Hyperon-Nucleon Scattering

In A Covariant Chiral Effective Field Theory Approach

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Contents

Hypernuclear physics

- Since 1953

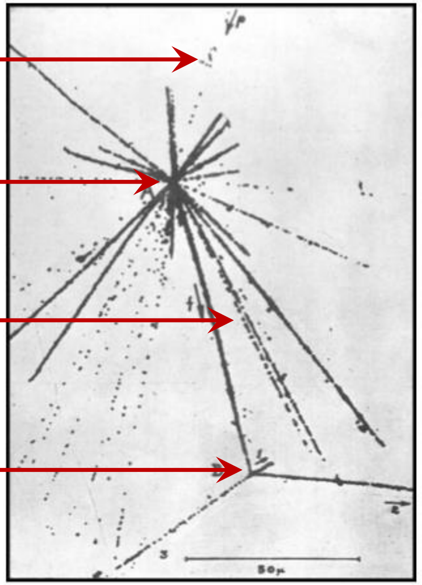
1947:	Rochester & Butler	First discovery of strange particle (Kaon)	Nature 160 (1947) 855
1953:	Gell-Mann	*Strangeness* was introduced	Phys. Rev. 92 (1953) 833
	Nakano & Nishijima		Prog. Theor. Phys. 10 (1953) 581
	Danysz & Pniewski	**First discovery of Λ-hypernucleus**	**Philos. Mag. Ser. 5 44 (1953) 348**
…			

Incoming high energy cosmic ray

Collision with the nucleus

Nuclear fragments that eventually stop in the emulsion

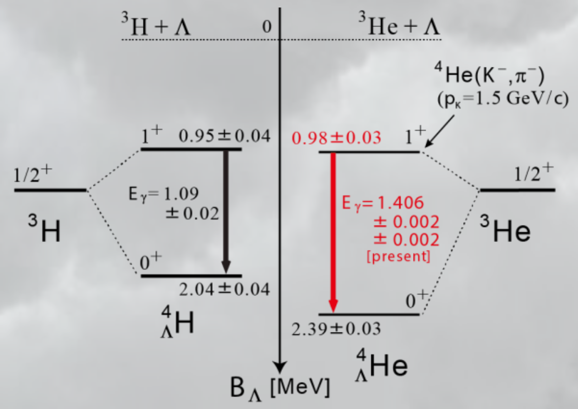
One fragment containing a hyperon disintegrates weakly



Hypernuclear physics

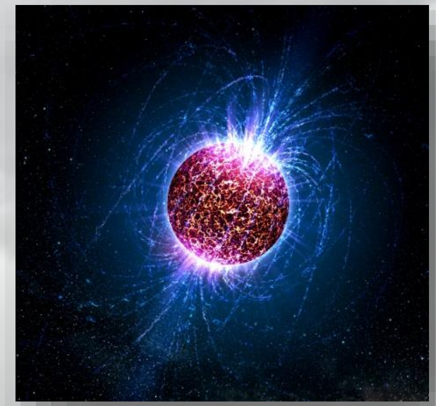
- We do not know in the present…

1. **Large CSB in A=4 hypernuclei?**
2. **A bound H-dibaryon?**
3. **Hyperon puzzle**



Yamamoto PRL 115 (2015) 222501...

Inoue PRL 106 (2011) 162002...



Lonardoni PRL 114 (2015) 092301...

4. Why is the Λ-nuclear spin-orbit splitting so small?
5. What is the role of three-body ΛNN interactions in hypernuclei and at neutron-star densities?
6. The Σ-nuclear interaction is established as being repulsive, but how repulsive?
7. Where is the onset of ΛΛ binding?
8. Do Ξ hyperons bind in nuclei and how broad are the single-particle levels given the ΞN → ΛΛ strong decay channel?
9. Where is the onset of Ξ stability?

…

Gal RMP 88 (2016) 035004

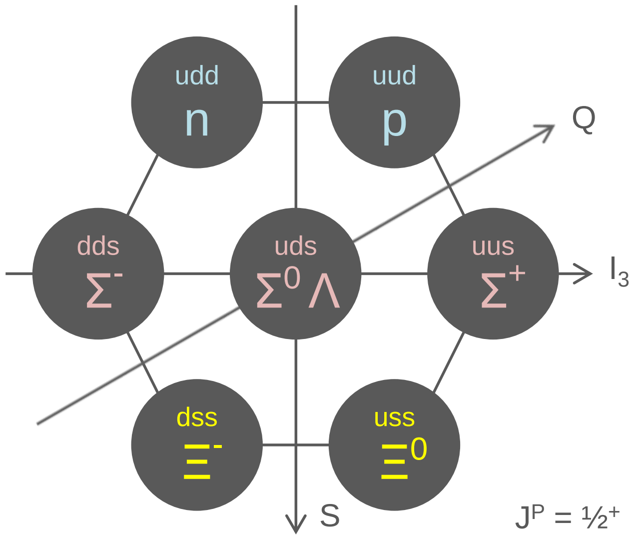
Baryon-baryon interactions

- Underlying these fascinating phenomena: baryon-baryon interactions

- Octet baryons

  Characterized by:

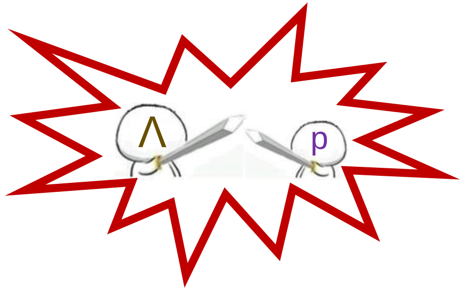
  - Charge (Q)
  - Strangeness (S)
  - Third component of isospin (I_3)



- Why baryon-baryon interactions?

Role of strangeness

SU(3)$_f$ symmetry



Hypernuclear physics

Astrophysics

Experimental status: YN

- Poor

1. Small quantity (36, $S = -1$, YN)
2. Age-old (1960s - 1970s)
3. Poor quality (large error bar)

- R. Engelmann, et al., Phys. Lett. 21 (1966) 587
- G. Alexander, et al., Phys. Rev. 173 (1968) 1452
- B. Sechi-Zorn, et al., Phys. Rev. 175 (1968) 1735
- F. Eisele, et al., Phys. Lett. 37B (1971) 204
- V. Hepp and H. Schleich, Z. Phys. 214 (1968) 71

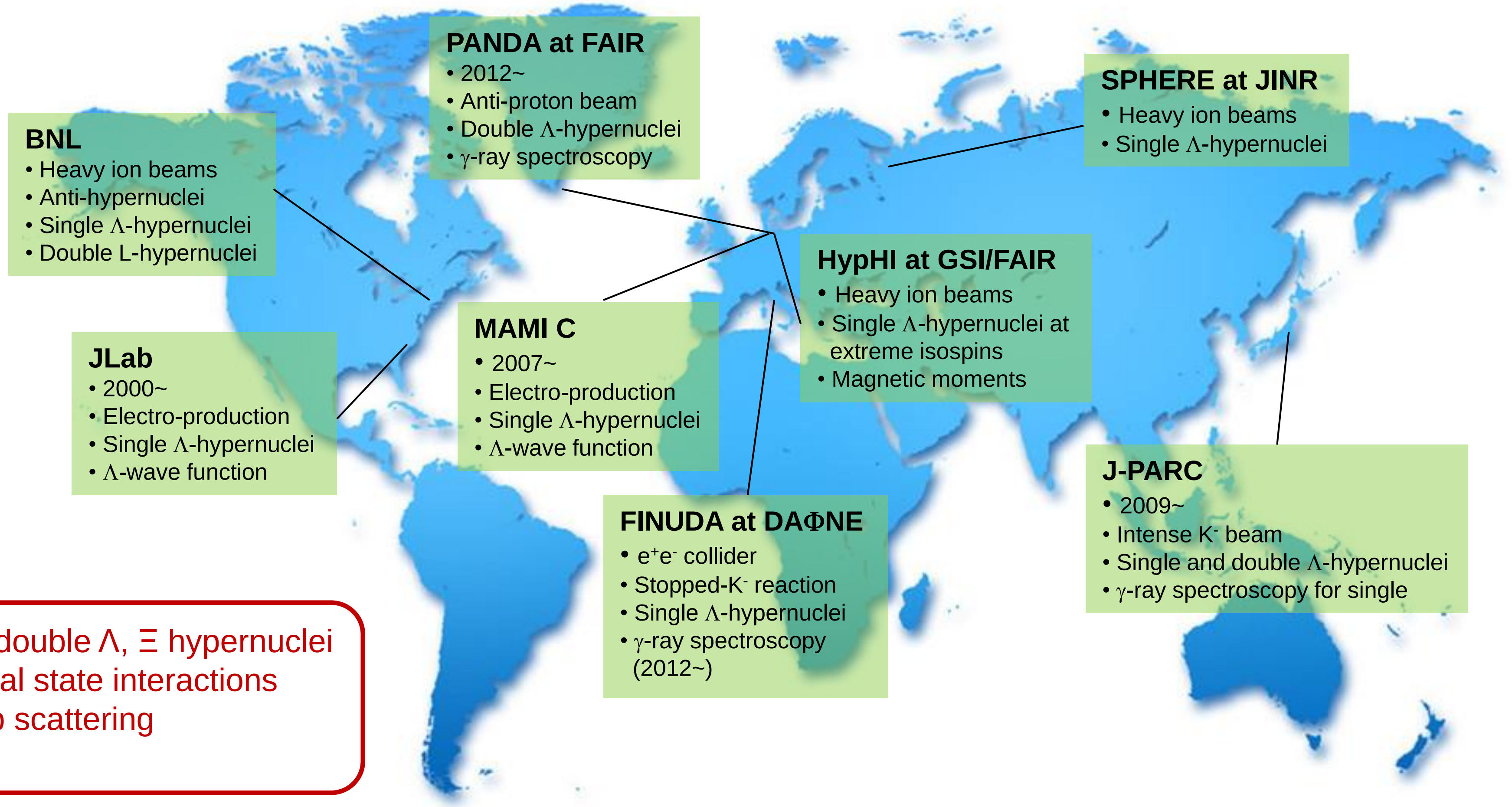
- Short lifetime of hyperons! ($\leq 10^{-10}$ s)

$$\Lambda \to p\pi^-,\ n\pi^0 \ldots \qquad \Sigma^- \to n\pi^- \ldots$$
$$\Sigma^+ \to p\pi^0,\ n\pi^+ \ldots \qquad \Xi^0 \to \Lambda\pi^0 \ldots$$
$$\Sigma^0 \to \Lambda\gamma,\ \Lambda\gamma\gamma \ldots \qquad \Xi^- \to \Lambda\pi^- \ldots$$

