

$Z_c(3900)$ from lattice QCD

based on Y. Ikeda et al., (HAL QCD), arXiv.1602.03465(hep-lat).

Yoichi IKEDA (RCNP, Osaka Univ.)

HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

S. Aoki, D. Kawai, T. Miyamoto, K. Sasaki (YITP, Kyoto Univ.)

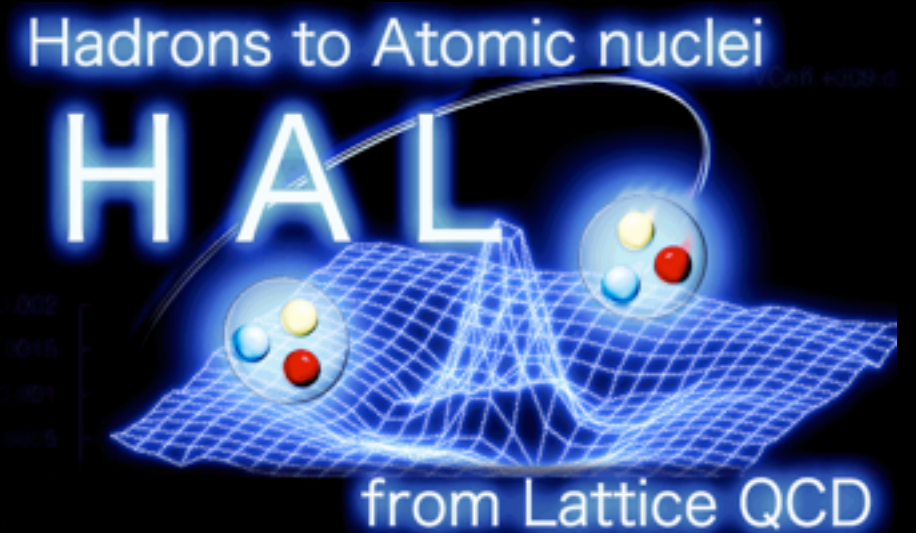
T. Doi, T. Hatsuda, T. Iritani (RIKEN)

S. Gongyo (Univ. Tours)

T. Inoue (Nihon Univ.)

Y. Ikeda, N. Ishii, K. Murano (RCNP, Osaka Univ.)

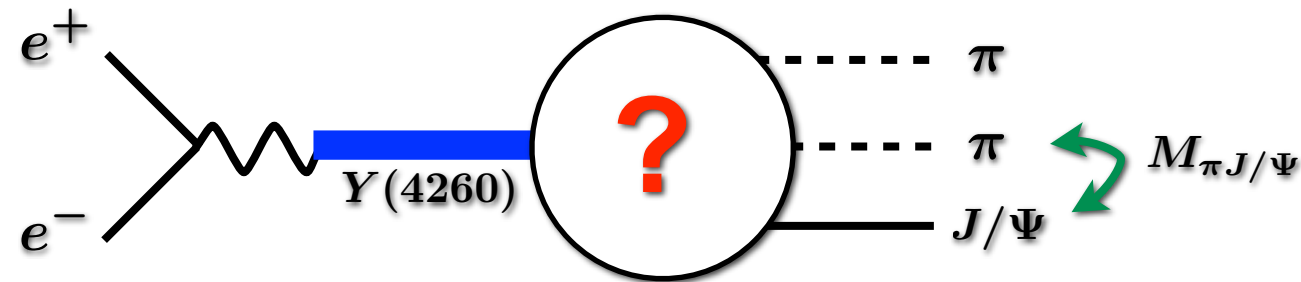
H. Nemura (Univ. Tsukuba)



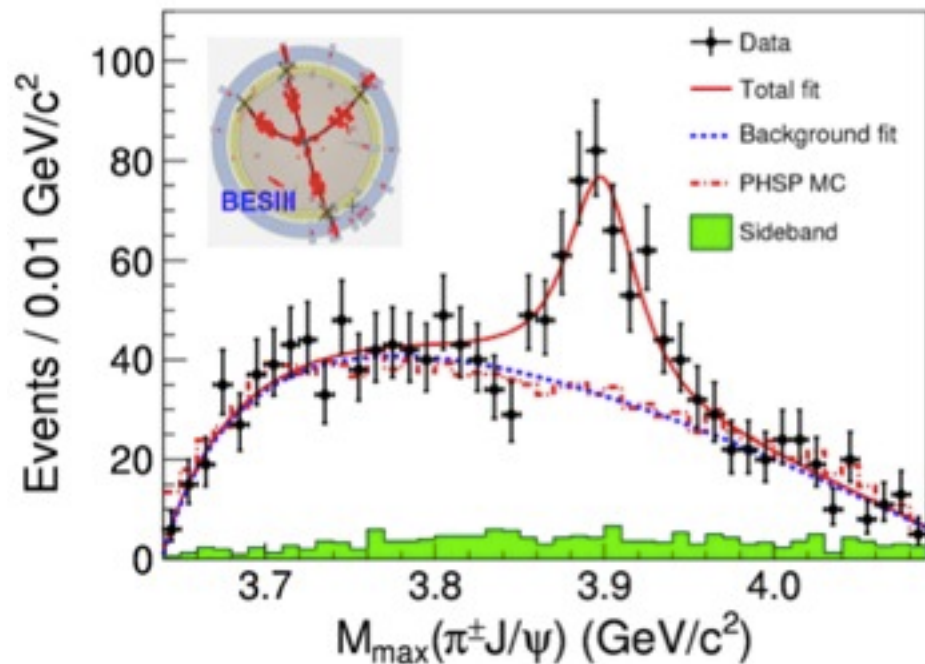
Intensive workshop in Realistic Hadron Interactions in QCD (RHIQCD2016)

@YITP, Kyoto (Nov. 28, 2016.)

