_n^2 \right) \psi_n(\vec{r}) = 0 \quad (|\vec{r}| > R) \quad \Rightarrow \text{S-matrix}$$

Nambu-Bethe-Salpeter wave function

Full details, see, [Aoki, Hatsuda, Ishii, PTP123, 89 \(2010\)](#).

Equal-time choice of NBS amplitudes (e.g., $\pi\pi$ scattering)

$$\begin{aligned}\Psi(\vec{x}_1, t; \vec{x}_2, t) &\equiv \langle 0 | \pi(x_1) \pi(x_2) | \pi(\vec{k}) \pi(-\vec{k}); in \rangle \\ &= \psi_{\pi\pi}(\vec{r}; W) e^{-iWt}\end{aligned}$$

$$W = 2\sqrt{m_\pi^2 + \vec{k}^2}$$

Outside interactions, NBS amplitudes satisfy non-interacting Klein-Gordon equations:

$$\begin{aligned}(\partial_t^2 - \nabla_i^2 + m_\pi^2) \Psi(\vec{x}_1, t; \vec{x}_2, t) &= 0 \quad (i = 1, 2) \\ (\nabla_r^2 + \vec{k}^2) \psi_{\pi\pi}(\vec{r}; W) &= 0\end{aligned}$$

🔊 NBS wave functions satisfy Helmholtz equation

🔊 Asymptotic form of NBS wave function:

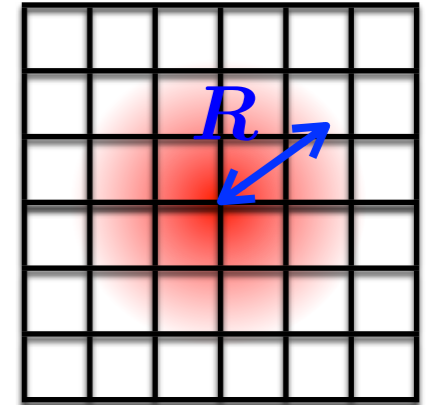
$$\psi_{\vec{k}}^{(l)}(r) \sim \frac{e^{i\delta_l(k)}}{kr} \sin(kr + \delta_l(k) - l\pi/2) \quad \text{--> faithful to scattering phase shift}$$

NBS wave function in quantum field theory is the best analogue to wave function in quantum mechanics

“Potential” as representation of S-matrix

- NBS wave func. in interacting region --> **half-off-shell T-matrix**

$$(\nabla^2 + \vec{k}_n^2)\psi_n(\vec{r}) = 2\mu\mathcal{K}_n(\vec{r}) \quad (|\vec{r}| < R)$$

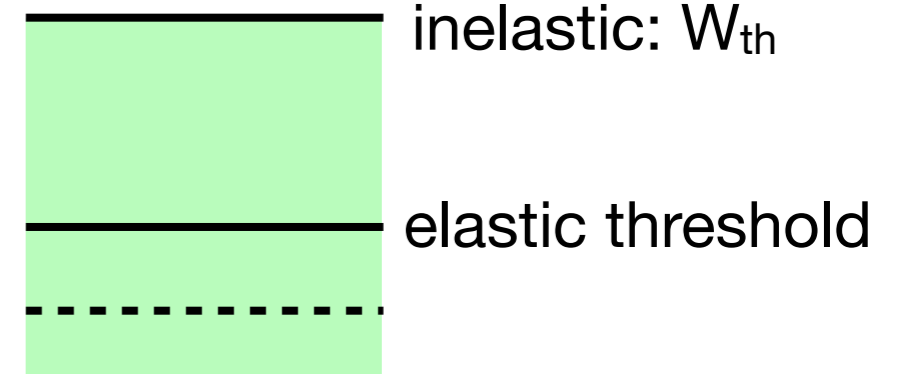


- Energy-independent** potentials (faithful to phase shifts)

$$U(\vec{r}, \vec{r}') = \sum_n^{W_{th}} \mathcal{K}_n(\vec{r}) \overline{\psi}_n(\vec{r}')$$



★ U(r,r') contains all 2PI contributions



above W_{th} : coupled-channel analysis

- “Potentials” become kernels of Schrödinger-type equations

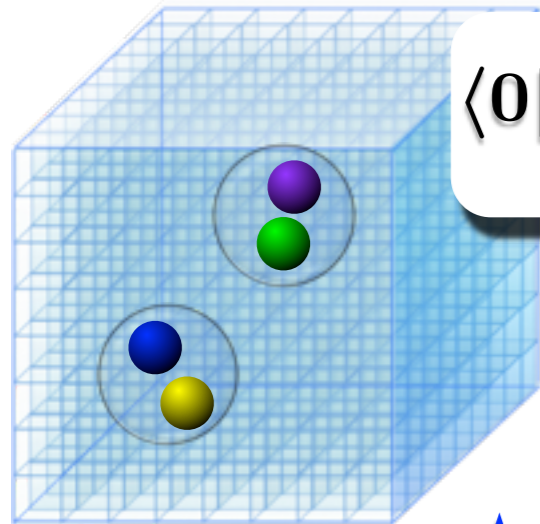
$$\left(\nabla^2 + \vec{k}_n^2\right)\psi_n(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r}, \vec{r}')\psi_n(\vec{r}')$$

[Ishii, Aoki, Hatsuda, PRL99, 02201 \(2007\).](#)

[Aoki, Hatsuda, Ishii, PTP123, 89 \(2010\).](#)

HAL QCD approach “potential” as representation of S-matrix

- ◆ HAL QCD approach: extract **energy-independent** interaction kernel
 - ➔ measure **spatial** correlation as well as temporal correlation

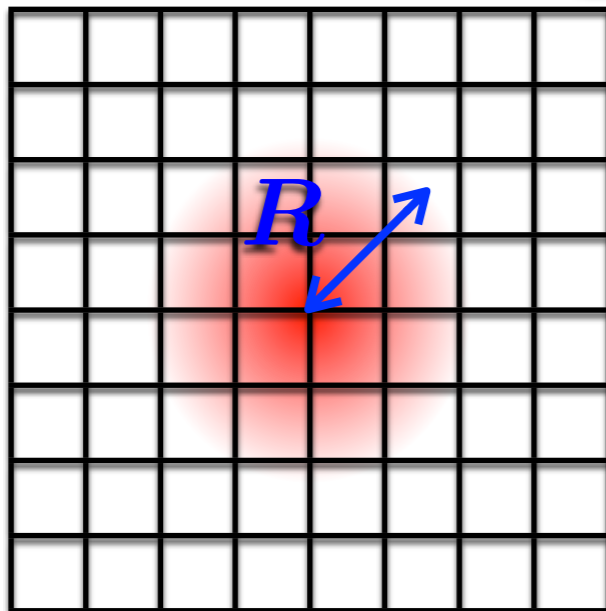


$$\langle 0 | \phi_1(\vec{x} + \vec{r}, \tau) \phi_2(\vec{x}, \tau) \Phi^\dagger(0) | 0 \rangle = \sqrt{Z_1 Z_2} \sum_n A_n \psi_n(\vec{r}) e^{-W_n \tau}$$

Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al, (HAL QCD), PLB712, 437(2012).

- ★ NBS wave functions inside interactions: **half-offshell T-matrix**

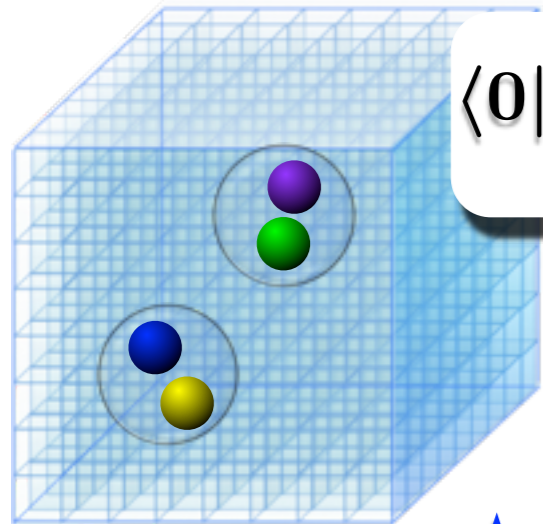
$$\left(\nabla^2 + \vec{k}_n^2 \right) \psi_n(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r}, \vec{r}') \psi_n(\vec{r}')$$



- $U(r, r')$ is faithful to S-matrix in elastic region
- $U(r, r')$ is energy-independent (until new threshold opens)
- $U(r, r')$ contains all 2PI contributions
- $U(r, r')$ is not an observable (applied to ab initio calc.)

Coupled-channel HAL QCD approach

- ◆ HAL QCD approach: extract **energy-independent** interaction kernel
 - ➔ measure **spatial** correlation as well as temporal correlation



$$\langle 0 | \phi_1^a(\vec{x} + \vec{r}, \tau) \phi_2^a(\vec{x}, \tau) \Phi^\dagger(0) | 0 \rangle = \sqrt{Z_1^a Z_2^a} \sum_n A_n \psi_n^a(\vec{r}) e^{-W_n \tau}$$

Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al, (HAL QCD), PLB712, 437(2012).

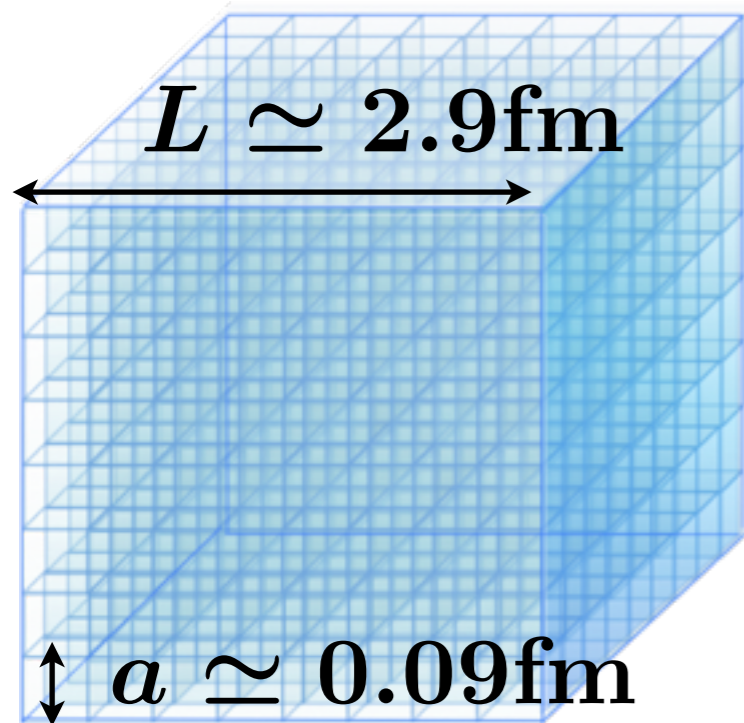
- ★ channel wave functions defined in asymptotic region: $\psi_n^a(r)$

$$\left(\nabla^2 + (\vec{k}_n^a)^2 \right) \psi_n^a(\vec{r}) = 2\mu^a \sum_b \int d\vec{r}' U^{ab}(\vec{r}, \vec{r}') \psi_n^b(\vec{r}')$$

- ★ **coupled-channel potential** $U^{ab}(r, r')$:

- $U^{ab}(r, r')$ is faithful to S-matrix in both elastic and inelastic regions
- $U^{ab}(r, r')$ is energy-independent (until new threshold opens)
- $U^{ab}(r, r')$ contains all 2PI contributions

Lattice QCD setup



★ $N_f=2+1$ full QCD

PACS-CS Coll., S. Aoki et al., PRD79, 034503, (2009).

- Iwasaki gauge & $O(a)$ -improved Wilson quark actions
- $a=0.0907(13) \text{ fm}$ $\rightarrow L \sim 2.9 \text{ fm}$ ($32^3 \times 64$)

★ Relativistic Heavy Quark action for charm

S. Aoki et al., PTP109, 383 (2003).

Y. Namekawa et al., PRD84, 074505 (2011).

- remove leading cutoff errors $O((m_c a)^n)$, $O(\Lambda_{\text{QCD}} a)$, ...

→ We are left with $O((a\Lambda_{\text{QCD}})^2)$ syst. error (\sim a few %)

Light meson mass (MeV)

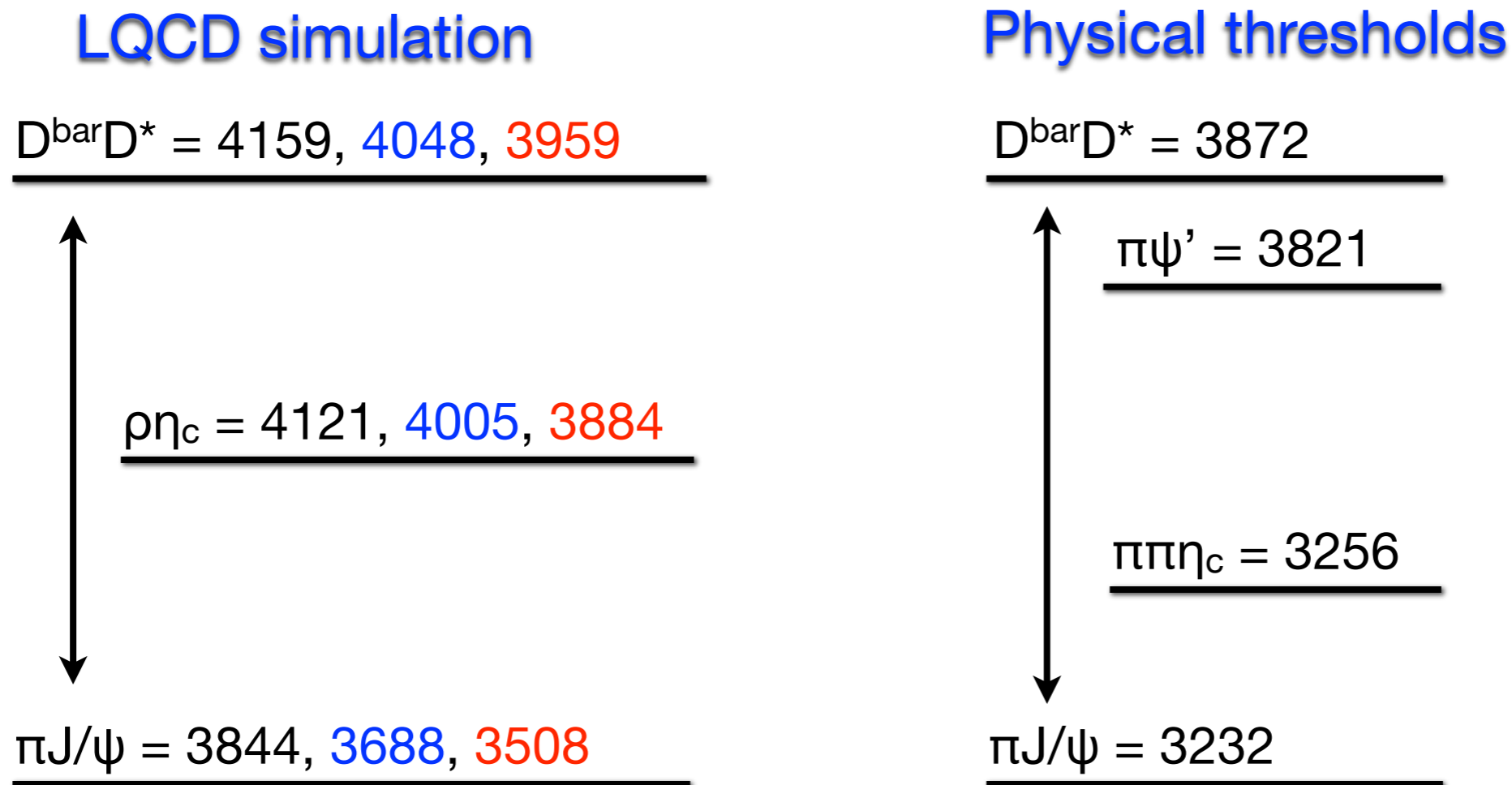
$$\begin{aligned} m_\pi &= 411(1), 572(1), 701(1) \\ m_K &= 635(2), 714(1), 787(1) \\ m_\rho &= 896(8), 1000(5), 1097(4) \end{aligned}$$

