Strangeness S=-2 baryon-baryon interactions from Lattice QCD

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for HAL QCD Collaboration

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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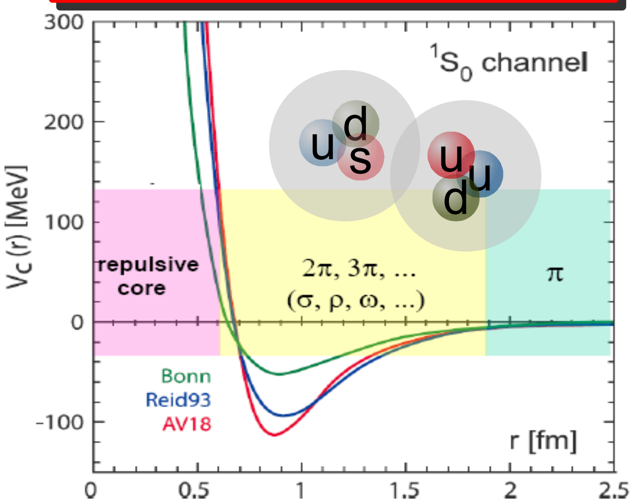
Introduction

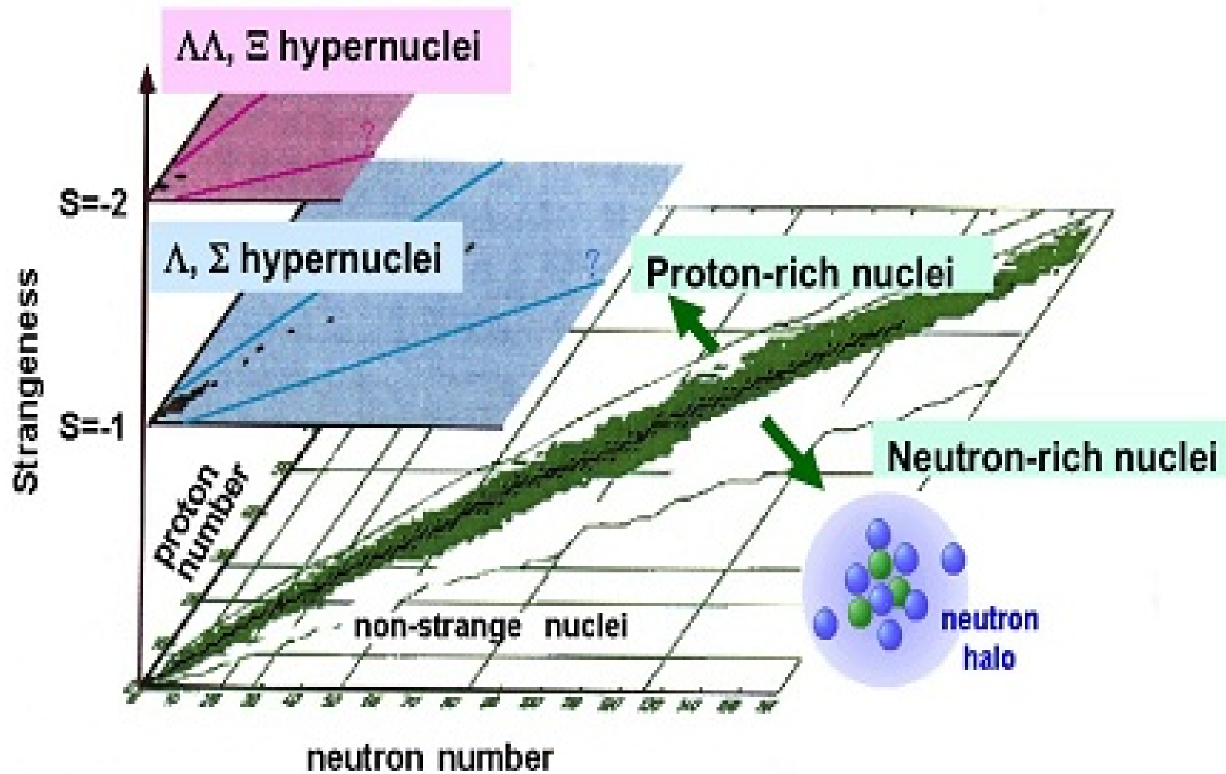
Introduction

BB interactions are crucial to investigate the nuclear phenomena

Once we obtain "proper" nuclear potentials,
we apply them to the structure of (hyper-) nucleus.

BB interaction (potential)





Properties of nuclaer potential

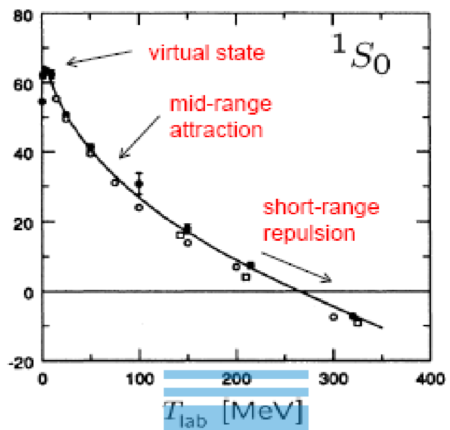
- **State dependence (spin, isospin)**
- **Long range attraction**
- **Short range repulsion**

How do we obtain the nuclear force?

Phenomenological descriptions

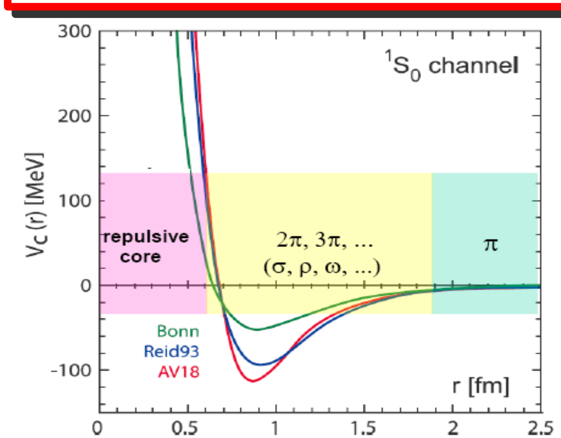
Traditional process to research the BB interaction / potential

Scattering observables



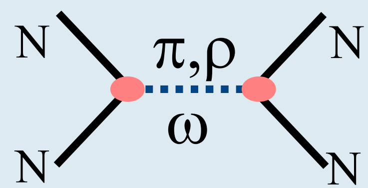
Model assumption

BB interaction (potential)



Meson exchange model

Described by hadron d.o.f.

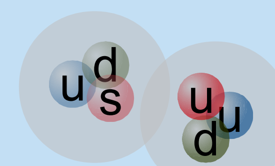


+ Phenomenological repul. core

H. Yukawa, PPMS17(1935)48
Th.A.Rijken, PTPS185(2010)14

Quark cluster model

Effective meson ex
+ quark anti-symmetrization

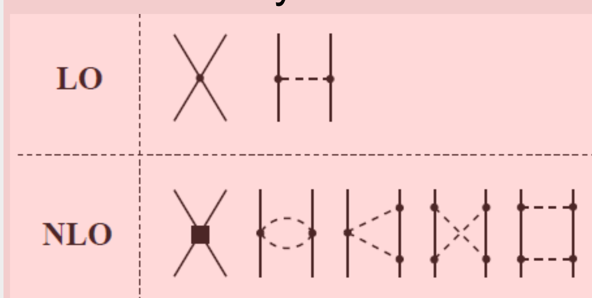


Quark Pauli effects
Color magnetic int.

M.Oka, PTPS137(2000)1
Y.Fujiwara, PPNP58(2007)439

Effective Field theory

Systematic calc. respecting with symmetry of QCD



Short range interaction is parametrized by contact term

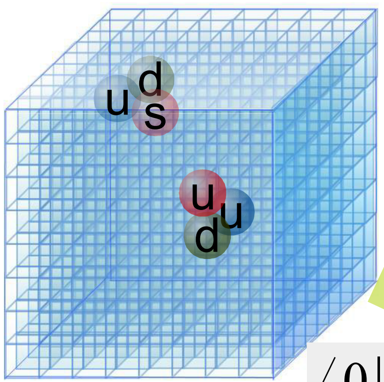
E. Epelbaum, RMP81(2009)1773
R. Machleidt, PRept.503(2011)1

The models would be highly ambiguous if experimental data are scarce!

Derivation of hadronic interaction from QCD

Start with the fundamental theory,QCD

Lattice QCD simulation



Lüscher's finite volume method

M. Lüscher, NPB354(1991)531

1. Measure the discrete energy spectrum, **E**
2. Put the E into the formula which connects E and **δ**

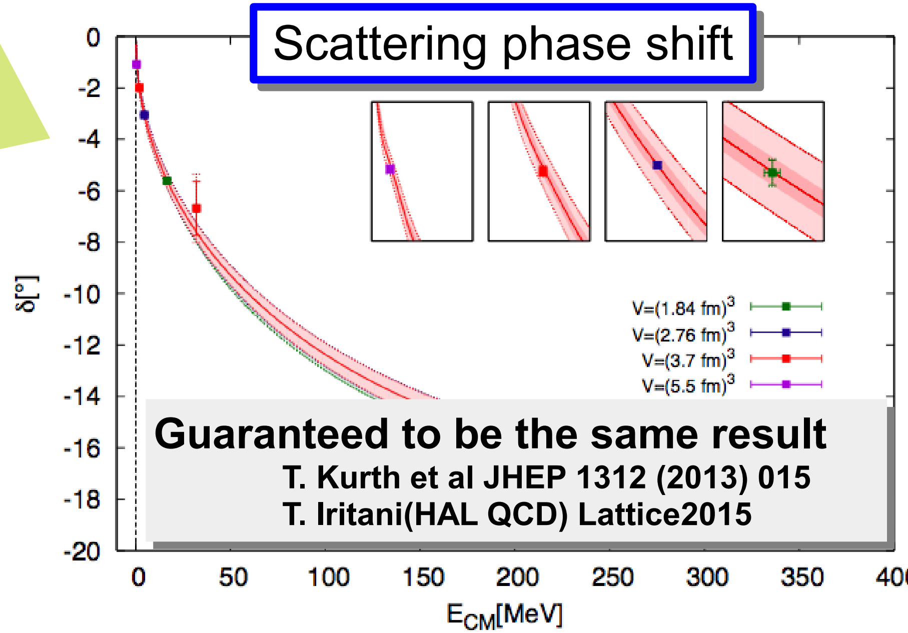
$$\langle 0|B_1 B_2(t,\vec{r})\bar{B}_2\bar{B}_1(t_0)|0\rangle = A_0 \Psi(\vec{r},E_0) e^{-E_0(t-t_0)} + \cdots$$

HAL QCD method

Ishii, Aoki, Hatsuda, PRL**99** (2007) 022001

1. Measure the NBS wave function, **Ψ**
2. Calculate potential, **V**, through Schrödinger eq.
3. Calculate observables by scattering theory

Scattering phase shift



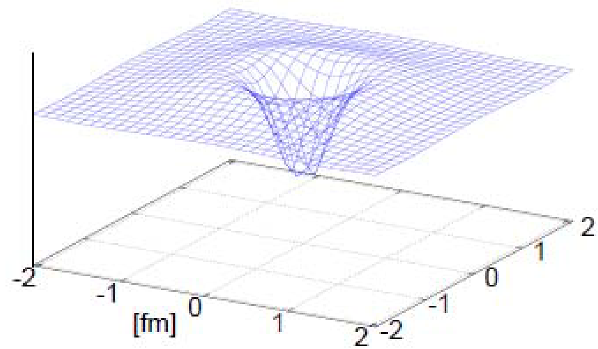
Guaranteed to be the same result
T. Kurth et al JHEP 1312 (2013) 015
T. Iritani(HAL QCD) Lattice2015

HAL QCD method

Nambu-Bethe-Salpeter wave function

Definition : equal time NBS w.f.

$$\Psi^{\alpha}(E,\vec{r})e^{-Et} = \sum_{\vec{x}} \langle 0|H_1^{\alpha}(t,\vec{x}+\vec{r})H_2^{\alpha}(t,\vec{x})|E\rangle$$



E : Total energy of the system

Local composite interpolating operators

$$B_{\alpha}=\epsilon^{abc}(q_a^T C\gamma_5 q_b)q_{c\alpha} \qquad D_{\mu\alpha}=\epsilon^{abc}(q_a^T C\gamma_{\mu} q_b)q_{c\alpha}$$

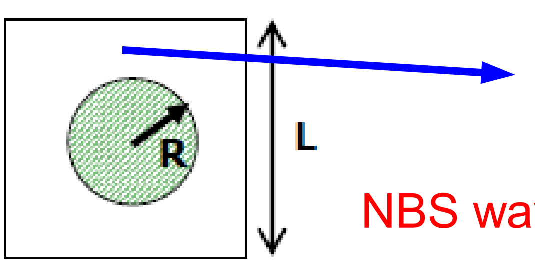
$$M=(\bar{q}_a\gamma_5 q_a)$$

Etc.....