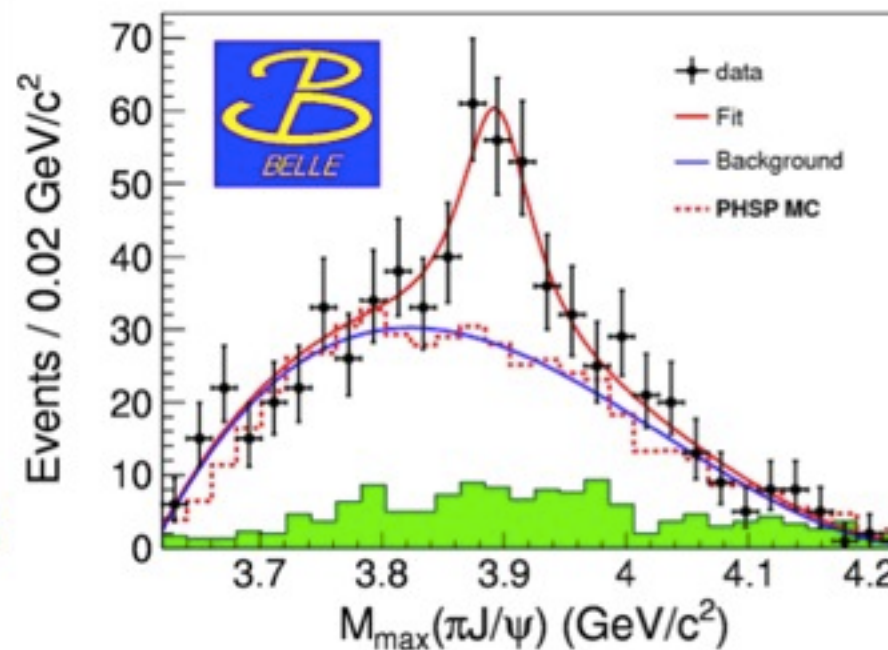
What is $Z_c(3900)$?



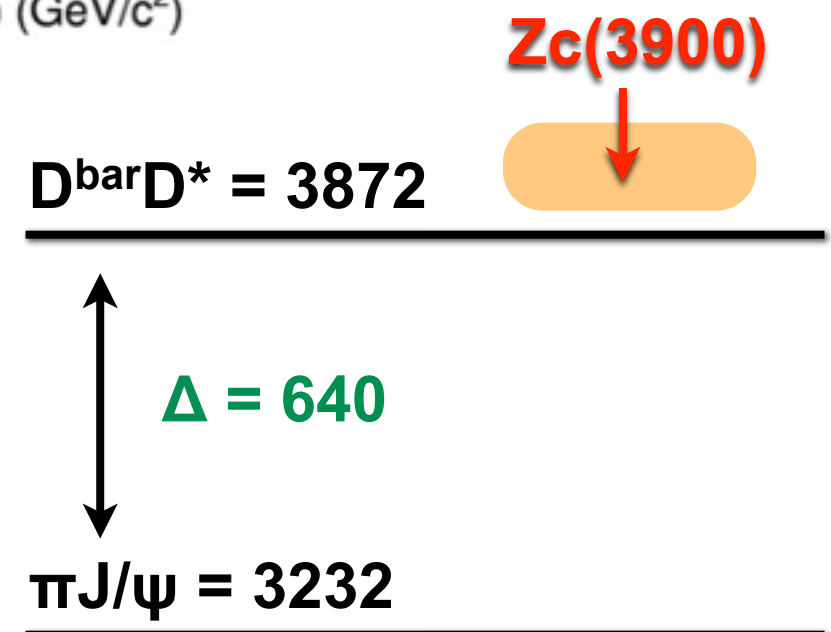
BESIII Coll., PRL110, 252001, (2013).



Belle Coll., PRL110, 252002, (2013).



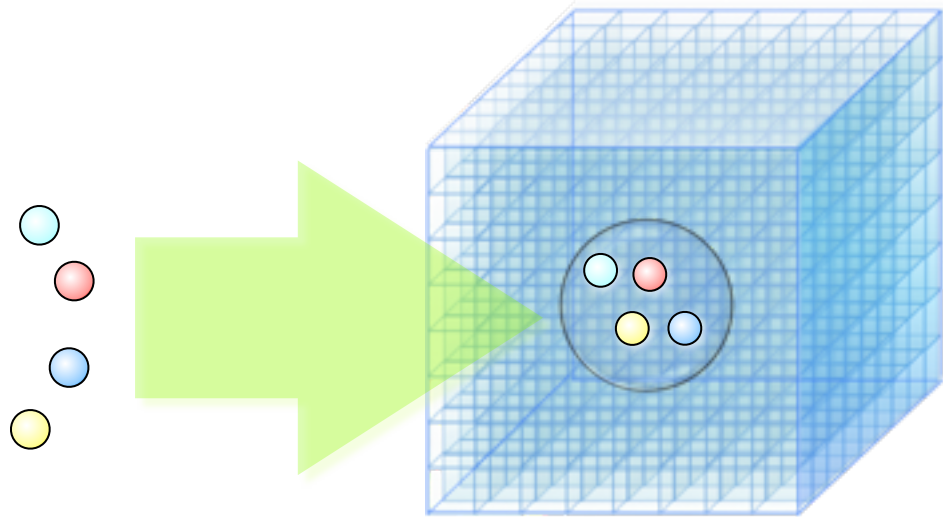
- observed in $\pi^\pm J/\psi$ invariant mass spectra
- require 4 quarks ($cc^{\text{bar}}ud^{\text{bar}}$)
- $J^P = 1^+$ most probable --> couple to s-wave $D^{\text{bar}}D^*$?
BESIII Coll., PRL112 (2014).
- peak confirmed by CLEO-c
Xiao et al., PLB727 (2013).



