A charming trap for soft pions

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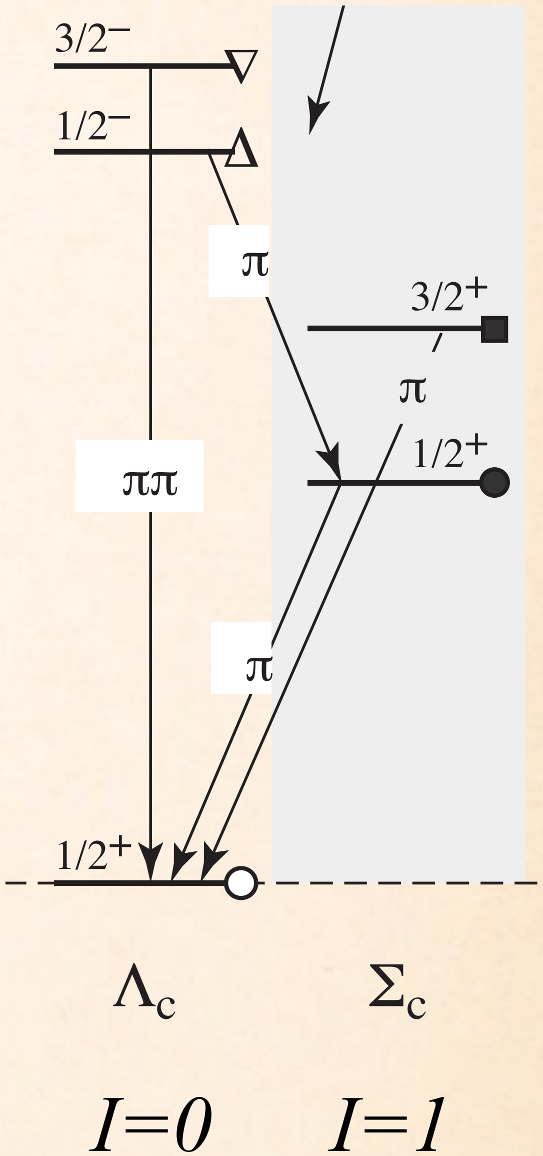
Outline

- Why do we study $\pi\Sigma_c$ system?
- What is ChEFT (in a nutshell)?
- How do we go about a system with very soft pions?
- What happens if we throw two pions at Σ_c?
- Have we seen the proposed $\pi\pi\Sigma_c$ resonance?

$\Lambda_c(2595)^+$ as an S-wave resonance in $\pi\Sigma_c$ channel

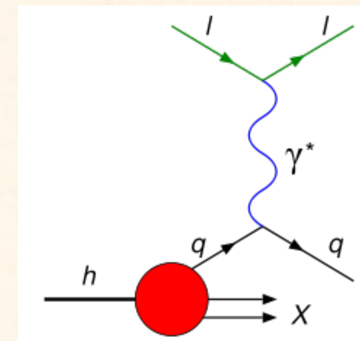
- 1~2 MeV above threshold — extremely shallow
- Width ~ 2MeV — narrow
- Strong attraction ($I=0$, $L=0$) between Σ_c and a very soft pion (Q ~ 20MeV)
- Pion mass diff. ignored for the moment
- Can a Σ_c trap two soft pions?



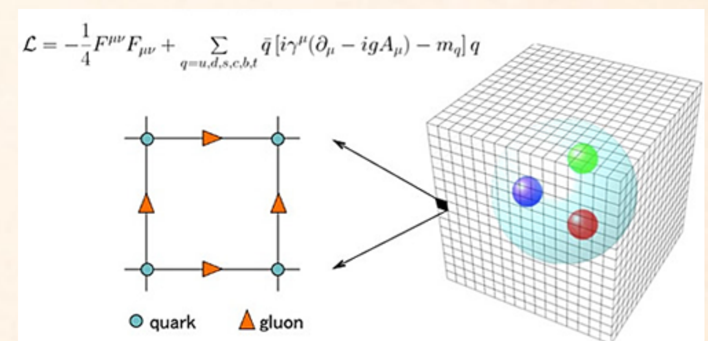
(very) Brief intro to Chiral EFT

3-momenta ↑

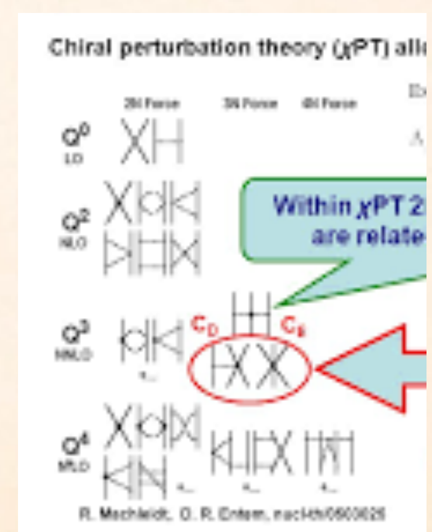
Few GeVs — QCD pert. theory



~ 1 GeV — Lattice QCD



~ Few MeVs — Chiral EFT



Chiral symmetry

- Approximate symmetry $SU(3)_L \times SU(3)_R$ of QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \sum_{f=u,d,s,\dots} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} \mathcal{G}_{a\mu\nu} \mathcal{G}_a^{\mu\nu}$$

Quark masses $m_f \longrightarrow 0$

$$\mathcal{L}^0_{\text{QCD}} = \sum_{l=u,d,s} (\bar{q}_{R,l} i\not{D}\, q_{R,l} + \bar{q}_{L,l} i\not{D}\, q_{L,l}) - \frac{1}{4} \mathcal{G}_{a\mu\nu} \mathcal{G}_a^{\mu\nu}$$

Invariant under

$$q_L \equiv \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \mapsto \Big(SU(3)_L \Big) \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \qquad q_R \equiv \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix} \mapsto \Big(SU(3)_R \Big) \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix}$$

Pions as Nambu-Goldstone bosons

- Switch to two flavors: *u* and *d*
- However, QCD vacuum (ground state) not invariant under chiral rotations, $SU(2)_A$, the axial part of $SU(2)_L \times SU(2)_R$
  $\Rightarrow$ spontaneous breaking of $SU(2)_A$
- Pions are Nambu-Goldstone bosons
- Would be massless if $m_{u,d} = 0$
- Couplings of pions to other particles (including self interactions) proportional to momenta, $\propto Q$, or squared mass, $\propto m_\pi^2$

Pion-baryon interactions

- Pion-baryon interactions constrained by spontaneous broken chiral symmetry
  - Coupling constants may be fixed, e.g. Weinberg-Tomozawa for Σ_c

$$\frac{i}{f_\pi^2}\Sigma^{a\,\dagger}\left(\pi^a\dot{\pi}^b-\pi^b\dot{\pi}^a\right)\Sigma^b$$

  - Coupling constants may NOT be fixed → Low Energy Constants (LEC)

$$i\frac{g_\Sigma}{f_\pi}\epsilon_{abc}\Sigma^{a\dagger}\vec{\sigma}\cdot\vec{\nabla}\pi^b\Sigma^c \qquad b_0\Sigma_a{}^\dagger\dot{\pi}_a\dot{\pi}_b\Sigma_b$$

