Strangeness S=-2 baryon-baryon interactions from Lattice QCD

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for HAL QCD Collaboration

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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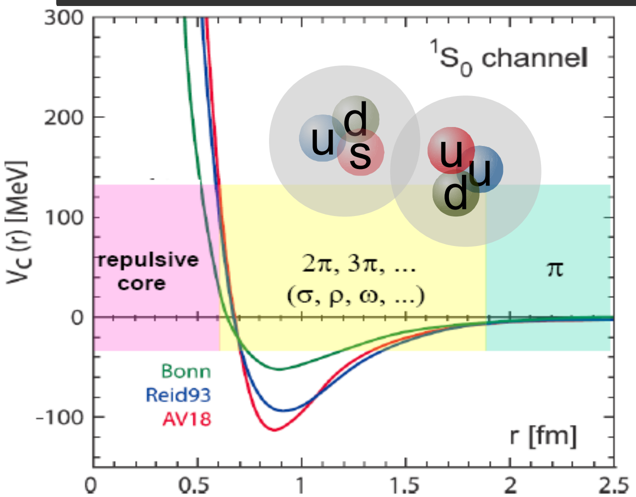
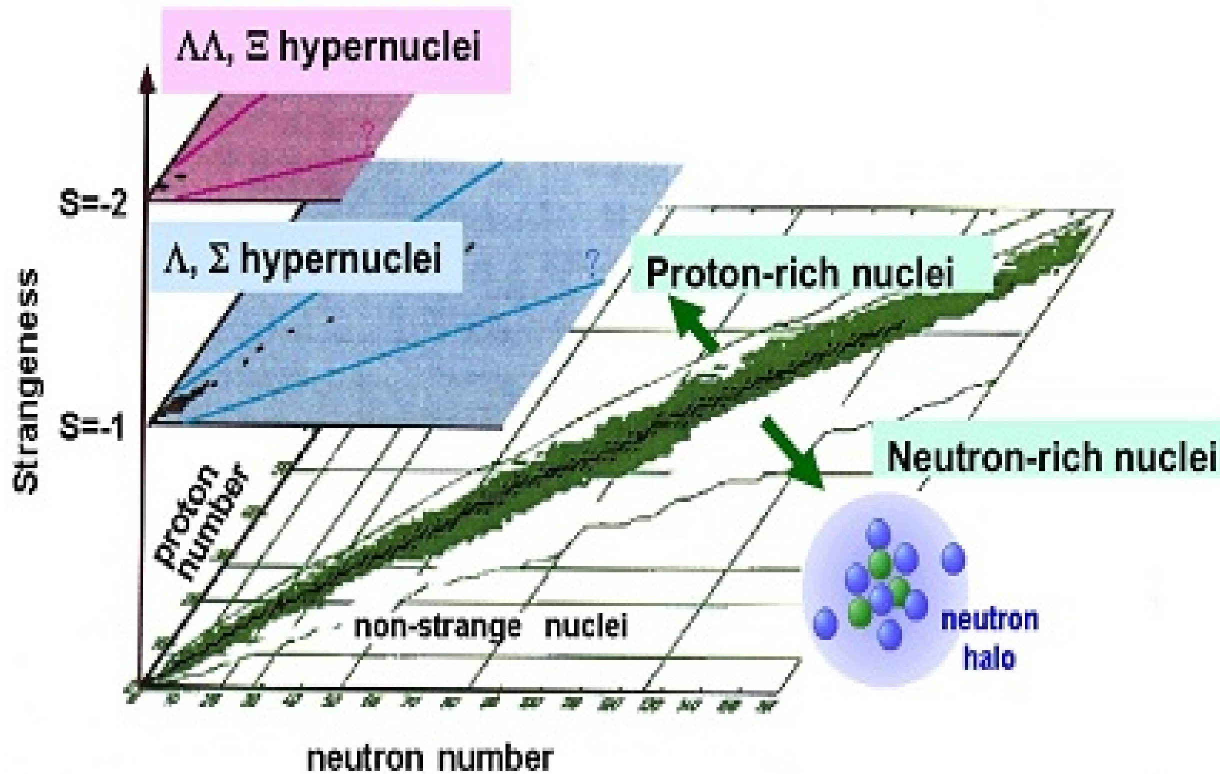
Introduction

BB interactions are crucial to investigate the nuclear phenomena

Once we obtain "proper" nuclear potentials,
we apply them to the structure of (hyper-) nucleus.

BB interaction (potential)





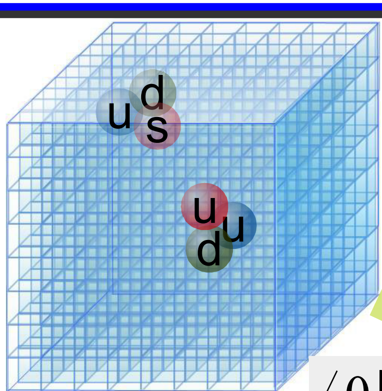
Properties of nuclaer potential
- **State dependence (spin, isospin)**
- **Long range attraction**
- **Short range repulsion**

How do we obtain the nuclear force?