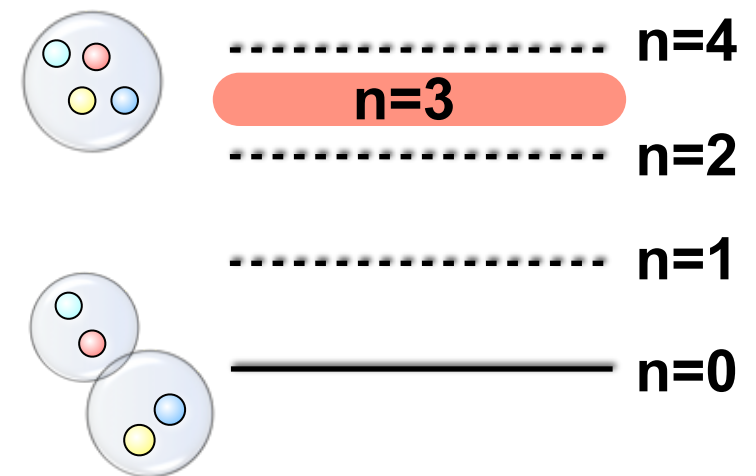
How to study $Z_c(3900)$ on the lattice?

◆ Conventional approach: LQCD spectrum

➔ identify all relevant $W_n(L)$ ($n=0,1,2, \dots$) from temporal correlations



$$\langle 0 | \Phi [c\bar{c}u\bar{d}] (\tau) | W_n \rangle = e^{-W_n \tau}$$



✓ No positive evidence for $Z_c(3900)$ in $J^P=1^+$

[S. Prelovsek et al., PLB 727, 172 \(2013\).](#)

[S.-H. Lee et al., PoS Lattice2014 \(2014\).](#)

[S. Prelovsek et al., PRD91, 014504 \(2015\).](#)

★ Why is the peak observed in expt.?

- Broad resonance? Threshold effect?
- Key is S-matrix elements w/ coupled-channel

➔ Lüscher's finite size formula in coupled-channel system

in practice, assumption about interaction kernels or K-matrices necessary

How to study $Z_c(3900)$ on the lattice?

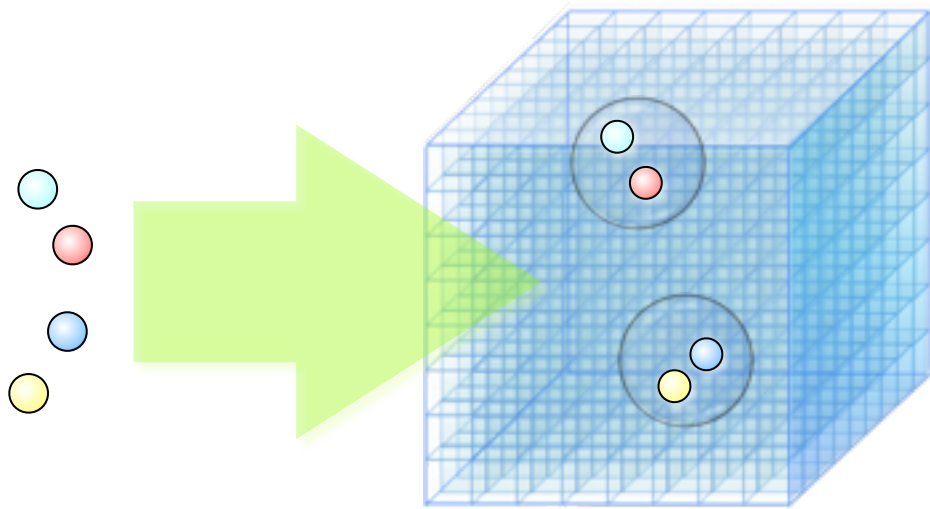
◆ HAL QCD approach: energy-independent interaction kernel

➔ measure not only temporal but also spatial correlation

[Ishii, Aoki, Hatsuda, PRL99, 02201 \(2007\).](#)

[Aoki, Hatsuda, Ishii, PTP123, 89 \(2010\).](#)

[Ishii et al.\(HAL QCD\), PLB712, 437\(2012\).](#)



$$\langle 0 | \phi_1(\vec{x} + \vec{r}, \tau) \phi_2(\vec{x}, \tau) | W_n \rangle = \psi_n(\vec{r}) e^{-W_n \tau}$$

★ **NBS wave functions: $\psi_n(r)$**

$$\left(\nabla^2 + \vec{k}_n^2 \right) \psi_n(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r}, \vec{r}') \psi_n(\vec{r}')$$

✓ **$U(r,r')$ is energy independent and contains all 2PI contributions**

➔ **Extension to coupled-channel system is straightforward**

- measure wave functions in each channel (asymptotic states --> S-matrix)
- extract potential matrix faithful to S-matrix
- calculate observables (mass spectrum, pole position,)

[Aoki et al. \[HAL QCD Coll.\], Proc. Jpn. Acad., Ser. B, 87 \(2011\); PTEP 2012, 01A105 \(2012\).](#)

Resonances from lattice QCD

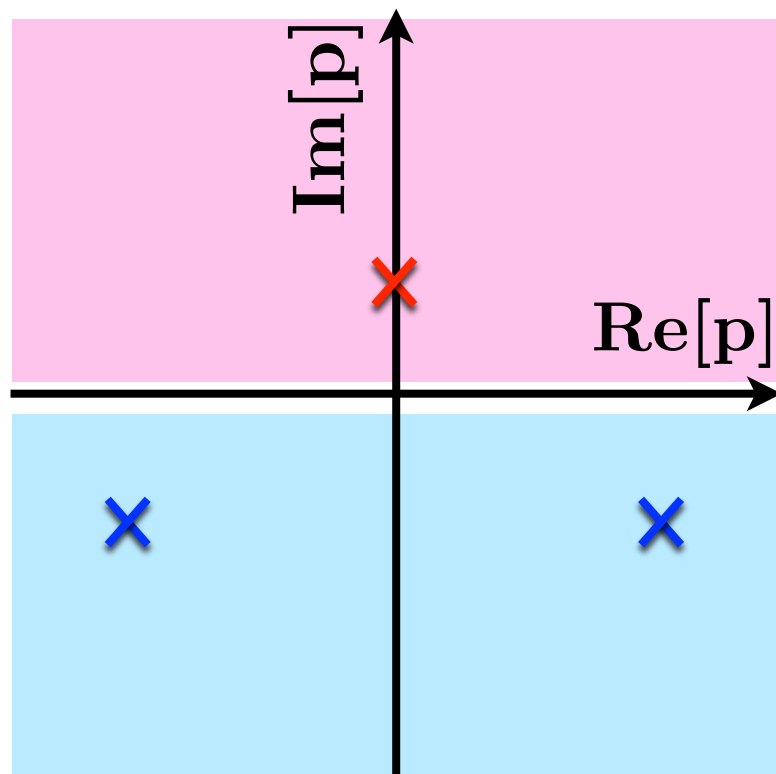
T-matrix in formal scattering theory (N/D method)

$$T^{-1}(\sqrt{s}) = V^{-1} + \frac{1}{2\pi} \int_{s_+}^{\infty} ds' \frac{\rho(s')}{s' - s}$$