- It satisfies the Helmholtz eq. in asymptotic region : $(p^2+\nabla^2)\Psi(E,\vec{r})=0$
- Using the reduction formula,

C.-J.D.Lin et al.,NPB619 (2001) 467.

$$\Psi^{\alpha}(E,\vec{r}) = \sqrt{Z_{H_1}}\sqrt{Z_{H_2}}\left(e^{i\vec{p}\cdot\vec{r}}+\int\frac{d^3q}{2E_q}\frac{T(q,p)}{4E_p(E_q-E_p-i\epsilon)}e^{i\vec{q}\cdot\vec{r}}\right)$$



$$\Psi(E,\vec{r})\simeq A\frac{\sin(pr+\delta(E))}{pr}$$

Phase shift is defined as

$$S\equiv e^{i\delta}$$

NBS wave function has a same asymptotic form with quantum mechanics.

(NBS wave function is characterized from phase shift)

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

Potential in HAL QCD method

We define potentials which satisfy Schrödinger equation

$$\left(p^2+\nabla^2\right)\Psi^{\alpha}(E,\vec{x}) \equiv \int d^3y\, U^{\alpha}_{\alpha}(\vec{x},\vec{y})\,\Psi^{\alpha}(E,\vec{y})$$

Energy independent potential

$$\left(p^2+\nabla^2\right)\Psi^{\alpha}(E,\vec{x}) = K^{\alpha}(E,\vec{x})$$

$$K^{\alpha}(E,\vec{x}) \equiv \int dE'\, K^{\alpha}(E',\vec{x}) \int d^3y\, \tilde{\Psi}^{\alpha}(E',\vec{y})\,\Psi^{\alpha}(E,\vec{y})$$

$$= \int d^3y \left[\int dE'\, K^{\alpha}(E',\vec{x})\,\tilde{\Psi}^{\alpha}(E',\vec{y})\right] \Psi^{\alpha}(E,\vec{y})$$

$$= \int d^3y\, U^{\alpha}_{\alpha}(\vec{x},\vec{y})\,\Psi^{\alpha}(E,\vec{y})$$

We can define an energy independent potential but it is fully non-local.

This potential automatically reproduce the scattering phase shift

Time-dependent method

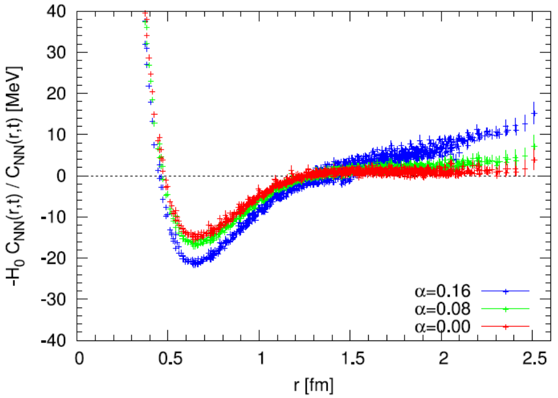
Start with the normalized four-point correlator.

$$R_I^{B_1 B_2}(t,\vec{r}) = F_{B_1 B_2}(t,\vec{r})\, e^{(m_1+m_2)t}$$
$$= A_0 \Psi(\vec{r},E_0) e^{-(E_0-m_1-m_2)t} + A_1 \Psi(\vec{r},E_1) e^{-(E_1-m_1-m_2)t} + \cdots$$

Each wave functions satisfies Schrödinger eq. with proper energy

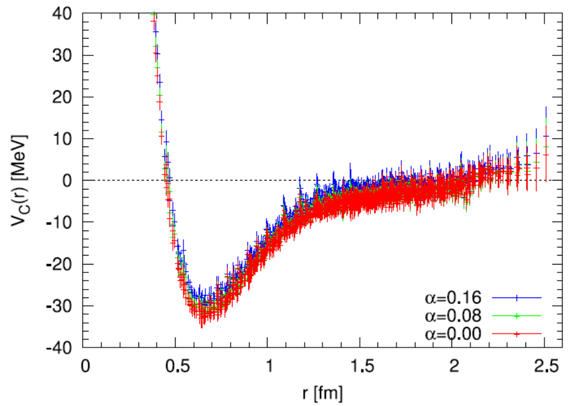
$$\left(\frac{p_0^2}{2\mu}+\frac{\nabla^2}{2\mu}\right)\Psi(\vec{r},E_0) = \int U(\vec{r},\vec{r}\,')\Psi(\vec{r}\,',E_0)\, d^3r'$$

$$\left(\frac{p_1^2}{2\mu}+\frac{\nabla^2}{2\mu}\right)\Psi(\vec{r},E_1) = \int U(\vec{r},\vec{r}\,')\Psi(\vec{r}\,',E_1)\, d^3r'$$



$$E_n - m_1 - m_2 \approx \frac{p_n^2}{2\mu}$$

$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right) R_I^{B_1 B_2}(t,\vec{r}) = \int U(\vec{r},\vec{r}\,') R_I^{B_1 B_2}(t,\vec{r})\, d^3r'$$



A single state saturation is not required!!

Treatment of non-local potential

$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1B_2}(t,\vec{r})=\int U(\vec{r},\vec{r}\,')R_I^{B_1B_2}(t,\vec{r})d^3r'$$

Derivative (velocity) expansion of U is performed to deal with its nonlocality.

- For the case of oct-oct system,

$$U(\vec{r},\vec{r}\,') = \underbrace{\left[V_C(r)+S_{12}V_T(r)\right]}_{\text{Leading order part}}+\left[\vec{L}\cdot\vec{S}_s V_{LS}(r)+\vec{L}\cdot\vec{S}_a V_{ALS}(r)\right]+O(\nabla^2)$$

- For the case of dec-oct and dec-dec system,

$$U(\vec{r},\vec{r}\,') = \underbrace{\left[V_C(r)+S_{12}V_{T_1}(r)+S_{ii}V_{T_2}(r)+O(Spin\,op^3)\right]}_{\text{Leading order part}}+O(\nabla^2)$$

$$\equiv \left[V_C^{eff}(r)\right]+O(\nabla^2)$$

$$((\vec{r}\cdot\vec{S_1})^2-\frac{\vec{r}^{\,2}}{3}\vec{S_1}^2+(\vec{r}\cdot\vec{S_2})^2-\frac{\vec{r}^{\,2}}{3}\vec{S_2}^2)V_{T^2}(r)$$

We consider the effective central potential which contains not only the genuine central potential but also tensor parts.

HAL QCD method

NBS wave function

$$\Psi(E,\vec{r})e^{-E(t-t_0)} = \sum_{\vec{x}} \langle 0|B_i(t,\vec{x}+\vec{r})B_j(t,\vec{x})|E,t_0\rangle$$

E : Total energy of system

- In asymptotic region : $(p^2+\nabla^2)\Psi(E,\vec{r})=0$
- In interaction region : $(p^2+\nabla^2)\Psi(E,\vec{r})=K(E,\vec{r})$

$$\Psi(E,\vec{r}) \simeq A\frac{\sin(pr+\delta(E))}{pr}$$

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

$$R_I^{B_1B_2}(t,\vec{r}) = \Psi_{B_1B_2}(\vec{r},t)e^{(m_1+m_2)t}$$

Modified Schrödinger equation

$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1B_2}(t,\vec{r}) = \int U(\vec{r},\vec{r}\,')R_I^{B_1B_2}(t,\vec{r})d^3r'$$

N. Ishii et al Phys. Lett. B712(2012)437

Derivative expansion

$$U(\vec{r},\vec{r}\,') = V_C(r)+S_{12}V_T(r)+\vec{L}\cdot\vec{S}_s V_{LS}(r)+\vec{L}\cdot\vec{S}_a V_{ALS}(r)+O(\nabla^2)$$

K. Murano et al Phys.Lett. B735 (2014) 19

Potential

$$V(\vec{r}) = \left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1B_2}(t,\vec{r})/R_I^{B_1B_2}(t,\vec{r})$$

HAL QCD method (coupled-channel)

NBS wave function

$$\Psi^{\alpha}(E_i,\vec{r})e^{-E_i t} = \langle 0|(B_1 B_2)^{\alpha}(\vec{r})|E_i\rangle$$

$$\Psi^{\beta}(E_i,\vec{r})e^{-E_i t} = \langle 0|(B_1 B_2)^{\beta}(\vec{r})|E_i\rangle$$

$$\int dr\, \tilde{\Psi}_{\beta}(E',\vec{r})\Psi^{\gamma}(E,\vec{r}) = \delta(E'-E)\delta_{\beta}{}^{\gamma}$$

$$R_E^{B_1 B_2}(t,\vec{r}) = \Psi_{B_1 B_2}(\vec{r},E)e^{(-E+m_1+m_2)t}$$

Leading order of velocity expansion and time-derivative method.

Modified coupled-channel Schrödinger equation

$$\begin{pmatrix} \left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\alpha}\right)R^{\alpha}_{E_0}(t,\vec{r}) \\ \left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\beta}\right)R^{\beta}_{E_0}(t,\vec{r}) \end{pmatrix} = \begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})\Delta^{\alpha}{}_{\beta}(t) \\ V^{\beta}{}_{\alpha}(\vec{r})\Delta^{\beta}{}_{\alpha}(t) & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E_0}(t,\vec{r}) \\ R^{\beta}_{E_0}(t,\vec{r}) \end{pmatrix}$$

$$\begin{pmatrix} \cdots \\ \left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\beta}\right)R^{\beta}_{E_1}(t,\vec{r}) \end{pmatrix} = \begin{pmatrix} \cdots & \cdots(\vec{r})\Delta^{\alpha}{}_{\beta}(t) \\ \cdots & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E_1}(t,\vec{r}) \\ R^{\beta}_{E_1}(t,\vec{r}) \end{pmatrix}$$

$$\Delta^{\alpha}_{\beta} = \frac{\exp\left(-(m_{\alpha_1}+m_{\alpha_2})t\right)}{\exp\left(-(m_{\beta_1}+m_{\beta_2})t\right)}$$

S.Aoki et al [HAL QCD Collab.] Proc. Jpn. Acad., Ser. B, 87 509
K.Sasaki et al [HAL QCD Collab.] PTEP no 11 (2015) 113B01

Considering two different energy eigen states

Potential

$$\begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})\Delta^{\alpha}_{\beta} \\ V^{\beta}{}_{\alpha}(\vec{r})\Delta^{\beta}_{\alpha} & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} = \begin{pmatrix} \left(\frac{\nabla^2}{2\mu_\alpha}-\frac{\partial}{\partial t}\right)R^{\alpha}_{E0}(t,\vec{r}) & \left(\frac{\nabla^2}{2\mu_\beta}-\frac{\partial}{\partial t}\right)R^{\alpha}_{E1}(t,\vec{r}) \\ \left(\frac{\nabla^2}{2\mu_\alpha}-\frac{\partial}{\partial t}\right)R^{\beta}_{E0}(t,\vec{r}) & \left(\frac{\nabla^2}{2\mu_\beta}-\frac{\partial}{\partial t}\right)R^{\beta}_{E1}(t,\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E0}(t,\vec{r}) & R^{\alpha}_{E1}(t,\vec{r}) \\ R^{\beta}_{E0}(t,\vec{r}) & R^{\beta}_{E1}(t,\vec{r}) \end{pmatrix}^{-1}$$

S=-2 BB interaction

Interests of S=-2 multi-baryon system

H-dibaryon

- The flavor singlet state with J=0 predicted by R.L. Jaffe.
  - Strongly attractive color magnetic interaction.
  - No quark Pauli principle for flavor singlet state.