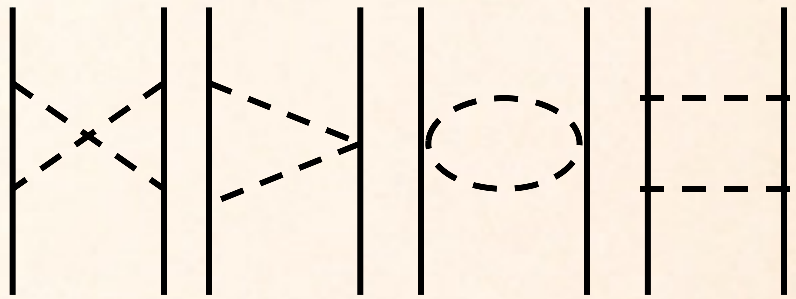
Σ_c axial coupling

$\pi\Sigma_c$ S-wave

Power counting for $Q \sim m_\pi$

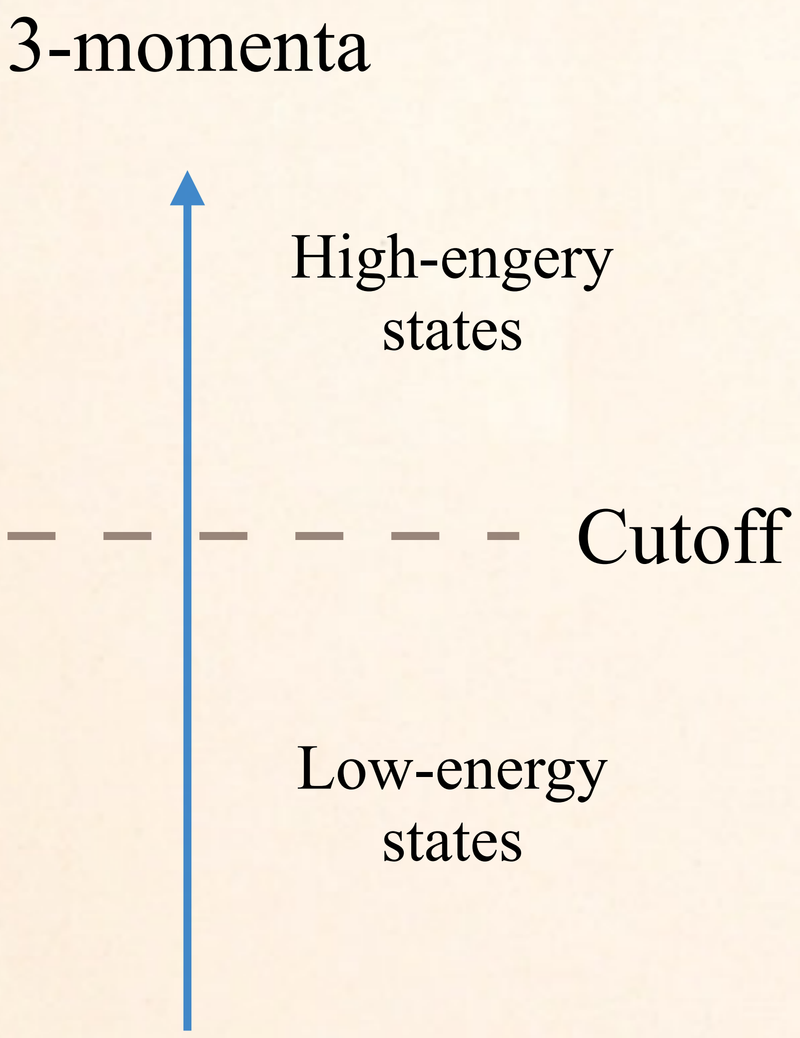
- Nucleon propagator — $1/Q$

Two-pion exchanges of nuclear forces



- Pion propagator — $1/Q^2$
- Loop integral — $Q^4/(16\pi^2)$
- A pion loop brings a suppression factor of $\left(\frac{Q}{4\pi f_\pi}\right)^2$
- Naive dimensional analysis assumed for undetermined LECs
  ⇒ Minimal number of LECs at a given order

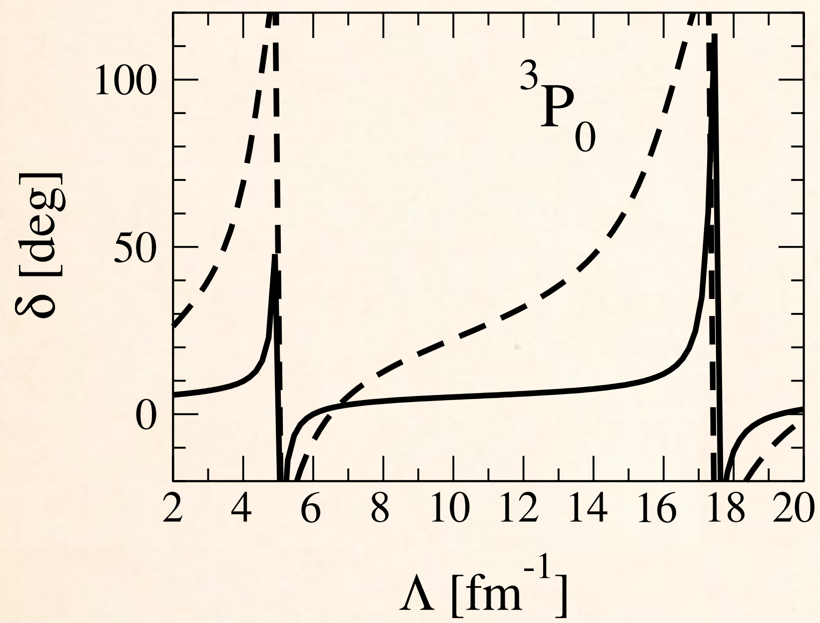
RG inv. constrains PC



- Cutoff $\rightarrow$ arbitrary separation between short and long-range physics
- Cutoff independence (RG invariance) $\Rightarrow$ free of modeling short-range physics
- Modify PC if it violates RG invariance

Nuclear forces in 3P0

The symptom:



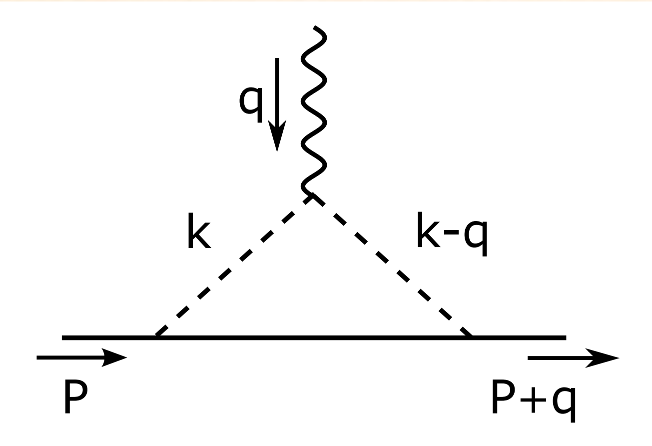
Solid: Tlab = 10 MeV, dashed: 50 MeV

(Nogga et. al; Birse; Pavon; Long & Yang; …)

The solution: promoting counterterms

LO	*C*
O(*Q*)	
O(	
O(	
O(	

Special kinematics may change PC



- $q^2 \simeq 4m_\pi^2 \Rightarrow$ unphysical region, $\vec{q}$ taking complex values

$$\vec{q}^{\,2} = -4m_\pi^2 + \mathcal{O}(\xi^2 m_\pi^2) \quad \text{and} \quad \xi \equiv m_\pi/m_N \ll 1$$

- Pions near shell $\Rightarrow$ $\vec{k} \sim \dfrac{\vec{q}}{2}$, cancellation between $\vec{k}^2$ and m_π^2

$$\sqrt{\vec{k}^2 + m_\pi^2} \sim \xi m_\pi \qquad \sqrt{(\vec{k}-\vec{q})^2 + m_\pi^2} \sim \xi m_\pi$$

- Pions extremely close to shell:

$$\sim \frac{1}{k^2 - m_\pi^2} \sim \frac{1}{(k-q)^2 - m_\pi^2} \sim \frac{1}{(\xi m_\pi)^2} \quad \text{vs.} \quad \sim \frac{1}{m_\pi^2}$$

- Overall, enhanced by $\sim m_N/m_\pi$, compared with standard ChPT counting

$$(\xi m_\pi)^4 \frac{1}{\xi m_\pi} \frac{1}{(\xi m_\pi)^2} \frac{1}{(\xi m_\pi)^2} \sim \frac{1}{\xi m_\pi}$$

(Lyu and Long ’16)

COUNTING!