$\Lambda p \to \Lambda p$		$\Lambda p \to \Lambda p$		$\Sigma^- p \to \Lambda n$	
$p_{\rm lab}^{\Lambda}$	$\sigma_{\rm exp}$	$p_{\rm lab}^{\Lambda}$	$\sigma_{\rm exp}$	$p_{\rm lab}^{\Sigma^-}$	$\sigma_{\rm exp}$
135 ± 15	209 ± 58	145 ± 25	180 ± 22	110 ± 5	174 ± 47
165 ± 15	177 ± 38	185 ± 15	130 ± 17	120 ± 5	178 ± 39
195 ± 15	153 ± 27	210 ± 10	118 ± 16	130 ± 5	140 ± 28
225 ± 15	111 ± 18	230 ± 10	101 ± 12	140 ± 5	164 ± 25
255 ± 15	87 ± 13	250 ± 10	83 ± 13	150 ± 5	147 ± 19
300 ± 30	46 ± 11	290 ± 30	57 ± 9	160 ± 5	124 ± 14
$\Sigma^+ p \to \Sigma^+ p$		$\Sigma^- p \to \Sigma^- p$		$\Sigma^- p \to \Sigma^0 n$	
$p_{\rm lab}^{\Sigma^+}$	$\sigma_{\rm exp}$	$p_{\rm lab}^{\Sigma^-}$	$\sigma_{\rm exp}$	$p_{\rm lab}^{\Sigma^-}$	$\sigma_{\rm exp}$
145 ± 5	123 ± 62	135 ± 5	184 ± 52	110 ± 5	396 ± 91
155 ± 5	104 ± 30	142.5 ± 5	152 ± 38	120 ± 5	159 ± 43
165 ± 5	92 ± 18	147.5 ± 5	146 ± 30	130 ± 5	157 ± 34
175 ± 5	81 ± 12	152.5 ± 5	142 ± 25	140 ± 5	125 ± 25
		157.5 ± 5	164 ± 32	150 ± 5	111 ± 19
		162.5 ± 5	138 ± 19	160 ± 5	115 ± 16
		167.5 ± 5	113 ± 16		
$\Sigma^- p$ inelastic capture ratio at rest			$r_R = 0.468 \pm 0.010$		

Units for p and σ: MeV/c and mb

Prospects: very promising



- Λ, double Λ, Ξ hypernuclei
- Final state interactions
- $\Sigma^{+}p$ scattering
- …

…

Basic map from Saito, HYP06

Theoretical status

- In about recent 2 decades

Group / Place	Model / Method	Reference
Phenomenological model		
Beijing-Tübingen	Chiral SU(3) quark cluster model	Zhang NPA 578 (1994) 573
Kyoto-Niigata:	SU(6) quark cluster model (FSS, fss2)	Fujiwara PRL 76 (1996) 2242
Nanjing:	Quark delocalization and color screening model	Ping NPA 657 (1999) 95
Nijmegen:	SU(3) meson exchange model (NSC, ESC…)	Rijken PRC 59 (1999) 21
Bonn-Jülich:	SU(6) meson exchange model (Jülich 94, 04)	Haidenbauer PRC 72 (2005) 044005
Valencia:	Meson exchange model (UChPT)	Sasaki PRC 74 (2006) 064002
…	…	…
Effective field theory		
Pecs-Groningen:	KSW approach	Korpa PRC 65 (2002) 015208
Bonn-Jülich:	Heavy baryon chiral effective field theory	Haidenbauer NPA 915 (2013) 24
Beihang-Peking:	Covariant chiral effective field theory	Li PRD 94 (2016) 014029
…	…	…
Lattice QCD simulation		
NPLQCD:	Lüscher’s finite volume method (phase shifts)	Beane NPA 794 (2007) 62
HAL QCD:	HAL QCD method (non-local potential)	Inoue PTP 124 (2010) 591
…	…	…

Some of the representative works

1. Background and significance

2. Chiral effective field theory

3. A covariant ChEFT approach

4. Results and discussion

5. Summary and outlook

Weinberg's approach

- Chiral Effective Field Theory

Advantages:

- ✓ Improve calculations systematically
- ✓ Estimate theoretical uncertainties
- ✓ Consistent three- and multi-baryon forces

First proposed
by
Steven Weinberg

Phys. Lett. B 251 (1990) 288

Nuclear forces from chiral lagrangians

Steven Weinberg [1]
Theory Group, Department of Physics, University of Texas, Austin, TX 78712, USA

Received 14 August 1990

Nucl. Phys. B 363 (1991) 3

EFFECTIVE CHIRAL LAGRANGIANS FOR NUCLEON–PION INTERACTIONS AND NUCLEAR FORCES

Steven WEINBERG*

Theory Group, Department of Physics, University of Texas, Austin, TX 78712, USA

Received 2 April 1991

In YN and YY interactions: Korpa '01, Polinder '06 '07, Haidenbauer '07 '10 '13 '15, Li '16...

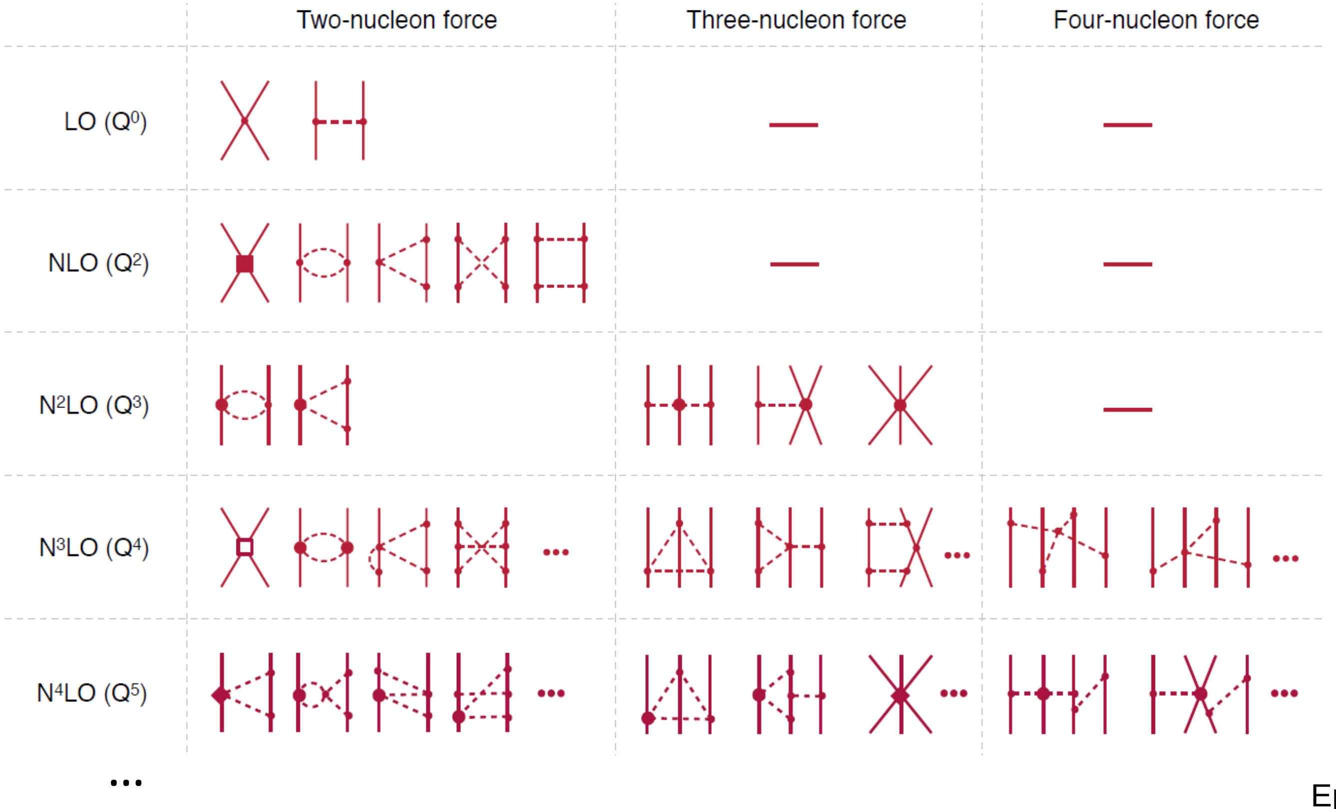
Weinberg's approach

Unsolved LECs

Fit to Exp. data

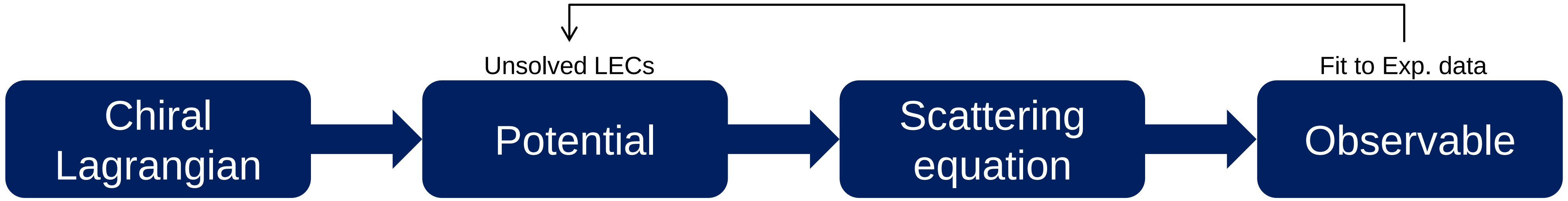
Chiral Lagrangian → Potential → Scattering equation → Observable

| ================ Power counting (systematic expansion) ================ |



Epelbaum, arXiv: 1510.07036 [nucl-th]