Charmed meson mass (MeV)

$$\begin{aligned} m_{\eta_c} &= 2988(1), 3005(1), 3024(1) \\ m_{J/\psi} &= 3097(1), 3118(1), 3143(1) \\ m_D &= 1903(1), 1947(1), 2000(1) \\ m_{D^*} &= 2056(3), 2101(2), 2159(2) \end{aligned}$$

Lattice QCD setup : thresholds

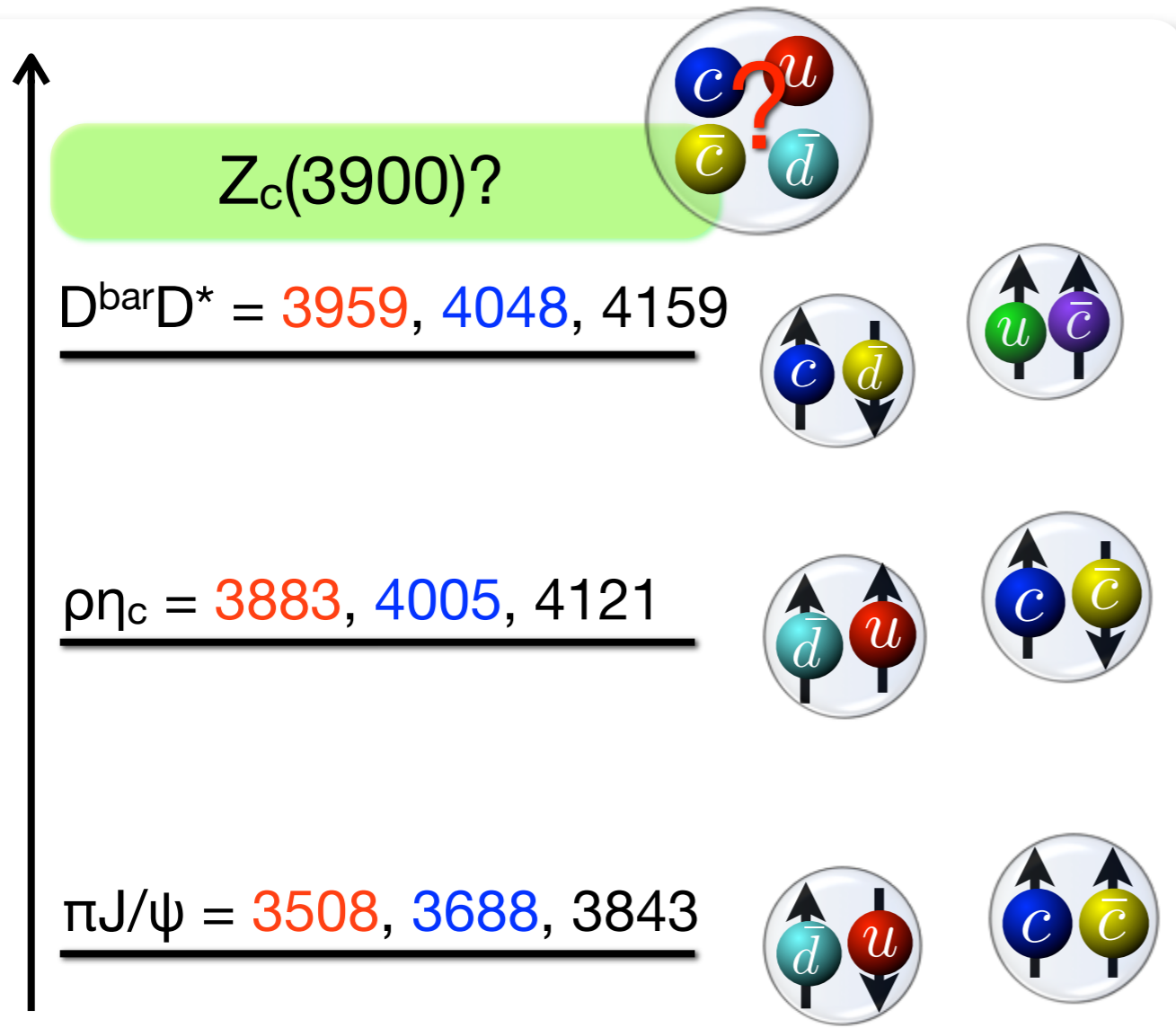
✦ S-wave meson-meson thresholds in $I^G J^{PC}=1+1^{+-}$ channel



- $M_{\pi\psi'} > M_{D^{\text{bar}}D^*}$ due to heavy pion mass
- $\rho \rightarrow \pi\pi$ decay not allowed w/ $L \sim 3\text{fm}$

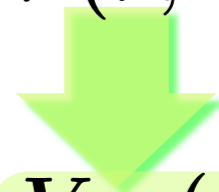
✦ **S-wave $\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$ coupled-channel analysis**
(reliable upto $D^{\text{bar}}D^*$ threshold)

Potential matrix in $I^G(J^{PC})=1^+(1^{+-})$: s-wave $\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$



✓ Velocity expansion:

$$U(\vec{r}, \vec{r}') = V(\vec{r}, \nabla) \delta(\vec{r} - \vec{r}')$$

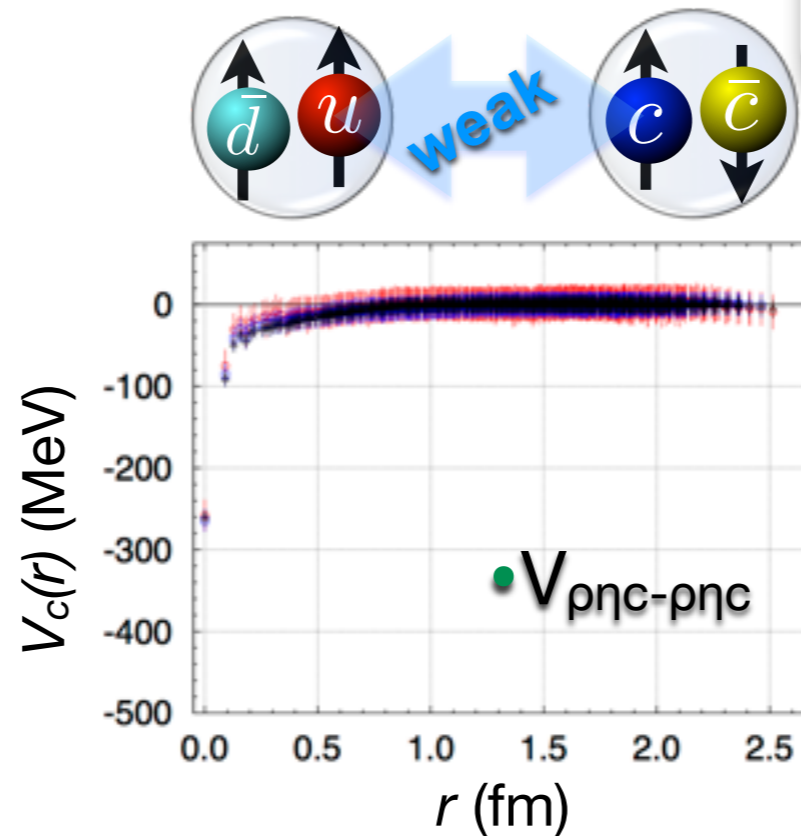
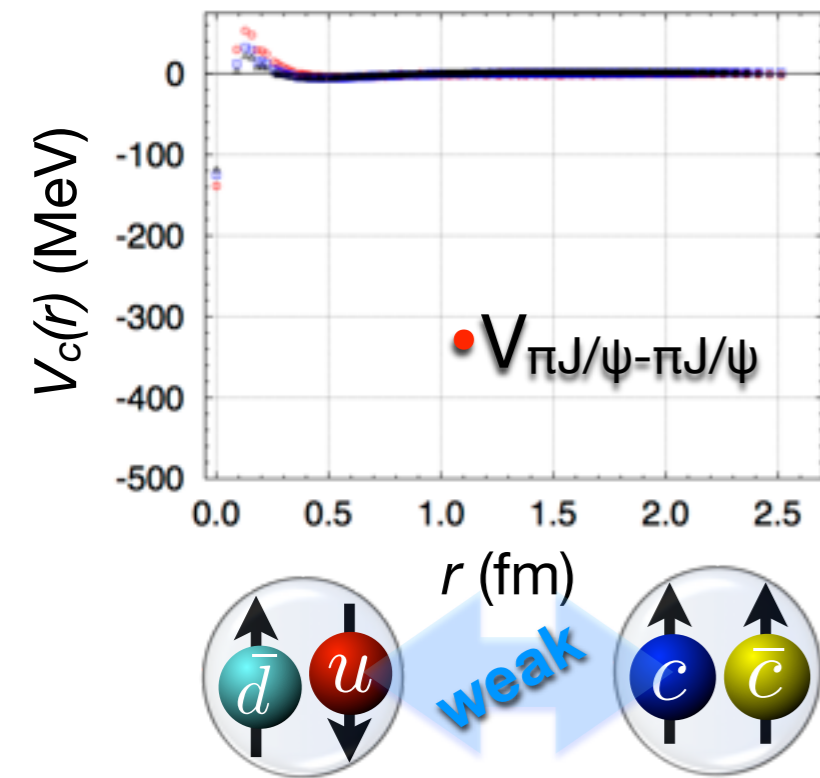


$$V(\vec{r}, \nabla) = V_{\text{LO}}(\vec{r}) + \mathcal{O}(\nabla)$$

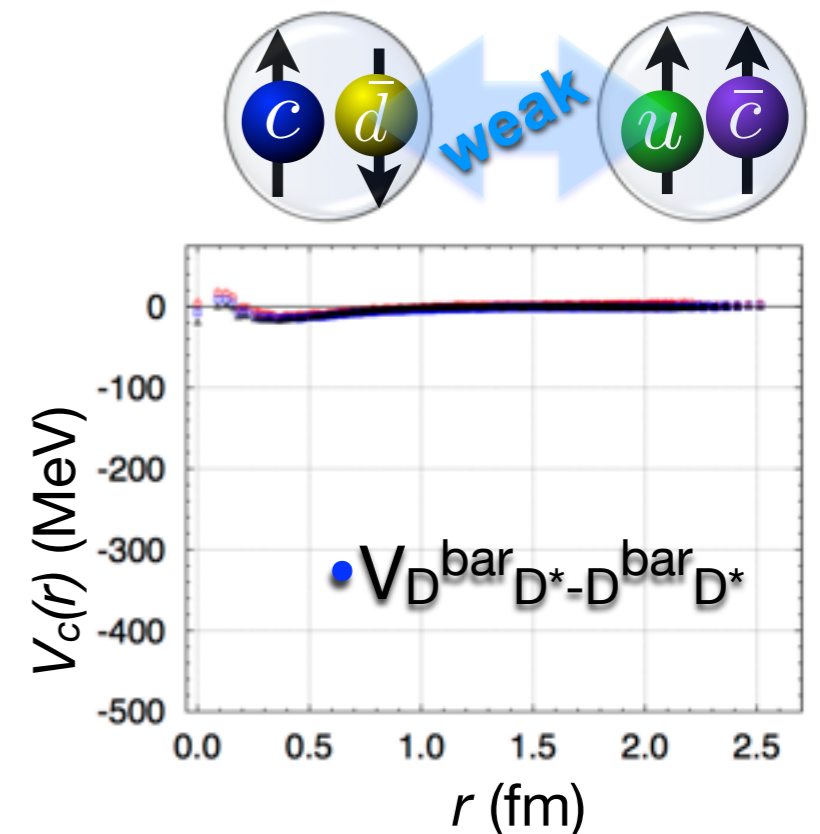
Extract (effective) LO potential :

$$\left(\nabla^2 + (\vec{k}_n^a)^2 \right) \psi_n^a(\vec{r}) = 2\mu^a \sum_b V^{ab}(\vec{r}) \psi_n^b(\vec{r})$$

Potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$)