$\Lambda_c(2595)^+$ as an S-wave resonance in $\pi\Sigma_c$ channel

- 1~2 MeV above threshold — extremely shallow
- Width ~ 2MeV — narrow
- Strong attraction ($I=0$, $L=0$) between Σ_c and a very soft pion (Q ~ 20MeV)
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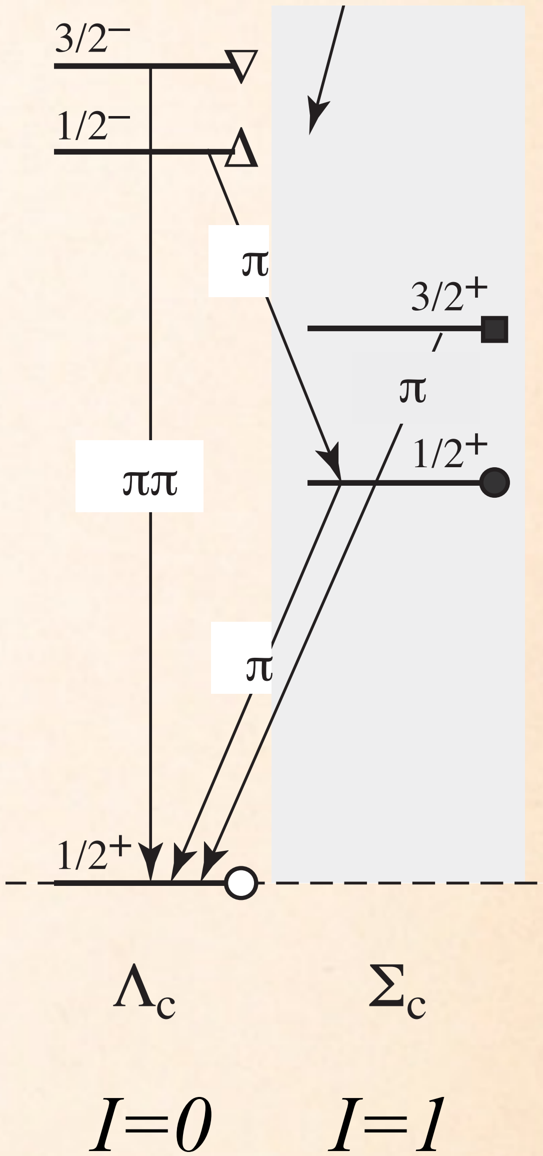
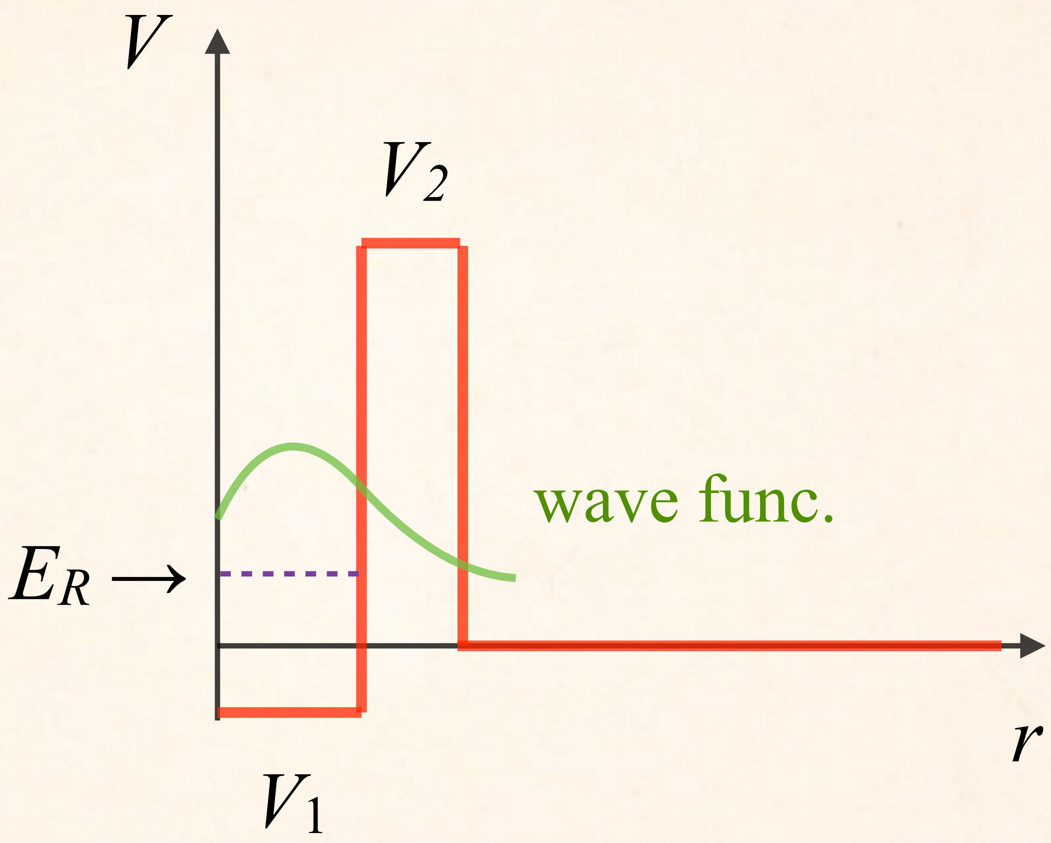


Shallow, narrow S-wave resonances



- Resonance ≈ a would-be bound state coupled to continuum
- Shallow ⇒ tuning V_1 so $E_R \to 0$
- Narrow