Weinberg's approach



| ================ Power counting (systematic expansion) ================ |

However,

(1) Lippmann-Schwinger equation

$$T(p,p') = V(p,p') + \int \frac{dp'' p''^2}{(2\pi)^3} V(p,p'') \frac{2\mu}{q_\nu^2 - p''^2 + i\epsilon} T(p'',p')$$

- Singular
  - Cutoff
  - Modified power counting

(2) Reductions

- The missing of relativistic effects

Weinberg's approach

Unsolved LECs | | | Fit to Exp. data

Chiral Lagrangian → Potential → Scattering equation → Observable

| ================ Power counting (systematic expansion) ================ |

However,

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- Singular
  - Cutoff
  - Modified power counting

(2) Reductions

- The missing of relativistic effects

Relativistic effects in
one-baryon and heavy-light systems

- Geng PRL 101 (2008) 222002
- Geng PRD 79 (2009) 094022
- Geng PRD 84 (2011) 074024
- Ren JHEP 12 (2012) 073
- Ren PRD 91 (2015) 051502
- …
- Geng PRD 82 (2010) 054022
- Geng PLB 696 (2011) 390
- Altenbuchinger PLB 713 (2012) 453
- …

Faster convergence!

Will it happen in the two-baryon system?

Power counting

- Naive dimensional analysis (Weinberg's proposal)

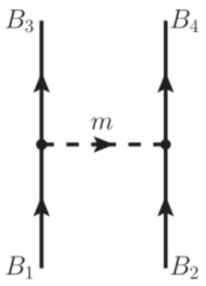
1. **Vertices from the k^{th} order Lagrangian ~ Q^k**
2. **Loop integration in n dimensions ~ Q^n**
3. **Meson propagator ~ Q^{-2}**
4. **Baryon propagator ~ Q^{-1}**

$$\nu = 2 - \frac{B}{2} + 2L + \sum_i v_i \Delta_i \qquad \Delta_i = d_i + \frac{1}{2} b_i - 2$$

- **ν – chiral order**
- B – number of external baryons
- L – number of goldstone boson loops
- i – number of types of the vertices
- v_i – number of vertices with dimension Δ_i
- d_i – number of derivatives
- b_i – number of internal baryon lines

Leading order ($\sim Q^{\nu=0}$) Feynman diagrams

B=4, L=0, i=1,
v=1, d=0, b=4.



B=4, L=0, i=1,
v=2, d=1, b=2.

Covariant chiral Lagrangians

Mesonic part

$$\mathcal{L}_M^{(2)} = \frac{f_0^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger \rangle + \frac{1}{2} B_0 f_0^2 \langle M U^\dagger + U M \rangle$$

$$U(x) = \mathrm{Exp}\left(i \frac{\phi(x)}{f_0} \right)$$

$$M \equiv \mathrm{diag}\,(m_u,\ m_d,\ m_s)$$

$$\phi(x) = \sum_{a=1}^{8} \phi_a(x) \lambda_a \equiv \begin{pmatrix} \pi^0 + \frac{\eta}{\sqrt{3}} & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{\eta}{\sqrt{3}} & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2\eta}{\sqrt{3}} \end{pmatrix}$$

Meson-baryon interaction

$$\mathcal{L}_{MB}^{(1)} = \left\langle \bar{B}\,(i\not{D} - M_B)\,B - \frac{D}{2}\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\} - \frac{F}{2}\bar{B}\gamma^\mu\gamma_5[u_\mu, B] \right\rangle$$

Covariant derivative:

$$D^\mu B = (\partial_\mu + \Gamma_\mu - i v_\mu^{(s)})B$$

$$B = \sum_a \frac{B_a \lambda_a}{\sqrt{2}} \equiv \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

Four-baryon contact terms

$$\mathcal{L}_{\mathrm{CT}}^1 = C_i^1 \langle \bar{B}_a \bar{B}_b (\Gamma_i B)_b (\Gamma_i B)_a \rangle \quad \mathcal{L}_{\mathrm{CT}}^2 = C_i^2 \langle \bar{B}_a (\Gamma_i B)_a \bar{B}_b (\Gamma_i B)_b \rangle \quad \mathcal{L}_{\mathrm{CT}}^3 = C_i^3 \langle \bar{B}_a (\Gamma_i B)_a \rangle \langle \bar{B}_b (\Gamma_i B)_b \rangle$$

Clifford algebra: $\Gamma_1 = 1,\quad \Gamma_2 = \gamma^\mu,\quad \Gamma_3 = \sigma^{\mu\nu},\quad \Gamma_4 = \gamma^\mu\gamma_5,\quad \Gamma_5 = \gamma_5.$

Leading order potentials (1st improvement)

- In Weinberg's approach

$$V_{\mathrm{LO}}(\boldsymbol{p}',\boldsymbol{p}) = C_S + C_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + N_1 N_2 \frac{\boldsymbol{\sigma}_1 \cdot (\boldsymbol{p}'-\boldsymbol{p})\, \boldsymbol{\sigma}_2 \cdot (\boldsymbol{p}'-\boldsymbol{p})}{(\boldsymbol{p}'-\boldsymbol{p})^2 + m^2 - i\epsilon}$$

Nonderivative four-baryon contact terms + One-pseudoscalar-meson-exchange

- Baryon spinors

Weinberg's approach

$$u_i(\boldsymbol{p},s) = \begin{pmatrix} \mathbb{1} \\ 0 \end{pmatrix} \chi_s$$

Covariant ChEFT approach

$$u_i(\boldsymbol{p},s) = \sqrt{\frac{E+M_i}{2M_i}} \begin{pmatrix} \mathbb{1} \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+M_i} \end{pmatrix} \chi_s$$

The 'small' components are NOT omitted!!!

Leading order potentials (1st improvement)

- Nonderivative four-baryon contact terms (helicity basis)

$$V_1^{\mathrm{CT}}(\boldsymbol{p}',\boldsymbol{p}) = C_1 \frac{(E_{p'}+M_B)(E_p+M_B)}{4M_B^2}\left(1-\frac{4|\boldsymbol{p}'||\boldsymbol{p}|\lambda_1'\lambda_1}{(E_{p'}+M_B)(E_p+M_B)}\right)\left(1-\frac{4|\boldsymbol{p}'||\boldsymbol{p}|\lambda_2'\lambda_2}{(E_{p'}+M_B)(E_p+M_B)}\right)\langle\lambda_1'\lambda_2'|\lambda_1\lambda_2\rangle$$

$$\begin{aligned}V_2^{\mathrm{CT}}(\boldsymbol{p}',\boldsymbol{p}) = C_2 \frac{(E_{p'}+M_B)(E_p+M_B)}{4M_B^2}&\left[\left(1+\frac{4|\boldsymbol{p}'||\boldsymbol{p}|\lambda_1'\lambda_1}{(E_{p'}+M_B)(E_p+M_B)}\right)\left(1+\frac{4|\boldsymbol{p}'||\boldsymbol{p}|\lambda_2'\lambda_2}{(E_{p'}+M_B)(E_p+M_B)}\right)\langle\lambda_1'\lambda_2'|\lambda_1\lambda_2\rangle\right.\\&\left.-\left(\frac{2|\boldsymbol{p}'|\lambda_1'}{E_{p'}+M_B}+\frac{2|\boldsymbol{p}|\lambda_1}{E_p+M_B}\right)\left(\frac{2|\boldsymbol{p}'|\lambda_2'}{E_{p'}+M_B}+\frac{2|\boldsymbol{p}|\lambda_2}{E_p+M_B}\right)\langle\lambda_1'\lambda_2'|\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2|\lambda_1\lambda_2\rangle\right]\end{aligned}$$

$$\begin{aligned}V_3^{\mathrm{CT}}(\boldsymbol{p}',\boldsymbol{p}) = 2C_3 \frac{(E_{p'}+M_B)(E_p+M_B)}{4M_B^2}&\left[\left(1-\frac{4|\boldsymbol{p}'||\boldsymbol{p}|\lambda_1'\lambda_1}{(E_{p'}+M_B)(E_p+M_B)}\right)\left(1-\frac{4|\boldsymbol{p}'||\boldsymbol{p}|\lambda_2'\lambda_2}{(E_{p'}+M_B)(E_p+M_B)}\right)\right.\\&\left.-\left(\frac{2|\boldsymbol{p}'|\lambda_1'}{E_{p'}+M_B}-\frac{2|\boldsymbol{p}|\lambda_1}{E_p+M_B}\right)\left(\frac{2|\boldsymbol{p}'|\lambda_2'}{E_{p'}+M_B}-\frac{2|\boldsymbol{p}|\lambda_2}{E_p+M_B}\right)\right]\langle\lambda_1'\lambda_2'|\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2|\lambda_1\lambda_2\rangle\end{aligned}$$