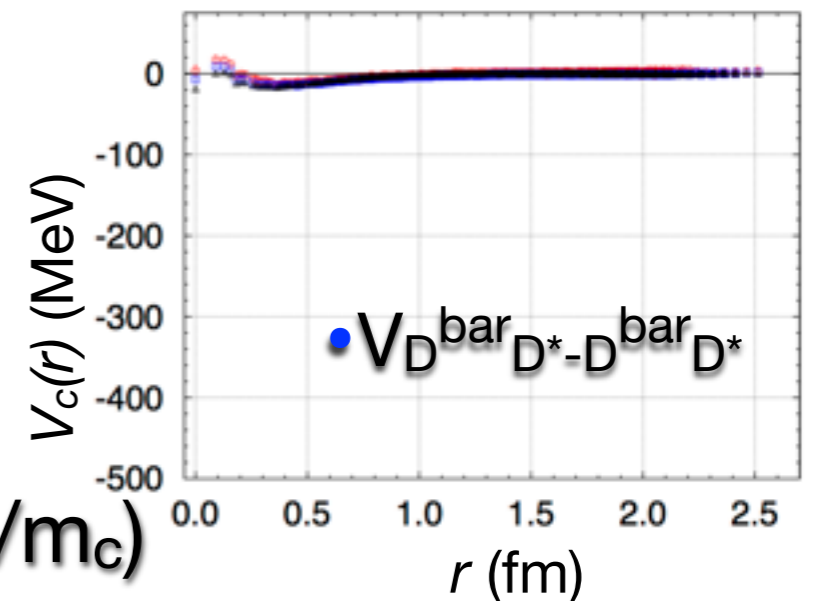
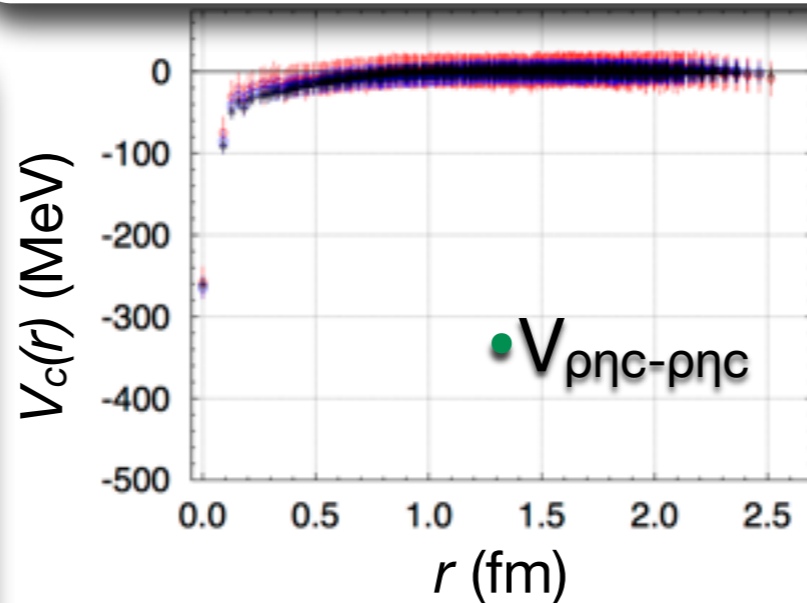
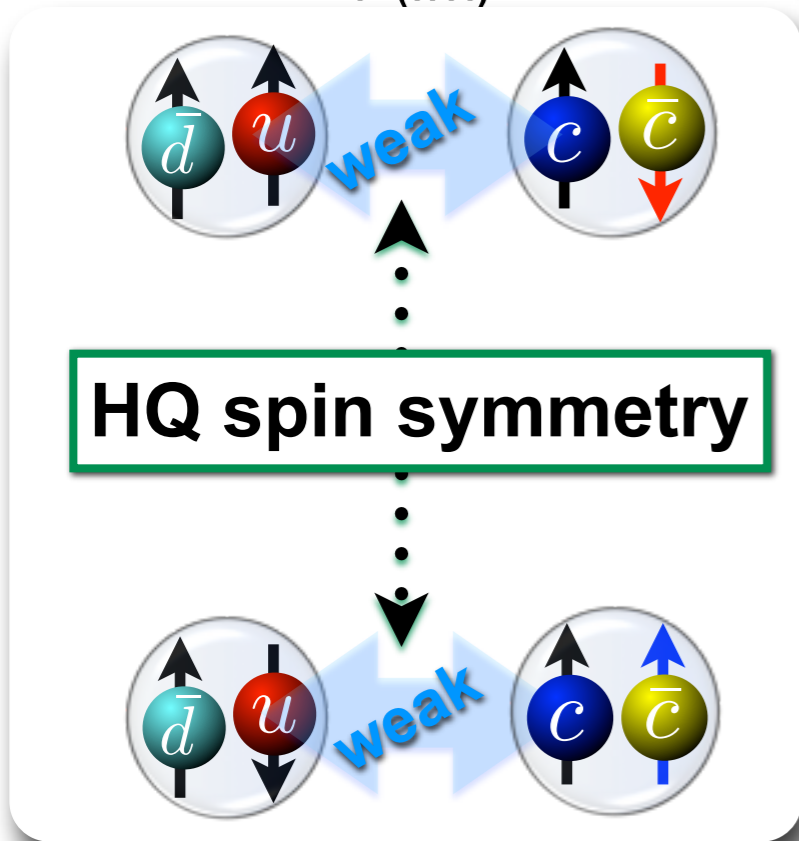
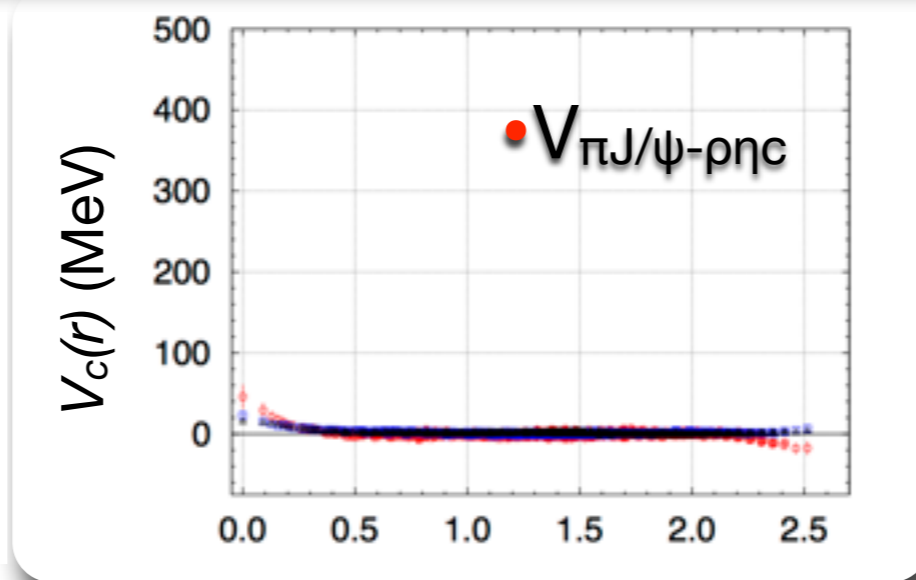
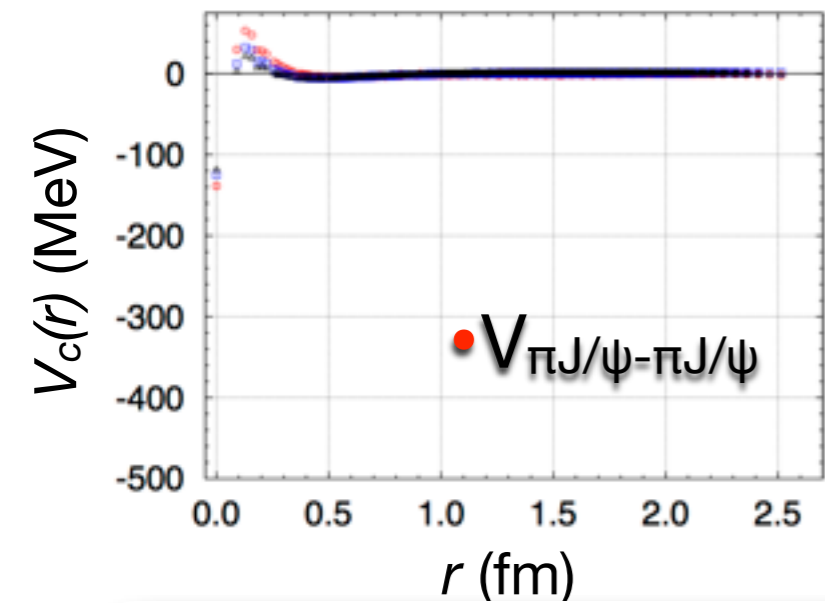
- $m_\pi = 410 \text{ MeV}$ (red)
- $m_\pi = 570 \text{ MeV}$ (blue)
- $m_\pi = 700 \text{ MeV}$ (black)



- All diagonal potentials are weak
- \Rightarrow no bound/resonant $\pi J/\psi$, $D^{\text{bar}}D^*$



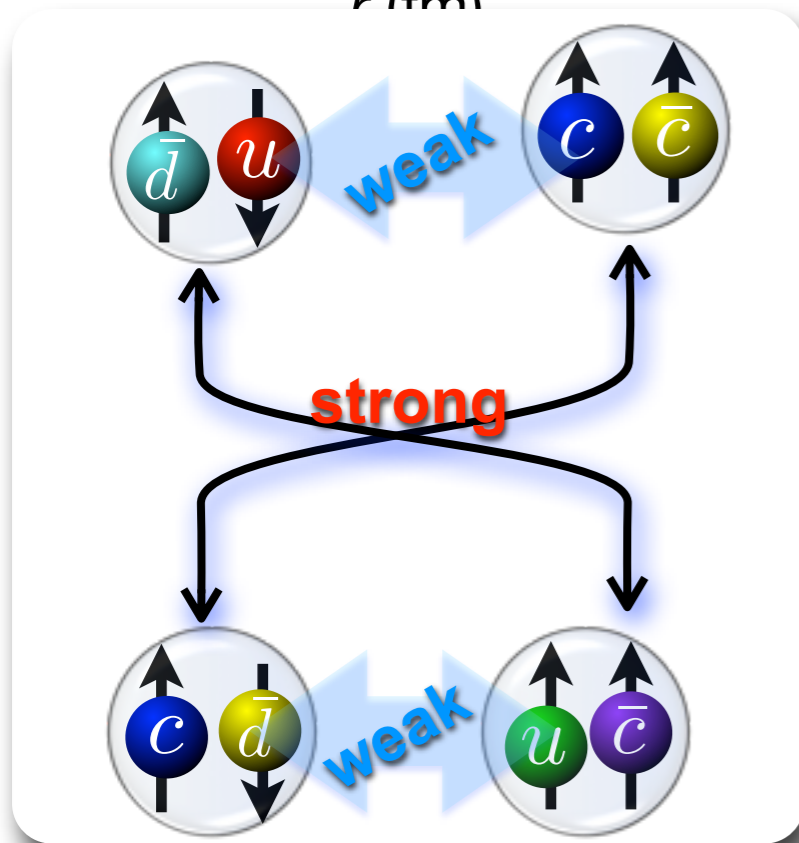
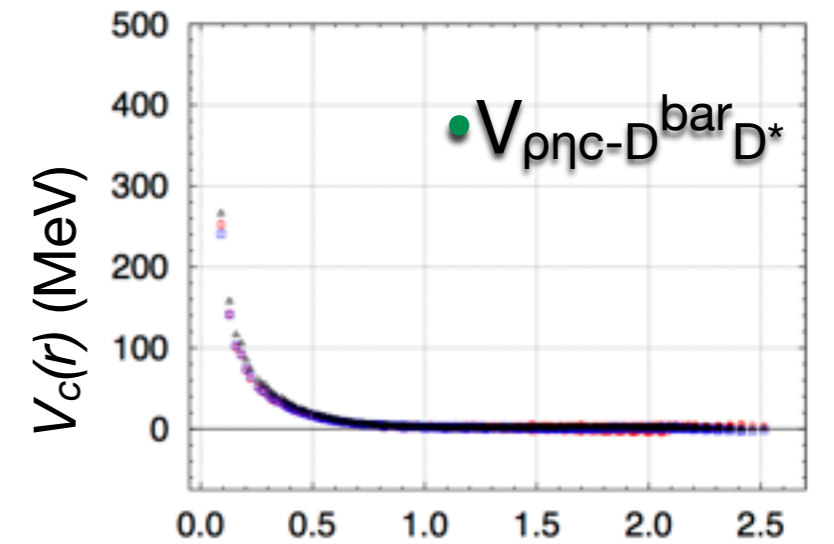
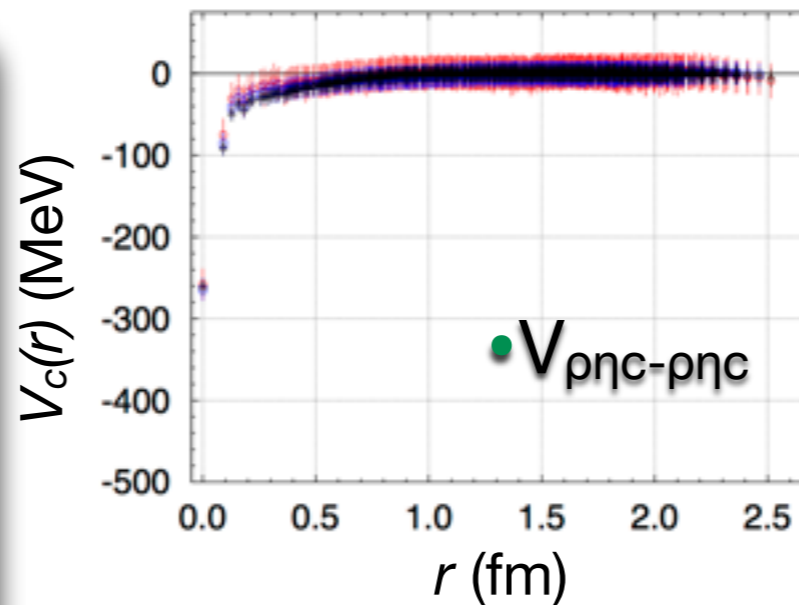
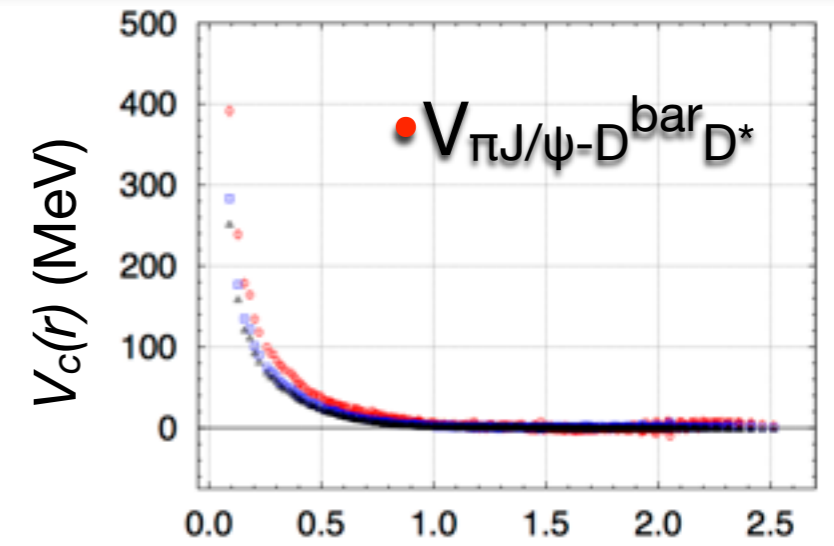
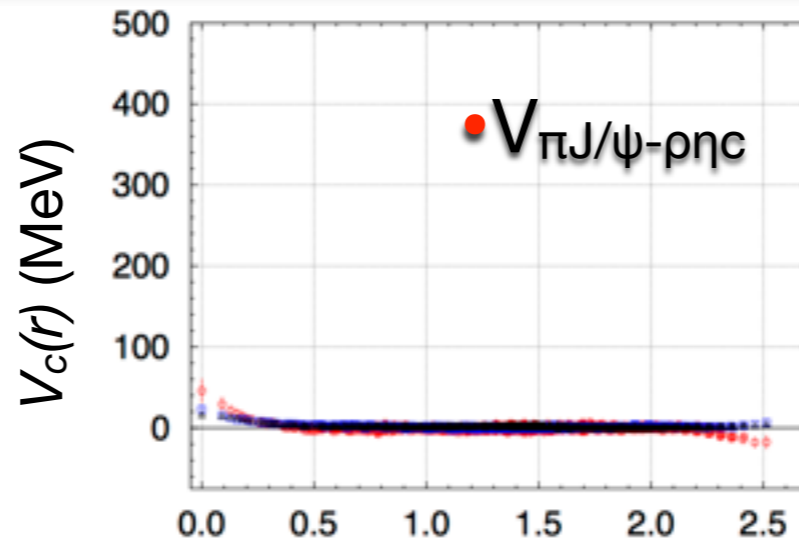
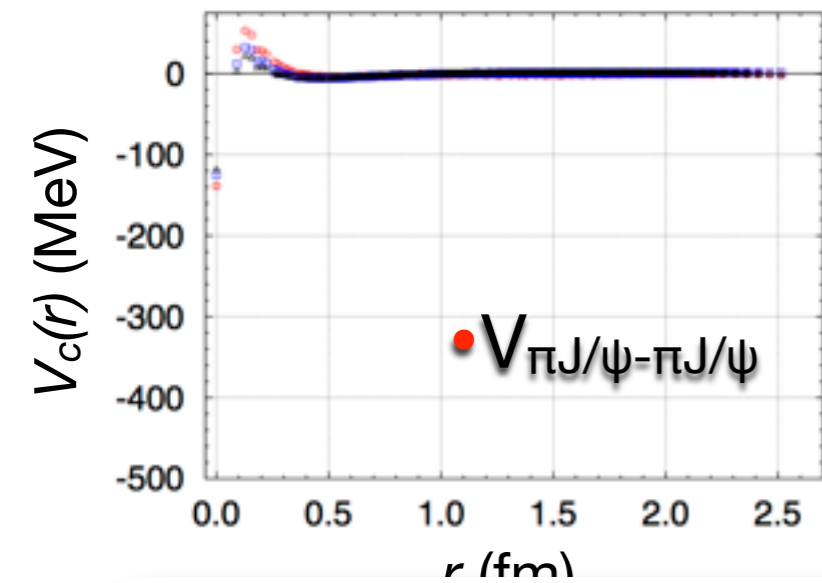
Potential matrix ($\pi J/\psi$ - $\rho \eta_c$ - $D^{\text{bar}} D^*$)



• Weak $\pi J/\psi - \rho \eta_c$ potential

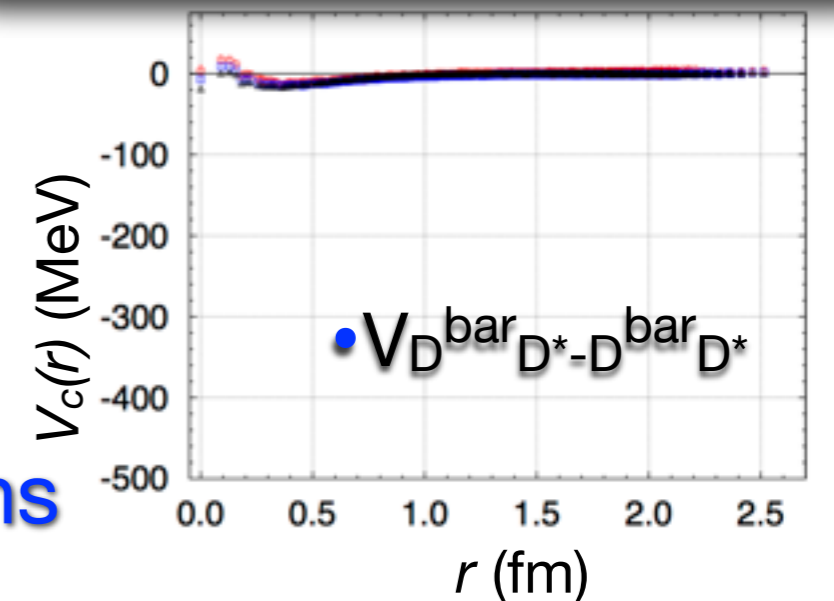
➡ charm quark spin-flip is suppressed by $O(1/m_c)$

Potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$)