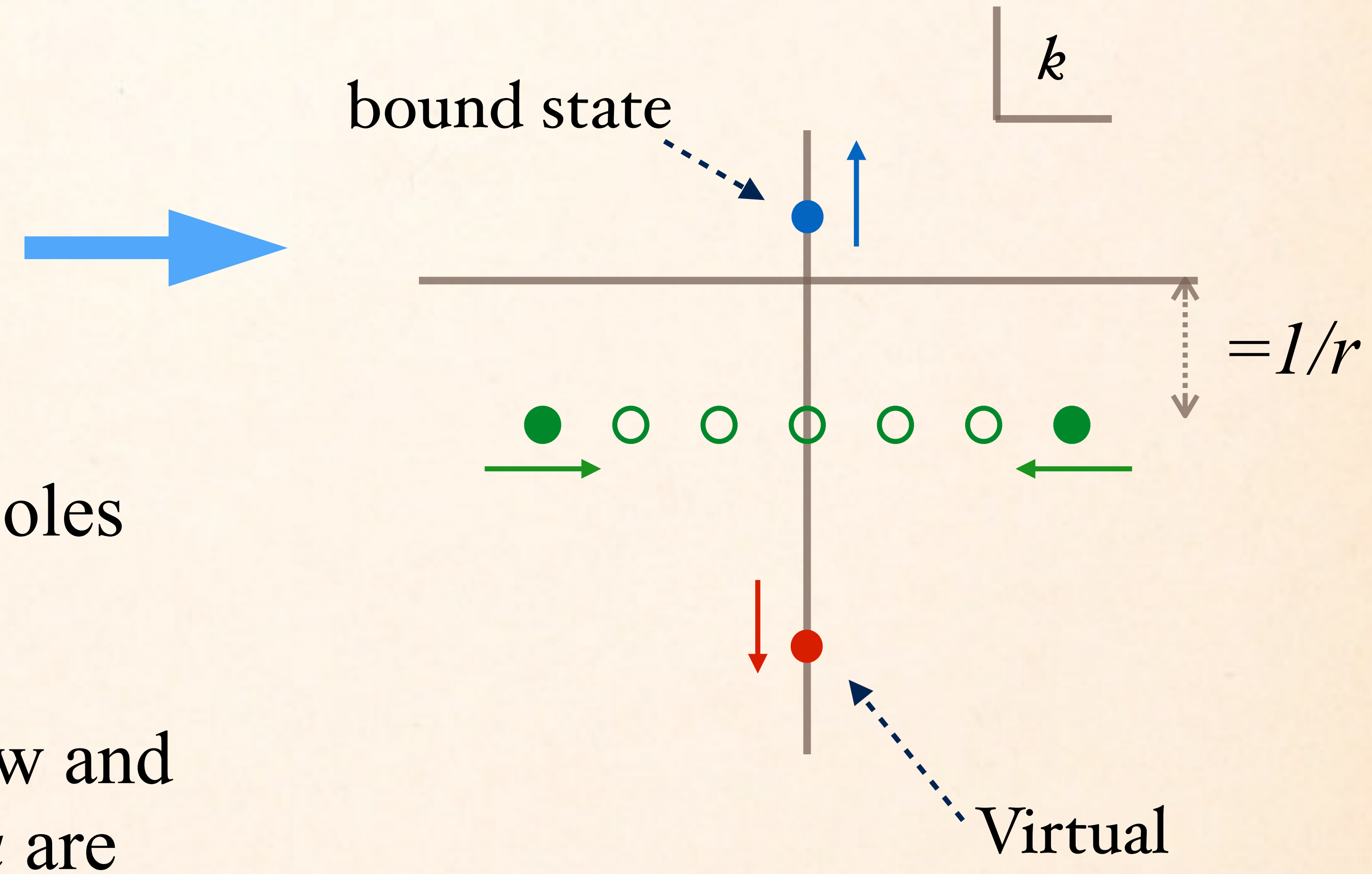
  ⇒ tuning V_2, weakly coupled to continuum, so width $\to 0$

- Less tuning for higher partial waves, because of centrifugal barriers

S-wave resonance poles

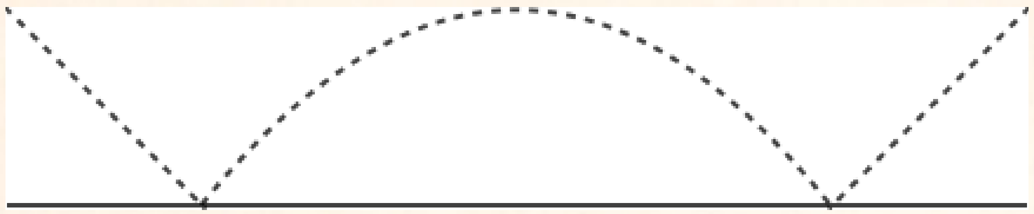
Effective range expansion : $$f^{(0)} = \frac{1}{-\frac{1}{a} + \frac{r}{2}k^2 - ik}$$ (Hyodo '13)

Fixing r and tuning a



- In higher waves, two poles meet at threshold
- $\Lambda_c(2595)^+$ (Λ_c^*) shallow and narrow ⇒ both r and a are large

Generated by Weinberg-Tomozawa?



$$\frac{i}{f_\pi^2}\Sigma^{a\dagger}\left(\pi^a\dot{\pi}^b - \pi^b\dot{\pi}^a\right)\Sigma^b$$

- WT ~ m_π/f_π^2
- A pion loop always suppressed by $\left(\frac{Q}{4\pi f_\pi}\right)^2$ → no good reason to resum
- A zero-range pot. subject to renormalization, $\delta^{(3)}(r)$, coupling constant not fixed by symmetry → resummation of WT facing issue of renormalization
- It's unlikely that WT alone can generate a near-threshold S-wave resonance
- "Subleading" $\pi\pi\Sigma\Sigma$ highly enhanced by QCD dynamics

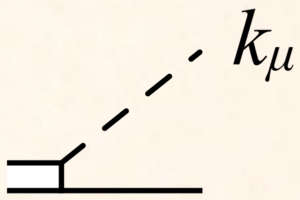
Explicit field of $\Lambda_c(2595)^+$

Ψ: Λ_c*

h: $O(1)$

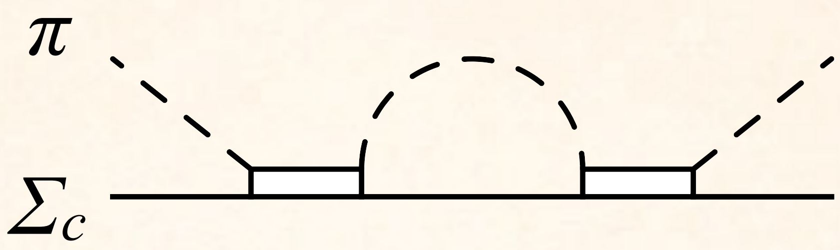
$$\frac{h}{\sqrt{3} f_\pi} \left(\Sigma^{a\,\dagger} \dot{\pi}^a \Psi + h.c. \right)$$

$$\Lambda_c^\star \to \pi \Sigma_c$$



- Ψ coupled to the S wave of $\pi\Sigma_c$ → time derivative on π (chiral symmetry, crucial!)
- $\delta \sim$ 1MeV above $\pi\Sigma_c$ threshold
- Small pion momenta, $Q \sim$ 20MeV → $k_0 = m_\pi + O(k^2/m_\pi)$
- Σ_c decay width ~ 2MeV, approximated as stable

Counting (very) soft pions



(BwL '15)

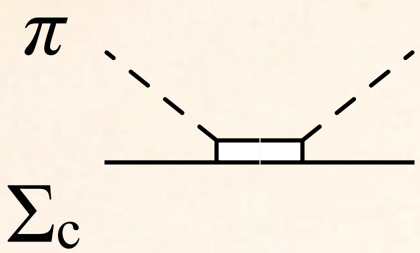
- pion prop. ~ $1/Q^2$

- baryon prop. ~ $1/(Q^2/m_\pi)$

$$\int \frac{d^4 l}{(2\pi)^4} \sim \frac{1}{4\pi} \frac{Q^5}{m_\pi}$$

nonrelativistic

Counting (very) soft pions



$$\frac{m_\pi}{f_\pi}\frac{1}{Q^2/m_\pi}\frac{m_\pi}{f_\pi} \sim \frac{m_\pi^3}{f_\pi^2 Q^2}$$

$$\frac{m_\pi^3}{f_\pi^2 Q^2}\frac{Q^5}{4\pi m_\pi}\frac{1}{Q^2}\frac{1}{Q^2/m_\pi}\frac{m_\pi^3}{f_\pi^2 Q^2}$$