Interaction part is not determined within scattering theory

➔ interactions faithful to S-matrix in QCD

Analyticity of T-matrix is uniquely determined



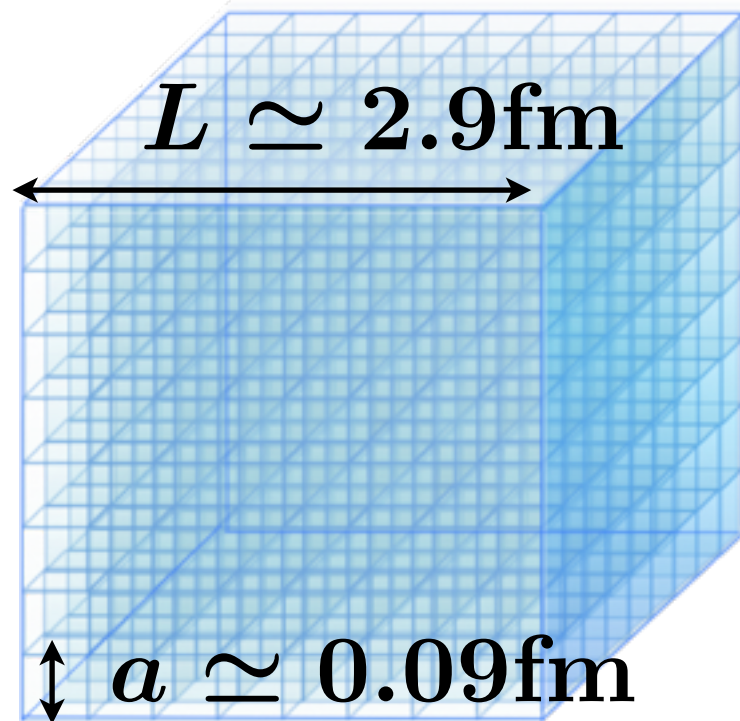
Bound states (physical sheet, 1st)

- binding energy --> T-matrix pole position
- coupling --> residue of pole

Resonance/virtual states (unphysical sheet, 2nd)

- Analytic continuation of T-matrix
- resonance energy --> T-matrix pole position
- coupling --> (complex) residue of pole?

Lattice QCD setup



★ $N_f=2+1$ full QCD

PACS-CS Coll., S. Aoki et al., PRD79, 034503, (2009).

- Iwasaki gauge & $O(a)$ -improved Wilson quark actions
- $a=0.0907(13) \text{ fm}$ $\rightarrow L \sim 2.9 \text{ fm}$ ($32^3 \times 64$)

★ Relativistic Heavy Quark action for charm

S. Aoki et al., PTP109, 383 (2003).

Y. Namekawa et al., PRD84, 074505 (2011).

- remove leading cutoff errors $O((m_c a)^n)$, $O(\Lambda_{\text{QCD}} a)$, ...

➔ We are left with $O((a\Lambda_{\text{QCD}})^2)$ syst. error (\sim a few %)

- three sets of full QCD gauge configs. used ($m_\pi \sim 410-700 \text{ MeV}$)

light hadron mass (MeV)

$$m_\pi = 411, 572, 701$$

$$m_K = 635, 714, 787$$

$$m_\rho = 896, 1000, 1097$$

$$m_N = 1215, 1411, 1583$$

Charm meson mass (MeV)