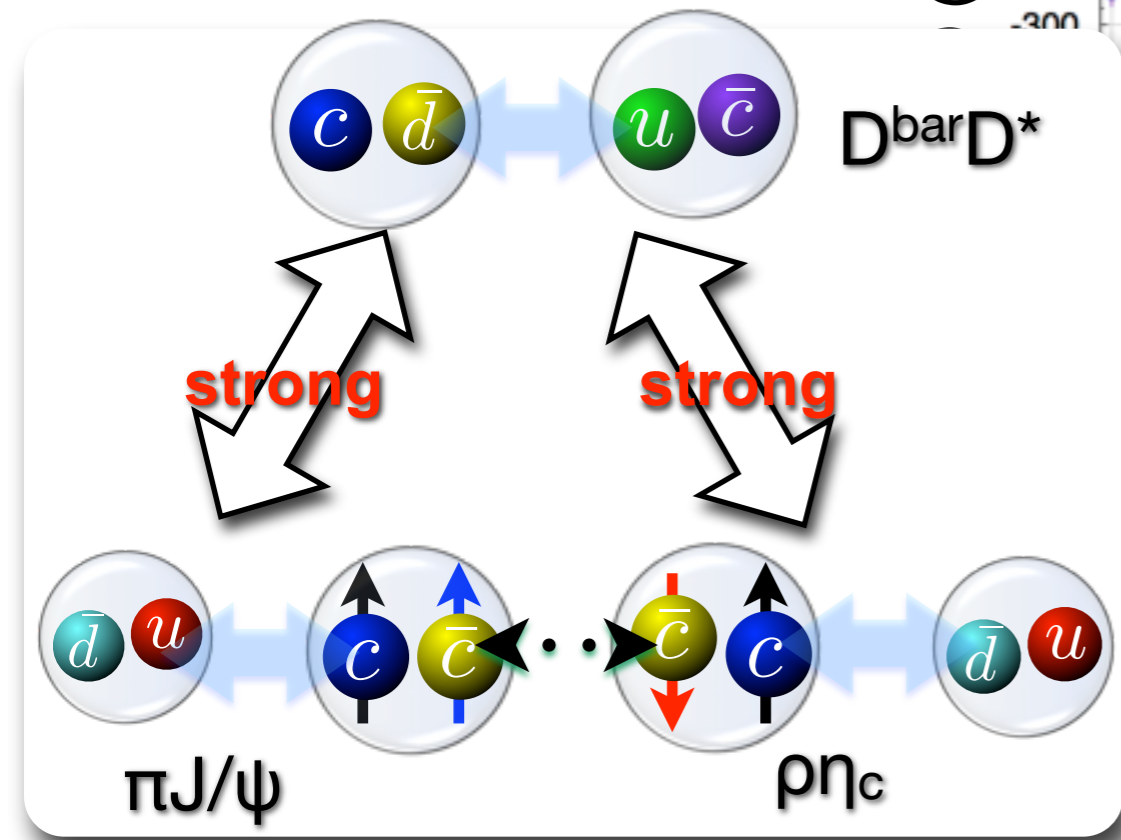
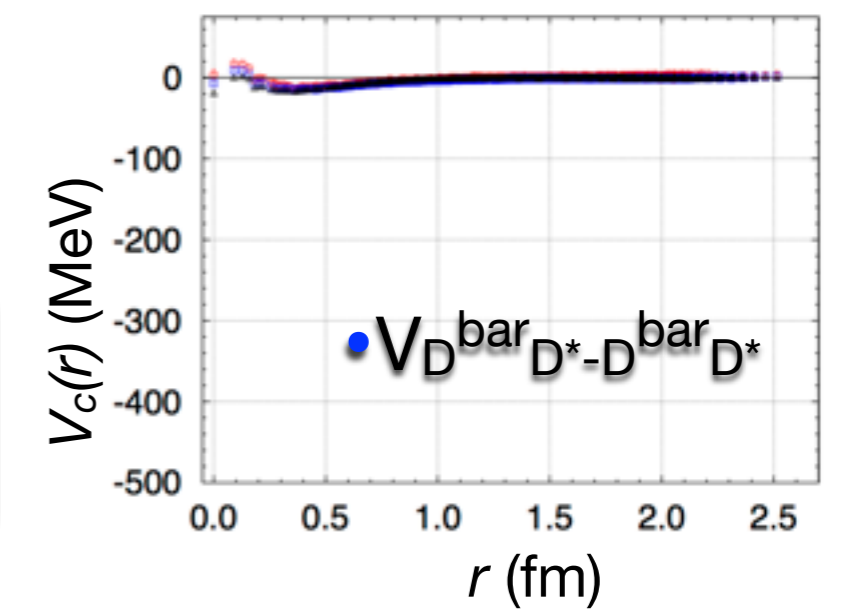
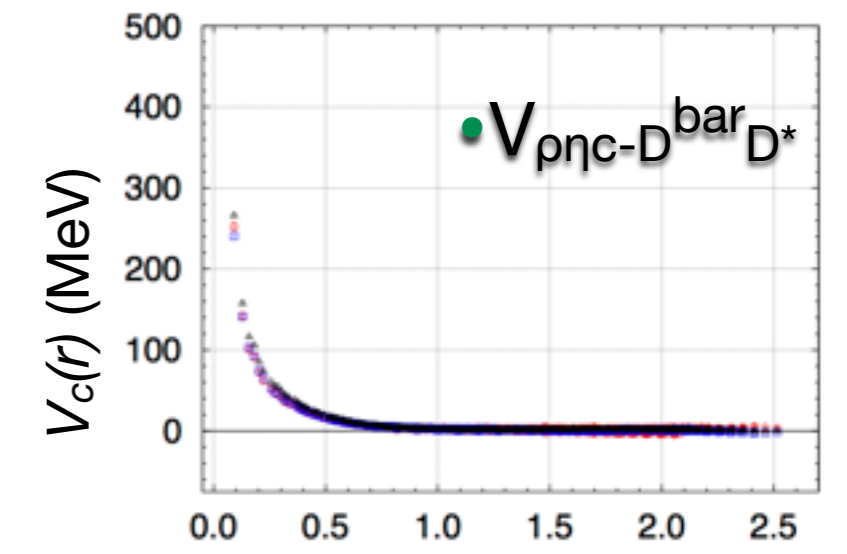
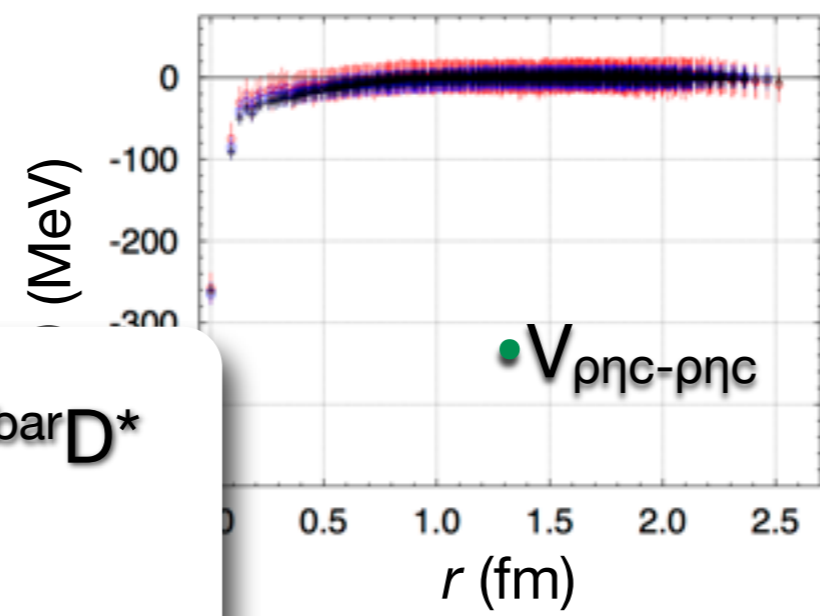
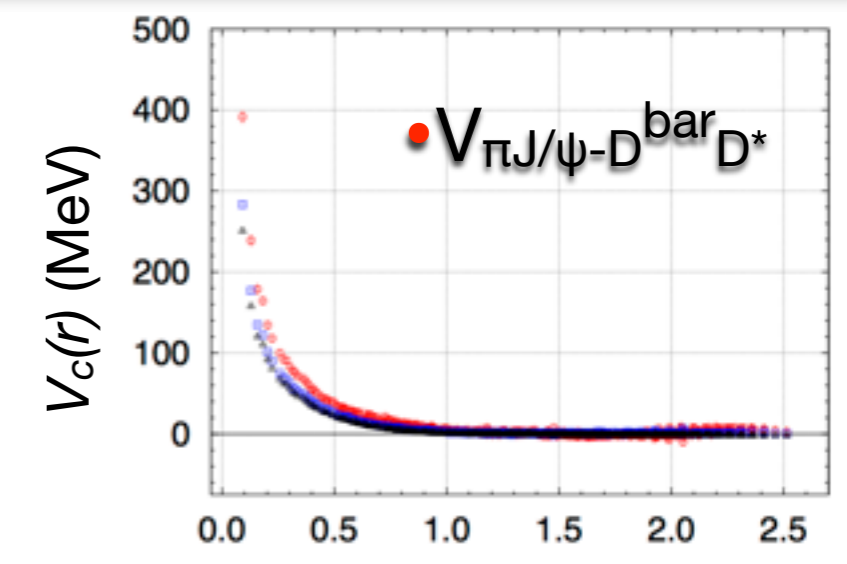
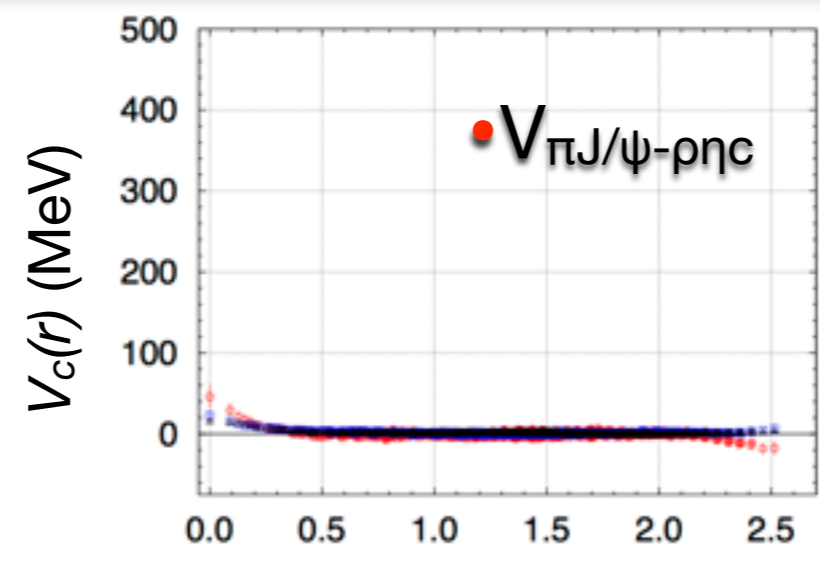
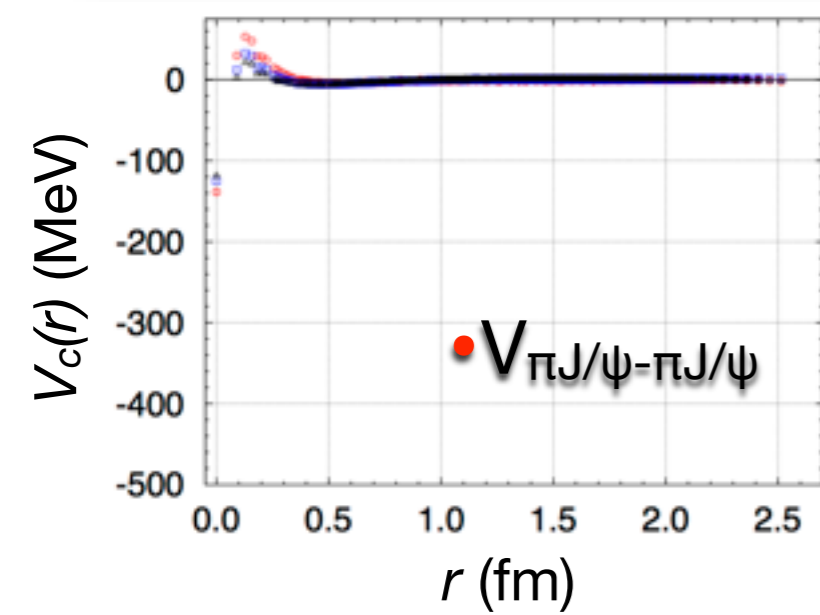


- Strong off-diagonal $D^{\text{bar}}D^*$ potentials

✓ strong charm-quark-exchange interactions



Potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$)

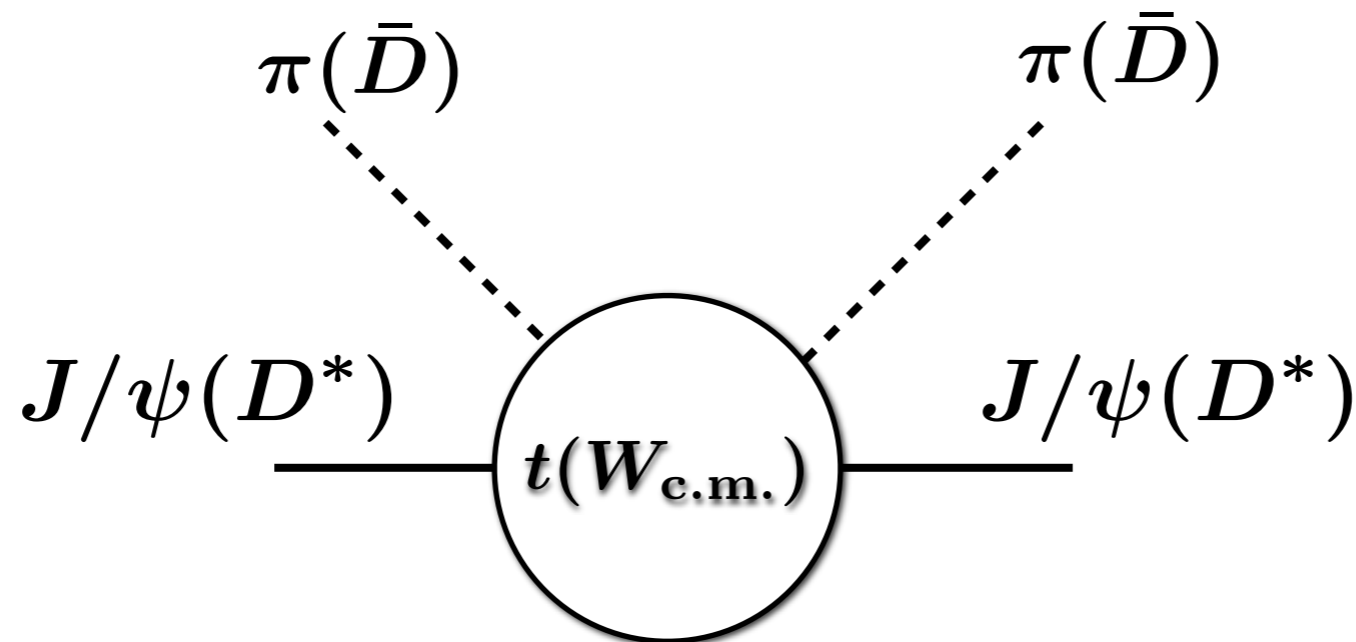


- \bullet $m_\pi = 410 \text{ MeV}$ (red dot)
- \bullet $m_\pi = 570 \text{ MeV}$ (blue dot)
- \bullet $m_\pi = 700 \text{ MeV}$ (black dot)

Two-body observables : structure of $Z_c(3900)$ in $I^G(J^{PC})=1^+(1^{+-})$

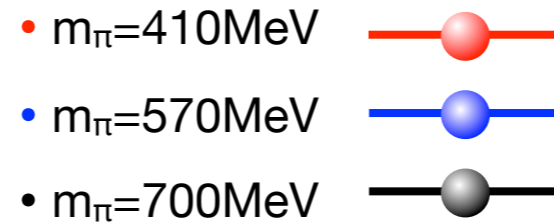
★ Two-body $\pi J/\psi$ & $D^{\text{bar}}D^*$ s-wave scattering