$$\begin{aligned}V_4^{\mathrm{CT}}(\boldsymbol{p}',\boldsymbol{p}) = C_4 \frac{(E_{p'}+M_B)(E_p+M_B)}{4M_B^2}&\left[\left(\frac{2|\boldsymbol{p}'|\lambda_1'}{E_{p'}+M_B}+\frac{2|\boldsymbol{p}|\lambda_1}{E_p+M_B}\right)\left(\frac{2|\boldsymbol{p}'|\lambda_2'}{E_{p'}+M_B}+\frac{2|\boldsymbol{p}|\lambda_2}{E_p+M_B}\right)\langle\lambda_1'\lambda_2'|\lambda_1\lambda_2\rangle\right.\\&\left.-\left(1+\frac{4|\boldsymbol{p}'||\boldsymbol{p}|\lambda_1'\lambda_1}{(E_{p'}+M_B)(E_p+M_B)}\right)\left(1+\frac{4|\boldsymbol{p}'||\boldsymbol{p}|\lambda_2'\lambda_2}{(E_{p'}+M_B)(E_p+M_B)}\right)\langle\lambda_1'\lambda_2'|\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2|\lambda_1\lambda_2\rangle\right]\end{aligned}$$

$$V_5^{\mathrm{CT}}(\boldsymbol{p}',\boldsymbol{p}) = C_5 \frac{(E_{p'}+M_B)(E_p+M_B)}{4M_B^2}\left(\frac{2|\boldsymbol{p}'|\lambda_1'}{E_{p'}+M_B}-\frac{2|\boldsymbol{p}|\lambda_1}{E_p+M_B}\right)\left(\frac{2|\boldsymbol{p}'|\lambda_2'}{E_{p'}+M_B}-\frac{2|\boldsymbol{p}|\lambda_2}{E_p+M_B}\right)\langle\lambda_1'\lambda_2'|\lambda_1\lambda_2\rangle$$

Leading order potentials (1st improvement)

- Nonderivative four-baryon contact terms (LSJ basis, all J = 0 & 1)

$$V^{\rm CT}_{B_1B_2}(^1S_0) = 4\pi X_0\left[(C_1+C_2-6C_3+3C_4)(1+A^2B^2)+(3C_2+6C_3+C_4+C_5)(A^2+B^2)\right]$$
$$\equiv 4\pi X_0\left[C^{B_1B_2}_{1S0}(1+A^2B^2)+\hat{C}^{B_1B_2}_{1S0}(A^2+B^2)\right]$$

$$V^{\rm CT}_{B_1B_2}(^3S_1) = 4\pi X_0\left[\frac{1}{9}(C_1+C_2+2C_3-C_4)(9+A^2B^2)+\frac{1}{3}(C_2+2C_3-C_4-C_5)(A^2+B^2)\right]$$
$$\equiv 4\pi X_0\left[\frac{1}{9}C^{B_1B_2}_{3S1}(9+A^2B^2)+\frac{1}{3}\hat{C}^{B_1B_2}_{3S1}(A^2+B^2)\right]$$

$$V^{\rm CT}_{B_1B_2}(^3P_1) = 4\pi X_0\left[-\frac{4}{3}(C_1-2C_2+4C_3+2C_4-C_5)AB\right] \equiv 4\pi X_0\left[-\frac{4}{3}C^{B_1B_2}_{3P1}AB\right]$$

with

$$X_0(\boldsymbol{p}',\boldsymbol{p}) = \frac{(E_{p'}+M_B)(E_p+M_B)}{4M_B^2},\qquad A(\boldsymbol{p}') = \frac{|\boldsymbol{p}'|}{E_{p'}+M_B},\qquad B(\boldsymbol{p}) = \frac{|\boldsymbol{p}|}{E_p+M_B}.$$

We choose the 5 LECs in $^1S_0$, $^3S_1$ and $^3P_1$ to be independent!

(Others in $^3P_0$, $^1P_1$, $^3S_1$-$^3D_1$, $^3D_1$-$^3S_1$, $^3D_1$ are not.)

Leading order potentials (1st improvement)

- Nonderivative four-baryon contact terms (LSJ basis, all J = 0 & 1)

$$V^{\rm CT}_{B_1B_2}(^3P_0) = 4\pi X_0\left[-2(C_1-4C_2-4C_4+C_5)AB\right]$$
$$= 4\pi X_0\left[-2(-C^{B_1B_2}_{1S0} - \hat{C}^{B_1B_2}_{1S0} + 2C^{B_1B_2}_{3S1} - 2\hat{C}^{B_1B_2}_{3S1})AB\right]$$

$$V^{\rm CT}_{B_1B_2}(^1P_1) = 4\pi X_0\left[-\frac{2}{3}(C_1+C_5)AB\right] = 4\pi X_0\left[-\frac{2}{3}(C^{B_1B_2}_{3S1} - \hat{C}^{B_1B_2}_{3S1})AB\right]$$

$$V^{\rm CT}_{B_1B_2}(^3S_1-{}^3D_1) = 4\pi X_0\left[\frac{2}{9}\sqrt{2}(C_1+C_2+2C_3-C_4)A^2B^2 + \frac{2}{3}\sqrt{2}(C_2+2C_3-C_4-C_5)B^2\right]$$
$$\equiv 4\pi X_0\left[\frac{2}{9}\sqrt{2}C^{B_1B_2}_{3S1}A^2B^2 + \frac{2}{3}\sqrt{2}\hat{C}^{B_1B_2}_{3S1}B^2\right]$$

$$V^{\rm CT}_{B_1B_2}(^3D_1-{}^3S_1) = 4\pi X_0\left[\frac{2}{9}\sqrt{2}(C_1+C_2+2C_3-C_4)A^2B^2 + \frac{2}{3}\sqrt{2}(C_2+2C_3-C_4-C_5)A^2\right]$$
$$\equiv 4\pi X_0\left[\frac{2}{9}\sqrt{2}C^{B_1B_2}_{3S1}A^2B^2 + \frac{2}{3}\sqrt{2}\hat{C}^{B_1B_2}_{3S1}A^2\right]$$

$$V^{\rm CT}_{B_1B_2}(^3D_1) = 4\pi X_0\left[\frac{8}{9}(C_1+C_2+2C_3-C_4)A^2B^2\right] \equiv 4\pi X_0\left[\frac{8}{9}C^{B_1B_2}_{3S1}A^2B^2\right]$$

Not independent LECs!

Leading order potentials (1st improvement)

- One-pseudoscalar-meson-exchange (helicity basis)

$$
\begin{aligned}
V^{\mathrm{OME}}(\boldsymbol{p}',\boldsymbol{p}) = & -N_1N_2\frac{(E_{p'}+M_B)(E_p+M_B)}{M_B^2} \\
& \times\left[(E_{p'}-E_p)\left(\frac{|\boldsymbol{p}'|\lambda_2'}{E_{p'}+M_B}+\frac{|\boldsymbol{p}|\lambda_2}{E_p+M_B}\right)-(|\boldsymbol{p}'|\lambda_2'-|\boldsymbol{p}|\lambda_2)\left(1+\frac{4|\boldsymbol{p}'||\boldsymbol{p}|\lambda_2'\lambda_2}{(E_{p'}+M_B)(E_p+M_B)}\right)\right] \\
& \times\left[(E_{p'}-E_p)\left(\frac{|\boldsymbol{p}'|\lambda_1'}{E_{p'}+M_B}+\frac{|\boldsymbol{p}|\lambda_1}{E_p+M_B}\right)-(|\boldsymbol{p}'|\lambda_1'-|\boldsymbol{p}|\lambda_1)\left(1+\frac{4|\boldsymbol{p}'||\boldsymbol{p}|\lambda_1'\lambda_1}{(E_{p'}+M_B)(E_p+M_B)}\right)\right] \\
& \times\frac{\langle\lambda_1'\lambda_2'|\lambda_1\lambda_2\rangle}{(E_{p'}-E_p)^2-(\boldsymbol{p}'-\boldsymbol{p})^2-m^2+i\epsilon} \\
\simeq & \; N_1N_2\frac{(E_{p'}+M_B)(E_p+M_B)}{M_B^2} \\
& \times\left[(E_{p'}-E_p)\left(\frac{|\boldsymbol{p}'|\lambda_2'}{E_{p'}+M_B}+\frac{|\boldsymbol{p}|\lambda_2}{E_p+M_B}\right)-(|\boldsymbol{p}'|\lambda_2'-|\boldsymbol{p}|\lambda_2)\left(1+\frac{4|\boldsymbol{p}'||\boldsymbol{p}|\lambda_2'\lambda_2}{(E_{p'}+M_B)(E_p+M_B)}\right)\right] \\
& \times\left[(E_{p'}-E_p)\left(\frac{|\boldsymbol{p}'|\lambda_1'}{E_{p'}+M_B}+\frac{|\boldsymbol{p}|\lambda_1}{E_p+M_B}\right)-(|\boldsymbol{p}'|\lambda_1'-|\boldsymbol{p}|\lambda_1)\left(1+\frac{4|\boldsymbol{p}'||\boldsymbol{p}|\lambda_1'\lambda_1}{(E_{p'}+M_B)(E_p+M_B)}\right)\right] \\
& \times\frac{\langle\lambda_1'\lambda_2'|\lambda_1\lambda_2\rangle}{(\boldsymbol{p}'-\boldsymbol{p})^2+m^2-i\epsilon}
\end{aligned}
$$

Energy-dependent term in the propagator is omitted, same as in the scattering equation!