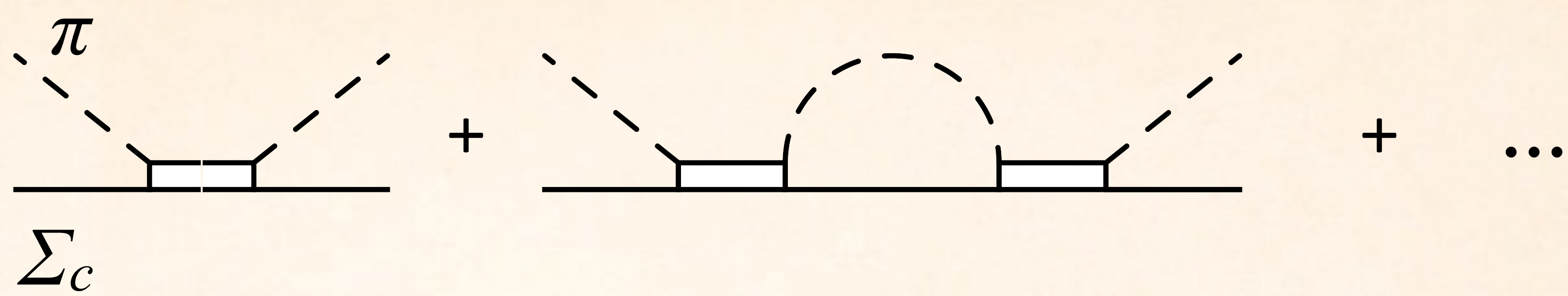
$$\sim \frac{m_\pi^3}{f_\pi^2 Q^2}\frac{\epsilon m_\pi}{Q}$$

$$\epsilon \equiv m_\pi^2/4\pi f_\pi^2 = 0.18$$

Resummation $\Leftrightarrow$ $\delta \sim Q^2/m_\pi \sim \epsilon^2 m_\pi$

- $\delta \sim 1\mathrm{MeV}$
- $Q \sim 20\mathrm{MeV}$

$\pi\Sigma_c$ scattering



$$a = \frac{h^2 m_\pi^2}{4\pi f_\pi^2}\frac{1}{m_\pi - \Delta} \sim \left(\frac{140\text{MeV}}{328\text{MeV}}\right)^2 \frac{1}{4\text{MeV}}$$

$$f^{(0)} = \frac{1}{-\frac{1}{a} + \frac{r}{2}k^2 - ik}$$

$$r = -\frac{4\pi f_\pi^2}{h^2 m_\pi^3} \sim \left(\frac{328\text{MeV}}{140\text{MeV}}\right)^2 \frac{1}{140\text{MeV}}$$

(Hyodo '13)
r = -19 fm
a = -10 fm
$\Longrightarrow$ h = 0.65

- r can be quite large when $\Delta \ll \sqrt{4\pi} f_\pi = 328\text{MeV}$
- a single fine-tuning $\Delta - m_\pi \to 0$ makes both a and r large

→ Chiral symmetry helps $\Lambda_c(2595)^+$ be shallow AND narrow

(BwL '15)

Breakdown of universality

- Universality : observables expected to scale w/ $m_\pi^\star - m_\pi \to 0$

- Additional large length scale of r → universality relations break down sooner than expected

E.g., binding energy when $m_\pi > m_\pi^\star$

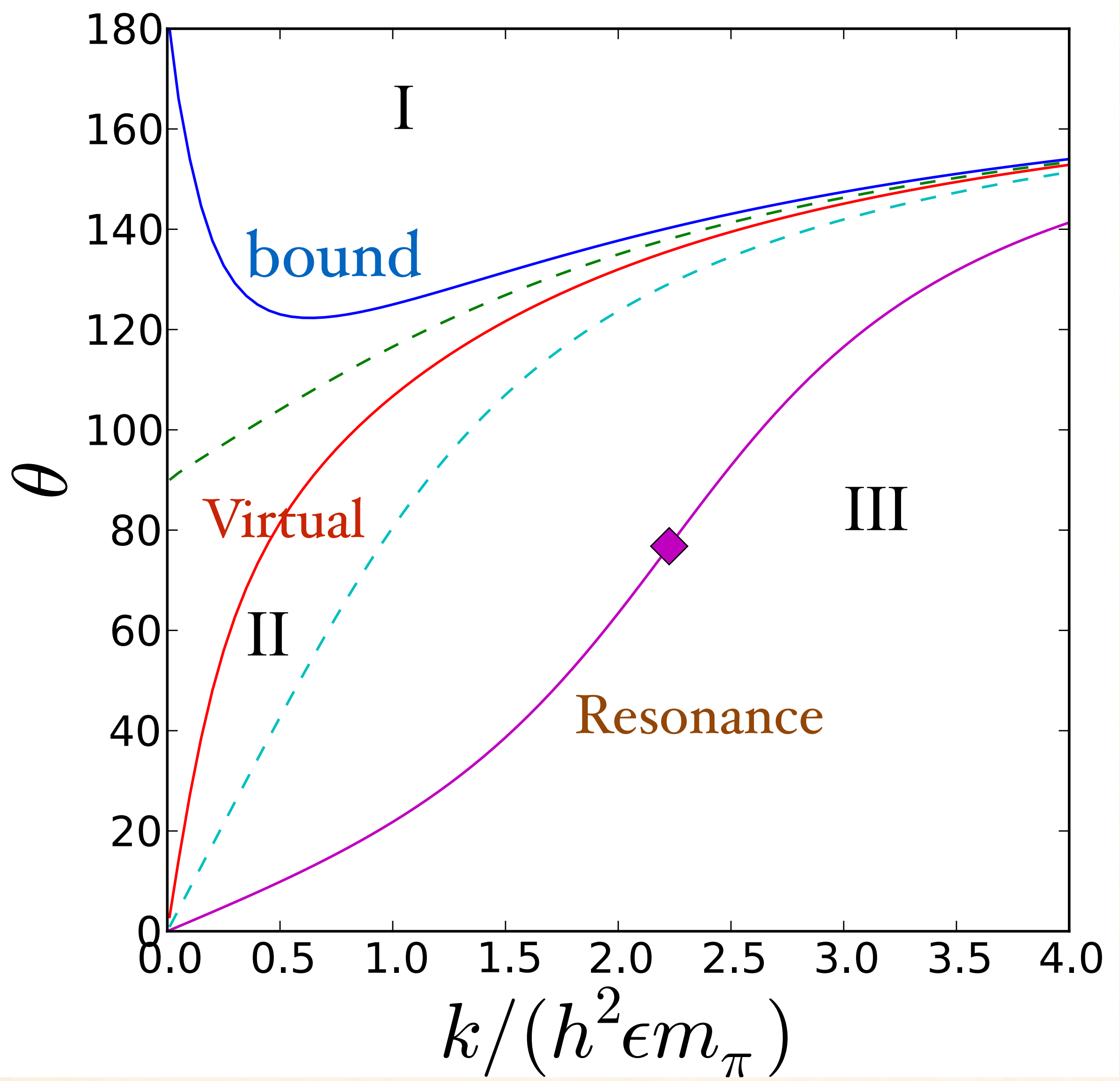
$$B_0(\delta; m_\pi) = \frac{h^4}{2}\epsilon^2 m_\pi \left(\sqrt{1 - \frac{2\delta}{h^4\epsilon^2 m_\pi}} - 1\right)^2 \qquad \delta = m_\pi^\star - m_\pi \qquad \epsilon \equiv \frac{m_\pi^2}{4\pi f_\pi^2}$$

- Universality recovered only in a tiny window of m_{pi}

$$B = \frac{\delta^2}{h^4\epsilon^2 m_\pi}\left[1 + \mathcal{O}\left(\frac{\delta}{h^2\epsilon^2 m_\pi}\right)\right] \text{ for } \left|\frac{m_\pi - m_\pi^\star}{m_\pi}\right| \ll \left(\frac{m_\pi^\star}{328\text{MeV}}\right)^4$$

(BwL '15)

Profile of phase shifts



$$\epsilon \equiv \frac{m_\pi^2}{4\pi f_\pi^2}$$

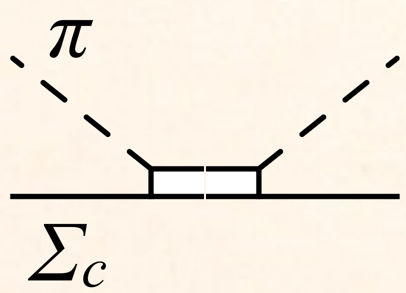
$$\tilde{\delta} \equiv \delta/(h^4\epsilon^2 m_\pi)$$

From top down

$$\tilde{\delta} = -0.2,\ 0.2,\ \text{and}\ 3$$

Can a Σ_c attract more pions?