➔ ideal scattering reaction to study structure of $Z_c(3900)$

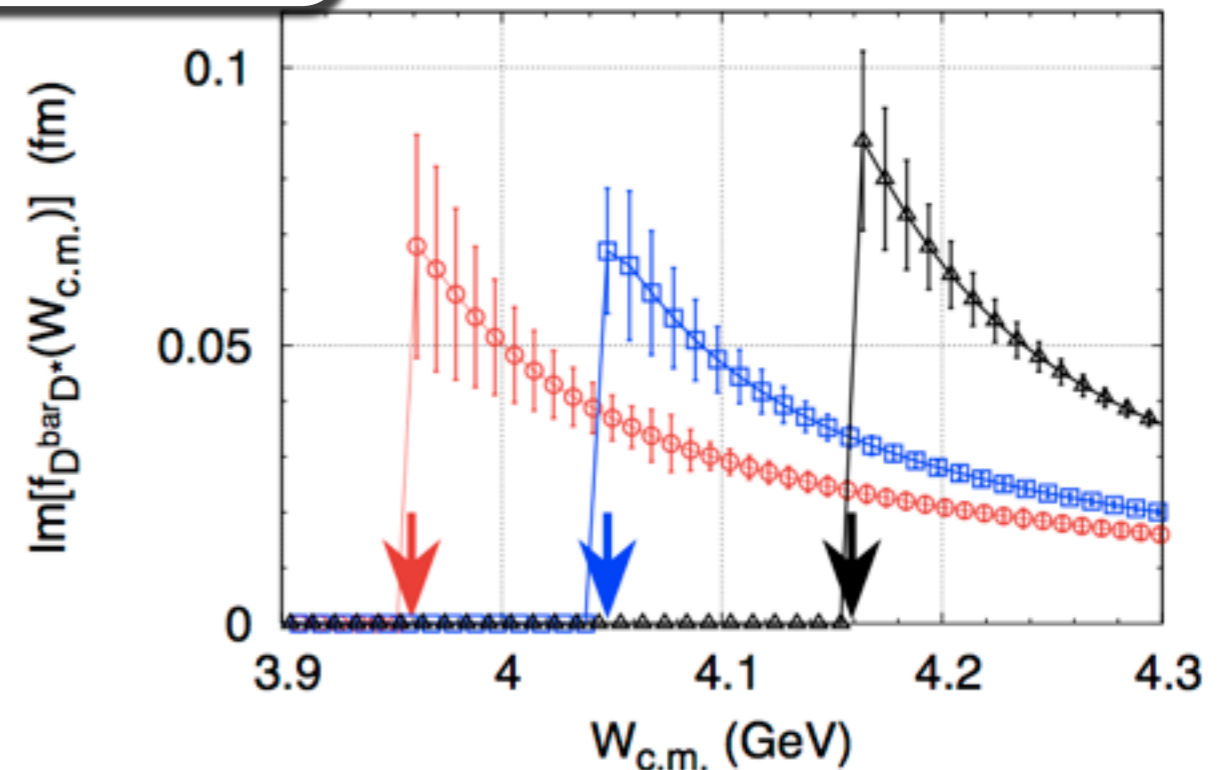
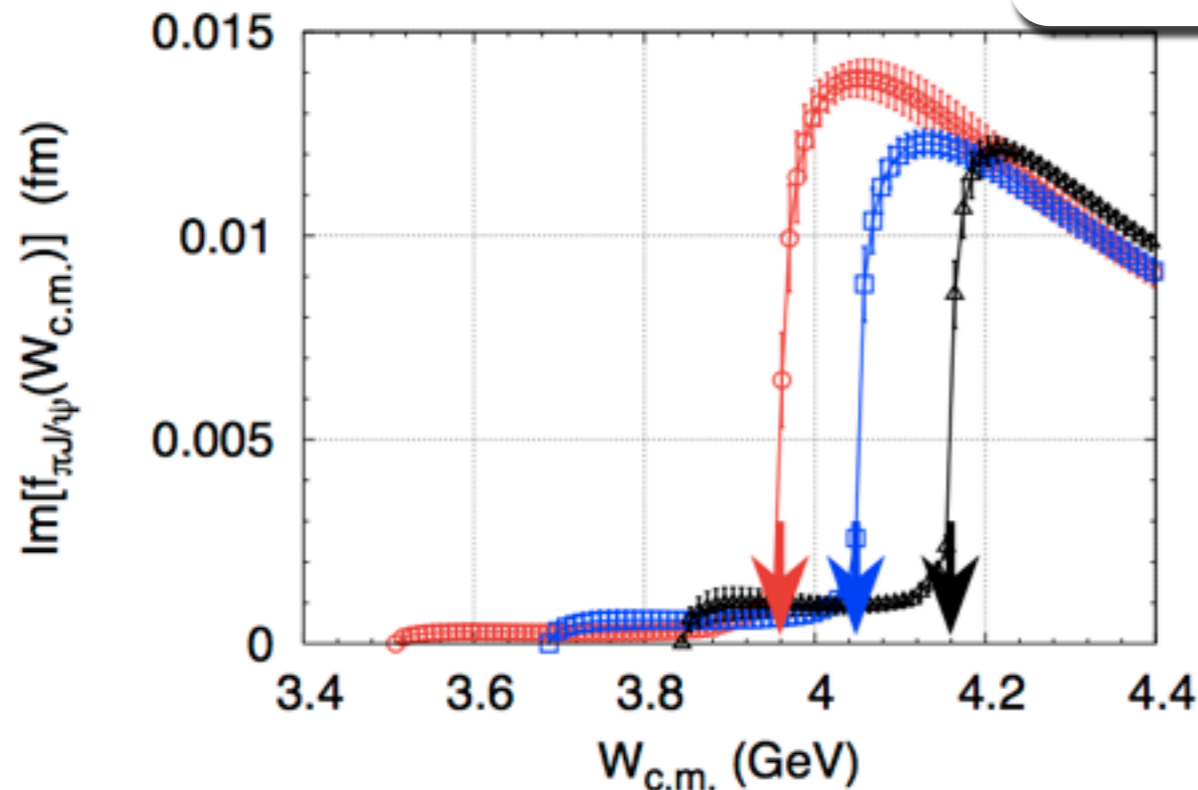


Invariant mass spectra of $\pi J/\psi$ & $D^{\text{bar}}D^*$

- $\pi J/\psi$ invariant mass



- $D^{\text{bar}}D^*$ invariant mass



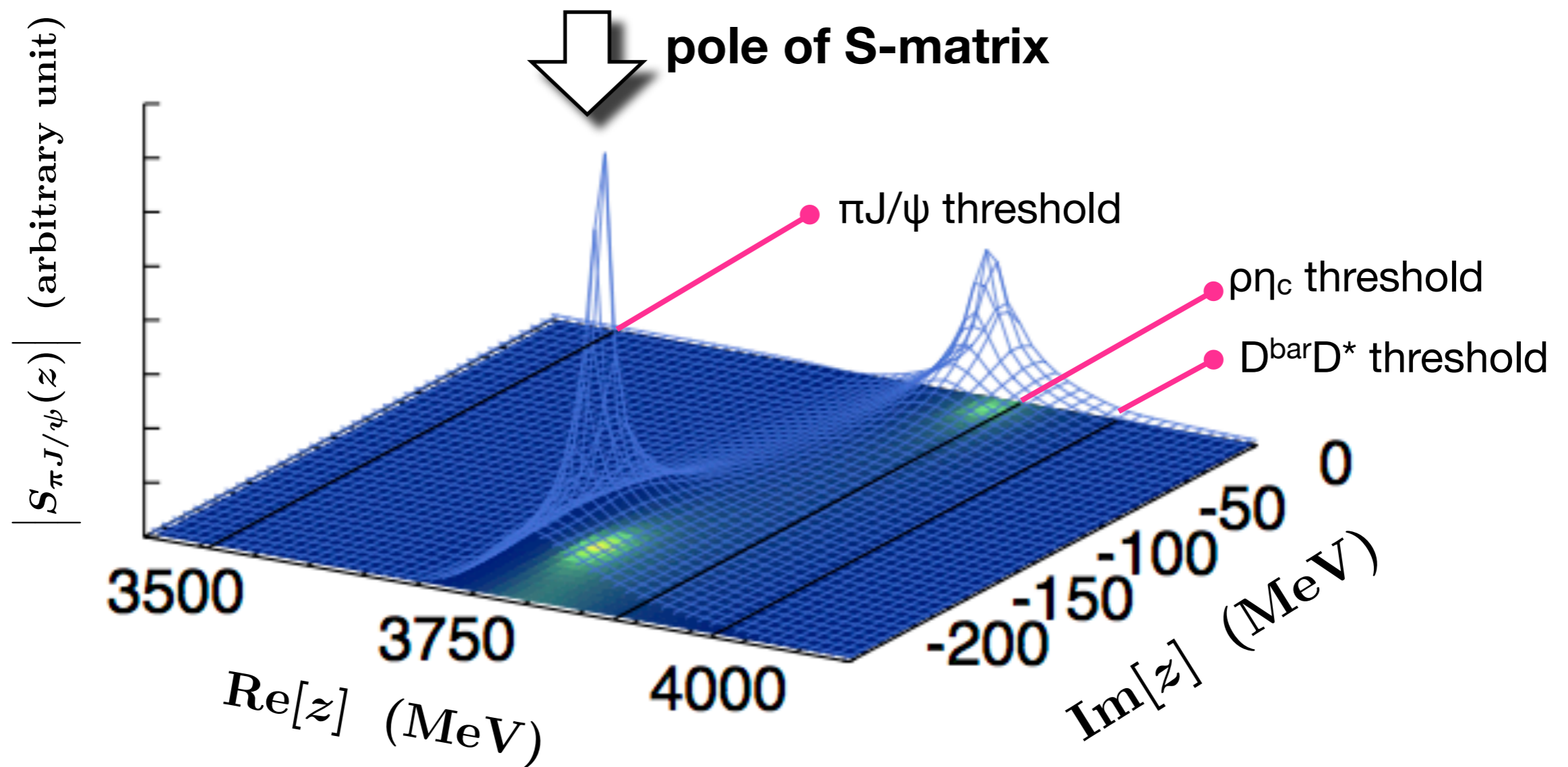
✓ Enhancement near $D^{\text{bar}}D^*$ threshold **due to large $\pi J/\psi$ - $D^{\text{bar}}D^*$ coupling**

- Peak in $\pi J/\psi$ invariant mass (**Not Breit-Wigner line shape**)
- Threshold enhancement in $D^{\text{bar}}D^*$ invariant mass (**cusp behavior**)

(No m_q dependence on qualitative behaviors of line shapes)

➡ Is $Z_c(3900)$ a conventional resonance?

Complex pole position ($\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $D^{\text{bar}}D^*$:2nd)



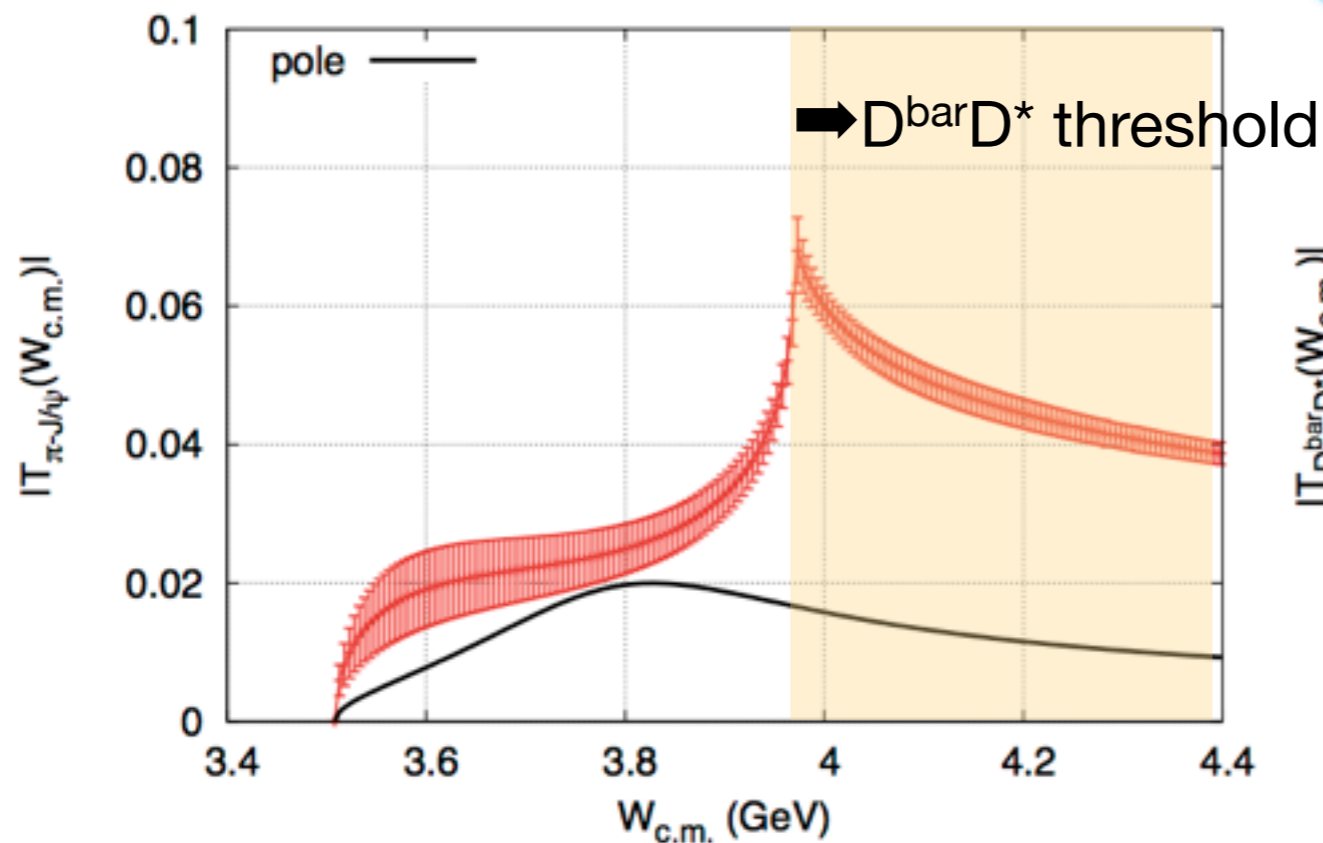
- “**Virtual**” pole on [2nd, 2nd, 2nd] sheet is found (far below $D^{\text{bar}}D^*$ threshold)
- No pole on other relevant sheets to $Z_c(3900)$
- $Z_c(3900)$ is not a conventional resonance
- **How large does the pole contribute to amplitudes?**