$$m_{\eta_c} = 2988, 3005, 3024$$

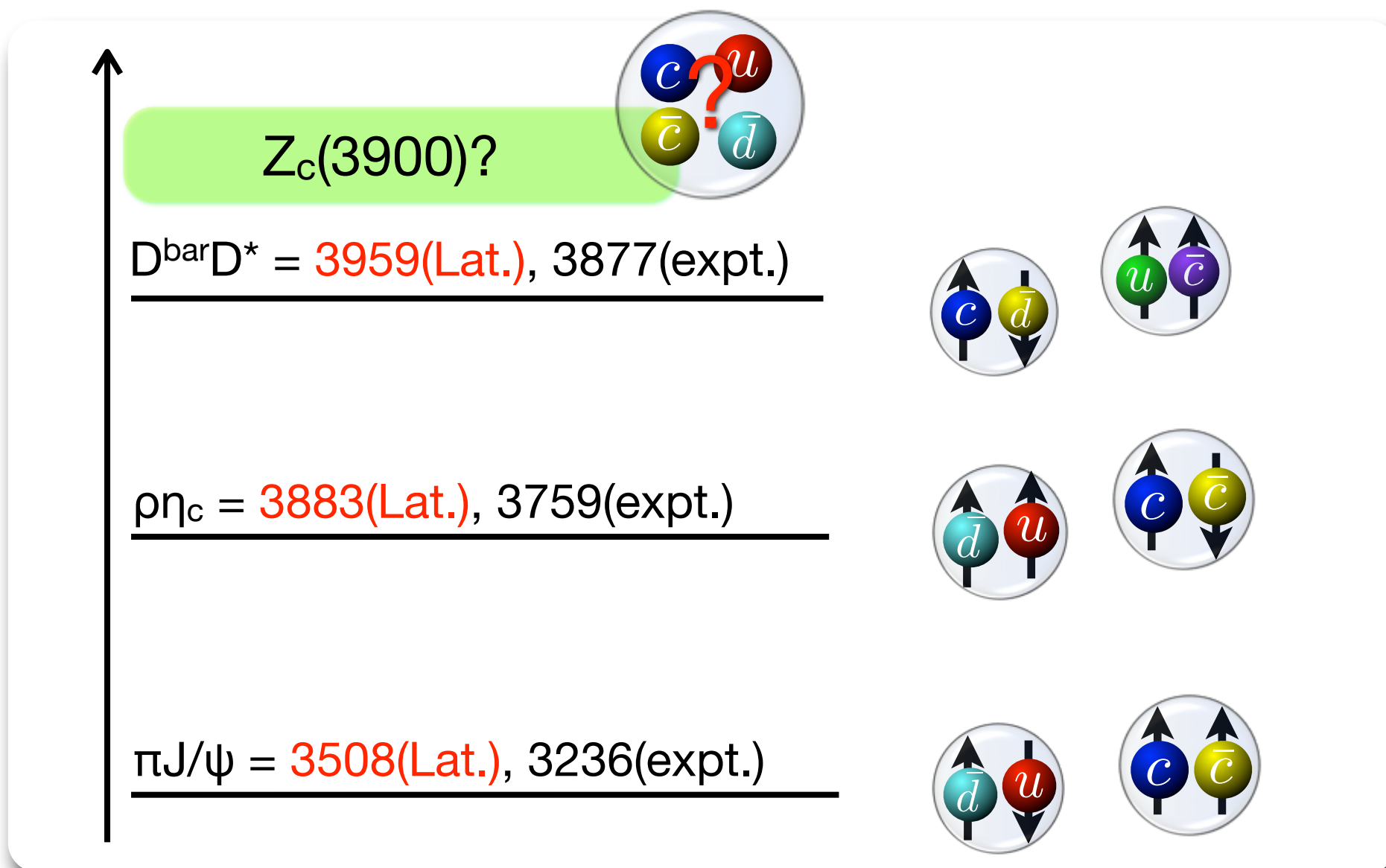
$$m_{J/\psi} = 3097, 3118, 3143$$

$$m_D = 1903, 1947, 2000$$

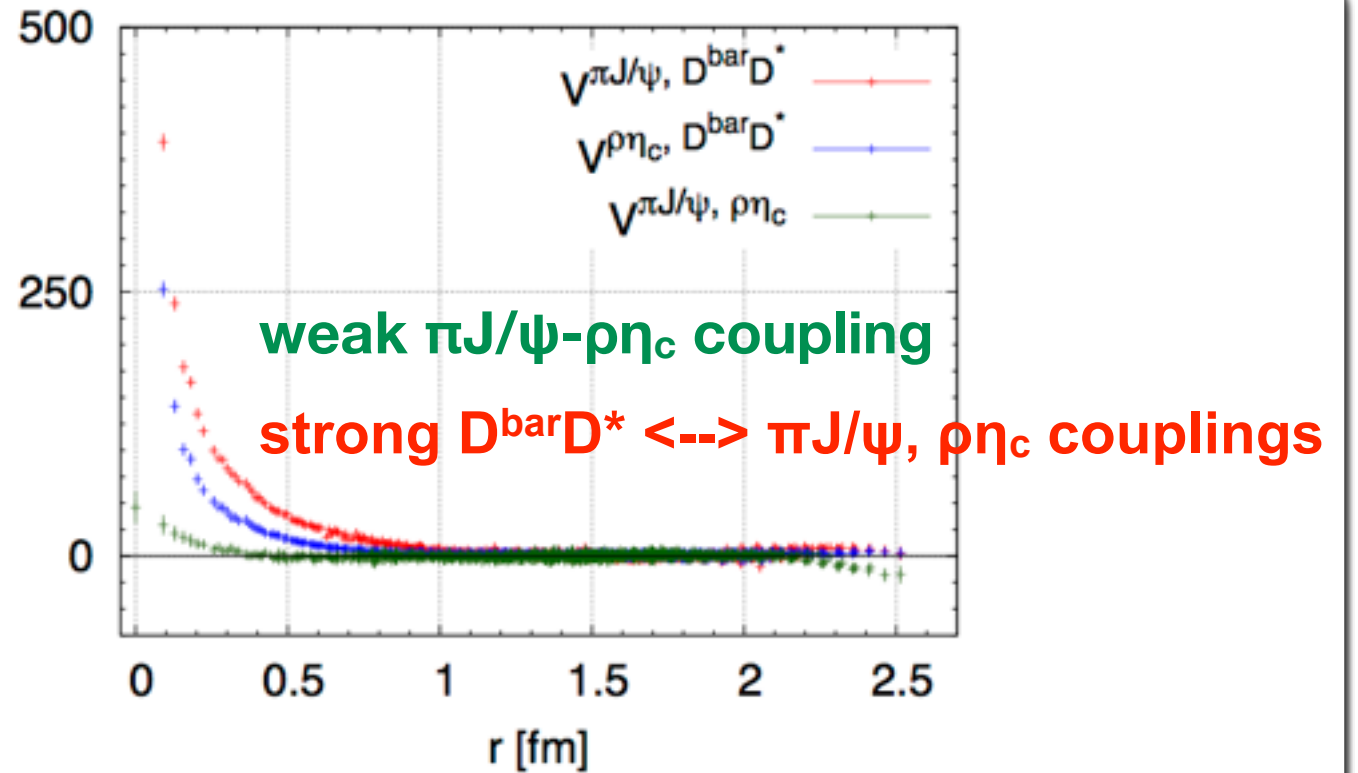
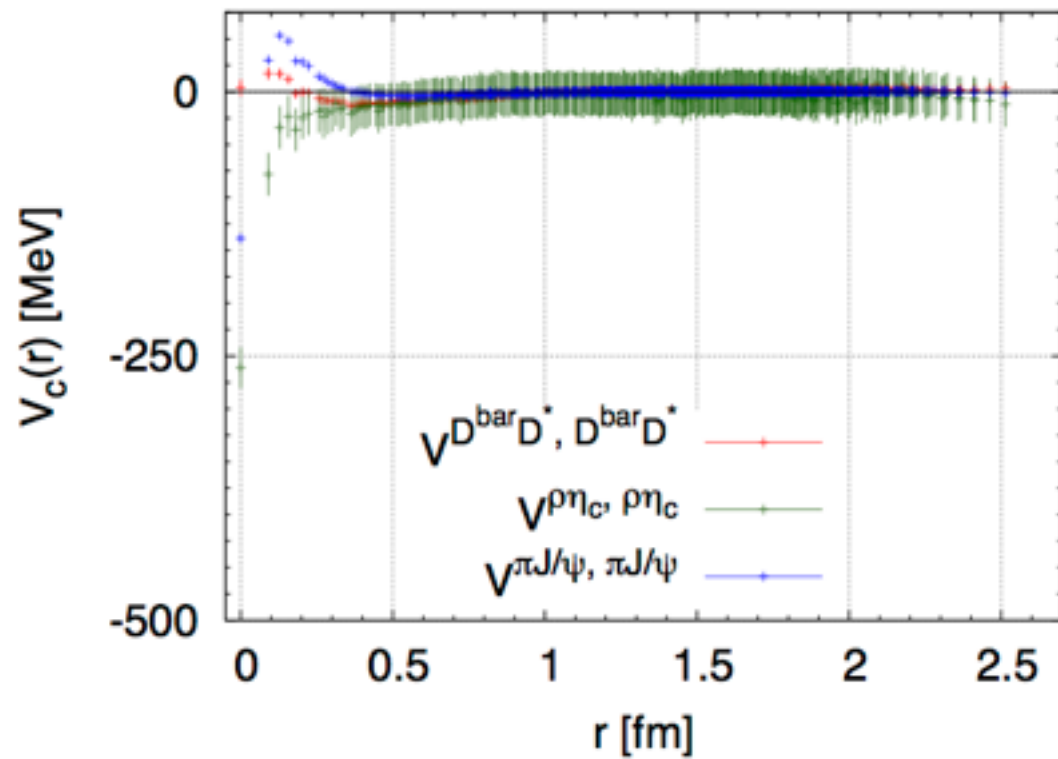
$$m_{D^*} = 2056, 2101, 2159$$