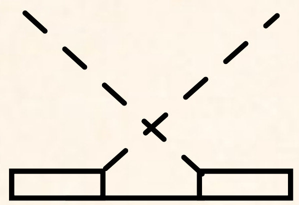
- very soft π's interact w/ other hadrons weakly
- $\pi\Sigma_c$ potential is energy-dependent
  $\rightarrow$ more complicated than independent-boson systems



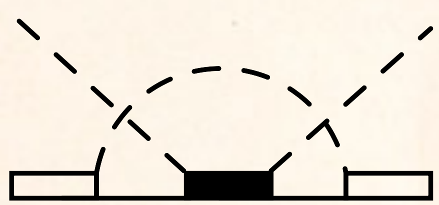
$$\frac{h^2 m_\pi^2}{f_\pi^2 (E-\delta)}$$

- Searching 3-body states by finding poles of $\pi\Lambda_c^*$ "scattering amplitude" (or any other correlation func. having same quantum numbers as $\langle 0|\pi_a \Psi \pi_a \Psi^\dagger|0\rangle$)

$\pi\Lambda_c^*$ scattering



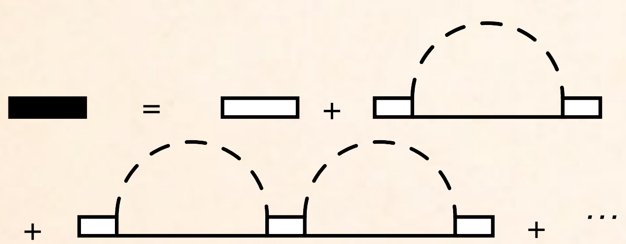
$$\frac{m_\pi}{f_\pi}\frac{1}{Q^2/m_\pi}\frac{m_\pi}{f_\pi} \sim \frac{m_\pi^3}{f_\pi^2 Q^2}$$



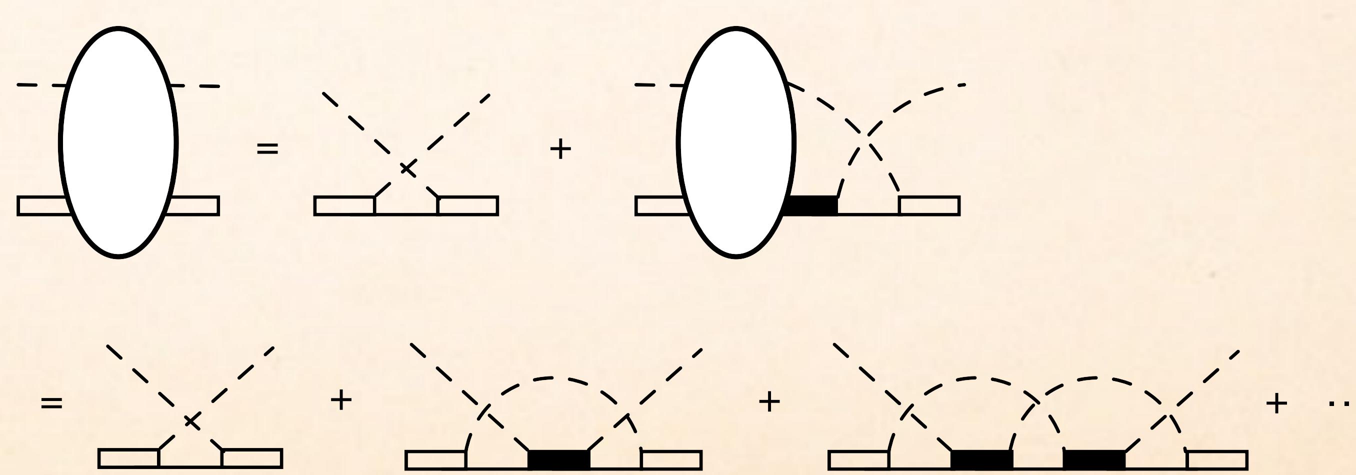
$$\frac{m_\pi^3}{f_\pi^2 Q^2}\frac{Q}{4\pi}\frac{m_\pi^3}{f_\pi^2 Q^2} \sim \frac{m_\pi^3}{f_\pi^2 Q^2}\frac{Q}{\epsilon m_\pi}$$

Comparable

$Q \sim \epsilon m_\pi$

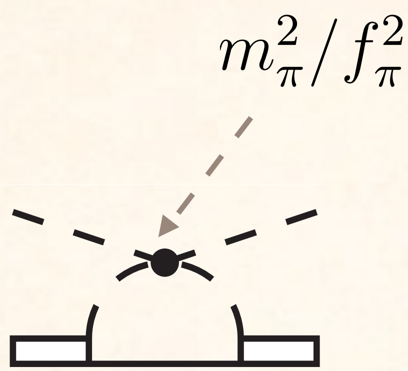


$\Rightarrow$ Solving

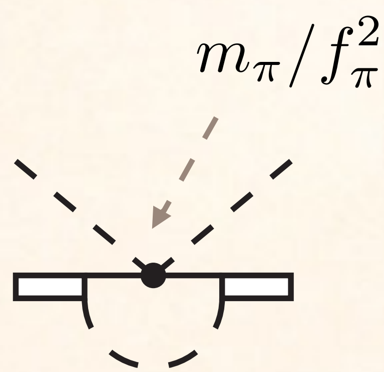


Estimating corrections

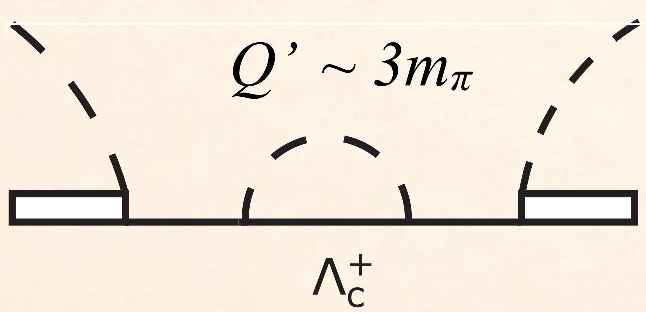
Pion s-wave interaction



Weinberg - Tomozawa



$$\sim \epsilon^2 \frac{m_\pi^3}{f_\pi^2 Q^3}$$



g.s. Λ_c^+

- (g.s. Λ_c^+) + π is more energetic, but still suppressed

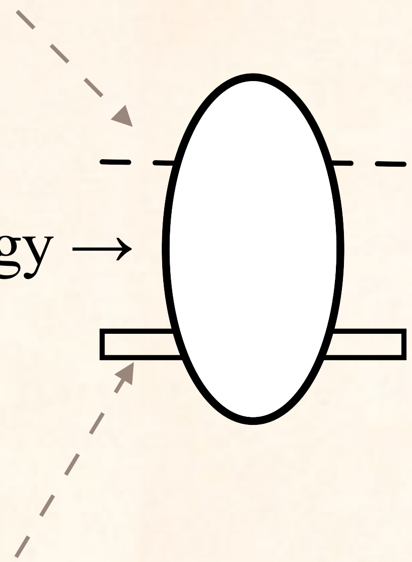
$$\epsilon^2 \left(\frac{Q'}{4\pi f_\pi}\right)^2 \frac{m_\pi^3}{f_\pi^2 Q^3}$$

Integral equation

q: 3-mom.