T-matrix of $\pi J/\psi$ & $D^{\text{bar}}D^*$

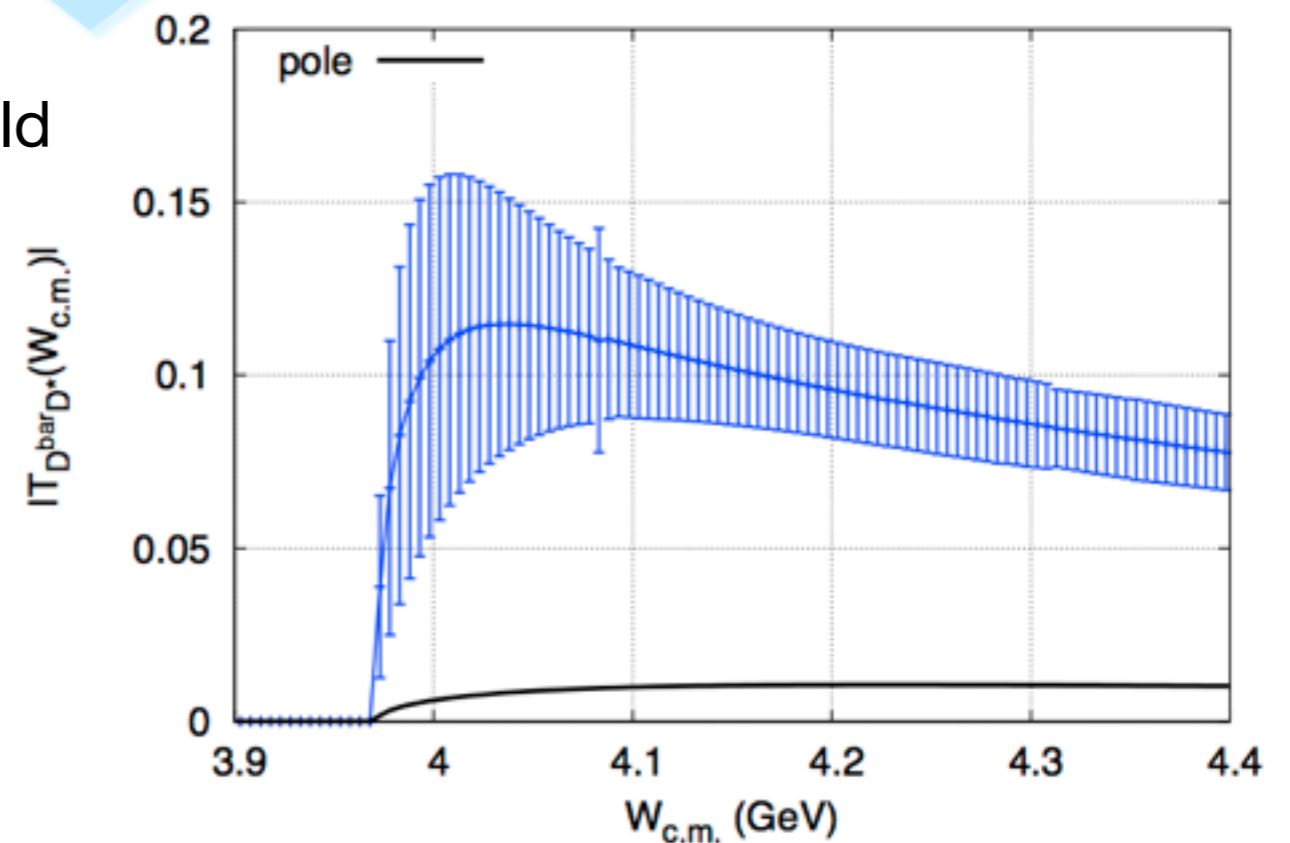
- calculate residues of T-matrices in $\pi J/\psi$ & $D^{\text{bar}}D^*$ channels

$$S(k) = 1 + 2iT(k)$$

- $\pi J/\psi$ - $\pi J/\psi$ T-matrix

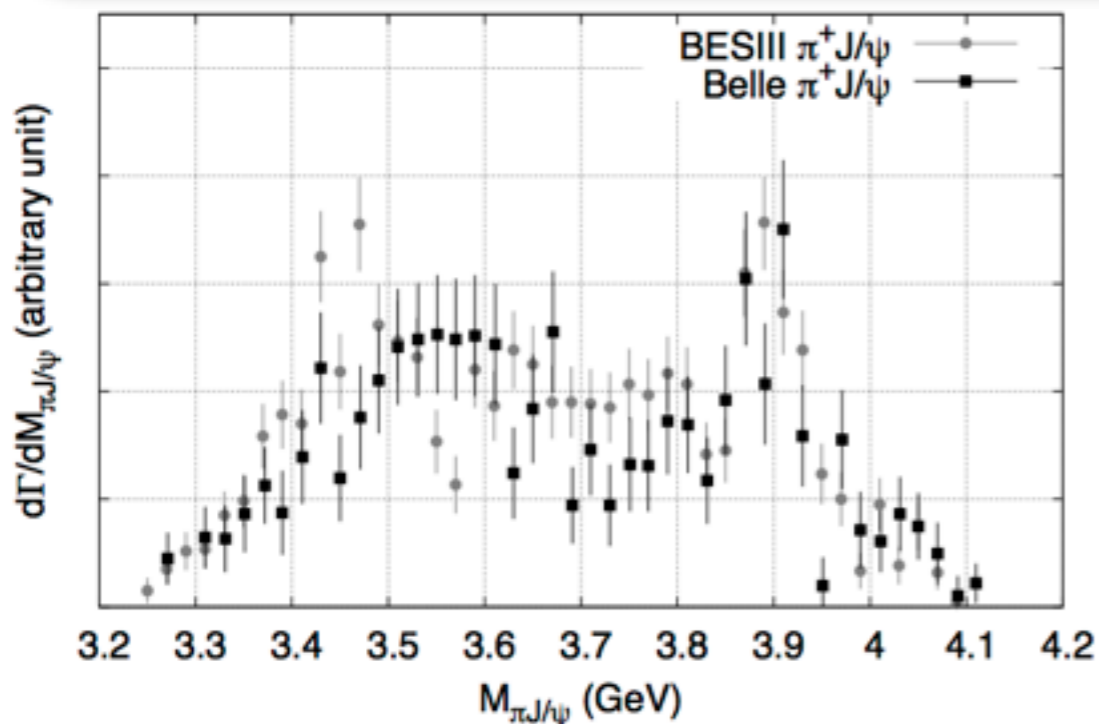
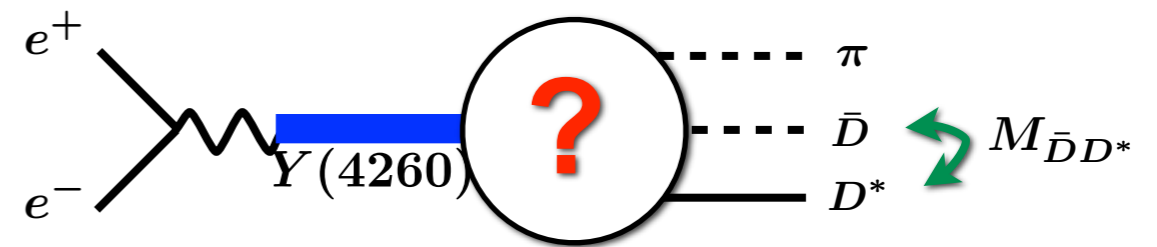
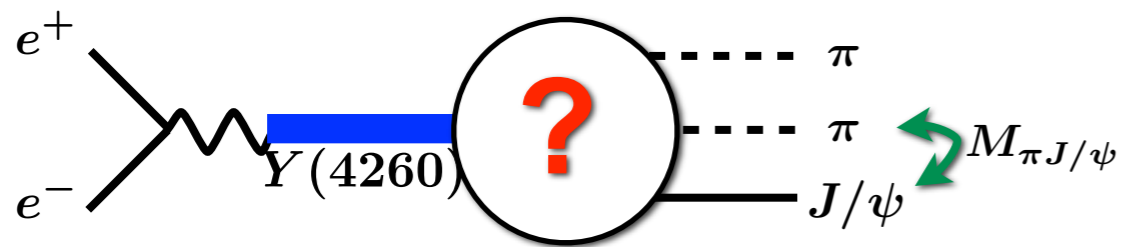


- $D^{\text{bar}}D^*$ - $D^{\text{bar}}D^*$ T-matrix



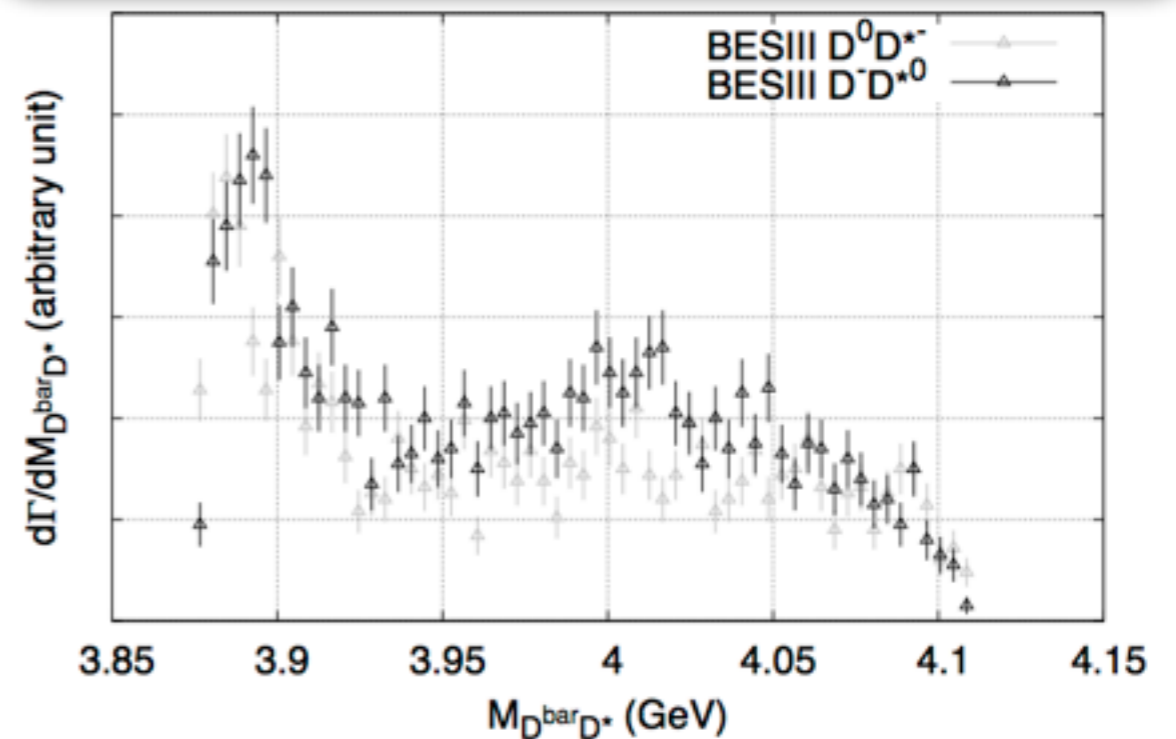
- contribution from virtual pole to T-matrix is small
- $Z_c(3900)$ is cusp at $D^{\text{bar}}D^*$ threshold induced by off-diagonal $V^{\pi\psi, D^{\text{bar}}D^*}$

Comparison with expt. data : Z_c(3900) production via Y(4260) decay



[BESIII Coll., PRL110, 252001, \(2013\).](#)

[Belle Coll., PRL110, 252002, \(2013\).](#)

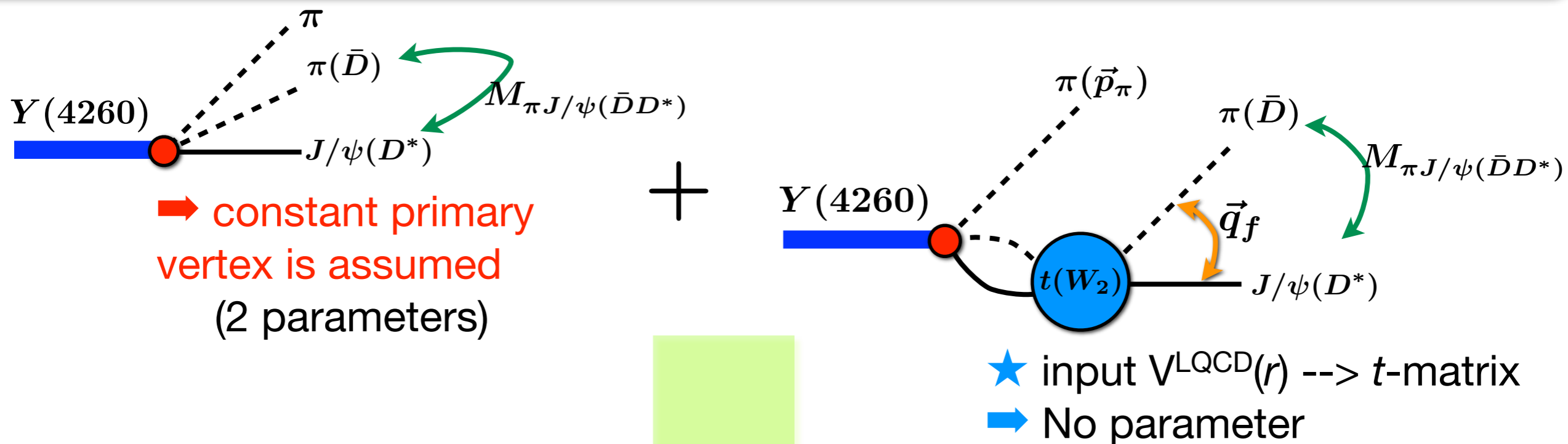


[BESIII Coll., PRL112, 022001, \(2014\).](#)

✓ check whether event distributions of $Y(4260)$ decays can be reproduced with HAL QCD coupled-channel potentials at $m_\pi=410$ MeV

Three-body decay of Y(4260)

$$d\Gamma_f \propto (2\pi)^4 \delta(W_3 - E_\pi(\vec{p}_\pi) - E_f(\vec{q}_f)) d^3 p_\pi d^3 q_f |T_f(\vec{p}_\pi, \vec{q}_f; W_3)|^2$$



✓ Three-body amplitudes

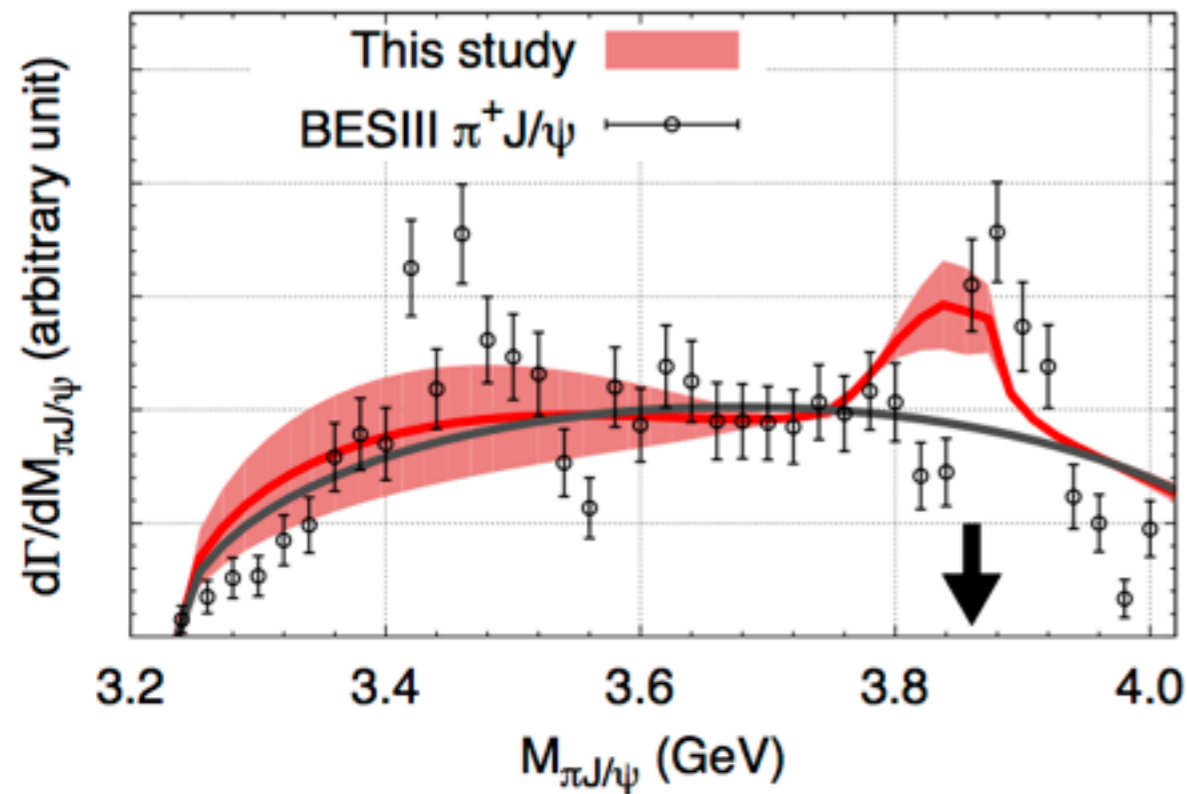
$$T_f(\vec{p}_\pi, \vec{q}_f; W_3) = \sum_{n=\pi\pi J/\psi, \pi\bar{D}D^*} C_n^{Y(4260)} \left[\delta_{nf} + \int d^3 q' \frac{t_{nf}(\vec{q}', \vec{q}_f, \vec{p}_\pi; W_3)}{W_3 - E_\pi(\vec{p}_\pi) - E_n(\vec{q}', \vec{p}_\pi) + i\epsilon} \right]$$

physical hadron masses employed to compare w/ expt. data

✓ fix decay vertex by Y(4260) --> $\pi\pi J/\psi$ expt. data

✓ predict Y(4260) --> $\pi\bar{D}D^*$ decay spectrum

Mass spectra ($\pi J/\psi$ w/ nonrelativistic kinematics)