Structure of $Z_c(3900)$

-- $\pi J/\psi$ - $\rho \eta_c$ - $D^{\text{bar}} D^*$ in $I^G(J^{PC})=1^+(1^{+-})$ --



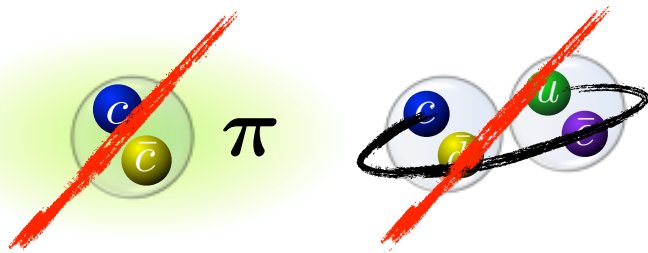
S-wave $\pi J/\psi - \rho \eta_c - D^{\text{bar}} D^*$ potential @ $m_\pi=410\text{MeV}$



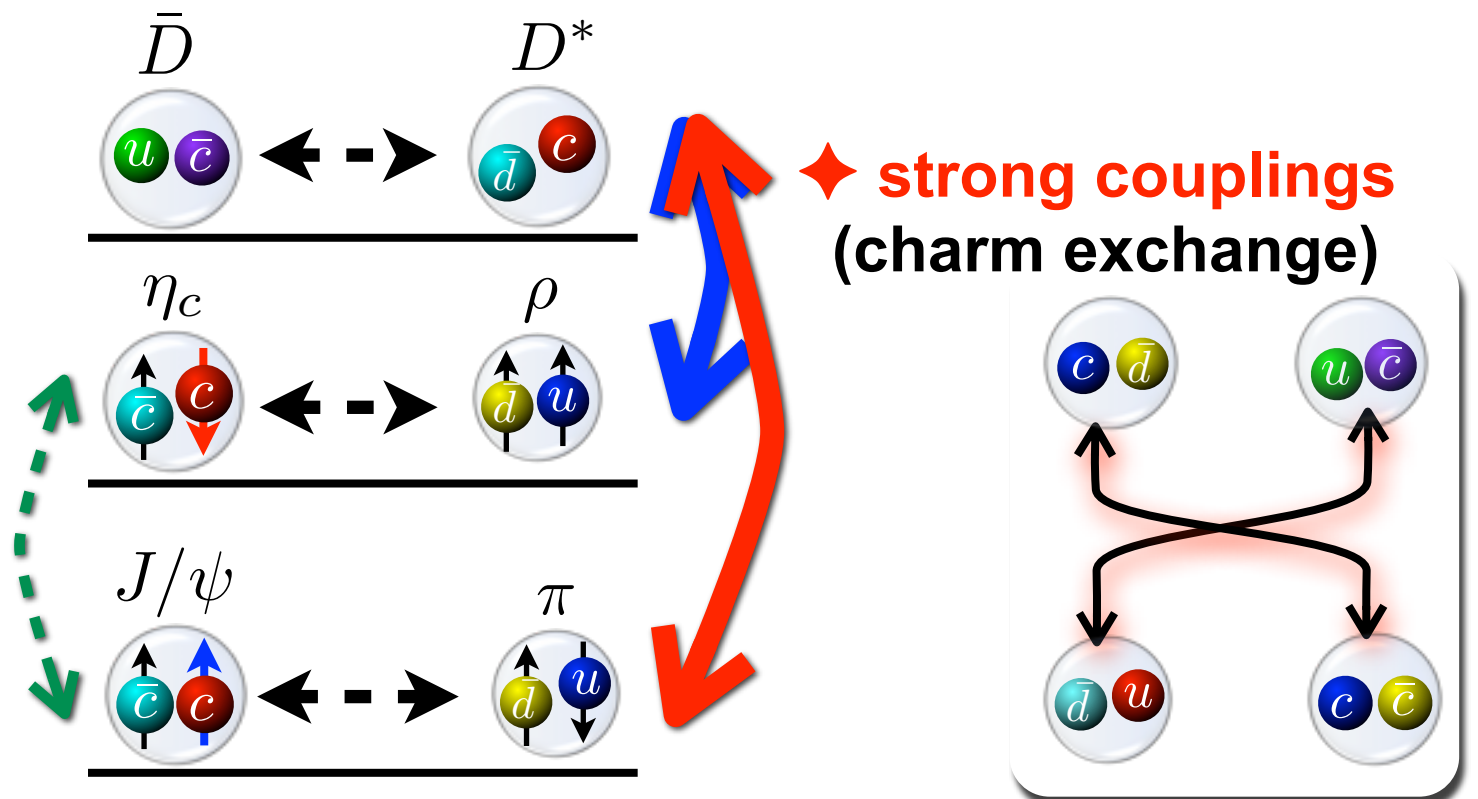
[Y. Ikeda et al., \(HAL QCD\), arXiv.1602.03465\(hep-lat\).](https://arxiv.org/abs/1602.03465)