Derivation of hadronic interaction from QCD

Start with the fundamental theory,QCD

Lattice QCD simulation



Lüscher's finite volume method

M. Lüscher, NPB354(1991)531

1. Measure the discrete energy spectrum, **E**
2. Put the E into the formula which connects E and **δ**

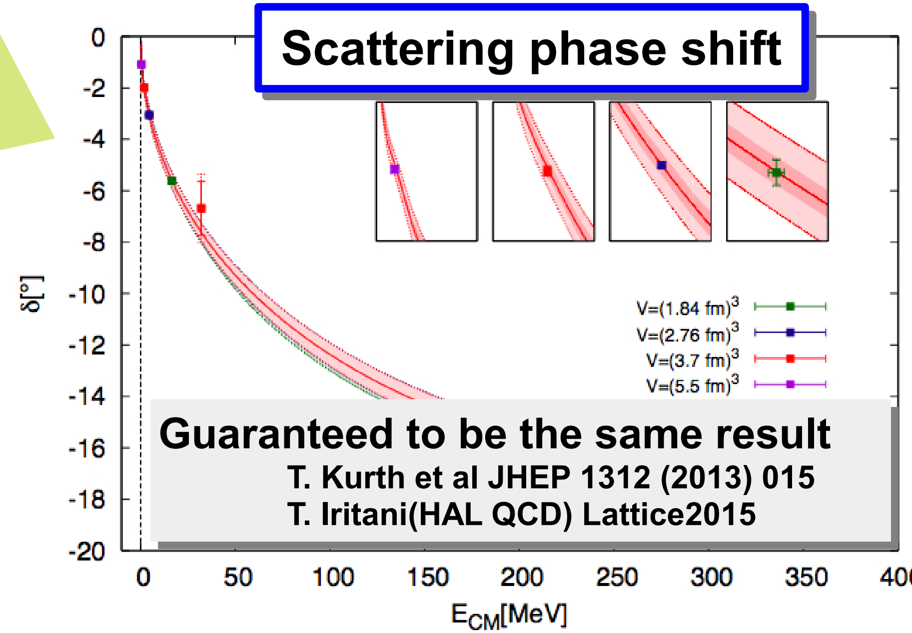
$$\langle 0|B_1 B_2(t,\vec{r})\bar{B}_2\bar{B}_1(t_0)|0\rangle = A_0 \Psi(\vec{r}, E_0) e^{-E_0(t-t_0)} + \cdots$$

HAL QCD method

Ishii, Aoki, Hatsuda, PRL**99** (2007) 022001

1. Measure the NBS wave function, **Ψ**
2. Calculate potential, **V**, through Schrödinger eq.
3. Calculate observables by scattering theory

Scattering phase shift



Guaranteed to be the same result
T. Kurth et al JHEP 1312 (2013) 015
T. Iritani(HAL QCD) Lattice2015

HAL QCD method

NBS wave function

$$\Psi(E,\vec{r})e^{-E(t-t_0)} = \sum_{\vec{x}} \langle 0|B_i(t,\vec{x}+\vec{r})B_j(t,\vec{x})|E,t_0\rangle$$

E : Total energy of system

- In asymptotic region : $(p^2+\nabla^2)\Psi(E,\vec{r})=0$
- In interaction region : $(p^2+\nabla^2)\Psi(E,\vec{r})=K(E,\vec{r})$

$$\Psi(E,\vec{r}) \simeq A\frac{\sin(pr+\delta(E))}{pr}$$

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

$$R_I^{B_1B_2}(t,\vec{r})=\Psi_{B_1B_2}(\vec{r},t)e^{(m_1+m_2)t}$$

Modified Schrödinger equation

$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1B_2}(t,\vec{r})=\int U(\vec{r},\vec{r}\,')R_I^{B_1B_2}(t,\vec{r})d^3r'$$

N. Ishii et al Phys. Lett. B712(2012)437

Derivative expansion

$$U(\vec{r},\vec{r}\,') = V_C(r)+S_{12}V_T(r)+\vec{L}\cdot\vec{S}_s V_{LS}(r)+\vec{L}\cdot\vec{S}_a V_{ALS}(r)+O(\nabla^2)$$

K. Murano et al Phys.Lett. B735 (2014) 19

Potential

$$V(\vec{r})=\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1B_2}(t,\vec{r})/R_I^{B_1B_2}(t,\vec{r})$$

HAL QCD method (coupled-channel)

NBS wave function

$$\Psi^{\alpha}(E_i,\vec{r})e^{-E_i t} = \langle 0|(B_1 B_2)^{\alpha}(\vec{r})|E_i\rangle$$

$$\Psi^{\beta}(E_i,\vec{r})e^{-E_i t} = \langle 0|(B_1 B_2)^{\beta}(\vec{r})|E_i\rangle$$

$$\int dr\,\tilde{\Psi}_{\beta}(E',\vec{r})\Psi^{\gamma}(E,\vec{r}) = \delta(E'-E)\delta_{\beta}{}^{\gamma}$$

$$R_E^{B_1 B_2}(t,\vec{r}) = \Psi_{B_1 B_2}(\vec{r},E)e^{(-E+m_1+m_2)t}$$

Leading order of velocity expansion and time-derivative method.

Modified coupled-channel Schrödinger equation

$$\begin{pmatrix} (-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\alpha})R^{\alpha}_{E_0}(t,\vec{r}) \\ (-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\beta})R^{\beta}_{E_0}(t,\vec{r}) \end{pmatrix} = \begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})\Delta^{\alpha}{}_{\beta}(t) \\ V^{\beta}{}_{\alpha}(\vec{r})\Delta^{\beta}{}_{\alpha}(t) & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E_0}(t,\vec{r}) \\ R^{\beta}_{E_0}(t,\vec{r}) \end{pmatrix}$$

$$\begin{pmatrix} \cdots \\ (-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\beta})R^{\beta}_{E_1}(t,\vec{r}) \end{pmatrix} \cdots \begin{pmatrix} \cdots(\vec{r})\Delta^{\alpha}{}_{\beta}(t) \\ V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E_1}(t,\vec{r}) \\ R^{\beta}_{E_1}(t,\vec{r}) \end{pmatrix}$$

$$\Delta^{\alpha}_{\beta} = \frac{\exp(-(m_{\alpha_1}+m_{\alpha_2})t)}{\exp(-(m_{\beta_1}+m_{\beta_2})t)}$$