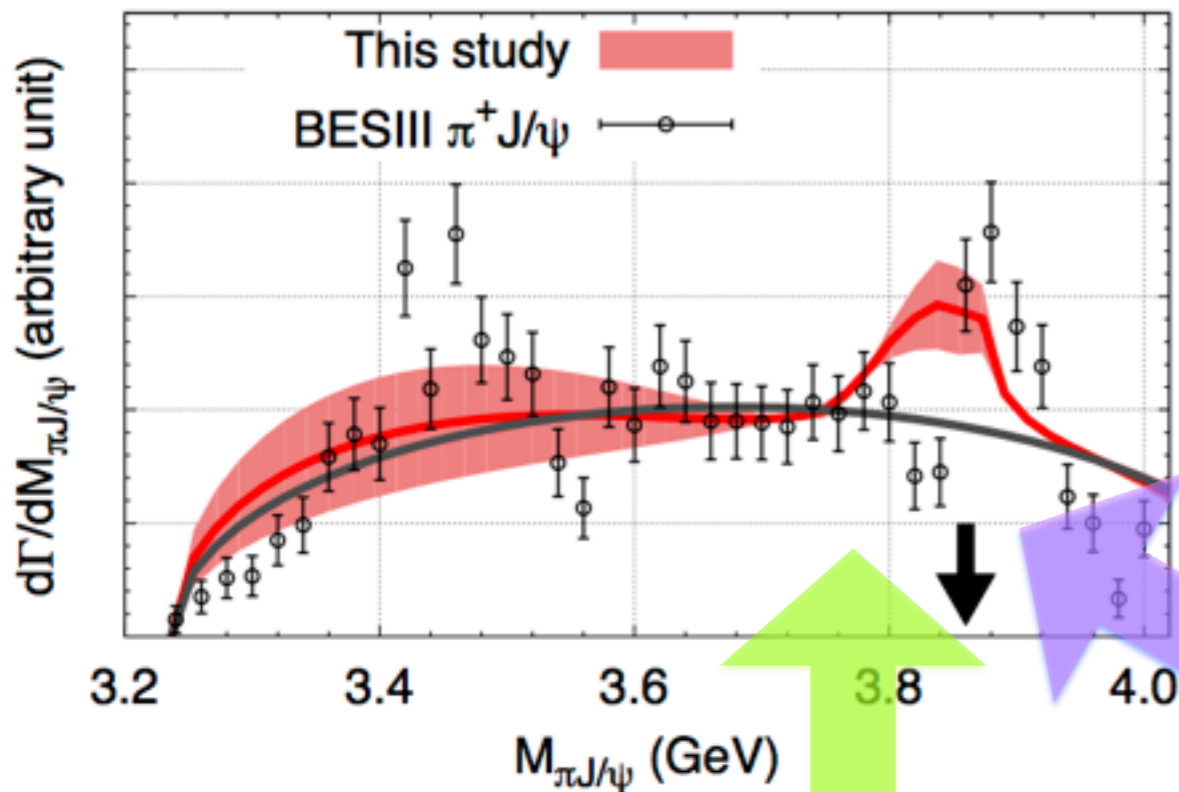


parameters: $C_{\pi D^{\text{bar}}} / C_{\pi J/\psi} = R e^{i\theta}$

--> $R=0.95(18)$, $\theta=-58(44)$ deg. (+overall factor)

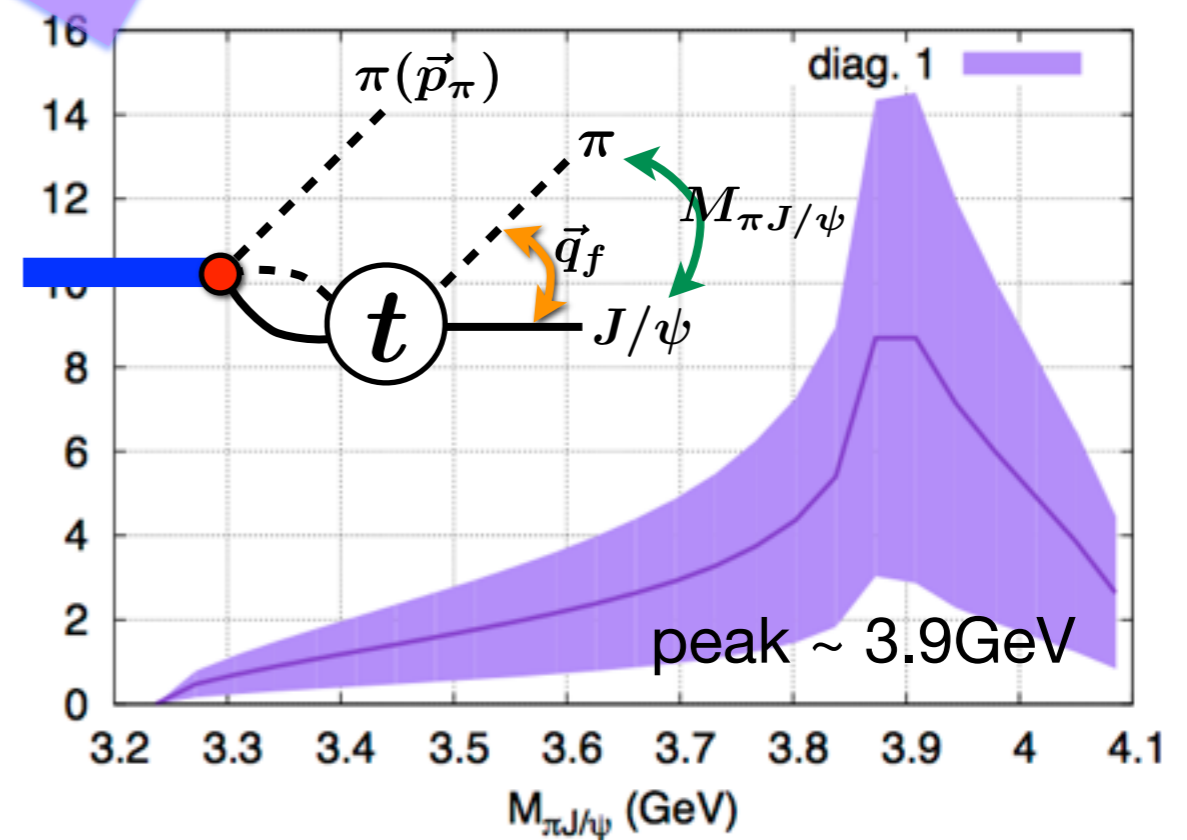
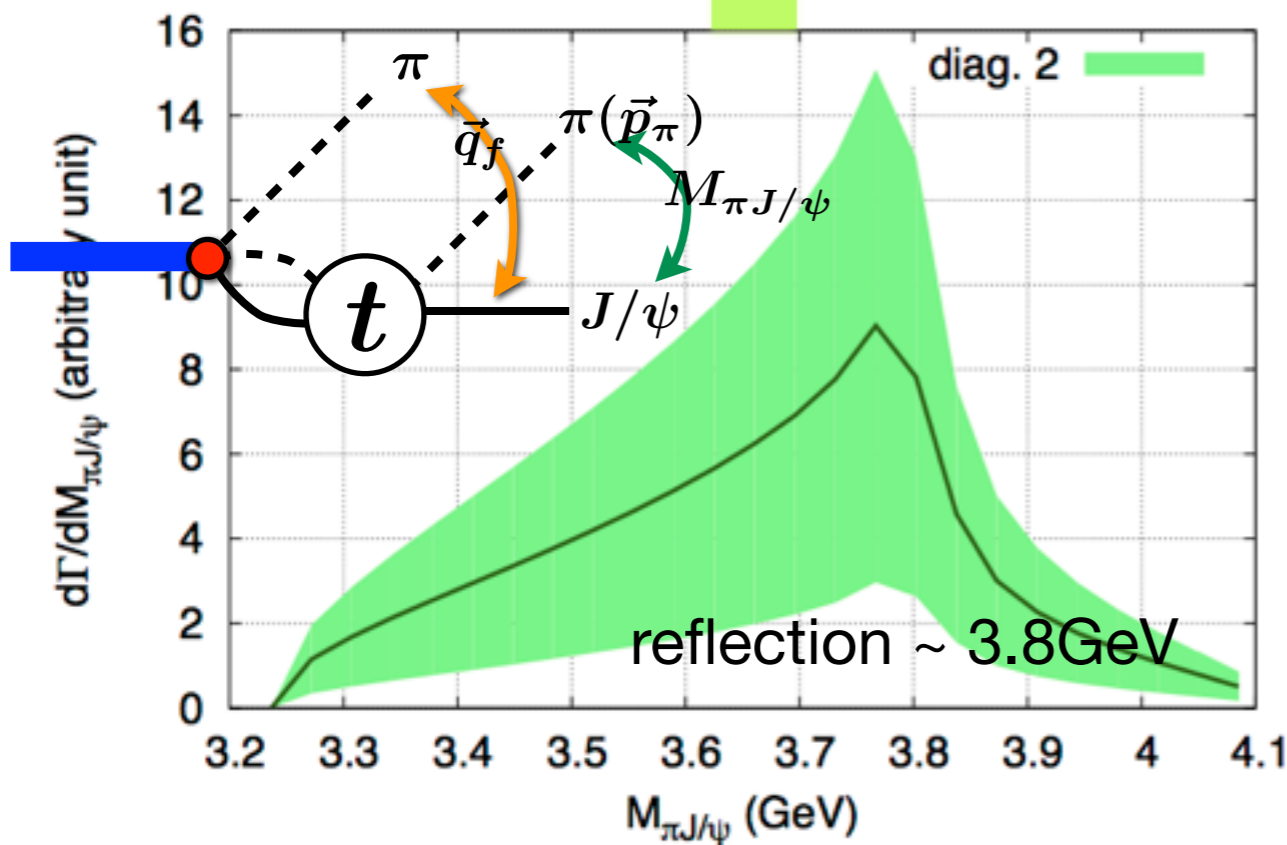
- ▶ peak nicely reproduced (a bit broad)
- ▶ peak induced by $V^{\pi J/\psi}$, $D^{\text{bar}} D^*$
- ▶ no reflection peak due to nonrelativistic kinematics

Mass spectra ($\pi J/\psi$ w/ nonrelativistic kinematics)

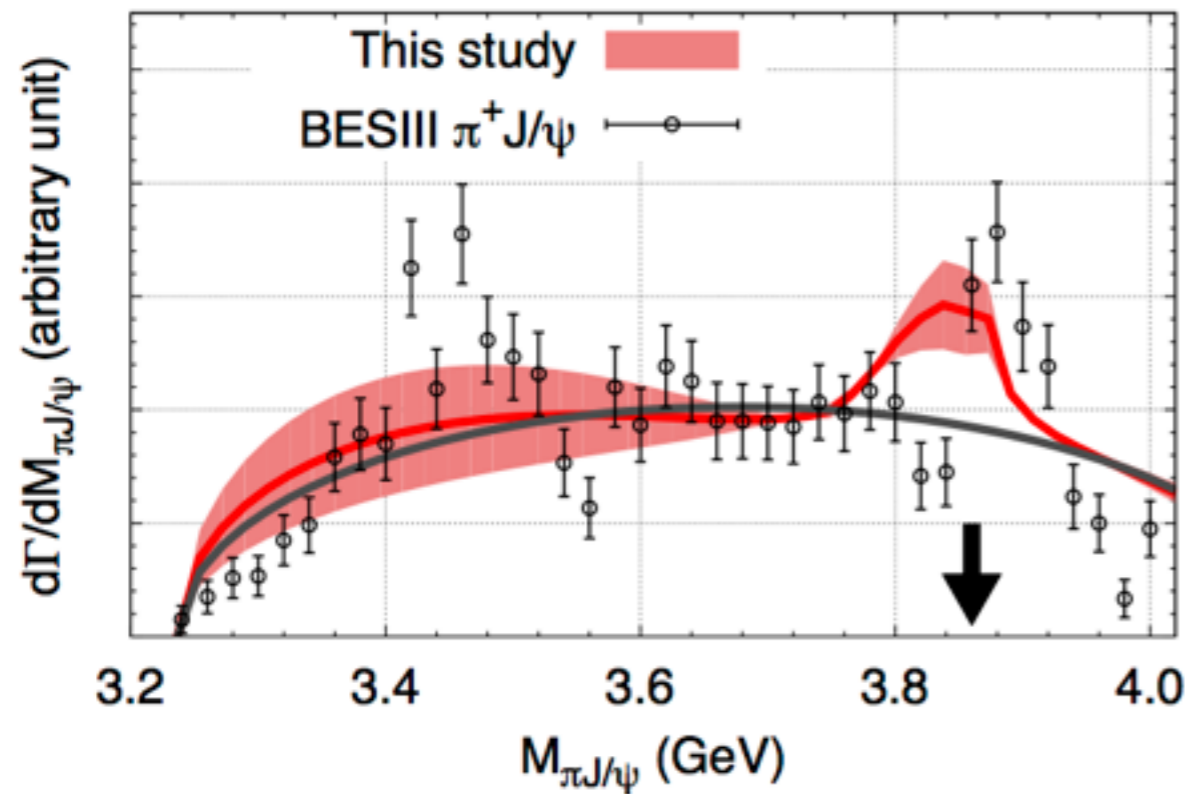


parameters: $C_{\pi D^{\text{bar}} D^*} / C_{\pi \pi J/\psi} = R e^{i\theta}$
 $\rightarrow R=0.95(18)$, $\theta=-58(44)$ deg. (+overall factor)

- ▶ peak nicely reproduced (a bit broad)
- ▶ peak induced by $V^{\pi J/\psi}$, $D^{\text{bar}} D^*$
- ▶ no reflection peak due to nonrelativistic kinematics

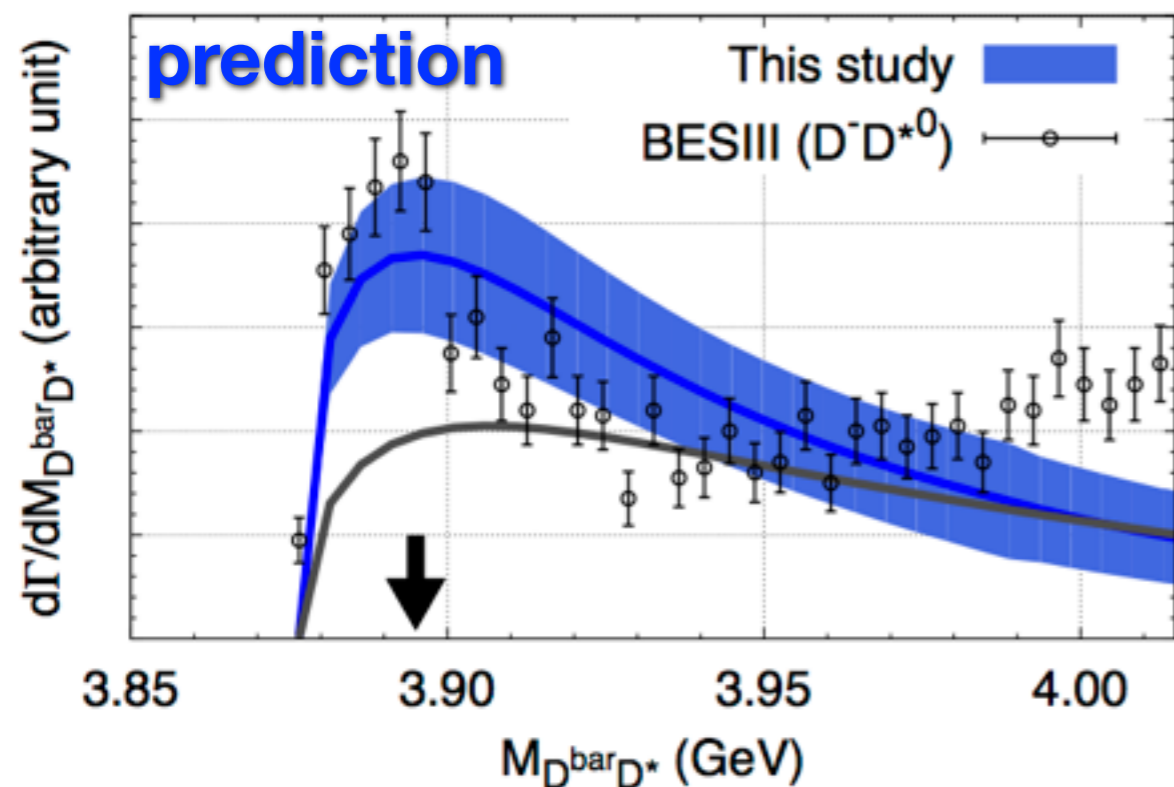


Mass spectra ($\pi J/\psi$ & $D^{\text{bar}} D^* \omega$ nonrelativistic kinematics)