★ weak diagonal potentials:

$Z_c(3900)$ is NOT simple $\pi J/\psi$ & $D^{\text{bar}} D^*$



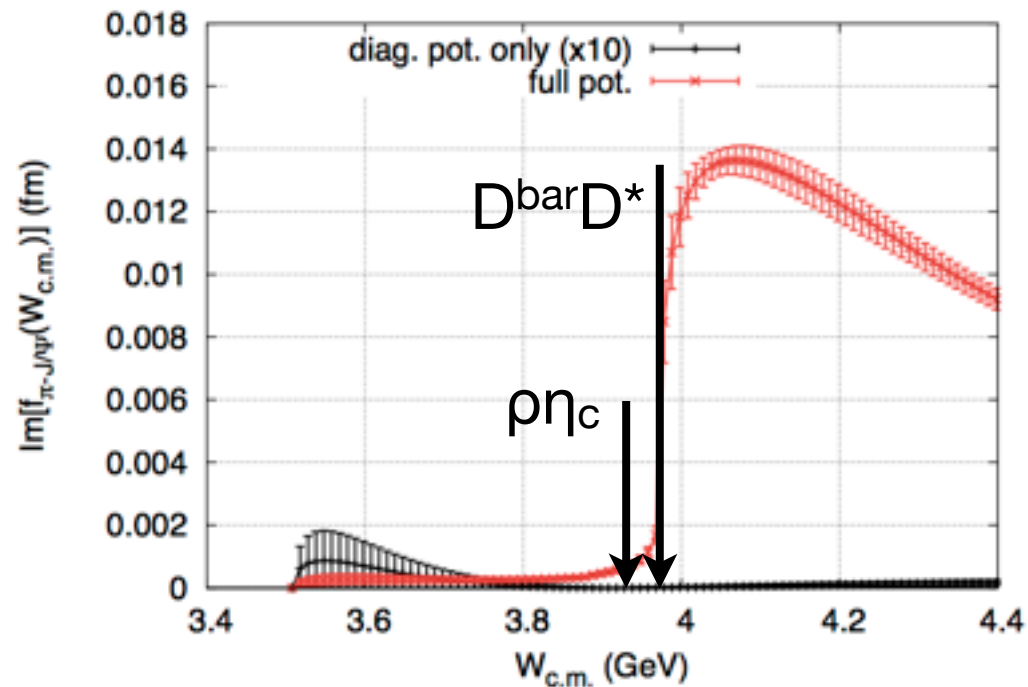
◆ heavy quark spin symmetry



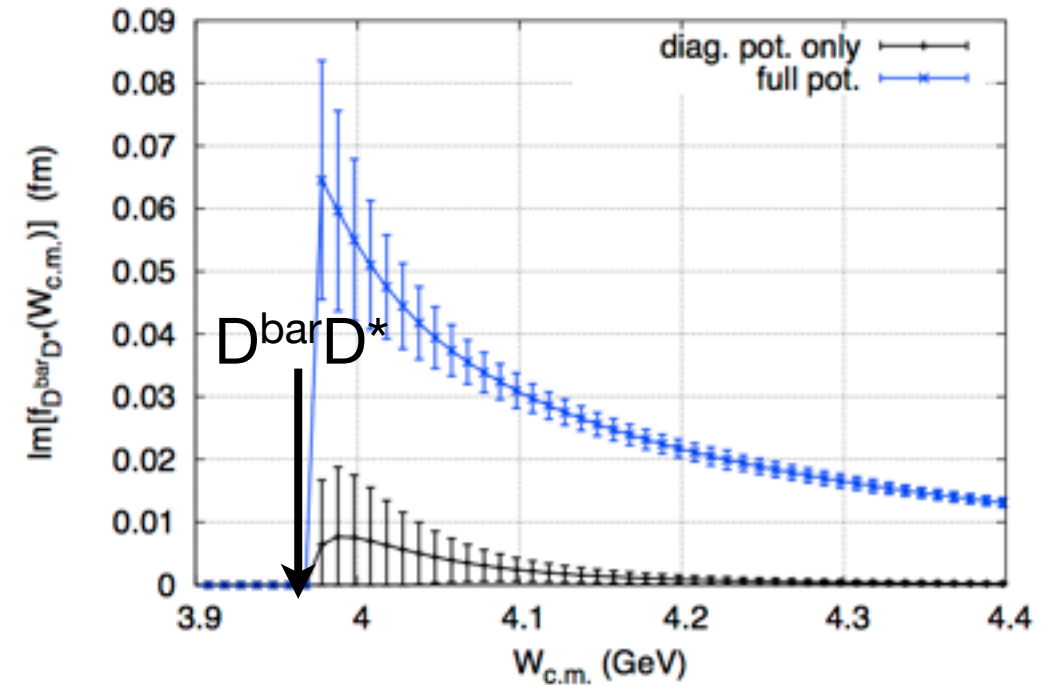
Invariant mass spectra of $\pi J/\psi$ & $D^{\text{bar}}D^*$