Double-Λ hypernucleus

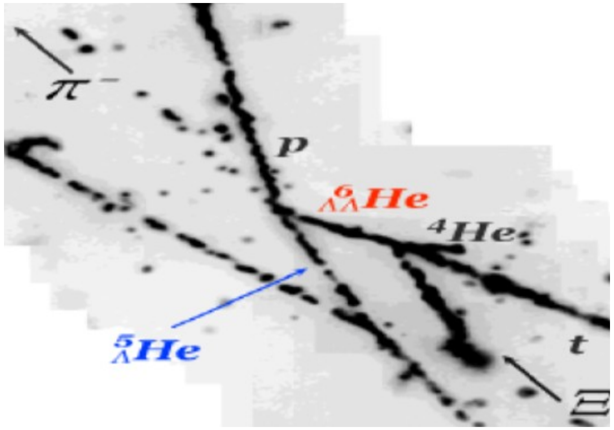
- Conclusions of the "NAGARA Event"

K.Nakazawa and KEK-E176 & E373 Collaborators

Λ–N attraction

Λ–Λ weak attraction

$m_H \geq 2m_\Lambda$ - 6.9MeV

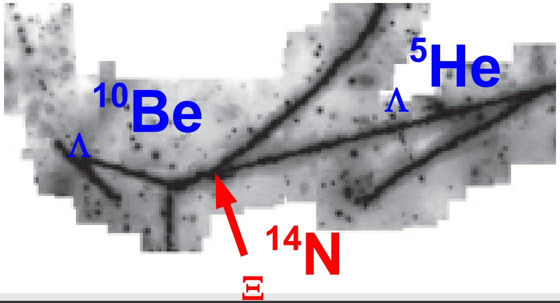


Ξ hypernucleus

- Conclusions of the "KISO Event"

K.Nakazawa and KEK-E373 Collaborators

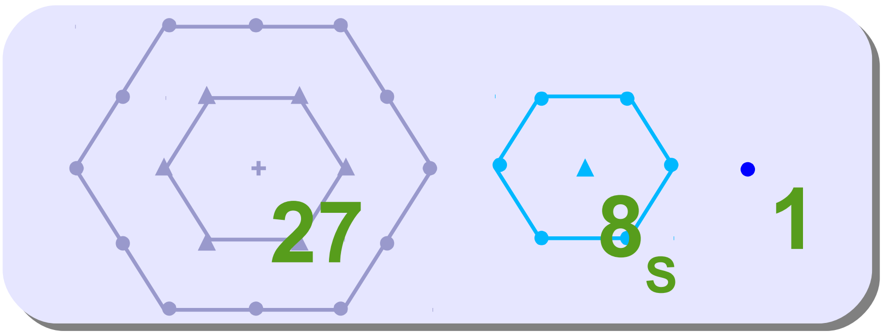
Ξ–N attraction

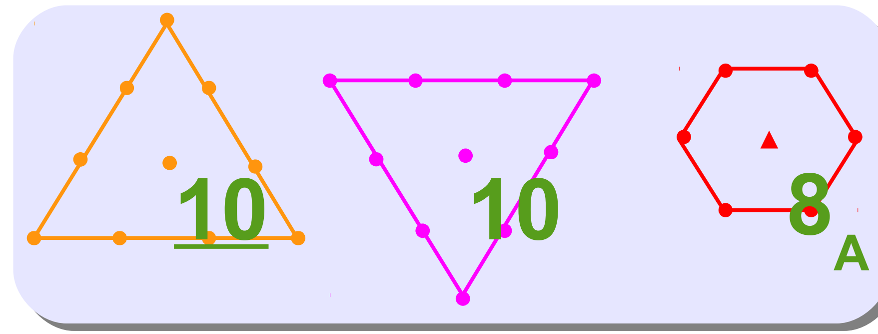


SU(3) feature of BB interaction

SU(3) classification

$8 \times 8 = 27 + 8_S + 1 + \overline{10} + 10 + 8_A$





In view of quark degrees of freedom

- **Short range repulsion in BB interaction could be a result of Pauli principle and color-magnetic interaction for the quarks.**

$$V_{OGE}^{CMI} \propto \frac{1}{m_{q1} m_{q2}} \langle \lambda_1 \cdot \lambda_2 \sigma_1 \cdot \sigma_2 \rangle f(r_{ij})$$

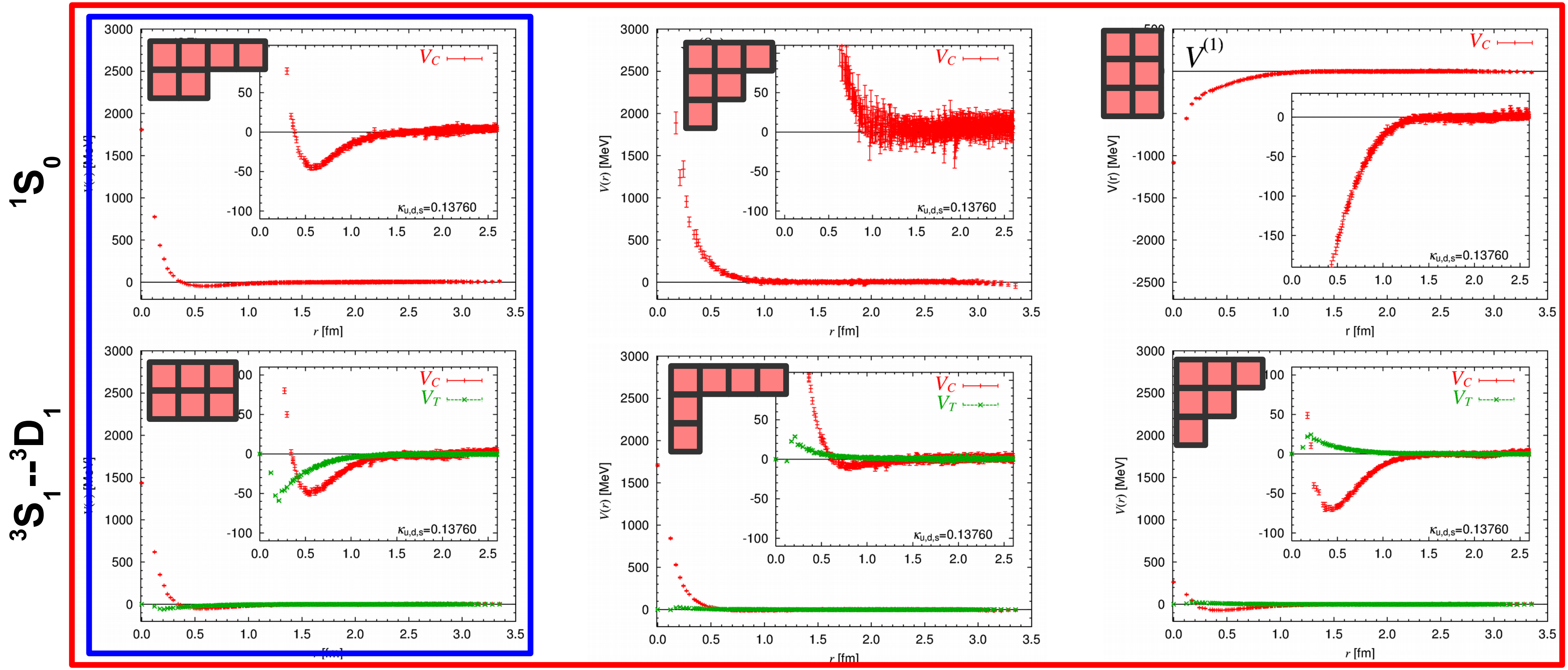
- **For the s-wave BB system, no repulsive core is predicted in flavor singlet state which is known as H-dibaryon channel.**

	Flavor symmetric states			Flavor anti-symmetric states		
	27	8	1	$\overline{10}$	10	8
Pauli	mixed	forbidden	allowed	mixed	forbidden	mixed
CMI	repulsive	repulsive	attractive	repulsive	repulsive	repulsive

Oka, Shimizu and Yazaki NPA464 (1987)

B-B potentials in SU(3) limit

m_π= 469MeV



Two-flavors   Three-flavors

- Quark Pauli principle can be seen at around short distances
  - No repulsive core in flavor singlet state
  - Strongest repulsion in flavor 8s state
- **Possibility of bound H-dibaryon in flavor singlet channel.**

Baryon-baryon system with S=-2

Spin singlet states

Isospin	BB channels		
I=0	$\Lambda\Lambda$	$N\Xi$	$\Sigma\Sigma$
I=1	$N\Xi$	$\Lambda\Sigma$	——
I=2	$\Sigma\Sigma$	——	——

Spin triplet states

Isospin	BB channels		
I=0	$N\Xi$	——	——
I=1	$N\Xi$	$\Lambda\Sigma$	$\Sigma\Sigma$

Relations between BB channels and SU(3) irreducible representations

$$8 \times 8 = 27 + 8_S + 1 + \overline{10} + 10 + 8_A$$

$J^P=0^+$, I=0

$$\begin{pmatrix} \Lambda\Lambda \\ N\Xi \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -\sqrt{5} & -\sqrt{8} & \sqrt{27} \\ \sqrt{20} & \sqrt{8} & \sqrt{12} \\ \sqrt{15} & -\sqrt{24} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix}$$

$J^P=0^+$, I=1

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \end{pmatrix} = \frac{1}{5} \begin{pmatrix} \sqrt{2} & -\sqrt{3} \\ \sqrt{3} & \sqrt{2} \end{pmatrix} \begin{pmatrix} 27 \\ 8 \end{pmatrix}$$

$J^P=0^+$, I=2

$$\Sigma\Sigma \Leftrightarrow 8$$

$J^P=1^+$, I=0

$$N\Xi \Leftrightarrow 8$$

$J^P=1^+$, I=1

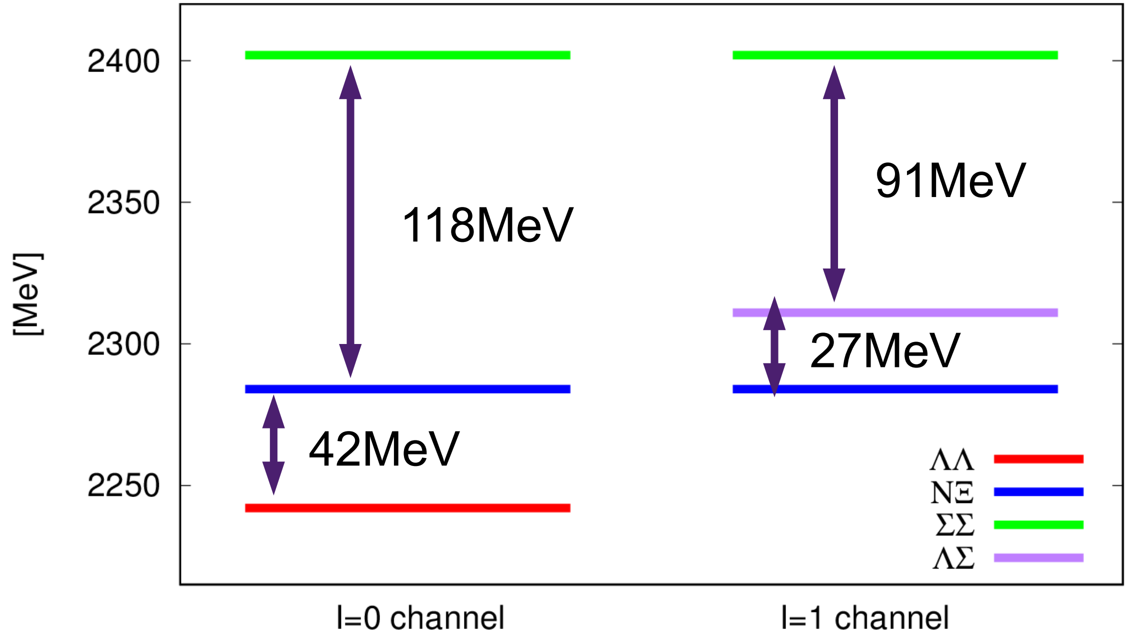
$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{2} & -\sqrt{2} & \sqrt{2} \\ \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & \sqrt{4} \end{pmatrix} \begin{pmatrix} 8 \\ 10 \\ \overline{10} \end{pmatrix}$$

Features of flavor singlet interaction is integrated into the S=-2 $J^P=0^+$, I=0 system.