S.Aoki et al [HAL QCD Collab.] Proc. Jpn. Acad., Ser. B, 87 509

K.Sasaki et al [HAL QCD Collab.] PTEP no 11 (2015) 113B01

Potential

Considering two different energy eigen states

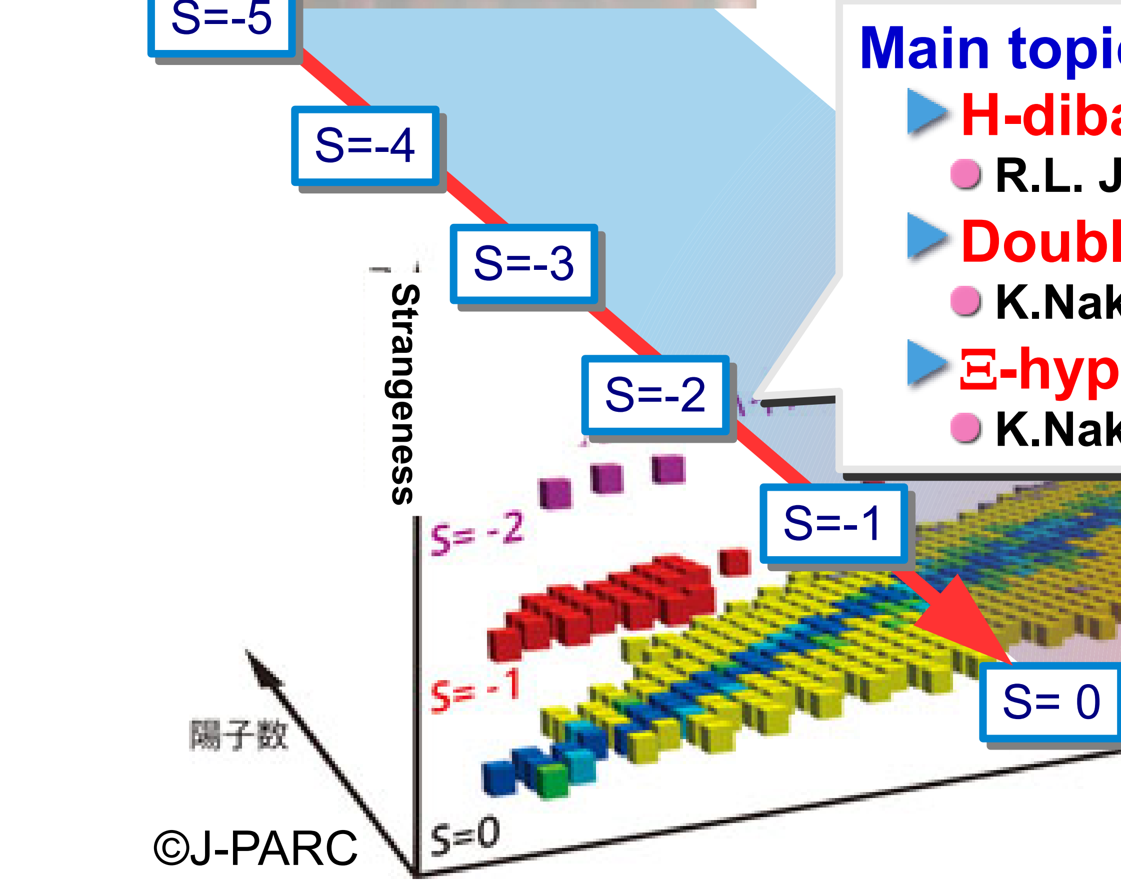
$$\begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})\Delta^{\alpha}_{\beta} \\ V^{\beta}{}_{\alpha}(\vec{r})\Delta^{\beta}_{\alpha} & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} = \begin{pmatrix} (\frac{\nabla^2}{2\mu_\alpha}-\frac{\partial}{\partial t})R^{\alpha}_{E0}(t,\vec{r}) & (\frac{\nabla^2}{2\mu_\beta}-\frac{\partial}{\partial t})R^{\alpha}_{E1}(t,\vec{r}) \\ (\frac{\nabla^2}{2\mu_\alpha}-\frac{\partial}{\partial t})R^{\beta}_{E0}(t,\vec{r}) & (\frac{\nabla^2}{2\mu_\beta}-\frac{\partial}{\partial t})R^{\beta}_{E1}(t,\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E0}(t,\vec{r}) & R^{\alpha}_{E1}(t,\vec{r}) \\ R^{\beta}_{E0}(t,\vec{r}) & R^{\beta}_{E1}(t,\vec{r}) \end{pmatrix}^{-1}$$

Introduction

BB interactions are crucial to investigate (hyper-)nuclear structures

Lattice QCD simulation

- **Advantageous for more strange quarks**
- **Signals getting worse as increasing the number of light quarks.**
- **Complementary role to experiment.**



Main topics of S=-2 multi baryon system

- **H-dibaryon**
  - R.L. Jaffe, PRL 38 (1977) 195
- **Double-Λ hypernuclei**
  - K.Nakazawa et al, KEK-E176 Collaboration
- **Ξ-hypernuclei**
  - K.Nakazawa et al, KEK-E373 Collaboration

Experiment

Baryon-baryon system with S=-2

Spin singlet states

Isospin	BB channels		
I=0	$\Lambda\Lambda$	$N\Xi$	$\Sigma\Sigma$
I=1	$N\Xi$	$\Lambda\Sigma$	––
I=2	$\Sigma\Sigma$	––	––

Spin triplet states

Isospin	BB channels		
I=0	$N\Xi$	––	––
I=1	$N\Xi$	$\Lambda\Sigma$	$\Sigma\Sigma$

Relations between BB channels and SU(3) irreducible representations

$$8 \times 8 = 27 + 8_S + 1 + \overline{10} + 10 + 8_A$$

$J^P=0^+$, I=0

$$\begin{pmatrix} \Lambda\Lambda \\ N\Xi \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -\sqrt{5} & -\sqrt{8} & \sqrt{27} \\ \sqrt{20} & \sqrt{8} & \sqrt{12} \\ \sqrt{15} & -\sqrt{24} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix}$$

$J^P=0^+$, I=1

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \end{pmatrix} = \frac{1}{5} \begin{pmatrix} \sqrt{2} & -\sqrt{3} \\ \sqrt{3} & \sqrt{2} \end{pmatrix} \begin{pmatrix} 27 \\ 8 \end{pmatrix}$$

$J^P=0^+$, I=2

$$\Sigma\Sigma \Leftrightarrow 8$$

$J^P=1^+$, I=0

$$N\Xi \Leftrightarrow 8$$

$J^P=1^+$, I=1

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{2} & -\sqrt{2} & \sqrt{2} \\ \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & \sqrt{4} \end{pmatrix} \begin{pmatrix} 8 \\ \overline{10} \\ 10 \end{pmatrix}$$

Features of flavor singlet interaction is integrated into the S=-2 $J^P=0^+$, I=0 system.