Scattering equation (2nd improvement)

- Lippmann-Schwinger equation (Weinberg's approach)

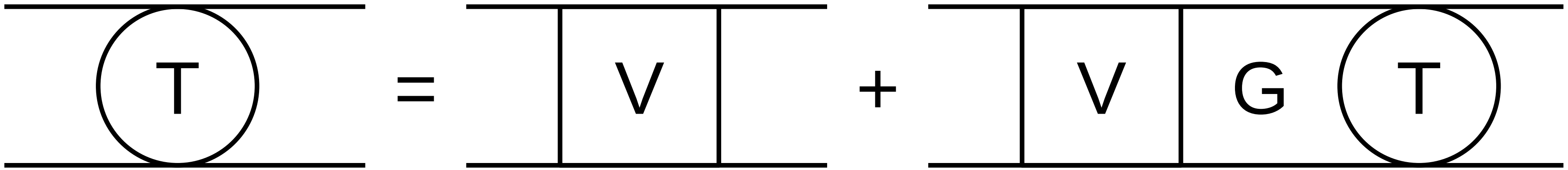
$$T^{\nu''\nu',J}_{\rho''\rho'}(p'',p';\sqrt{s}) = V^{\nu''\nu'}_{\rho''\rho'}(p'',p') + \sum_{\rho,\nu}\int_0^\infty \frac{dp\,p^2}{(2\pi)^3} V^{\nu''\nu}_{\rho''\rho}(p'',p)\frac{2\mu_\nu}{q_\nu^2 - p^2 + i\epsilon} T^{\nu\nu',J}_{\rho\rho'}(p,p';\sqrt{s})$$

ρ: partial wave $\quad$ ν: particle channel

- Kadyshevsky equation* (More relativistic effects involved)

$$T^{\nu''\nu',J}_{\rho''\rho'}(p'',p';\sqrt{s}) = V^{\nu''\nu'}_{\rho''\rho'}(p'',p') + \sum_{\rho,\nu}\int_0^\infty \frac{dp\,p^2}{(2\pi)^3} \frac{2\mu_\nu^2\, V^{\nu''\nu}_{\rho''\rho}(p'',p)\, T^{\nu\nu',J}_{\rho\rho'}(p,p';\sqrt{s})}{(p^2+4\mu_\nu^2)(\sqrt{q_\nu^2+4\mu_\nu^2} - \sqrt{p^2+4\mu_\nu^2} + i\epsilon)}$$

A 3-dimensional reduction of the relativistic Bethe-Salpeter equation



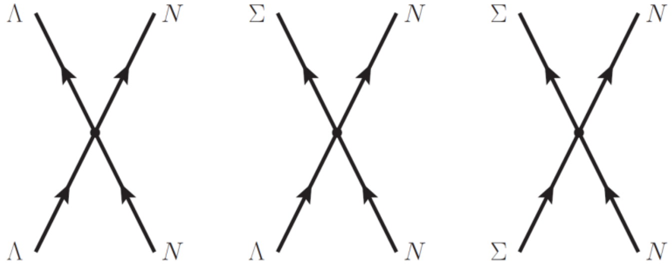
*Kadyshevsky, NPB 6 (1968) 125

ΛN and ΣN systems

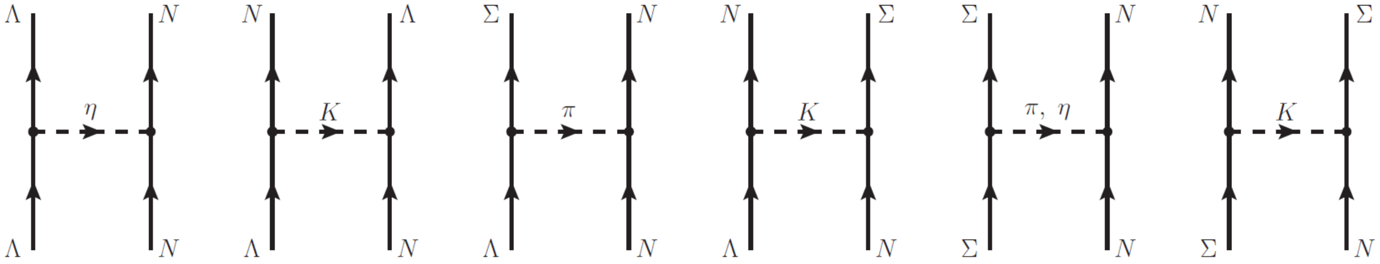
- S = -1; I = 3/2, 1/2

$\Sigma^+ p$	Λp, $\Sigma^+ n$, $\Sigma^0 p$	Λn, $\Sigma^0 n$, $\Sigma^- p$	$\Sigma^- n$	
+3/2	+1/2	-1/2	-3/2	I_3

- Nonderivative four-baryon contact terms (LO):



- One-pseudoscalar-meson-exchange (LO)

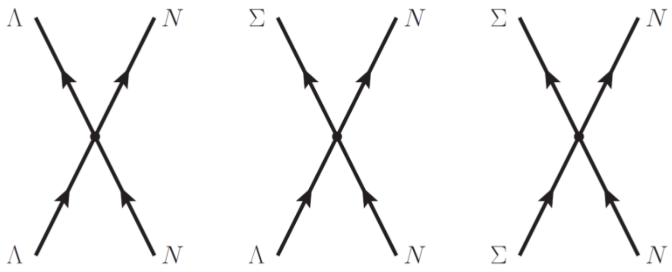


ΛN and ΣN systems

- S = -1; I = 3/2, 1/2

Σ^+p	Λp, Σ^+n, Σ^0p	Λn, Σ^0n, Σ^-p	Σ^-n	I_3
+3/2	+1/2	-1/2	-3/2	

- Nonderivative four-baryon contact terms (LO):



Strict SU(3) symmetry is imposed, 12 low energy constants (LECs)

	$C^{\Lambda\Lambda}_{1S0}$	$\hat{C}^{\Lambda\Lambda}_{1S0}$	$C^{\Sigma\Sigma}_{1S0}$	$\hat{C}^{\Sigma\Sigma}_{1S0}$	
$C^{\Lambda\Lambda}_{3S1}$	$\hat{C}^{\Lambda\Lambda}_{3S1}$	$C^{\Sigma\Sigma}_{3S1}$	$\hat{C}^{\Sigma\Sigma}_{3S1}$	$C^{\Lambda\Sigma}_{3S1}$	$\hat{C}^{\Lambda\Sigma}_{3S1}$
		$C^{\Lambda\Lambda}_{3P1}$	$C^{\Sigma\Sigma}_{3P1}$		

Fitting procedure

- 36 YN scattering data
- Λ-hypertriton: a further constraint

  However, we cannot in the present
  - Λp $^1S_0$: sensitive to the hypertriton
  - Λp $^3S_1$: sensitive to the scattering data
  - **The Λp *S*-wave scattering lengths are considered**
- Σ-nucleus: repulsive (I=3/2, ΣN, $^3S_1$)
- Then a combined fit of NN & YN?

$\Lambda p \to \Lambda p$		$\Lambda p \to \Lambda p$		$\Sigma^- p \to \Lambda n$	
$p_{\rm lab}^{\Lambda}$	$\sigma_{\rm exp}$	$p_{\rm lab}^{\Lambda}$	$\sigma_{\rm exp}$	$p_{\rm lab}^{\Sigma^-}$	$\sigma_{\rm exp}$
135 ± 15	209 ± 58	145 ± 25	180 ± 22	110 ± 5	174 ± 47
165 ± 15	177 ± 38	185 ± 15	130 ± 17	120 ± 5	178 ± 39
195 ± 15	153 ± 27	210 ± 10	118 ± 16	130 ± 5	140 ± 28
225 ± 15	111 ± 18	230 ± 10	101 ± 12	140 ± 5	164 ± 25
255 ± 15	87 ± 13	250 ± 10	83 ± 13	150 ± 5	147 ± 19
300 ± 30	46 ± 11	290 ± 30	57 ± 9	160 ± 5	124 ± 14
$\Sigma^+ p \to \Sigma^+ p$		$\Sigma^- p \to \Sigma^- p$		$\Sigma^- p \to \Sigma^0 n$	
$p_{\rm lab}^{\Sigma^+}$	$\sigma_{\rm exp}$	$p_{\rm lab}^{\Sigma^-}$	$\sigma_{\rm exp}$	$p_{\rm lab}^{\Sigma^-}$	$\sigma_{\rm exp}$
145 ± 5	123 ± 62	135 ± 5	184 ± 52	110 ± 5	396 ± 91
155 ± 5	104 ± 30	142.5 ± 5	152 ± 38	120 ± 5	159 ± 43
165 ± 5	92 ± 18	147.5 ± 5	146 ± 30	130 ± 5	157 ± 34
175 ± 5	81 ± 12	152.5 ± 5	142 ± 25	140 ± 5	125 ± 25
		157.5 ± 5	164 ± 32	150 ± 5	111 ± 19
		162.5 ± 5	138 ± 19	160 ± 5	115 ± 16
		167.5 ± 5	113 ± 16		
$\Sigma^- p$ inelastic capture ratio at rest		$r_R = 0.468 \pm 0.010$			

Units for p and σ: MeV/c and mb

Fitting procedure

- Combined fit of NN & YN? - NO

SU(3) relations for the various contact potentials in the isospin basis

	Channel	I	$V(\xi)$		
			$\xi = {}^1S_0, {}^3P_0, {}^3P_1, {}^3P_2$	$\xi = {}^3S_1, {}^3S_1{-}{}^3D_1, {}^1P_1$	$\xi = {}^1P_1{-}{}^3P_1$
$S=0$	$NN \to NN$	0	–	$C_\xi^{10^*}$	–
	$NN \to NN$	1	C_ξ^{27}	–	–
$S=-1$	$\Lambda N \to \Lambda N$	$\frac{1}{2}$	$\frac{1}{10}(9C_\xi^{27}+C_\xi^{8_s})$	$\frac{1}{2}(C_\xi^{8_a}+C_\xi^{10^*})$	$\frac{-1}{\sqrt{20}}C_\xi^{8_s8_a}$
	$\Lambda N \to \Sigma N$	$\frac{1}{2}$	$\frac{3}{10}(-C_\xi^{27}+C_\xi^{8_s})$	$\frac{1}{2}(-C_\xi^{8_a}+C_\xi^{10^*})$	$\frac{-3}{\sqrt{20}}C_\xi^{8_s8_a}$
	$\Sigma N \to \Lambda N$				$\frac{1}{\sqrt{20}}C_\xi^{8_s8_a}$
	$\Sigma N \to \Sigma N$	$\frac{1}{2}$	$\frac{1}{10}(C_\xi^{27}+9C_\xi^{8_s})$	$\frac{1}{2}(C_\xi^{8_a}+C_\xi^{10^*})$	$\frac{3}{\sqrt{20}}C_\xi^{8_s8_a}$
	$\Sigma N \to \Sigma N$	$\frac{3}{2}$	C_ξ^{27}	C_ξ^{10}	–

$$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$$

- The χ^2 goes up to 244!
- Overestimated Σ^+p cross sections
- A near threshold bound state in Σ^+p channel