E: total CM energy →

$\mathcal{E} \equiv 2m_\pi E$



E_Λ: Λ_c^* energy

$\mathcal{B} \equiv -2m_\pi E_\Lambda$

$$t(q;\mathcal{E},\mathcal{B}) = \frac{8\pi/|r|}{3(q^2+\mathcal{B})} + \frac{2}{3\pi}\int_{\Sigma_l} dl \frac{l^2}{q^2-\mathcal{E}+l^2+i0} \times \frac{t(l;\mathcal{E},\mathcal{B})}{-\frac{1}{a}-\frac{|r|}{2}(\mathcal{E}-l^2)+\sqrt{l^2-\mathcal{E}-i0}-i0},$$

$$-1/a = \frac{\delta}{\epsilon h^2}, \quad r = -\left(\epsilon h^2 m_\pi\right)^{-1}$$

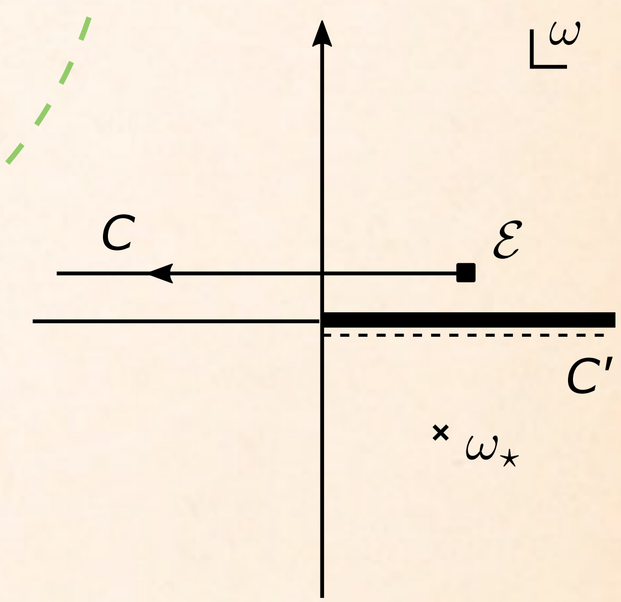
- $t(q) \to 1/q^2$, so integral converges as $l \to \infty$; and cutoff independent
- 3-body resonances

  = poles of $t(q;E,E_\Lambda)$ as a function of E

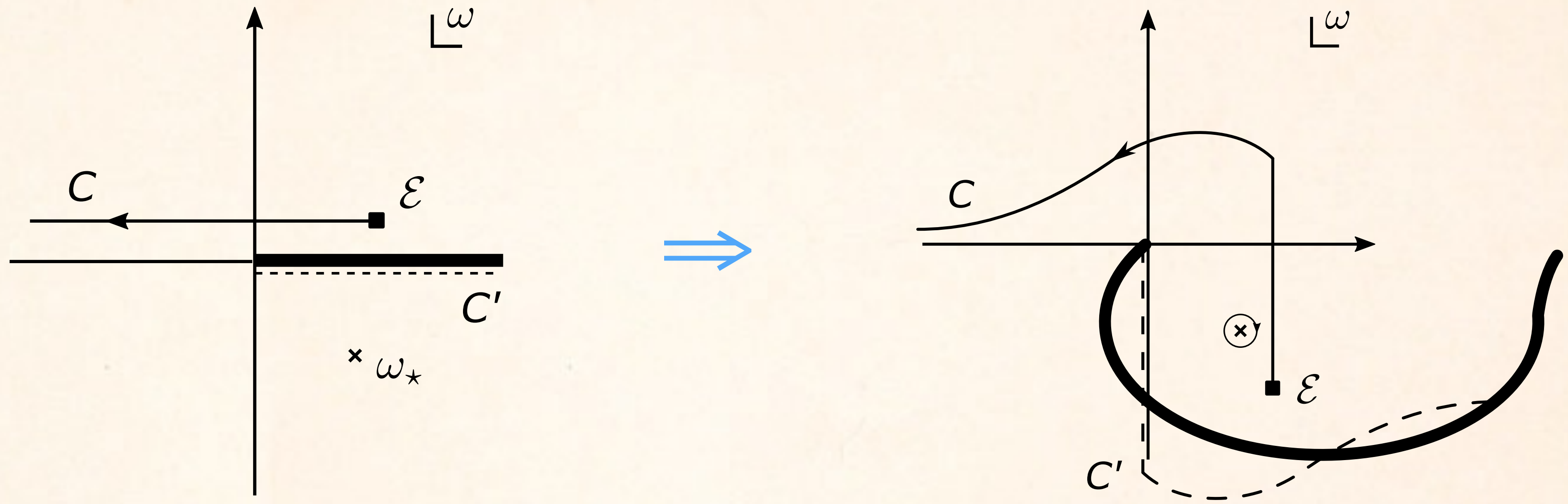
$$t(q;\mathcal{E},\mathcal{B}) = \frac{8\pi/|r|}{3(q^2+\mathcal{B})} + \frac{2}{3\pi}\int_{\Sigma_l} dl \frac{l^2}{q^2-\mathcal{E}+l^2+i0} \times \frac{t(l;\mathcal{E},\mathcal{B})}{-\frac{1}{a}-\frac{|r|}{2}(\mathcal{E}-l^2)+\sqrt{l^2-\mathcal{E}-i0}-i0},$$

- Deform the contour so as NOT to cross any singularities of the integrand

- Instead of l^2, looking at $\omega_l \equiv \mathcal{E} - l^2$
- Poles of $(q^2-\mathcal{E}+l^2)^{-1} = (\mathcal{E}-\omega_l-\omega_q)^{-1}$
- Poles of dressed Λ_c^* prop.
- Branch cut $\sqrt{l^2-\mathcal{E}} = \sqrt{-\omega_l}$
- l - singularities of $t(l;\mathcal{E},\mathcal{B})$