Keys to understand H-dibaryon

A strongly bound state predicted by Jaffe in 1977 using MIT bag model.

H-dibaryon state is

- **SU(3) flavor singlet [uuddss], strangeness S=-2.**
- **spin and isospin equals to zero, and $J^P = 0^+$**

▶ **Strongly attractive interaction is expected in flavor singlet channel.**

- **Short range one-gluon exchange contributions**

  Strongly attractive Color Magnetic Interaction

- **Symmetry of two-baryon system (Pauli principle)**

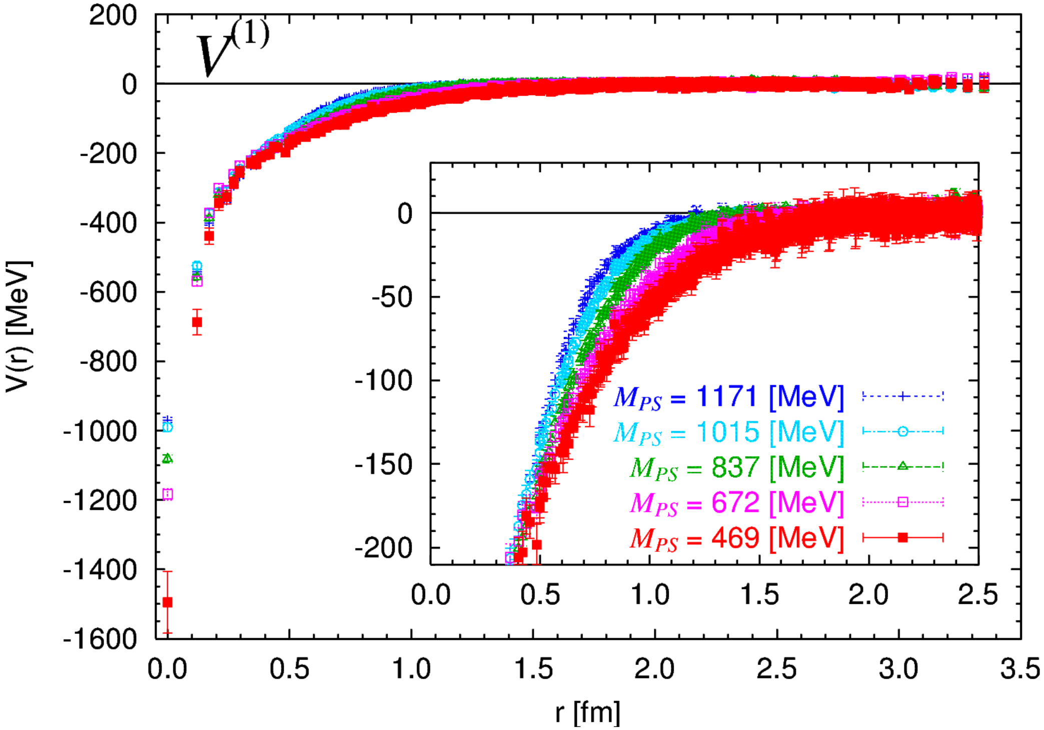
  Flavor singlet channel is free from Pauli blocking effect

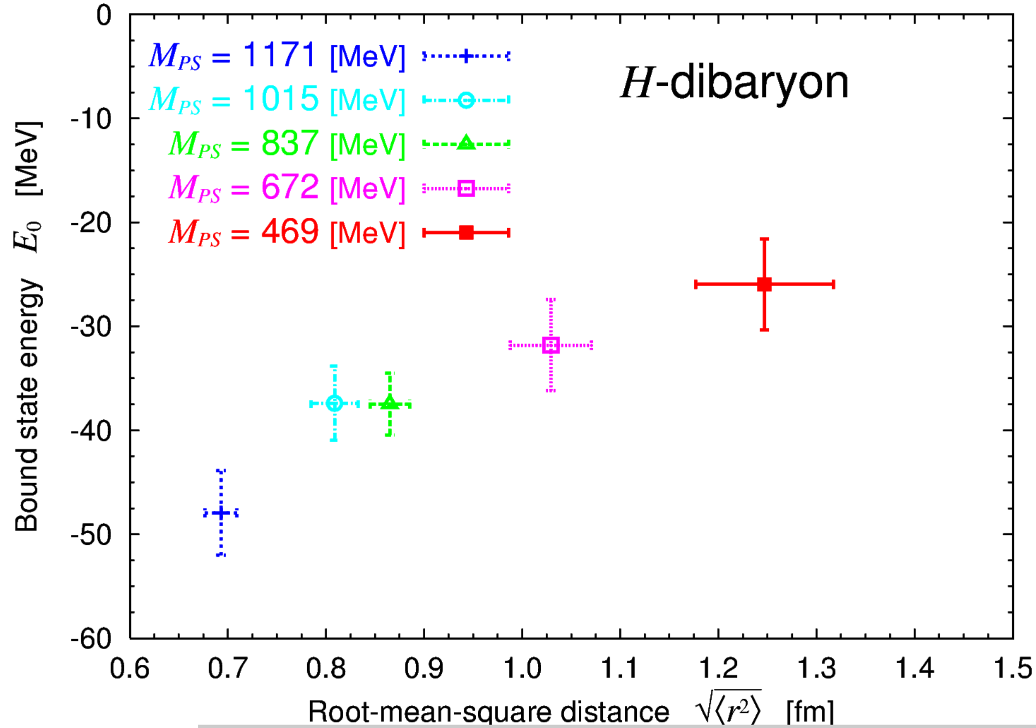
	27	8	1	$\overline{10}$	10	8
Pauli	mixed	forbidden	allowed	mixed	forbidden	mixed
CMI	repulsive	repulsive	attractive	repulsive	repulsive	repulsive

Oka, Shimizu and Yazaki NPA464 (1987)

Hunting for H-dibaryon in SU(3) limit

Strongly attractive interaction is expected in flavor singlet channel.