Numerical setup

- 2+1 flavor gauge configurations.
  - Iwasaki gauge action & O(a) improved Wilson quark action
  - a = 0.086 [fm], a^{-1} = 2.300 GeV.
  - 96^3x96 lattice, L = 8.24 [fm].
  - 414 confs x 28 sources x 4 rotations.
- Flat wall source is considered to produce S-wave B-B state.

	Mass [MeV]
π	146
K	525
m_π/m_K	0.28
N	956±12
Λ	1121±4
Σ	1201±3
Ξ	1328±3



Lists of channels

I=0 states

Strong attraction (H-dibaryon)

Spin	BB channels			SU(3) representation		
$^1S_0$	$\Lambda\Lambda$	$N\Xi$	$\Sigma\Sigma$	1	8s	27
$^3S_1$	--	$N\Xi$	--	8a	--	--

I=1 states

Attraction

Strong repulsion

Spin	BB channels			SU(3) representation		
$^1S_0$	$N\Xi$	--	$\Lambda\Sigma$	--	8s	27
$^3S_1$	$N\Xi$	$\Sigma\Sigma$	$\Lambda\Sigma$	8a	10	10*

Similar to The NN potential

Repulsion

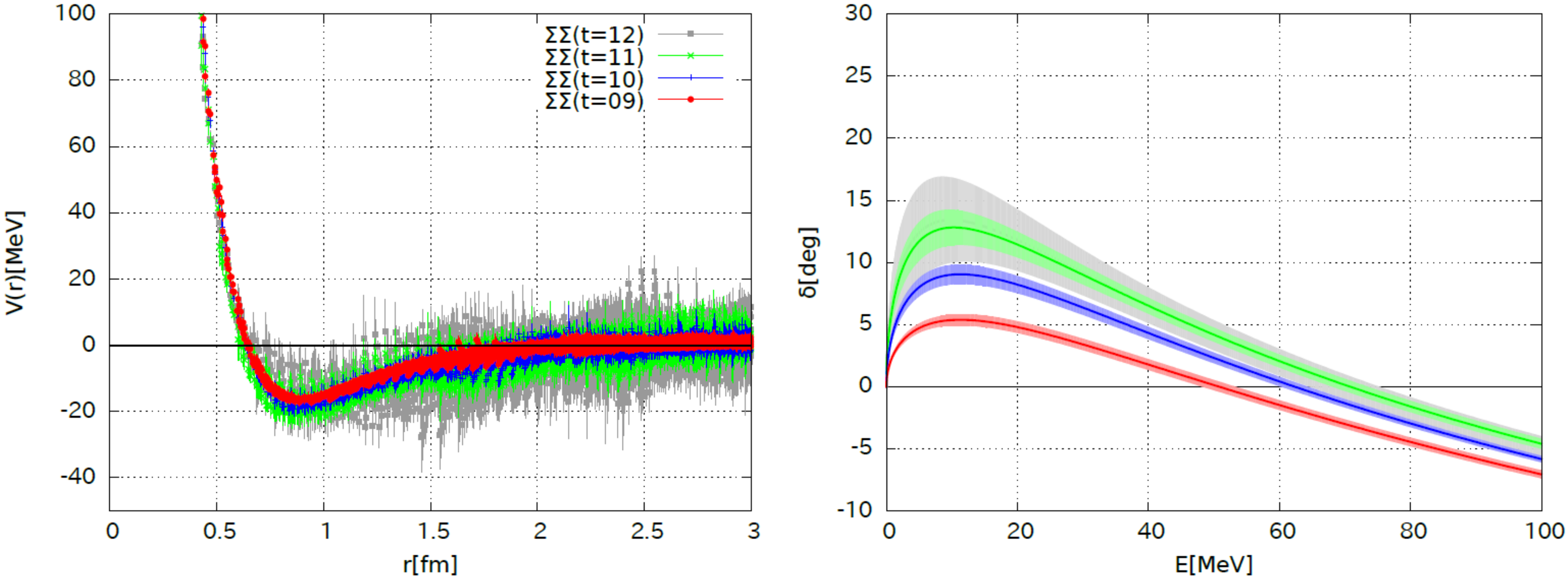
I=2 states

Spin	BB channels	SU(3) representation		
$^1S_0$	$\Sigma\Sigma$	--	--	27
~~$^3S_1$~~				

$\Sigma\Sigma$ (I=2) $^1S_0$ channel

Belongs to 27plet

Preliminary!

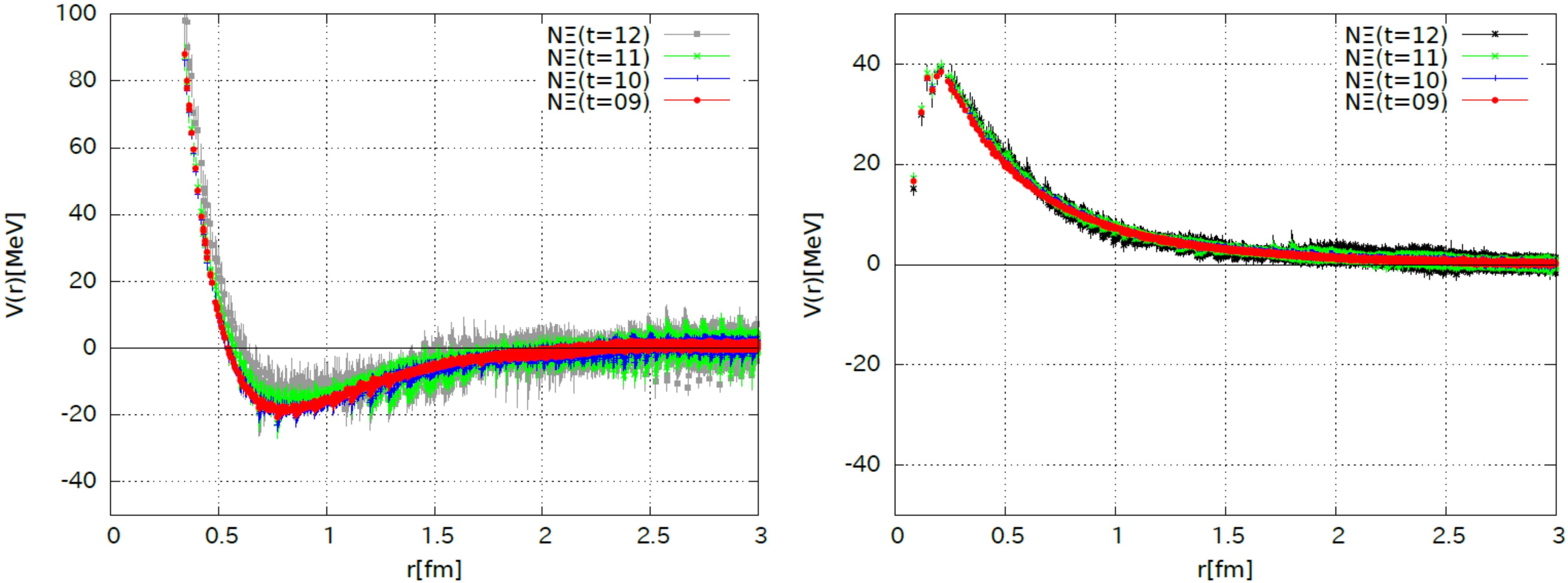


- $\Sigma\Sigma$ (I=2) potential belongs to the 27plet in flavor SU(3) limit.
- The potential has an attractive pocket and repulsive core.
- In this time range, potentials are qualitatively similar.
- Potential saturation is achieved at after t=11??

NΞ (I=0) $^3S_1$-$^3D_1$ channel

Belongs to 8a-plet

Preliminary!

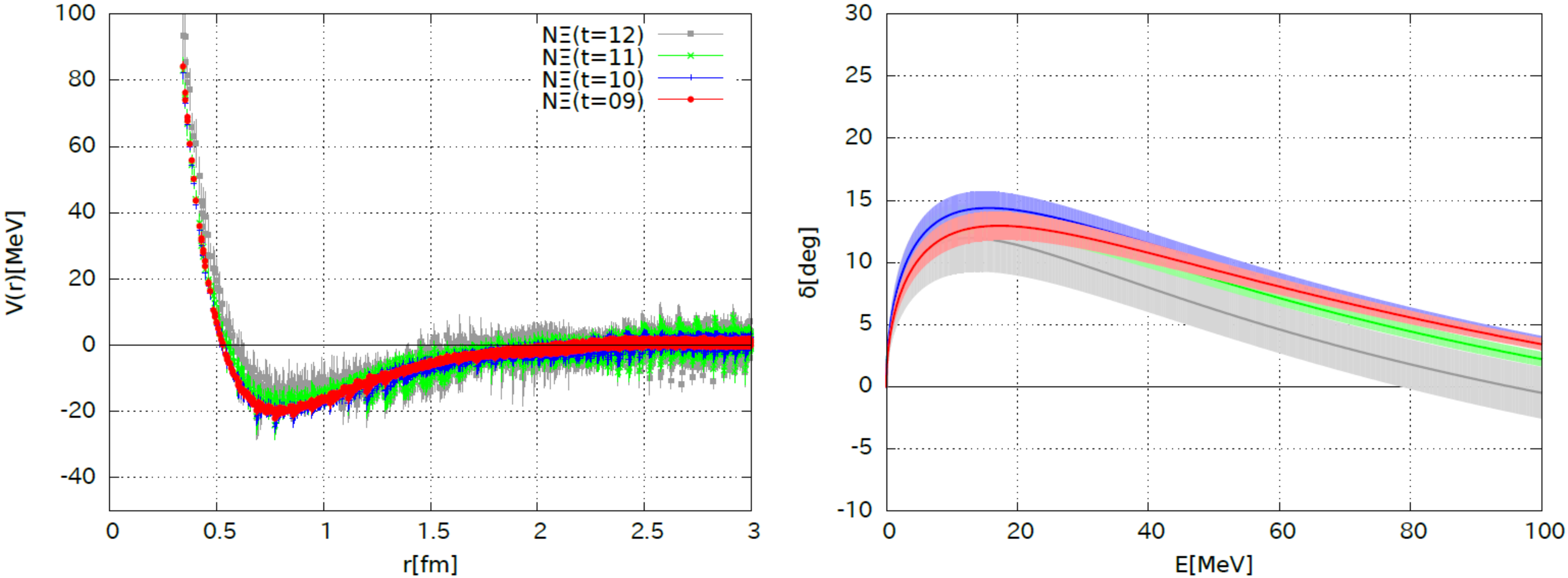


- **NΞ (I=0,spin triplet) potential belongs to the 8plet in flavor SU(3) limit.**
- **The potential has an attractive pocket and repulsive core.**
- **Tensor potential is weaker than the phenomenological NN tensor potential.**

NΞ (I=0) $^3S_1$ channel

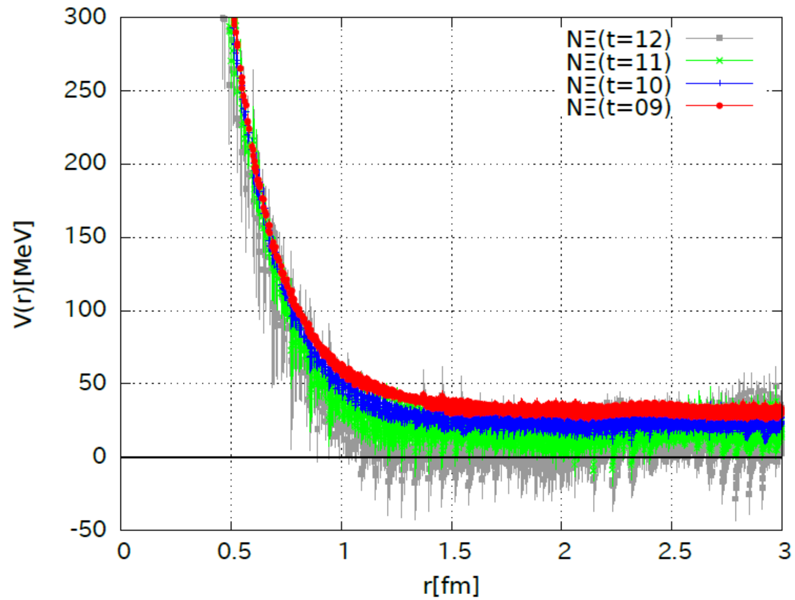
Belongs to 8a-plet

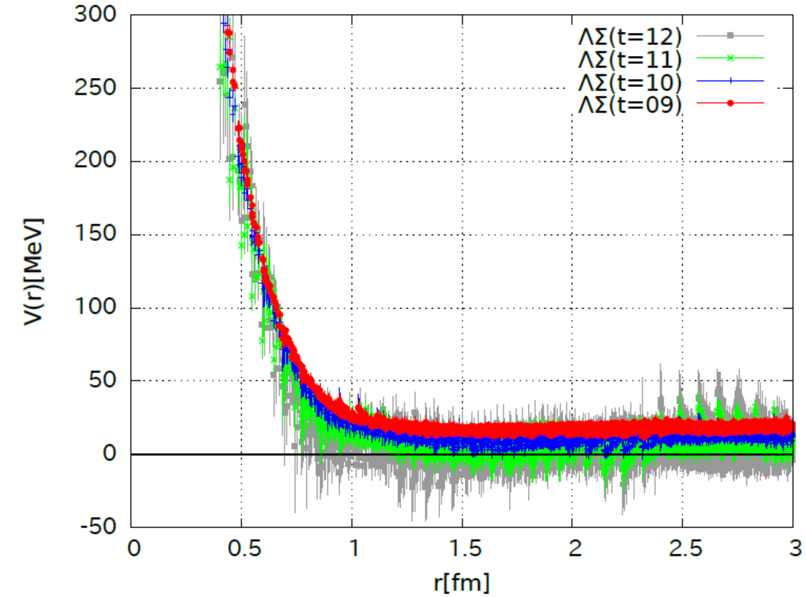
Preliminary!