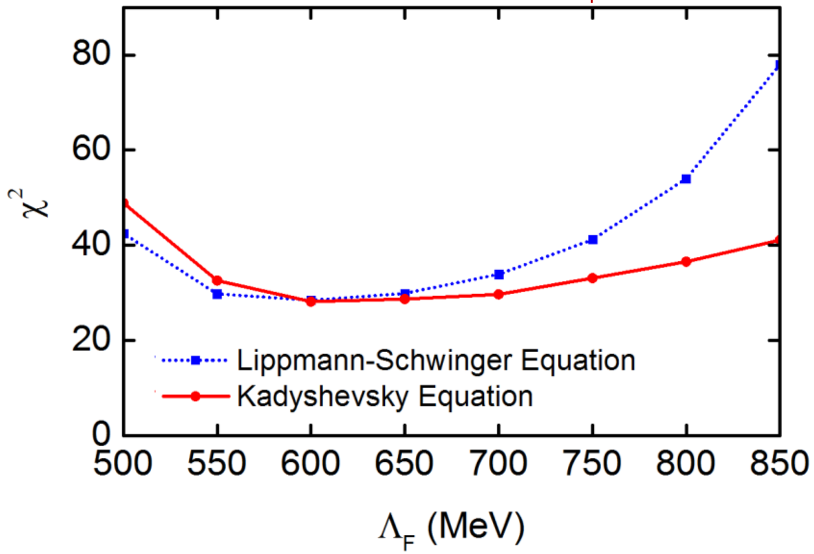
1. Background and significance
2. Chiral effective field theory
3. A covariant ChEFT approach
4. Results and discussion
5. Summary and outlook

Relativistic effects in the scattering equation (EG approach)

- χ^2 in the fit (nonrelativistic potentials, 36 YN data)

$$f^{\Lambda_F}(p,p') = \exp\left[-\left(\frac{p}{\Lambda_F}\right)^{2n} - \left(\frac{p'}{\Lambda_F}\right)^{2n}\right]$$

Cutoff dependence ($\Lambda_F \sim m_\rho$) of χ^2



1. Best description of the experimental data: qualitatively similar!

Li, PRD 94 (2016) 014029

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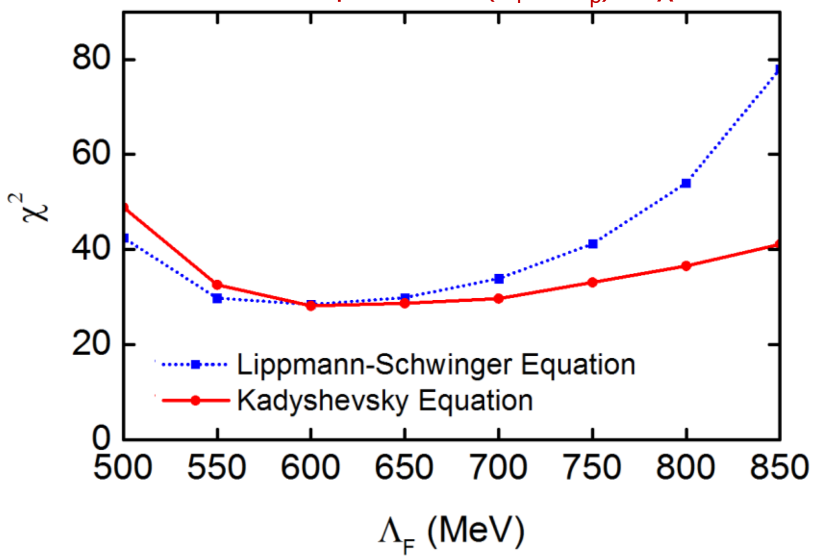
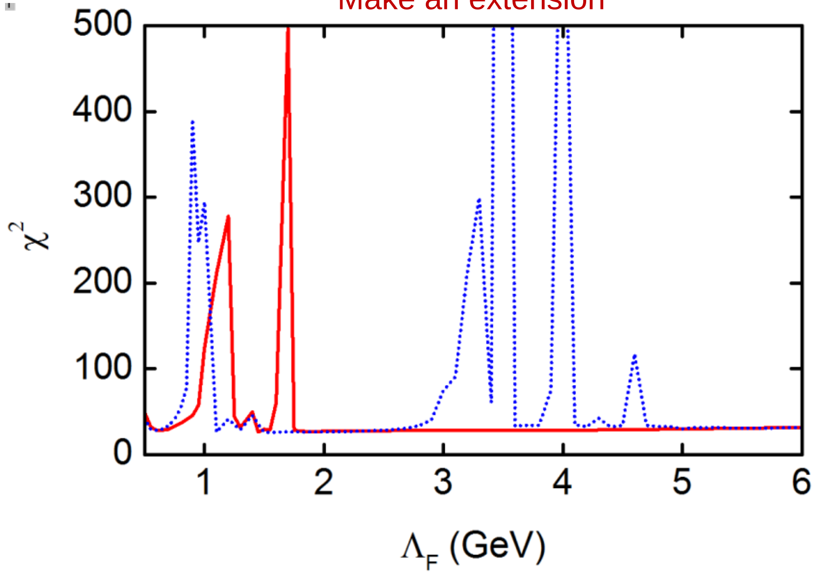


Make an extension



1. Best description of the experimental data: qualitatively similar!
2. Less peaks in using Kadyshevsky equation (EG approach)

🤔 **But where do these peaks come from?**

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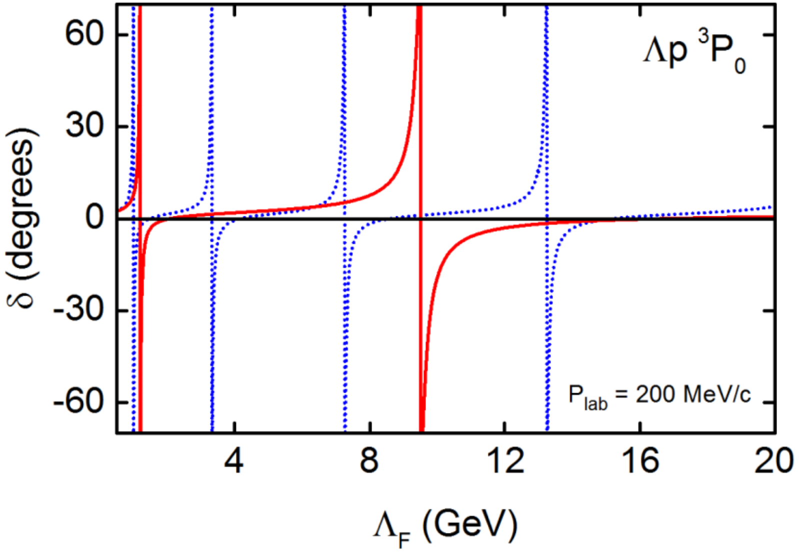
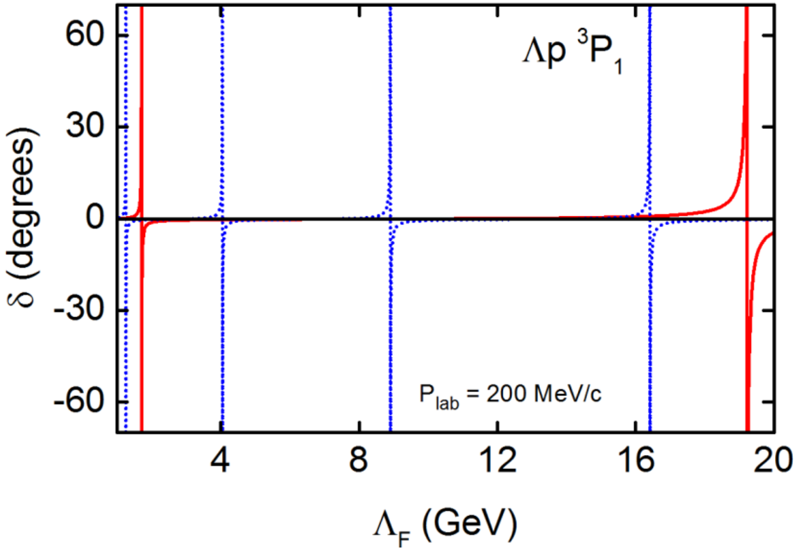
Relativistic effects in the scattering equation (EG approach)