★ 2-body scattering (ideal setting to understand $Z_c(3900)$ structure)

• $\pi J/\psi$ invariant mass



• $D^{\text{bar}}D^*$ invariant mass



[Y. Ikeda et al., \[HAL QCD\], arXiv.1602.03465 \[hep-lat\] \(2016\).](#)

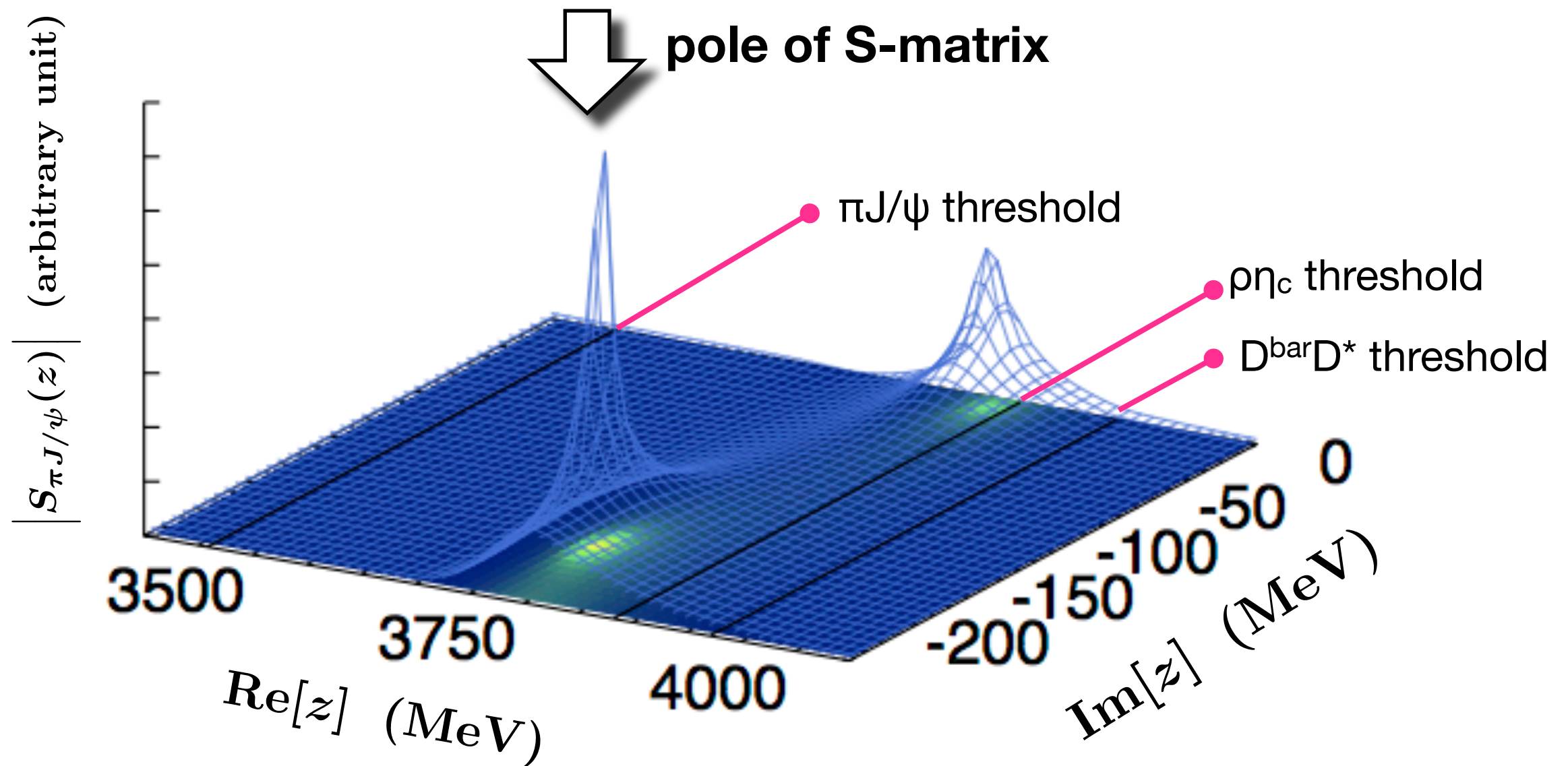
✓ Enhancement near $D^{\text{bar}}D^*$ threshold due to strong $V_{\pi J/\psi, D^{\text{bar}}D^*}$

- Peak in $\pi J/\psi$ (not Breit-Wigner line shape)

- Threshold enhancement in $D^{\text{bar}}D^*$

✓ Is $Z_c(3900)$ a conventional resonance? --> pole position

Complex pole position ($\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $D^{\text{bar}}D^*$:2nd)



- “Virtual” pole on [2nd, 2nd, 2nd] sheet is found (far below $D^{\text{bar}}D^*$ threshold)
- No pole on other relevant sheets to $Z_c(3900)$
- $Z_c(3900)$ is not a conventional resonance
- $Z_c(3900)$ is cusp induced by off-diagonal $V^{\pi\psi, D^{\text{bar}}D^*}$

Summary

◆ $Z_c(3900)$ in $I^G(J^P)=1^+(1^+)$ channel on the lattice@ $m_\pi>400\text{MeV}$

- ★ Large channel coupling between $\pi J/\psi$ and $D^{\text{bar}}D^*$ is a key
 - ★ Enhancement at $D^{\text{bar}}D^*$ threshold in mass spectra
 - ★ Heavy quark spin symmetry is observed in c.c. potentials
 - ▶ $Z_c(3900)$ is neither simple $D^{\text{bar}}D^*$ molecule nor hadro-charmonium
 - ▶ Virtual pole on complex energy plane is found (very far from $D^{\text{bar}}D^*$ threshold)
- ➔ **$Z_c(3900)$ is threshold effect induced by $D^{\text{bar}}D^*-\pi J/\psi$ coupling**

✿ Physical point simulation is the next step

✿ Future plans

- ▶ other systems : $X(3872)$
- ▶ extension to bottom systems