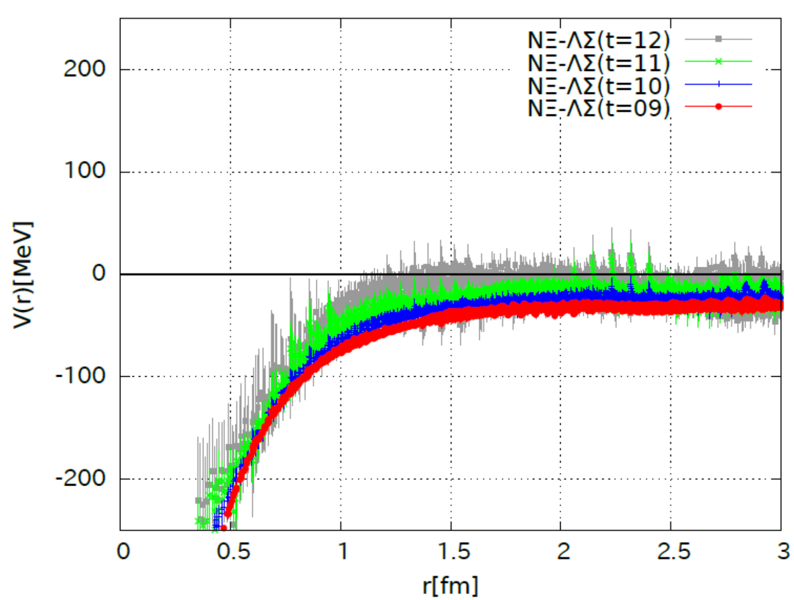
- **Effective NΞ (I=0,spin triplet) central potential is plotted.**
- **From this figure, we find that the tensor potential effects are small.**
- **Phase shifts are same within the error bars.**

$N\Xi$, $\Lambda\Sigma$ (I=1) $^1S_0$ channel





Preliminary!

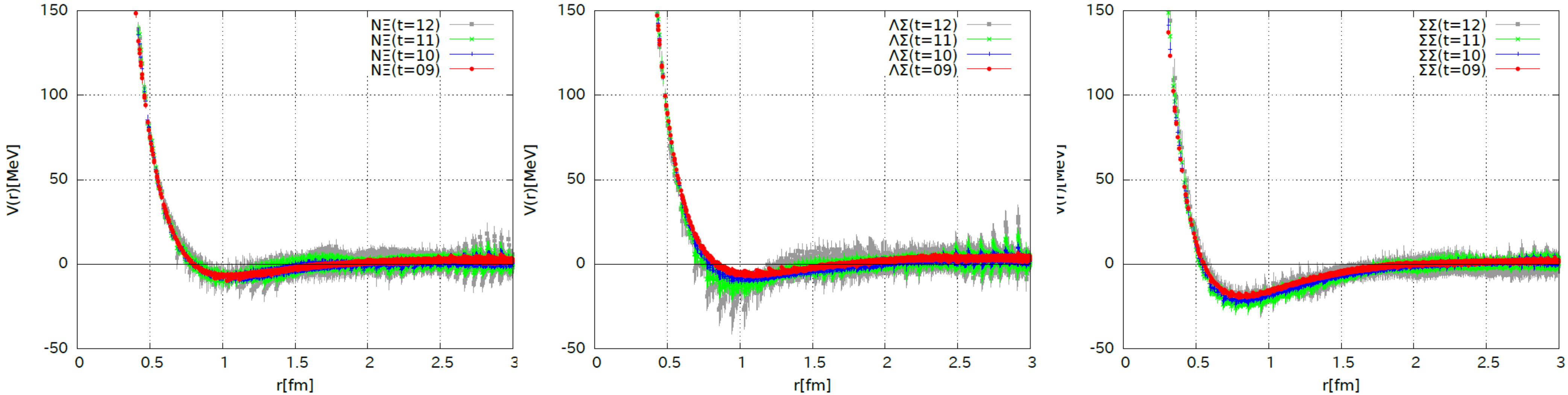


- **Diagonal elements are repulsive in whole range.**
- **Diagonal $N\Xi$ potential is strongly repulsive.**
  - **It means that the $N\Xi$ potential is strongly depend on the channel.**
- **Potentials are not saturated in this time range.**

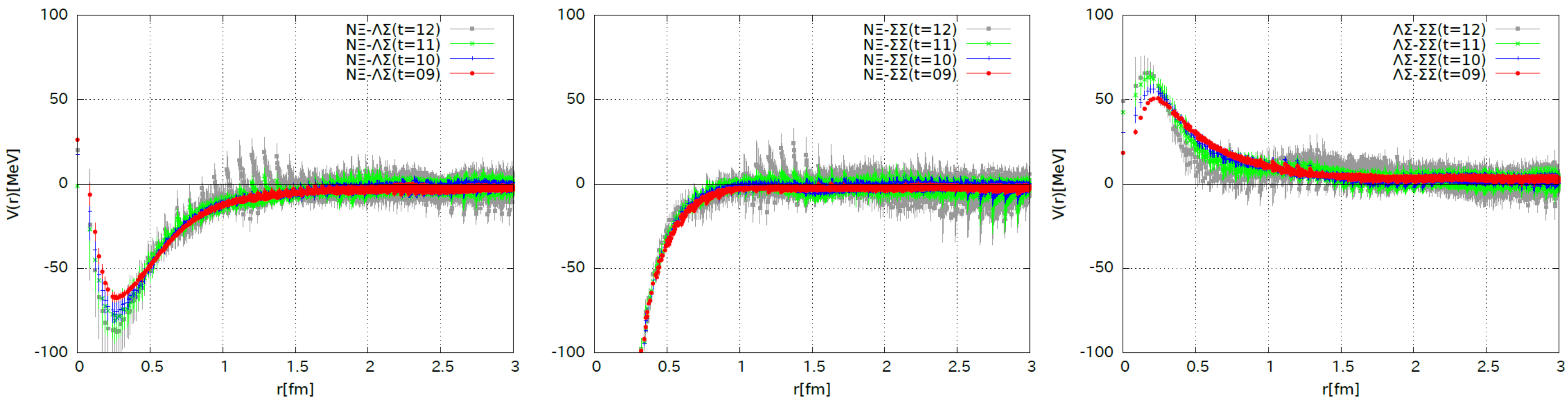
$N\Xi$, $\Lambda\Sigma$, $\Sigma\Sigma$ (I=1) $^3S_1$ channel

Diagonal elements

Preliminary!



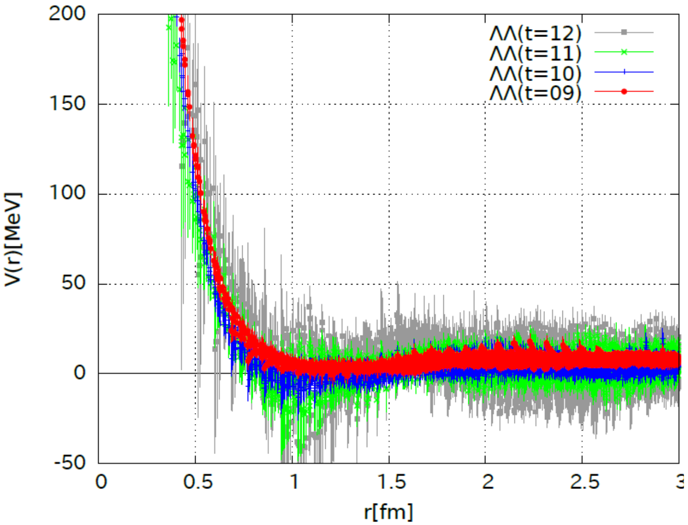
Off-diagonal elements

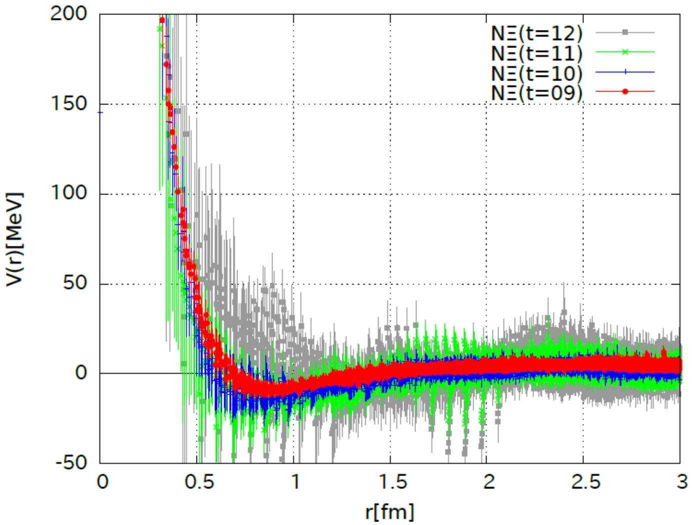


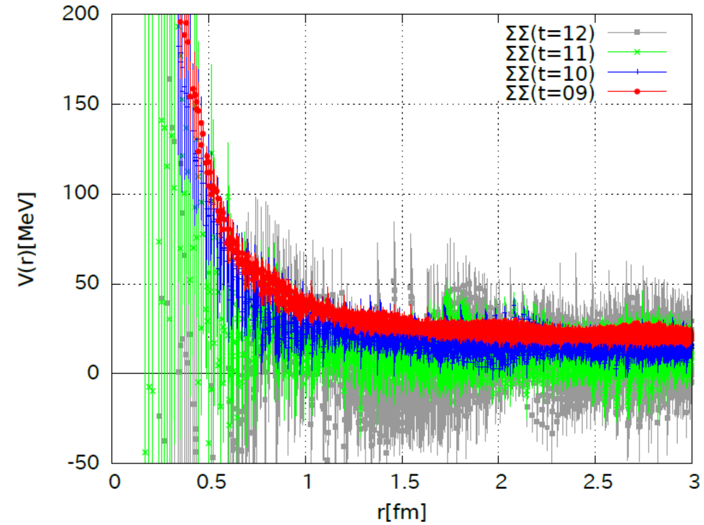
$\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$ $(I=0)$ $^1S_0$ channel

Diagonal elements

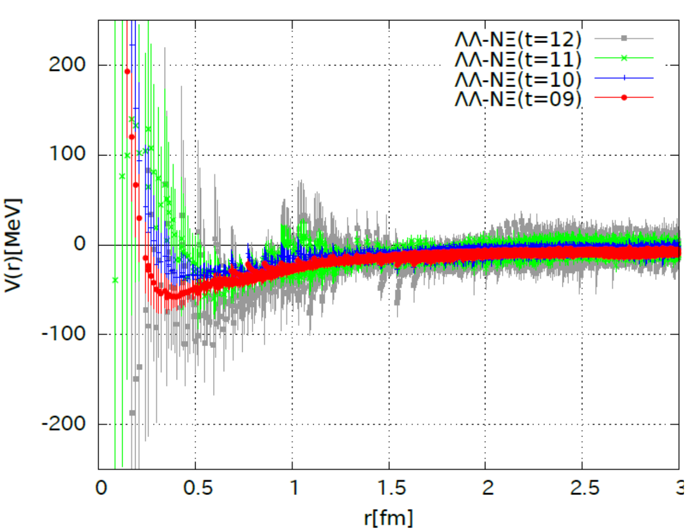
Preliminary!