- Limit-cycle-like behaviors in the phase shifts

Cutoff dependence in Λp $^3P_0$

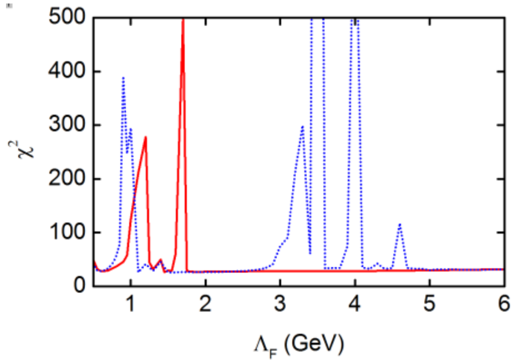


Cutoff dependence in Λp $^3P_1$



1. Limit-cycle-like behaviors appear
2. Kadyshevsky equation: cutoff dependence is mitigated

Divergent phase shifts ⟷ Very large χ^2

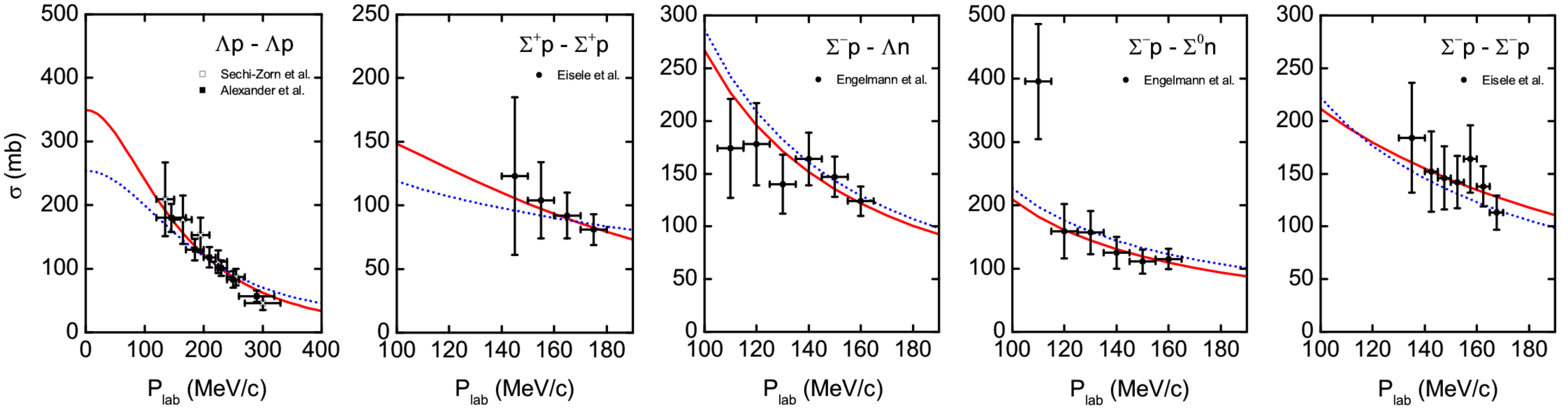


Li, PRD 94 (2016) 014029

Relativistic effects in the potentials

- Description of experimental data (cross sections)

Λ_F = 600 MeV



Red solid line: Covariant ChEFT (LO)   Blue dotted line: Weinberg's approach (LO)

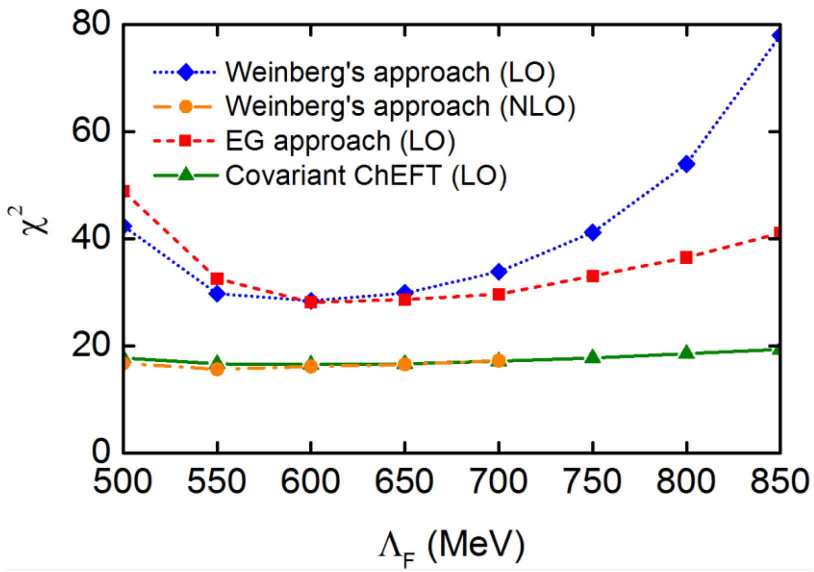
36 YN data	Weinberg's approach		Covariant ChEFT	NSC97f$
No. of LECs (or parameters)	5 (LO*)	23 (NLO#)	12 (LO)	29
χ^2	28.3	16.2	16.6	16.7

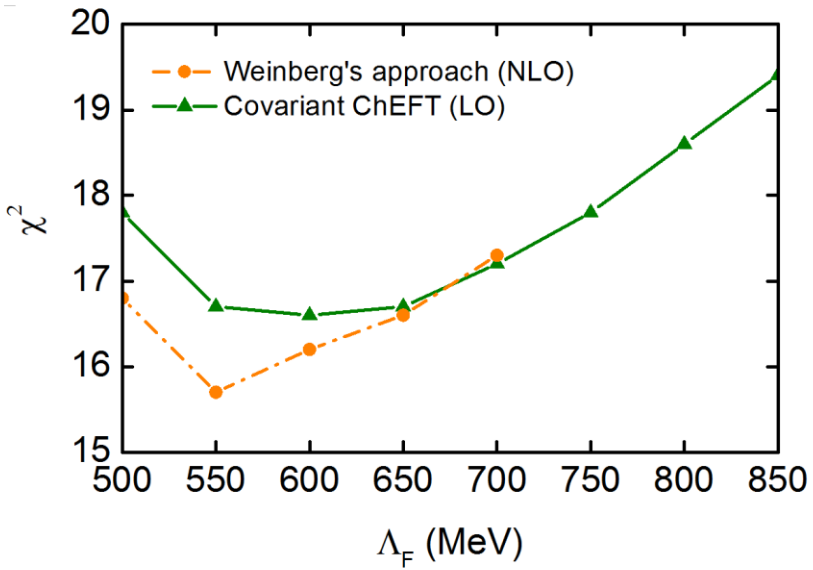
*Polinder NPA 799 (2006) 244   #Haidenbauer NPA 915 (2013) 24 $Rijken PRC 59 (1999) 21

Li, Ren and Geng. In preperation

Relativistic effects in the potentials

- Cutoff dependence of χ^2





1. Clear improvement of χ^2 and cutoff dependence
2. Renormalization group invariance is NOT realized

Li, Ren and Geng. In preperation

Summary and outlook

- Summary

1. Hyperon-nucleon scattering is studied in a covariant ChEFT approach at leading order
   - Covariant chiral Lagrangians
   - Relativistic potentials
   - (Semi-)Relativistic scattering equation
2. Relativistic effects in the scattering equation: cutoff dependence is mitigated
3. Relativistic effects in the potentials: better description of experimental data

Summary and outlook

- Outlook

1. Strangeness S = -2, -3, -4 systems
   - ΛΛ, ΣΛ, ΣΣ, ΞN (-2)
   - ΞΛ, ΞΣ (-3)
   - ΞΞ (-4)

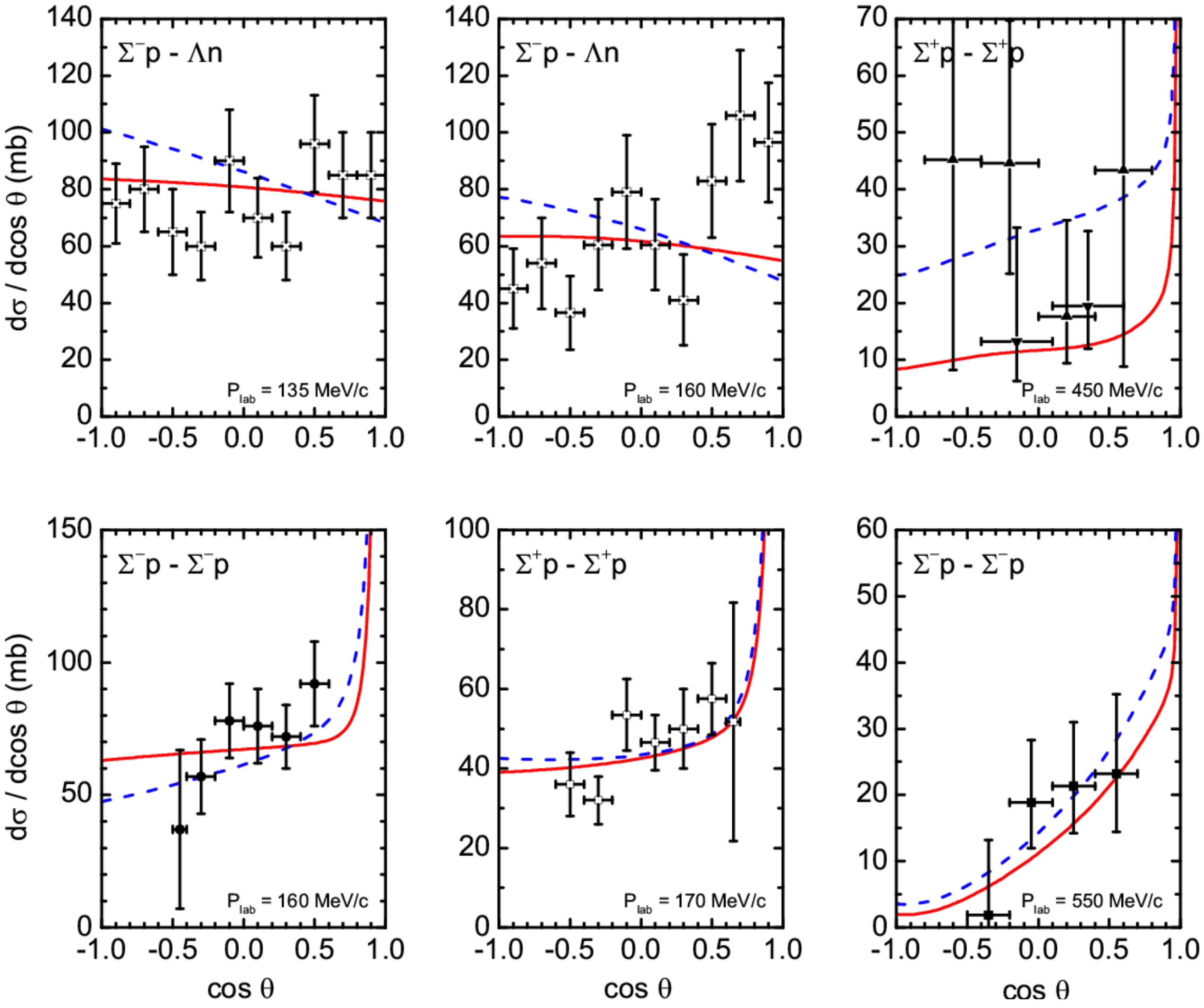
2. Few/Many-body calculations
   - As further constraints to pin down the LECs
   - Predictions: new Λ/ΛΛ/Ξ hypernuclei?