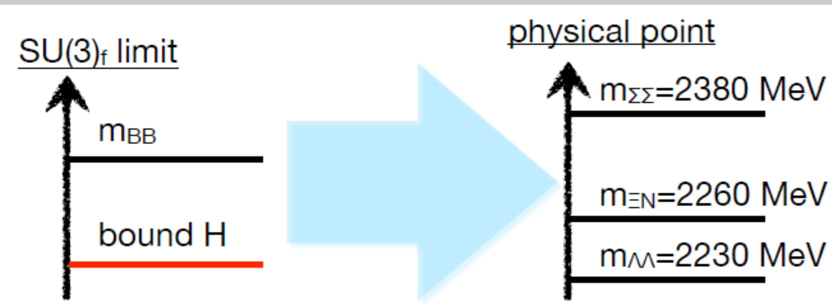
T.Inoue et al[HAL QCD Coll.] NPA881(2012) 28

- **Strongly attractive potential was found in the flavor singlet channel.**
- **Bound state was found in this mass range with SU(3) symmetry.**

What happens at the physical point?

▶ **SU(3) breaking effects**

- **Threshold separation**
- **Changes of interactions**



Non-trivial contributions

Works on H-dibaryon state

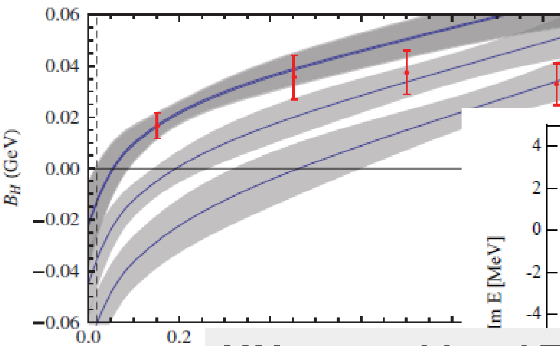
Theoretical status

Several sort of calculations and results
(bag models, NRQM, Quenched LQCD….)

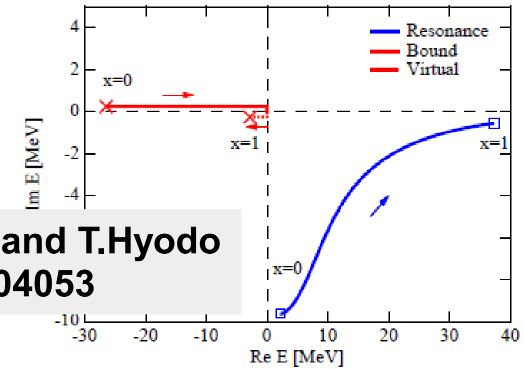
→ **There were no conclusive result.**

Chiral extrapolations of recent LQCD data

→ **Unbound or resonance**



P. E. Shanahan et al
PRL 107(2011) 092004



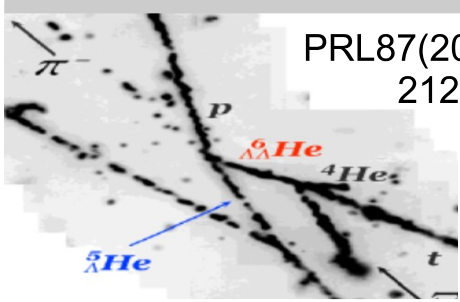
Y.Yamaguchi and T.Hyodo
hep-ph:1607.04053

Experimental status

"NAGARA Event"

K.Nakazawa et al
KEK-E176 & E373 Coll.

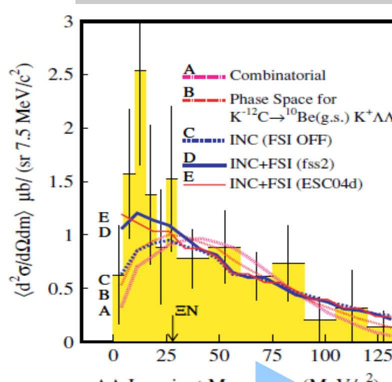
PRL87(2001) 212502



- Deeply bound dibaryon state is ruled out

"$^{12}C(K^-,K^+\Lambda\Lambda)$ reaction"

C.J.Yoon et al KEK-PS E522 Coll.



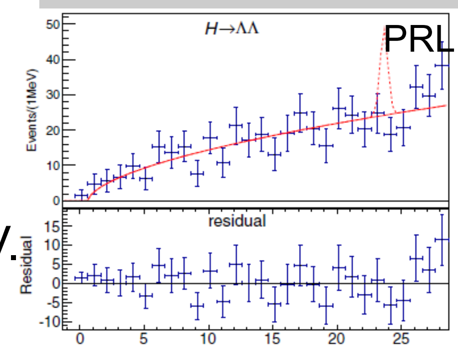
PRC75(2007) 022201(R)

- Significance of enhancements below 30 MeV.

→ **Larger statistics**
J-PARC E42

"Y(1S) and Y(2S) decays"

B.H. Kim et al Belle Coll.