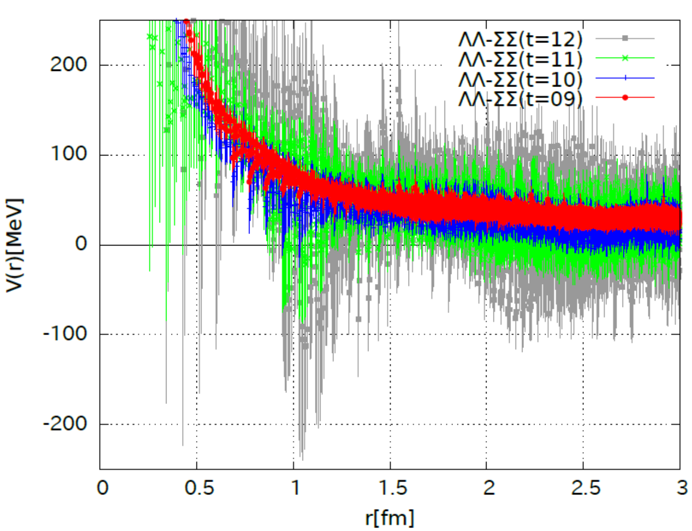


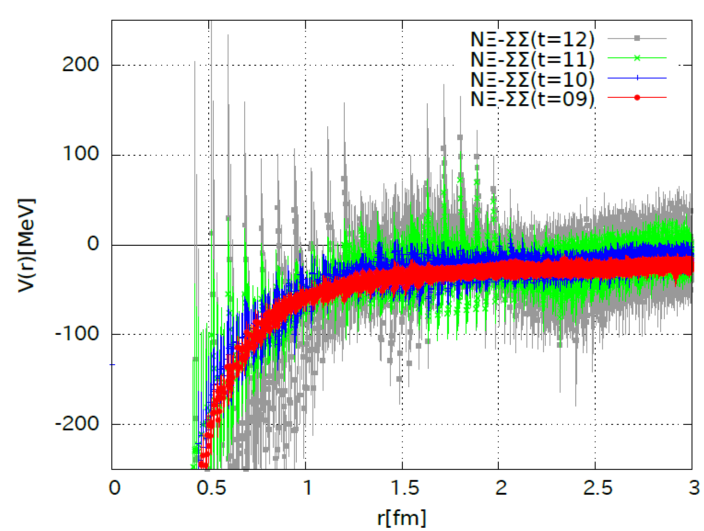




Off-diagonal elements







Comparison of potential matrices

Transformation of potentials
from the particle basis to the SU(3) irreducible representation (irrep) basis.

SU(3) Clebsh-Gordan coefficients

$$\begin{pmatrix} |1\rangle \\ |8\rangle \\ |27\rangle \end{pmatrix} = U \begin{pmatrix} |\Lambda\Lambda\rangle \\ |N\Xi\rangle \\ |\Sigma\Sigma\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{N\Xi} & V^{\Lambda\Lambda}_{\Sigma\Sigma} \\ V^{N\Xi}_{\Lambda\Lambda} & V^{N\Xi} & V^{N\Xi}_{\Sigma\Sigma} \\ V^{\Sigma\Sigma}_{\Lambda\Lambda} & V^{\Sigma\Sigma}_{N\Xi} & V^{\Sigma\Sigma} \end{pmatrix} U^t \longrightarrow \begin{pmatrix} V_1 & & \\ & V_8 & \\ & & V_{27} \end{pmatrix}$$

$$\begin{pmatrix} |\Sigma\Sigma\rangle \\ |N\Xi\rangle \\ |\Lambda\Lambda\rangle \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -1 & -\sqrt{24} & \sqrt{15} \\ \sqrt{12} & \sqrt{8} & \sqrt{20} \\ \sqrt{27} & -\sqrt{8} & -\sqrt{5} \end{pmatrix} \begin{pmatrix} |27\rangle \\ |8_s\rangle \\ |1\rangle \end{pmatrix}$$

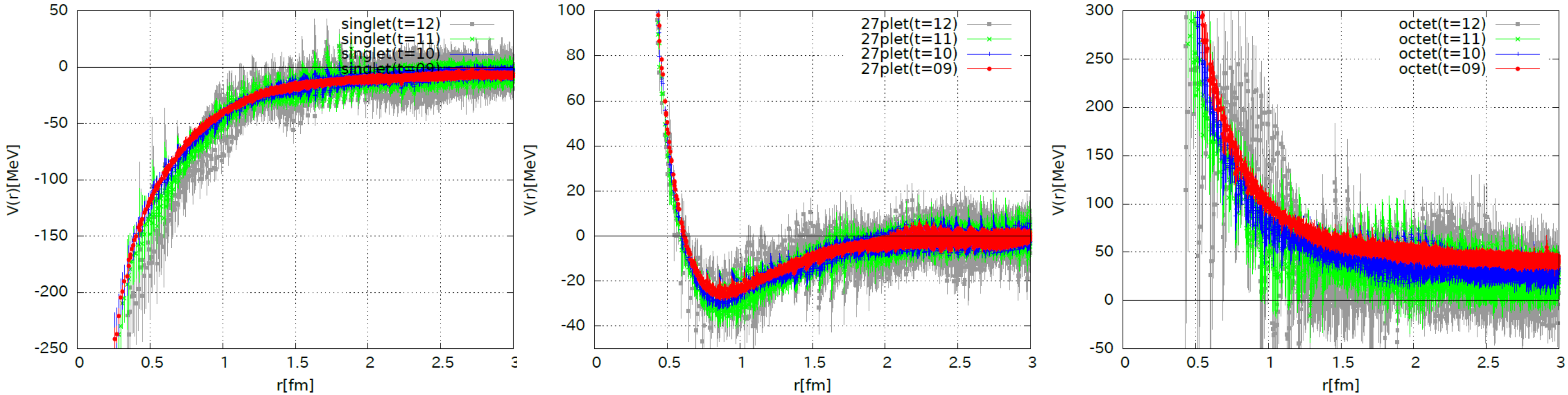
In the SU(3) irreducible representation basis,
the potential matrix should be diagonal in the SU(3) symmetric configuration.

Off-diagonal part of the potential matrix in the SU(3) irrep basis
would be an effective measure of the SU(3) breaking effect.

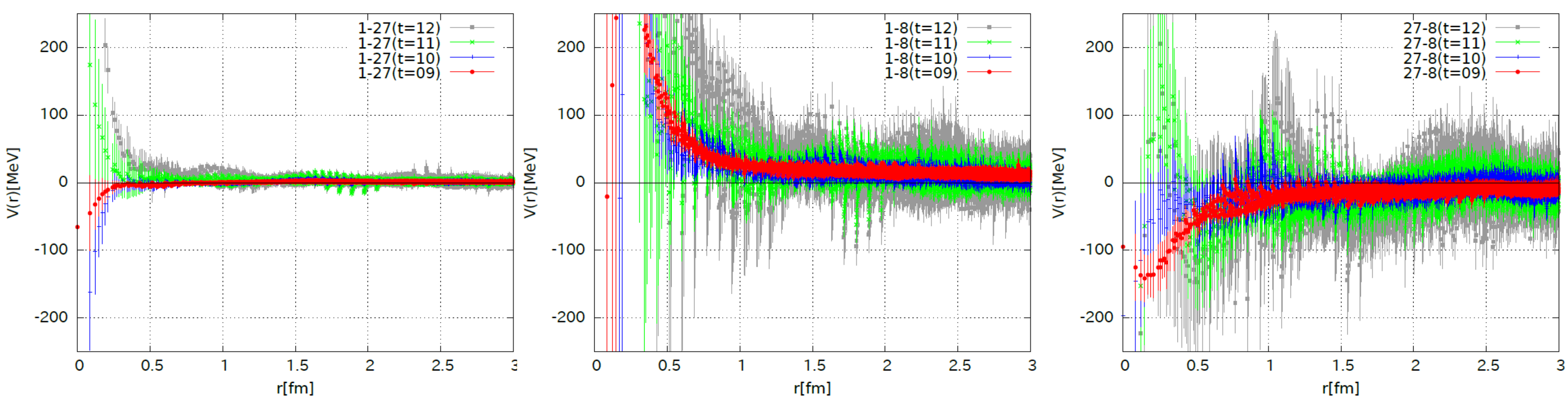
1, 8, 27plet (I=0) $^1S_0$ channel

Diagonal elements

Preliminary!

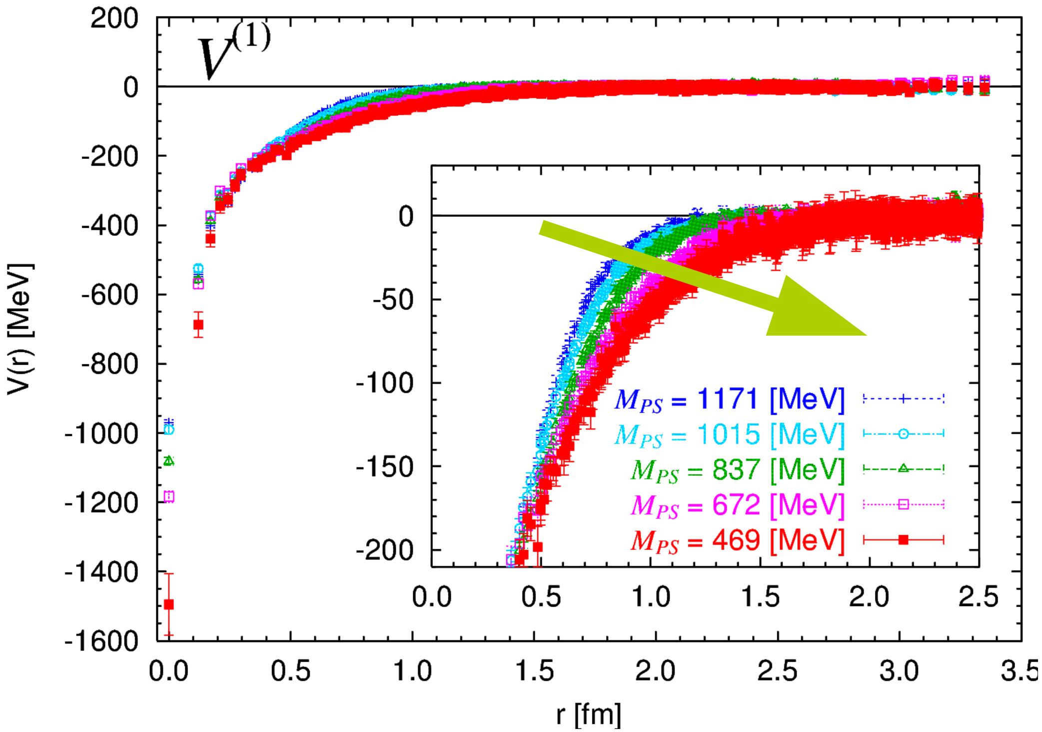


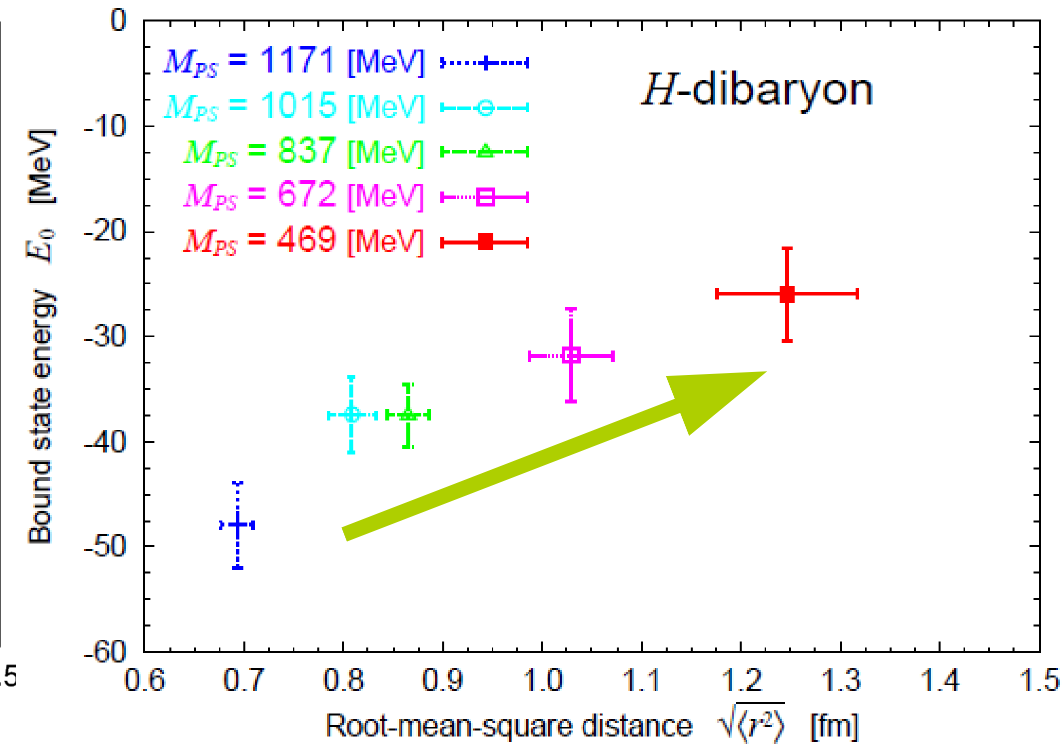
Off-diagonal elements



H-dibaryon

H-dibaryon (flavor SU(3) situations)