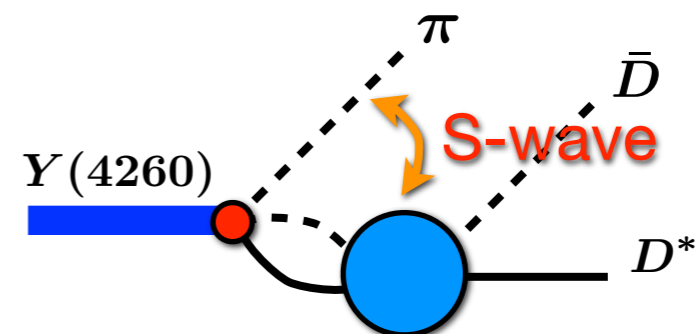
parameters: $C_{\pi D^{\text{bar}} D^*} / C_{\pi \pi J/\psi} = R e^{i\theta}$
 $\rightarrow R=0.95(18)$, $\theta=-58(44)$ deg. (+overall factor)

- ▶ peak nicely reproduced (a bit broad)
- ▶ peak induced by $V^{\pi J/\psi}$, $D^{\text{bar}} D^*$
- ▶ no reflection peak due to nonrelativistic kinematics

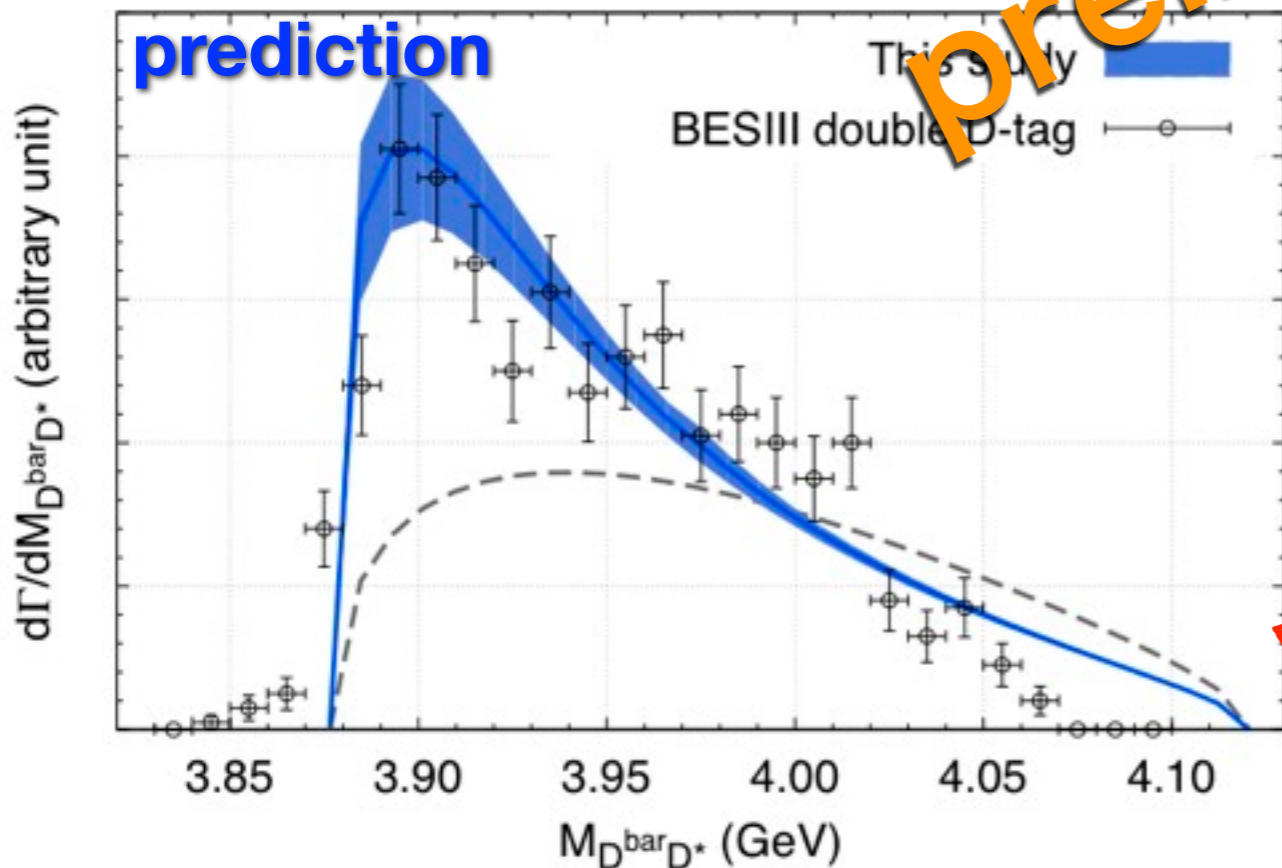
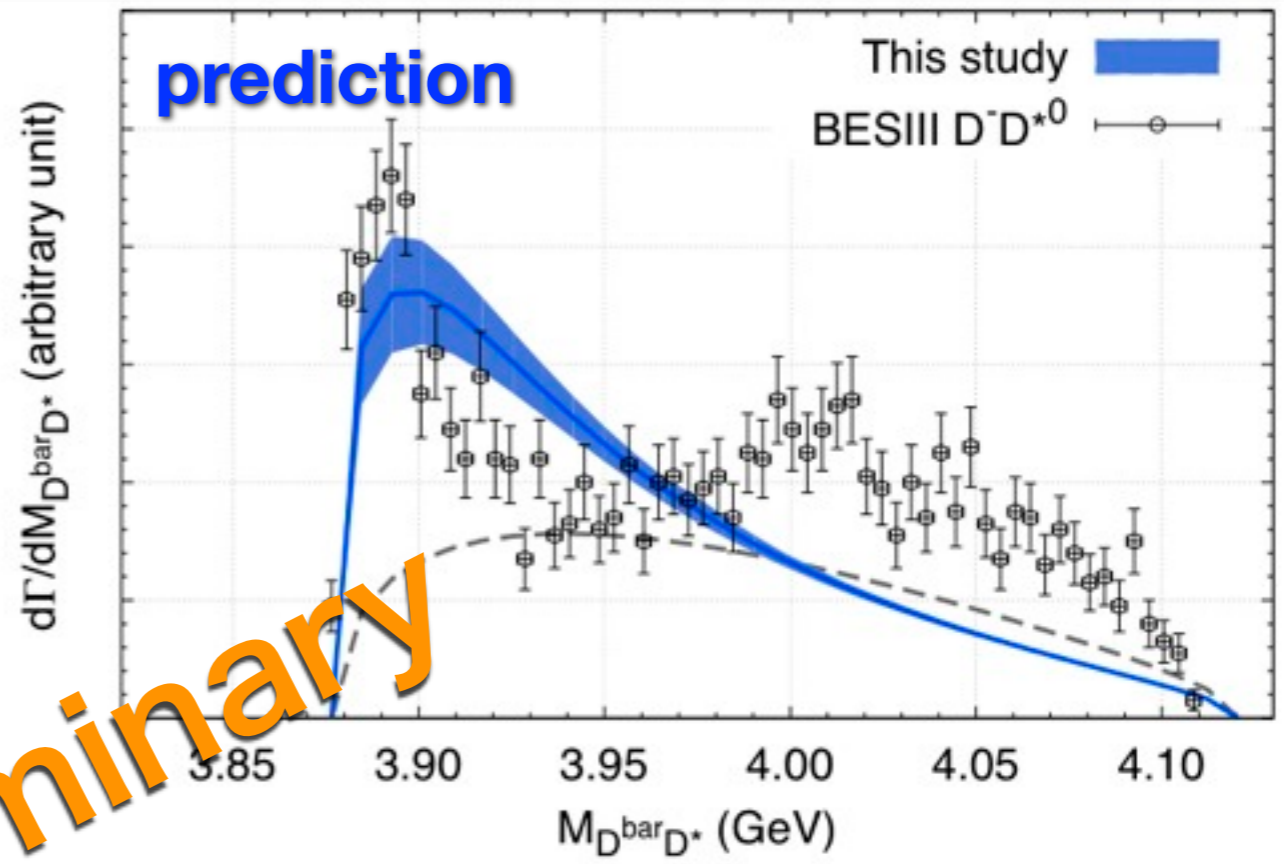
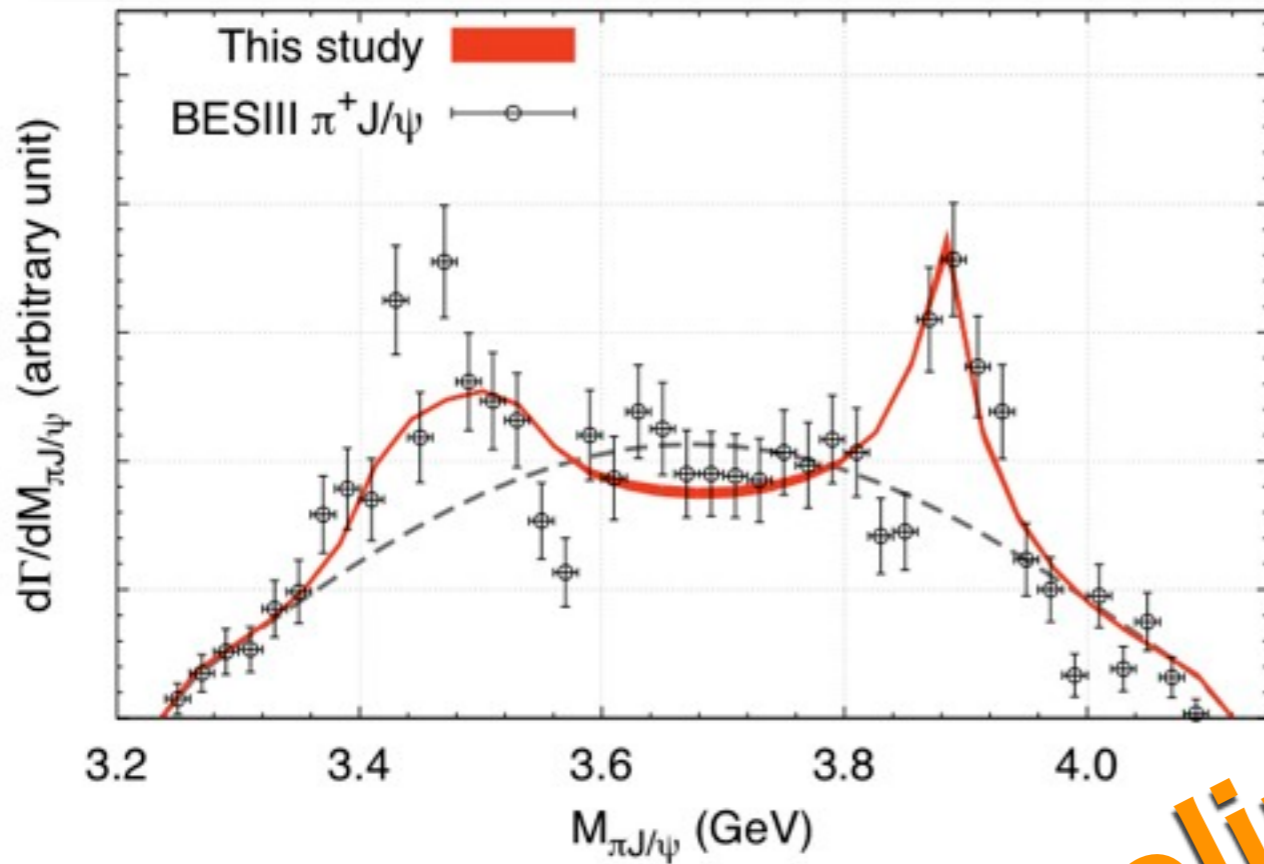


★ Good agreement around 3.9 GeV

- Deviation from expt. data at high energies
- ▶ large B.G. for single-D tag data?
- ▶ explicit $D^{\text{bar}} D^*$ channel coupling?
- ▶ higher partial wave?



Mass spectra ($\pi J/\psi$ & $D^{\text{bar}}D^*$ w/ relativistic kinematics)



- Deviation from expt. data at high energies due to large B.G. for **single-D tag** data
- good agreement with **double-D tag** data

**BESIII Coll., PRD92, 9, 092006, (2015).
 B. Wang, MENU2016**

Summary

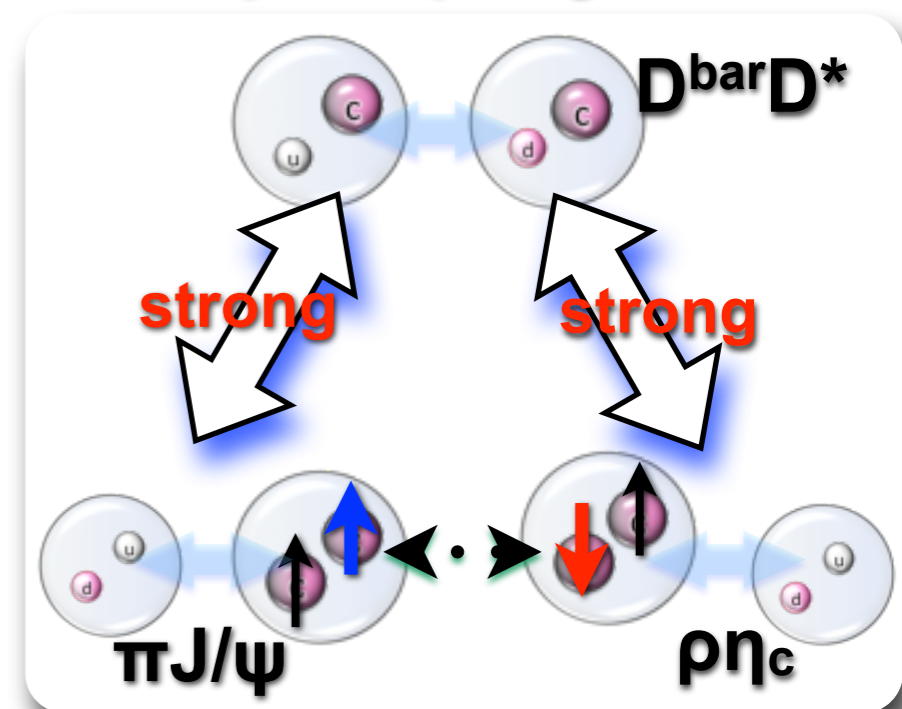
◆ $Z_c(3900)$ in $I^G(J^P)=1^+(1^+)$ channel on the lattice@ $m_\pi>400\text{MeV}$

- ★ Large channel coupling between $\pi J/\psi$ and $D^{\text{bar}}D^*$ is a key
 - ★ Enhancement at $D^{\text{bar}}D^*$ threshold in mass spectra
 - ★ Heavy quark spin symmetry is observed in c.c. potentials
 - ▶ $Z_c(3900)$ is neither simple $D^{\text{bar}}D^*$ molecule nor hadro-charmonium
 - ▶ Virtual pole on complex energy plane is found (very far from $D^{\text{bar}}D^*$ threshold)
- ➔ **$Z_c(3900)$ is threshold effect induced by $D^{\text{bar}}D^*-\pi J/\psi$ coupling**

✿ Physical point simulation is the next step

✿ Future plans

- ▶ other systems : $X(3872)$
- ▶ extension to bottom systems



Thank you for your attention!!