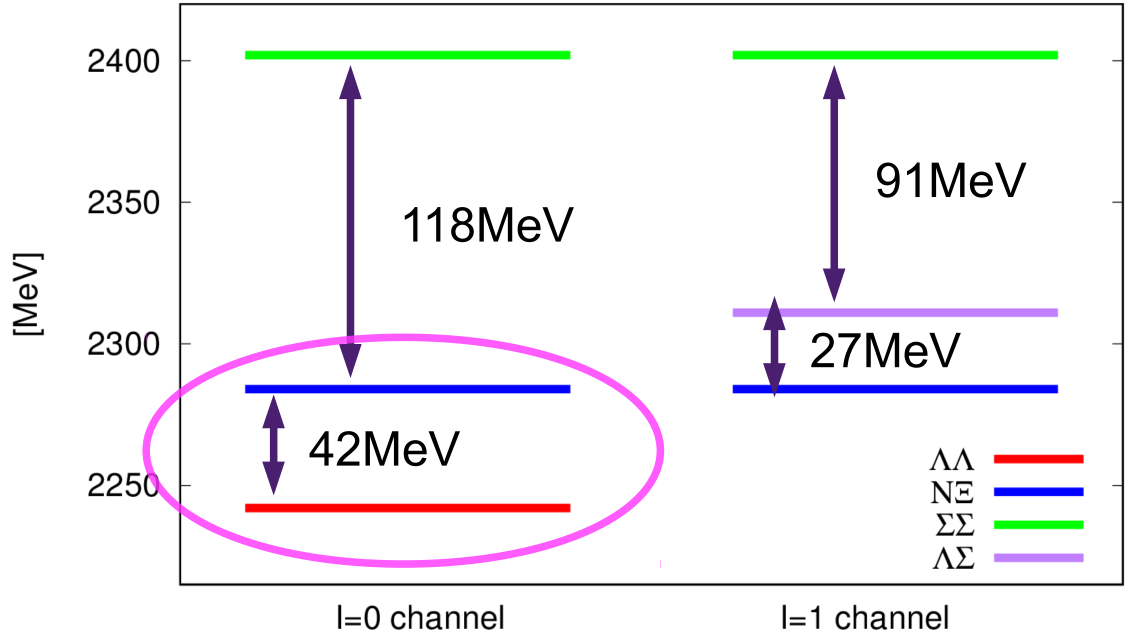
PRL110(2013) 222002

- There is no sign of near threshold enhancement.

Numerical setup

- $2+1$ flavor gauge configurations.
  - Iwasaki gauge action & O(a) improved Wilson quark action
  - a = 0.086 [fm], a^{-1} = 2.300 GeV.
  - $96^3 x96$ *lattice*, L = 8.24 [fm].
  - 414 confs x 28 sources x 4 rotations.
- Flat wall source is considered to produce S-wave B-B state.

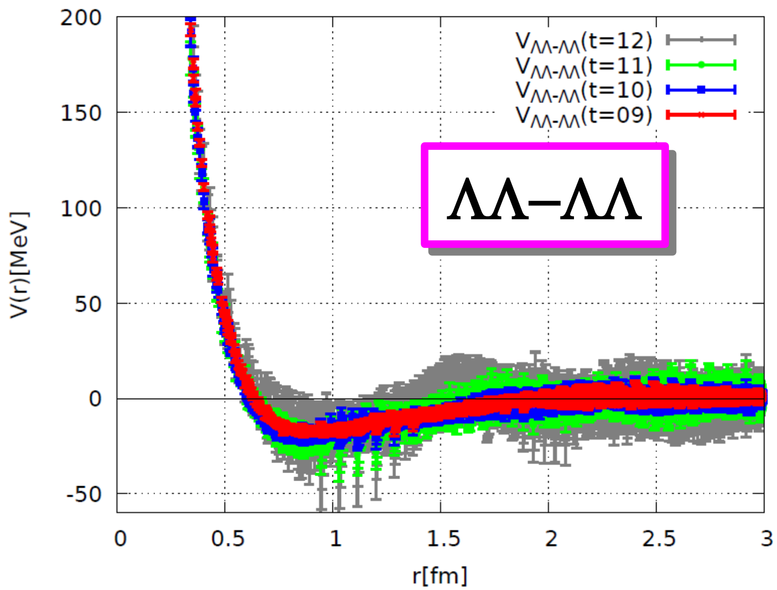
	Mass [MeV]
π	146
K	525
m_π/m_K	0.28
N	956±12
Λ	1121±4
Σ	1201±3
Ξ	1328±3

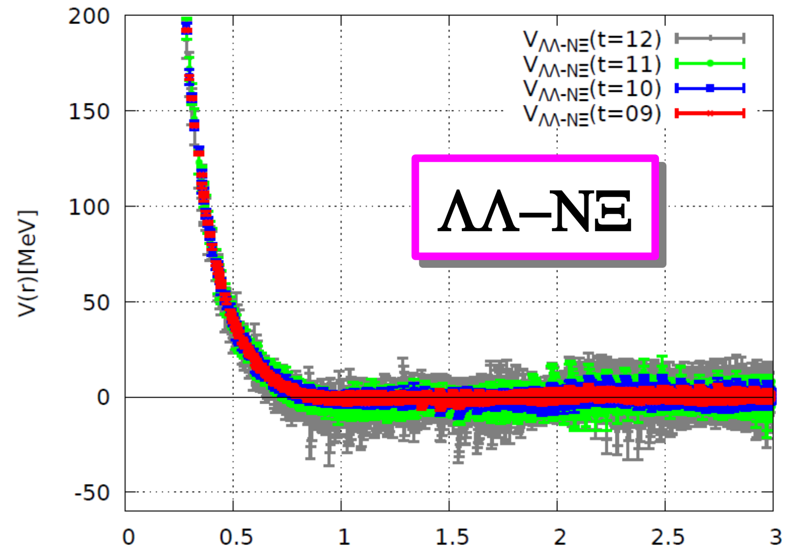


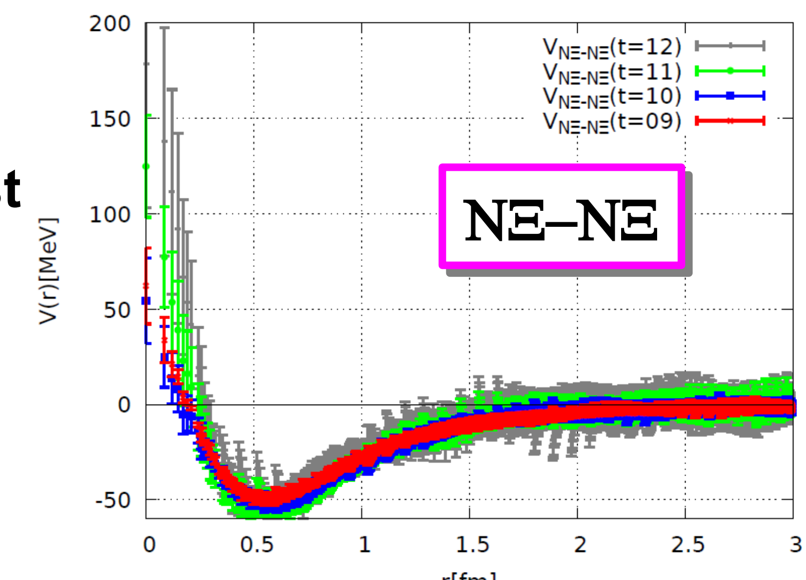
$\Lambda\Lambda$, $N\Xi$ (I=0) $^1S_0$ potential (2ch calc.)