Deforming contour

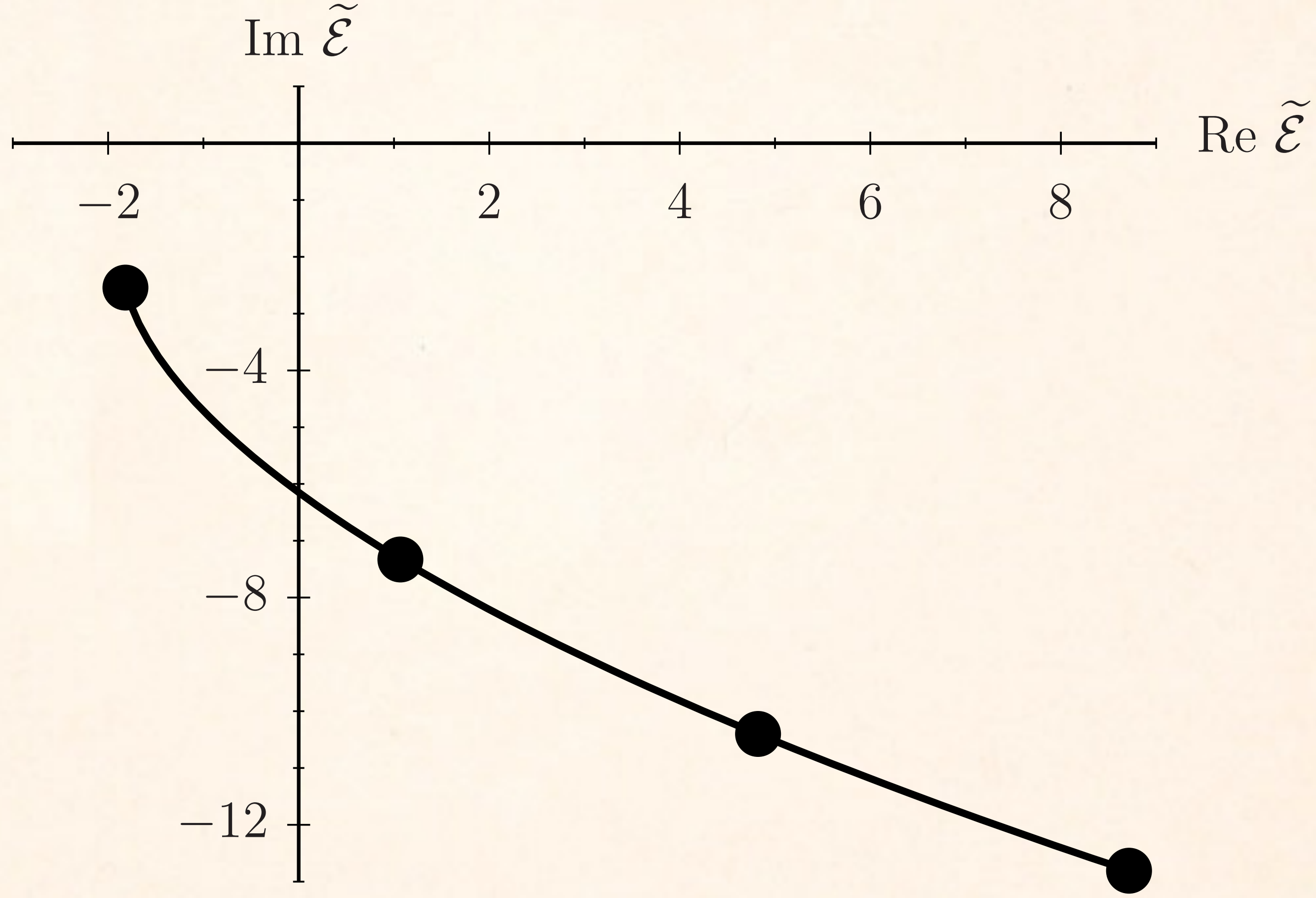


- Solid line: contour in omega plane
- Thick line: square root cut
- Dashed line: $t(l; \mathcal{E}, \mathcal{B})$ cut as a func. of l
- Cross: poles of dressed prop.
- Be wary of "standard" procedures (e.g. Peace & Afnan)

3-body resonance pole

(BwL '16)

$$\widetilde{\mathcal{E}} \equiv 2m_\pi E r^2$$



- 3B pole trajectory as *a* varies, with $|r|^{-1}$ as unit
- $|r|/a$ = -4 ~ -1

Results

(BwL ’16)

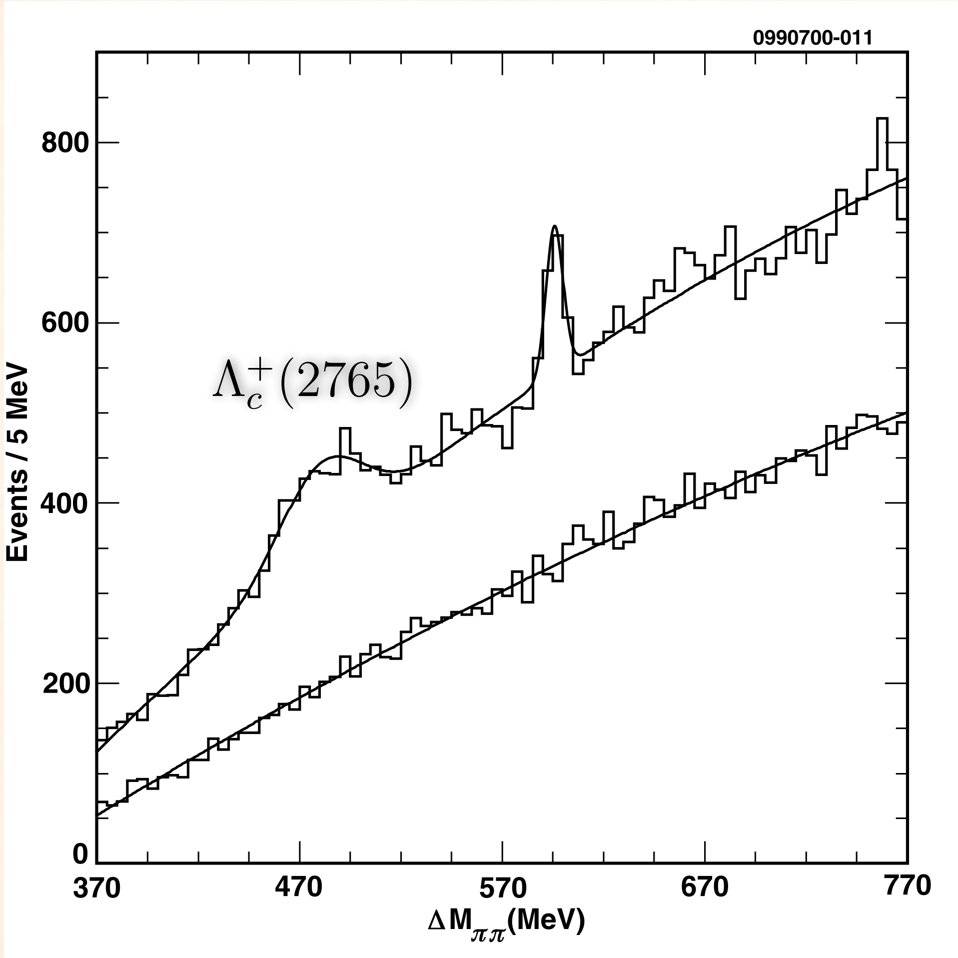
$$M_{\Lambda_c^\star} - M_{\Lambda_c^+} = 305.8\,\text{MeV}\,, \quad h^2 = \frac{3}{2} \times 0.36$$ (CDF ’11)

$\Longrightarrow$ $$M_{\Sigma(\pi\pi\Sigma_c,\frac{1}{2})} - (M_{\Sigma_c} + 2m_\pi) = (-0.45 - 0.02i)\text{MeV}$$

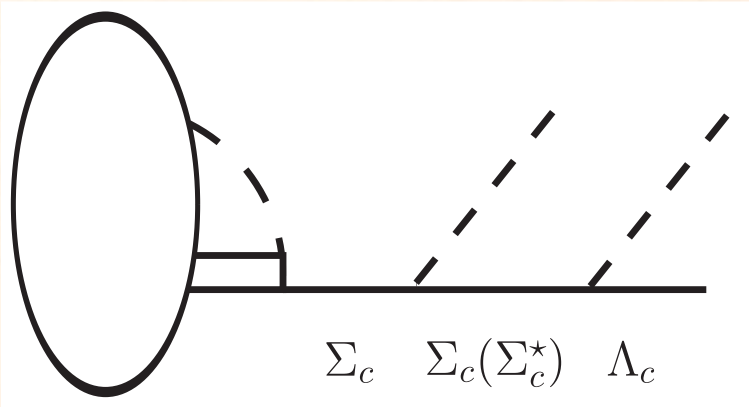
$$M_{\Lambda_c^\star} - M_{\Lambda_c^+} = 308.7\,\text{MeV}\,, \quad h^2 = \frac{3}{2} \times 0.30$$ (Chiladze & Falk ’97)

$\Longrightarrow$ $$M_{\Sigma(\pi\pi\Sigma_c,\frac{1}{2})} - (M_{\Sigma_c} + 2m_\pi) = (4.00 - 5.72i)\text{MeV}$$

$\Lambda_c^+(2765)$?



The decay of $\Sigma(\pi\pi\Sigma_c, \frac{1}{2})$ into $\Lambda_c^+\pi^-\pi^+$



Observation of New States Decaying into $\Lambda_c^+\pi^-\pi^+$

(CLEO '01)

Summary

- $\Lambda_c(2595)^+$ a near-threshold *S*-wave resonance coupled to $\pi\Sigma_c$

- Strong attraction of very soft pions to Σ_c : A extremely rare realization of S-wave resonant interaction with both large *a* and *r*

- Thanks to chiral symmetry, only one fine-tuning needed

- It helps form a shallow $\pi\pi\Sigma_c$ resonance

- More molecular states with this soft-pion attraction?