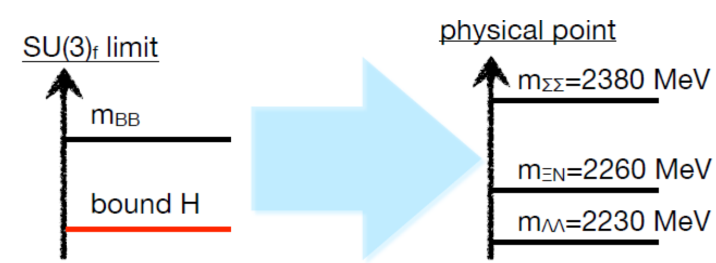


HAL : PRL106(2011)162002
NPL : PRL106(2011)162001

- The bound H-dibaryon state in heavy pion region.
- Potential in flavor singlet channel is getting more attractive as decreasing quark masses

Does the H-dibaryon state survive on the physical situation?

→ **Go to the SU(3) broken world.**



Works on H-dibaryon state

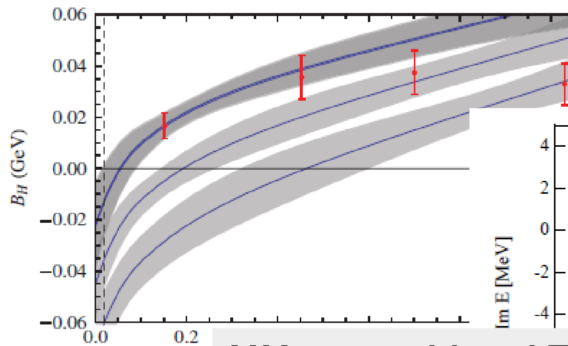
Theoretical status

Several sort of calculations and results
(bag models, NRQM, Quenched LQCD….)

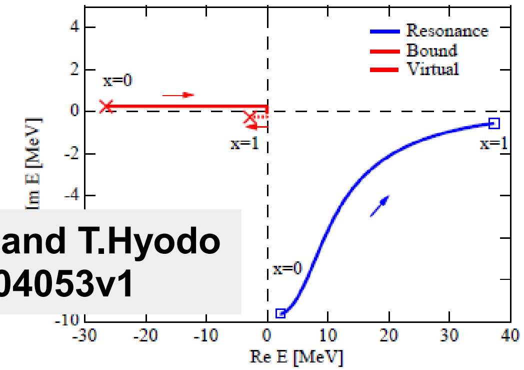
→ There were no conclusive result.

Chiral extrapolations of recent LQCD data

→ **Unbound or resonance**



P. E. Shanahan et al
PRL 107(2011) 092004



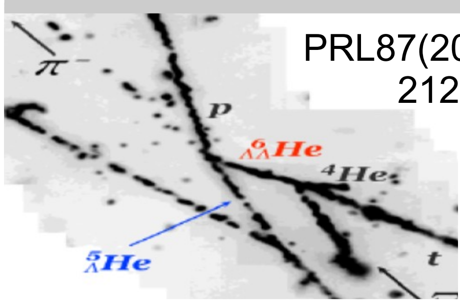
Y.Yamaguchi and T.Hyodo
hep-ph:1607.04053v1

Experimental status

"NAGARA Event"

K.Nakazawa et al
KEK-E176 & E373 Coll.

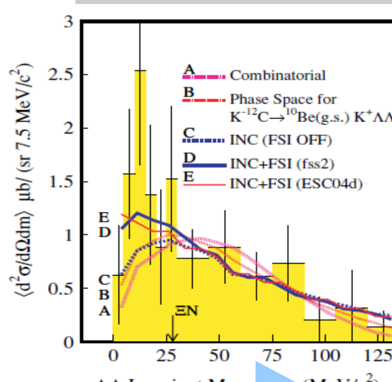
PRL87(2001) 212502



- Deeply bound dibaryon state is ruled out

"^{12}C(K$^-$,K$^+$ΛΛ) reaction"

C.J.Yoon et al KEK-PS E522 Coll.



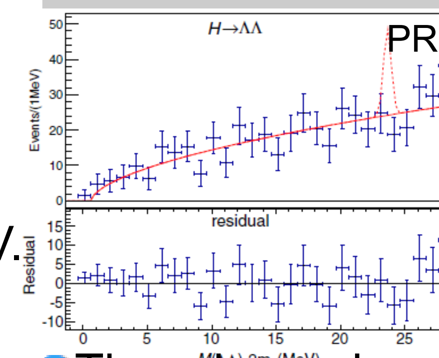
PRC75(2007) 022201(R)

- Significance of enhancements below 30 MeV.

→ **Larger statistics**
J-PARC E42

"Υ(1S) and Υ(2S) decays"

B.H. Kim et al Belle Coll.



PRL110(2013) 222002

- There is no sign of near threshold enhancement.

Effective two channel potential

Original coupled channel equation

$$\begin{pmatrix} \left(E^{\Lambda\Lambda}-H_0^{\Lambda\Lambda}\right)R^{\Lambda\Lambda}(\vec{r},t) \\ \left(E^{\Xi N}-H_0^{\Xi N}\right)R^{\Xi N}(\vec{r},t) \\ \left(E^{\Sigma\Sigma}-H_0^{\Sigma\Sigma}\right)R^{\Sigma\Sigma}(\vec{r},t) \end{pmatrix} = \begin{pmatrix} V^{\Lambda\Lambda}_{\Lambda\Lambda}(\vec{r}) & V^{\Lambda\Lambda}_{\Xi N}(\vec{r}) & V^{\Lambda\Lambda}_{\Sigma\Sigma}(\vec{r}) \\ V^{\Xi N}_{\Lambda\Lambda}(\vec{r}) & V^{\Xi N}_{\Xi N}(\vec{r}) & V^{\Xi N}_{\Sigma\Sigma}(\vec{r}) \\ V^{\Sigma\Sigma}_{\Lambda\Lambda}(\vec{r}) & V^{\Sigma\Sigma}_{\Xi N}(\vec{r}) & V^{\Sigma\Sigma}_{\Sigma\Sigma}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\Lambda\Lambda}(\vec{r},t) \\ R^{\Xi N}(\vec{r},t) \\ R^{\Sigma\Sigma}(\vec{r},t) \end{pmatrix}$$

Truncation of ΣΣ channel

Reduced coupled channel equation

$$\begin{pmatrix} \left(E^{\Lambda\Lambda}-H_0^{\Lambda\Lambda}\right)R^{\Lambda\Lambda}(\vec{r},t) \\ \left(E^{\Xi N}-H_0^{\Xi N}\right)R^{\Xi N}(\vec{r},t) \end{pmatrix} = \begin{pmatrix} \overline{V^{\Lambda\Lambda}_{\Lambda\Lambda}}(\vec{r}) & \overline{V^{\Lambda\Lambda}_{\Xi N}}(\vec{r}) \\ \overline{V^{\Xi N}_{\Lambda\Lambda}}(\vec{r}) & \overline{V^{\Xi N}_{\Xi N}}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\Lambda\Lambda}(\vec{r},t) \\ R^{\Xi N}(\vec{r},t) \end{pmatrix}$$

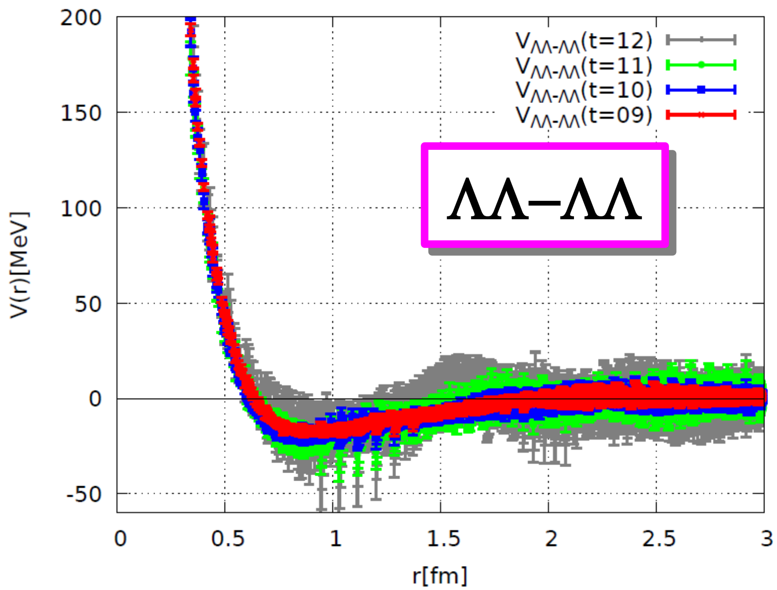
Effective ΛΛ–NΞ potential

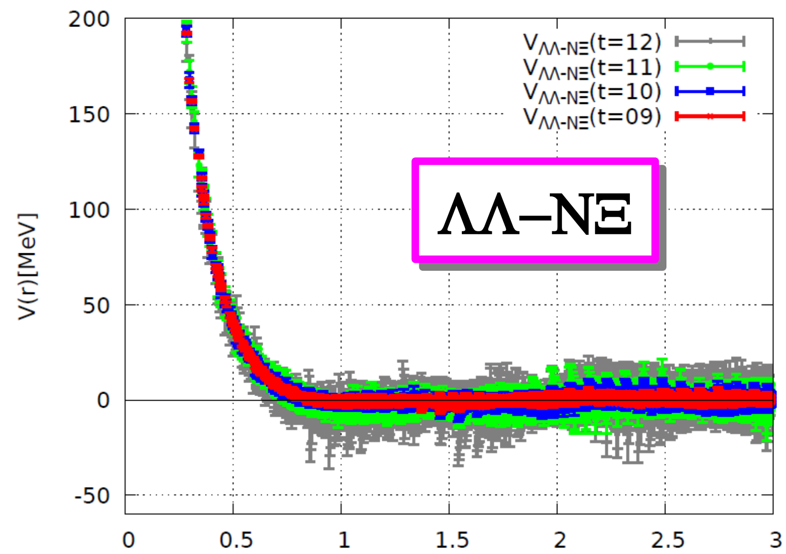
- The same scattering phase shift would be expected in a low energy region.
- Non-locality (energy dependence, higher derivative contribution) of potential matrix could be enhanced.

$\Lambda\Lambda$, $N\Xi$ (I=0) $^1S_0$ potential (Effective 2ch calc.)