- N_f = 2+1 full QCD with L = 8fm, ***m_π = 146 MeV***

Preliminary!







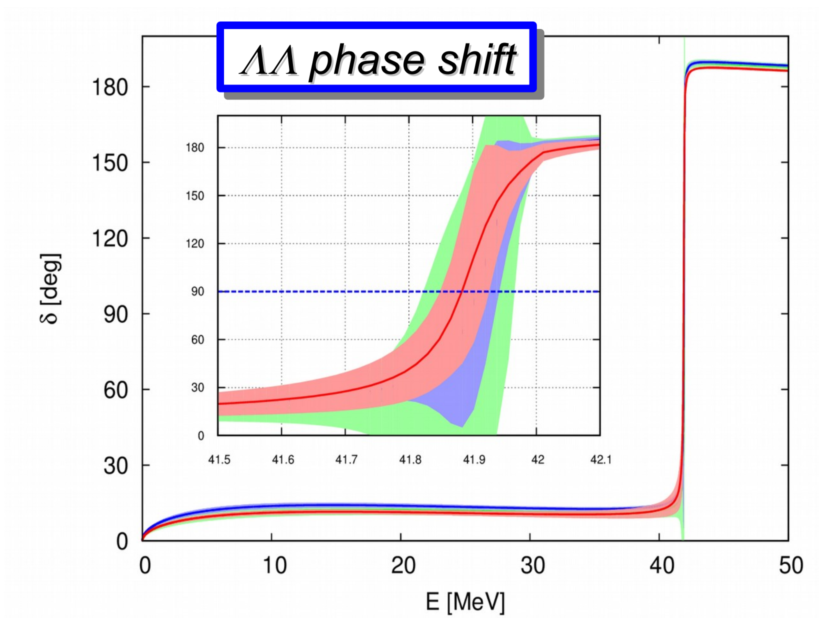
- **Potential calculated by only using $\Lambda\Lambda$ and $N\Xi$ channels.**
- **Long range part of potential is almost stable against the time slice.**
- **Short range part of $N\Xi$ potential changes as time t goes.**
- **$\Lambda\Lambda$–$N\Xi$ transition potential is quite small in r > 0.7fm region**

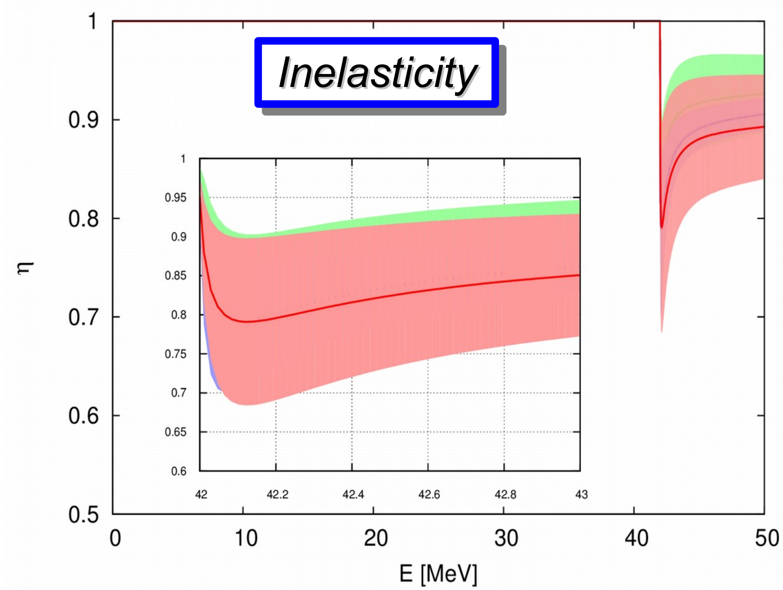
ΛΛ and NΞ phase shift and inelasticity

t=09
t=10
t=11

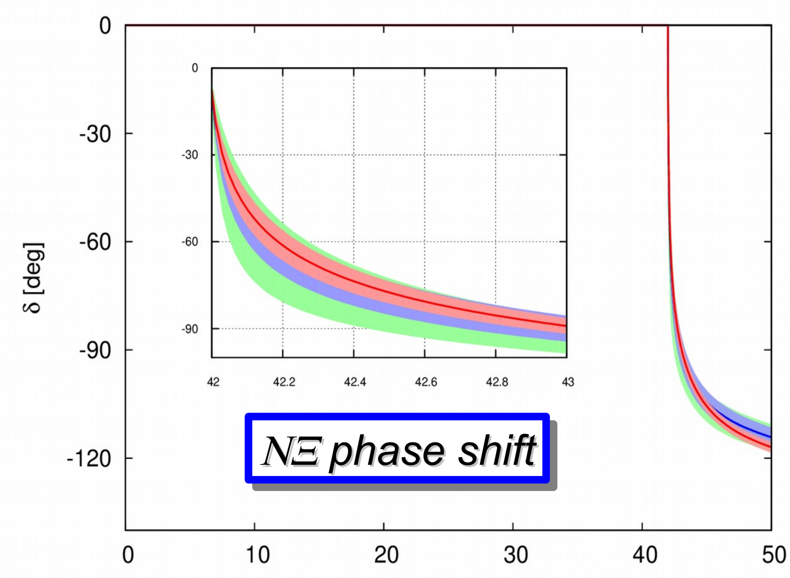
- N_f = 2+1 full QCD with L = 8fm, ***m_π = 146 MeV***

Preliminary!