thanks

Differential cross sections

Λ_F = 600 MeV

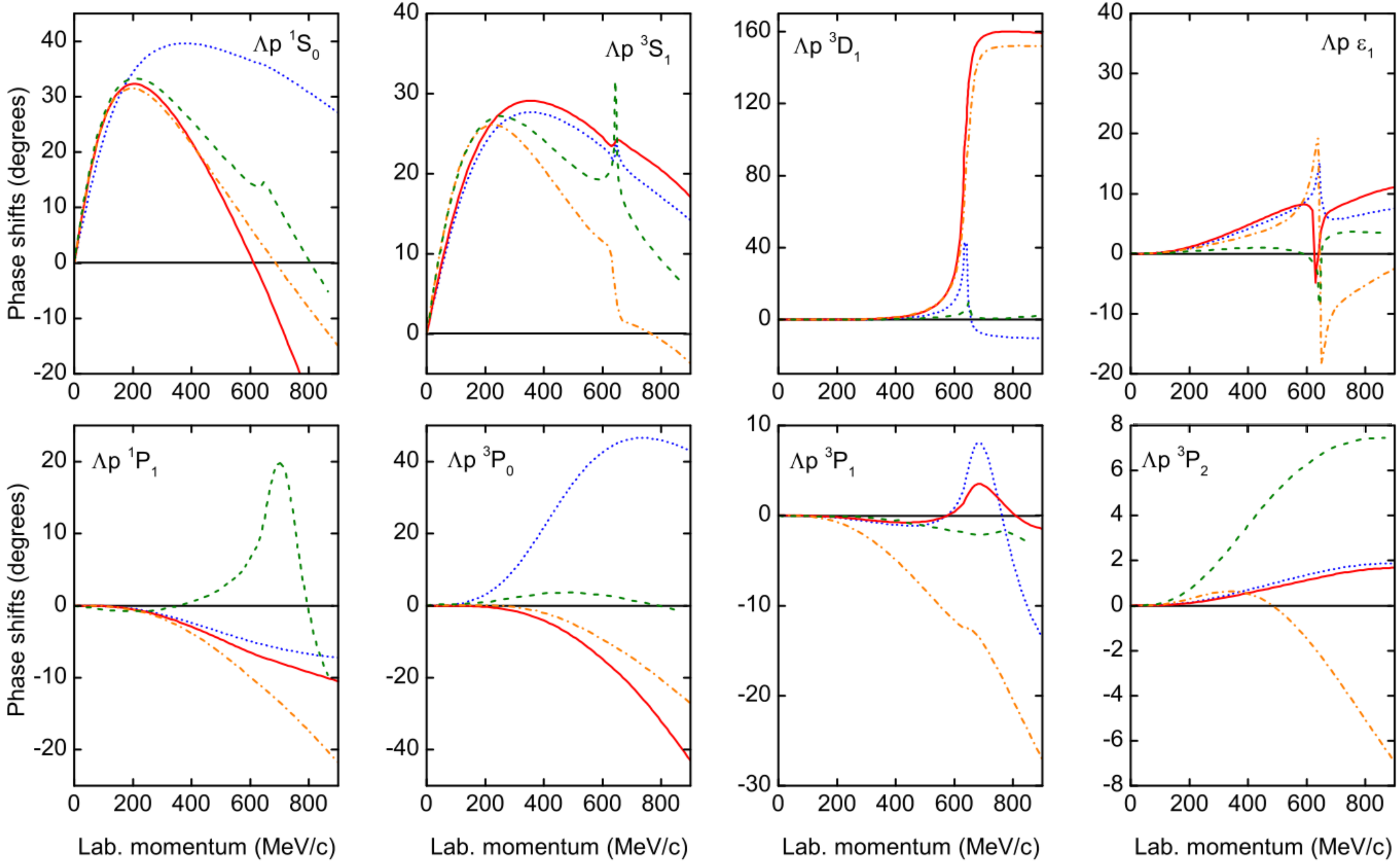
Red: Covariant ChEFT   Blue: Weinberg's approach



Phase shifts

Λ_F = 600 MeV

Red: Covariant ChEFT Blue: Weinberg's approach Green: Jülich 04 Orange: NSC97f



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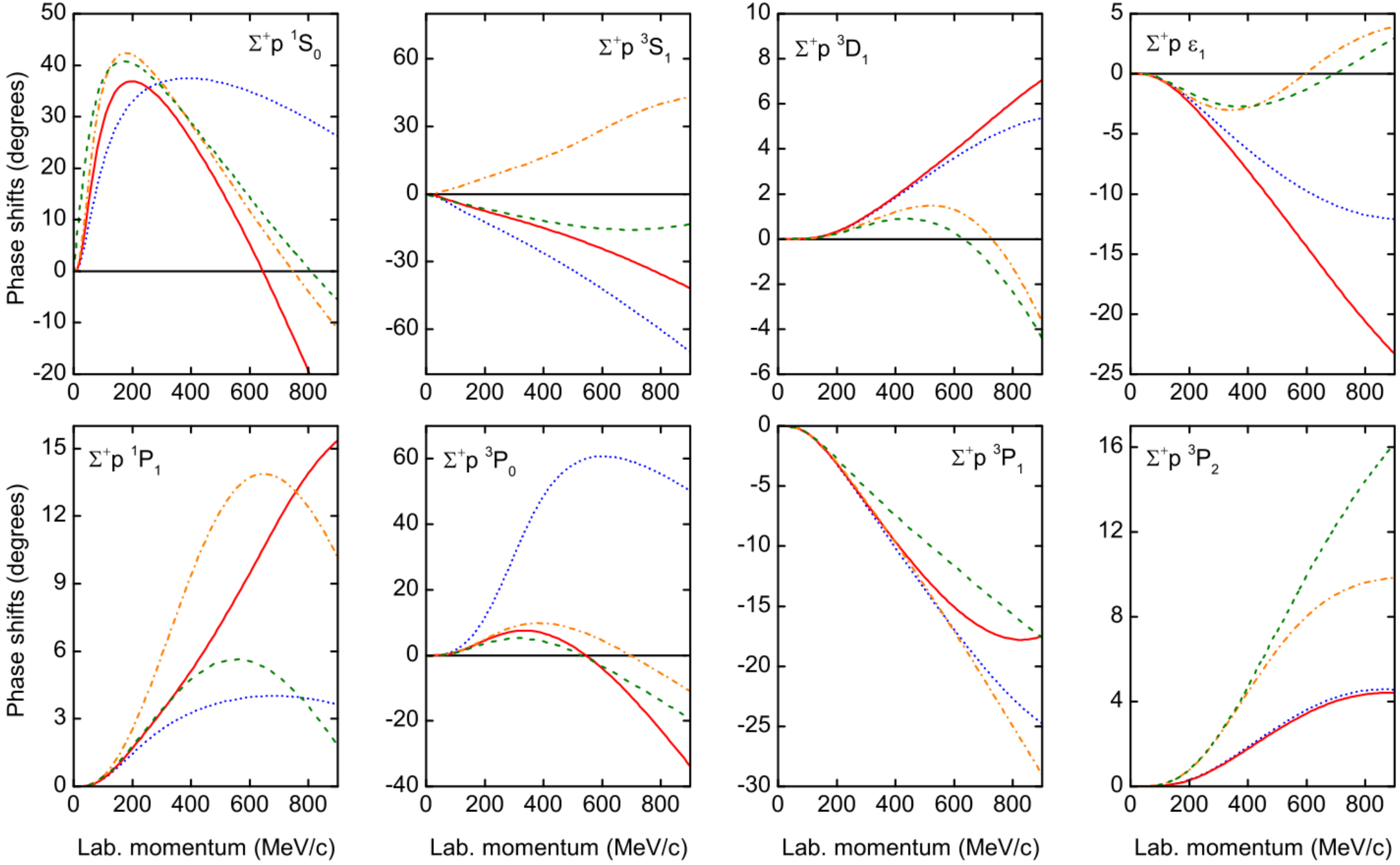

Scattering lengths

Λp	Weinberg's approach		Covariant ChEFT	NSC97f
$^1S_0$	-1.91 (LO)	-2.91 (NLO)	-2.45	-2.60
$^3S_1$	-1.23	-1.54	-1.32	-1.72

$$a_s = -1.8\begin{cases} +2.3 \text{ fm} \\ -4.2 \text{ fm} \end{cases} \quad \text{and} \quad a_t = -1.6\begin{cases} +1.1 \text{ fm} \\ -0.8 \text{ fm}, \end{cases}$$

A. Gasparyan PRC 69 (2004) 034006, extract from final-state interaction

$\Sigma^+ p$	Weinberg's approach		Covariant ChEFT	NSC97f
$^1S_0$	-2.32 (LO)	-3.56 (NLO)	-4.15	-4.35
$^3S_1$	0.65	0.49	0.38	-0.25