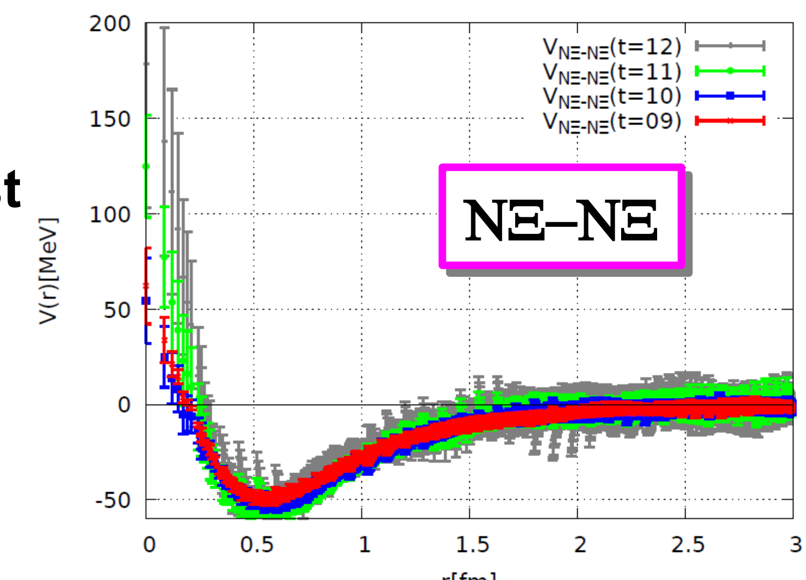
- N_f = 2+1 full QCD with L = 8fm, m_π = 146 MeV

Preliminary!





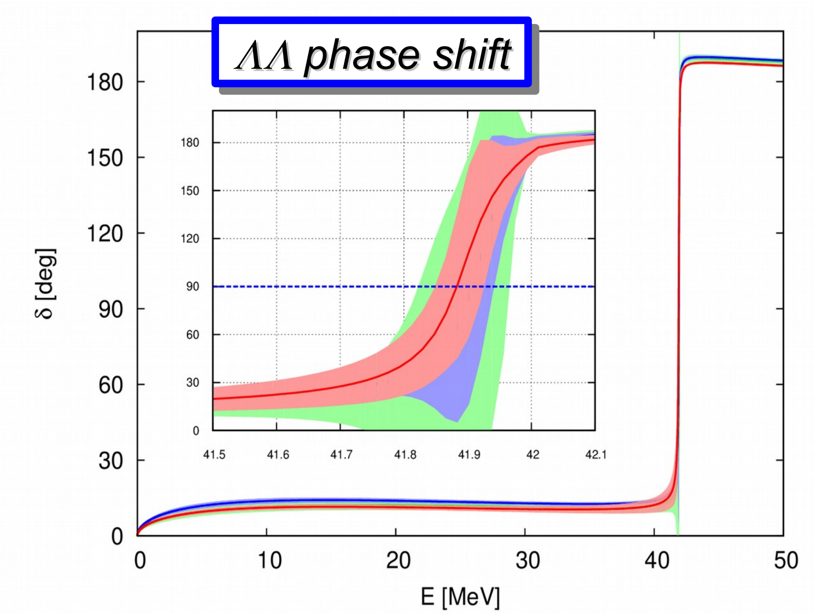
- **Potential calculated by only using $\Lambda\Lambda$ and $N\Xi$ channels.**
- **Long range part of potential is almost stable against the time slice.**
- **Short range part of $N\Xi$ potential changes as time t goes.**
- **$\Lambda\Lambda$–$N\Xi$ transition potential is quite small in r > 0.7fm region**

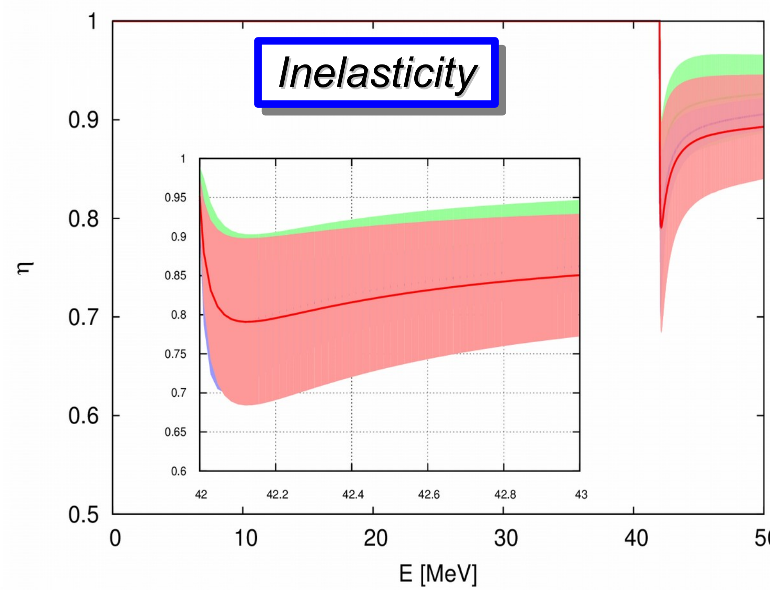


$\Lambda\Lambda$ and $N\Xi$ phase shift and inelasticity T-dep

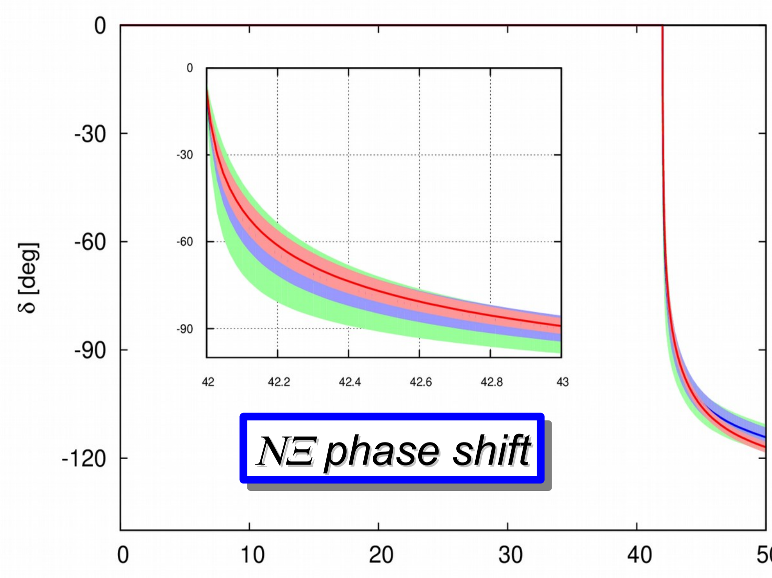
▶ N_f = 2+1 full QCD with L = 8fm, m_π = 146 MeV

Preliminary!





t=09
t=10
t=11

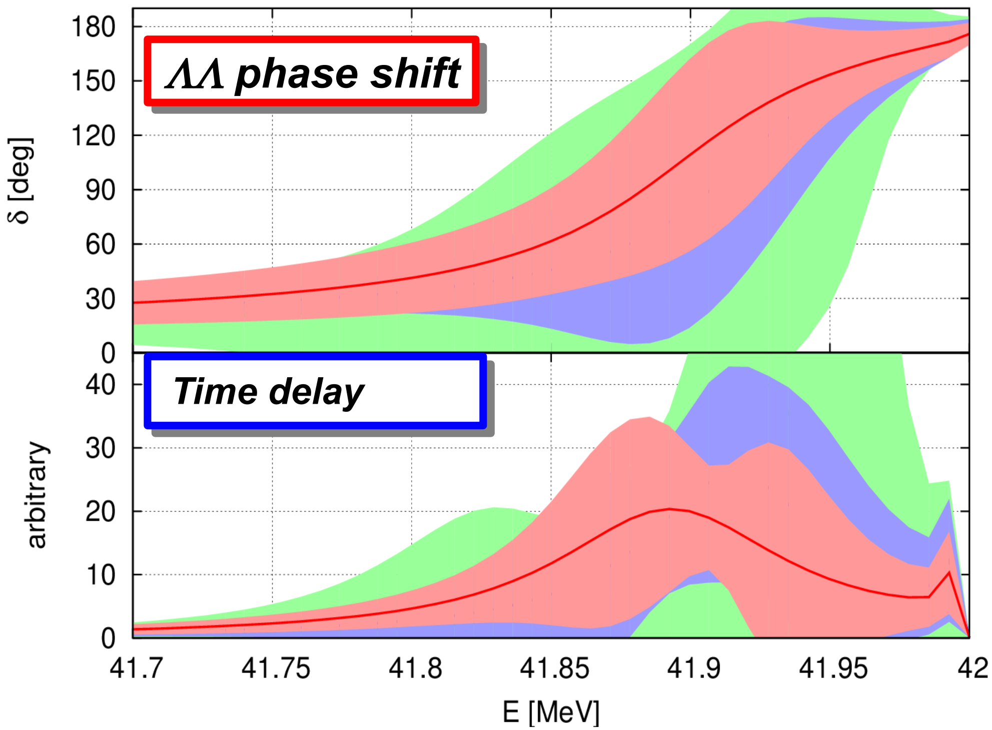


- $\Lambda\Lambda$ and $N\Xi$ phase shift is calculated
  by using 2ch effective potential.
- A sharp resonance is found
  just below the $N\Xi$ threshold.
- Inelasticity is small.

Breit-Wigner mass and width

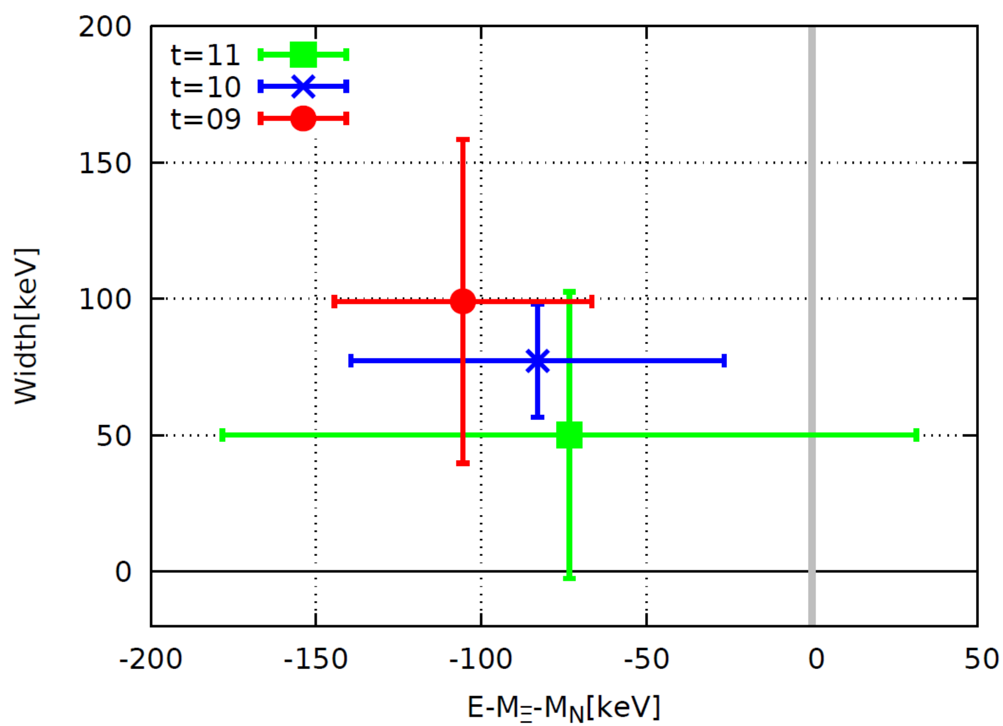
▶ $N_f = 2+1$ full QCD with L = 8fm, $m_\pi = 146$ MeV

Preliminary!



- **Fitting the time delay of ΛΛ scattering by the Breit-Wigner type finction,**

Resonance enargy and width



- **In the vicinity of resonance point,**

$$\delta(E) = \delta_B - \arctan\left(\frac{\Gamma/2}{E - E_r}\right)$$

thus

$$\frac{d\,\delta(E)}{d\,E} = \frac{\Gamma/2}{(E - E_r)^2 + (\Gamma/2)^2}$$

Summary and outlook

- **We have investigated coupled channel baryonic interactions from lattice QCD.**

- **We find that**
  - **Potential in ΛΛ $^1S_0$ channel is weakly attractive.**
  - **NΞ potential is largely depends on its channel.**
  - **Potential in flavor singlet $^1S_0$ channel is strongly attractive.**

- **We have studied dibaryon candidate states**
  - **H-dibaryon channel**
    - **We perform ΛΛ–NΞ coupled channel calculation.**
    - **Sharp resonance is found just below the NΞ threshold (Time slice saturation is not achieved yet.)**

- **We continue to study it by using higher statistical data.**