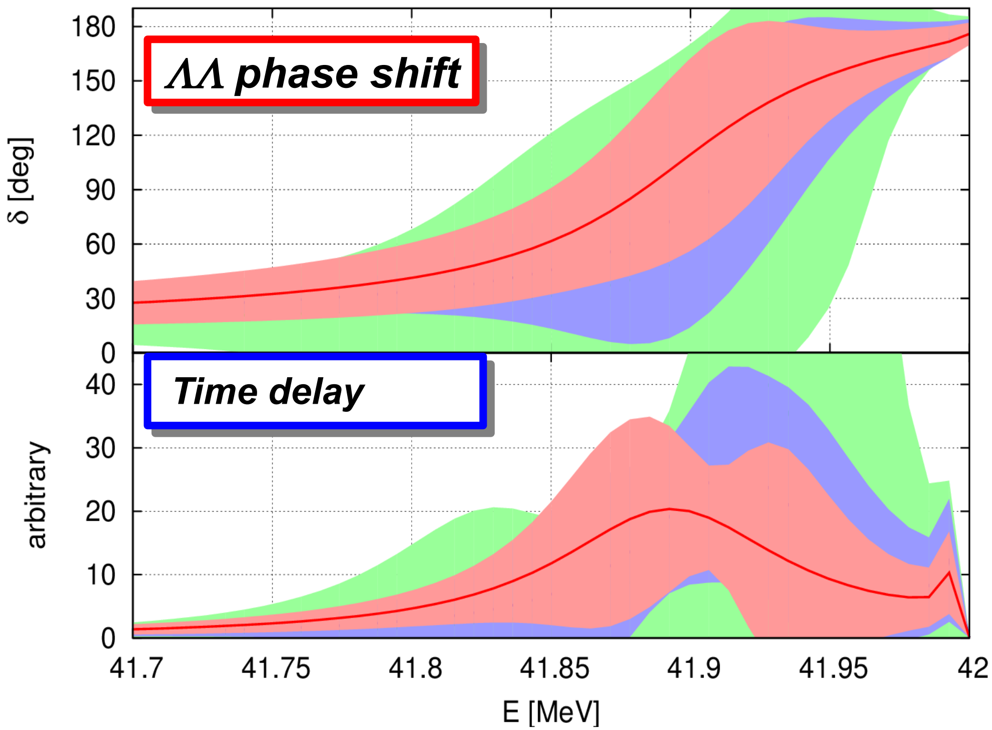
- **ΛΛ and NΞ phase shift is calculated**
  by using 2ch effective potential.
- **A sharp resonance is found**
  just below the NΞ threshold.
- **Inelasticity is small.**



Breit-Wigner mass and width

▶ N_f = 2+1 full QCD with L = 8fm, m_π = 146 MeV

Preliminary!



- **In the vicinity of resonance point,**

$$\delta(E) = \delta_B - \arctan\left(\frac{\Gamma/2}{E - E_r}\right)$$

thus

$$\frac{d\,\delta(E)}{d\,E} = \frac{\Gamma/2}{(E - E_r)^2 + (\Gamma/2)^2}$$

- **Fitting the time delay of ΛΛ scattering by the Breit-Wigner type finction,**

Resonance enargy and width

t=09

$$E_R - E_{\Lambda\Lambda} = 41.894 \pm 0.039 [MeV]$$
$$\Gamma = 0.099 \pm 0.059 [MeV]$$

t=10

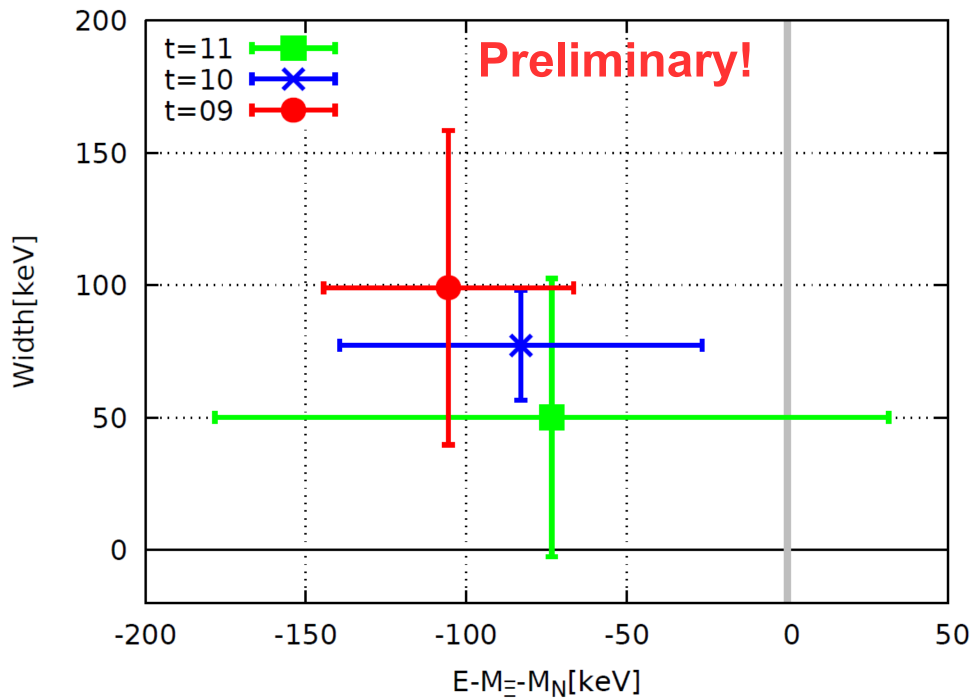
$$E_R - E_{\Lambda\Lambda} = 41.917 \pm 0.056 [MeV]$$
$$\Gamma = 0.077 \pm 0.021 [MeV]$$

t=11

$$E_R - E_{\Lambda\Lambda} = 41.927 \pm 0.105 [MeV]$$
$$\Gamma = 0.050 \pm 0.053 [MeV]$$

Summary

- H-dibaryon state is investigated using 414confs x 28src x 4rot.
- We perform ΛΛ–NΞ coupled channel calculation.
- Sharp resonance is found just below the NΞ threshold.
  - Resonance position and width from Breit-Wigner type fit



- We continue to study it by using higher statistical data.