Kenji Sasaki (YITP, Kyoto University) for HAL QCD Collaboration

**HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration**

<table>
<thead>
<tr>
<th>Name</th>
<th>Institution</th>
<th>Location</th>
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<tbody>
<tr>
<td>S. Aoki</td>
<td>YITP</td>
<td>(YITP)</td>
</tr>
<tr>
<td>T. Doi</td>
<td>RIKEN</td>
<td>(RIKEN)</td>
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<tr>
<td>F. Etminan</td>
<td>Birjand U.</td>
<td>(U. of Tours)</td>
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<td>S. Gongyo</td>
<td>RCNP</td>
<td>(RCNP)</td>
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<td>T. Hatsuda</td>
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<td>(RIKEN)</td>
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<td>Y. Ikeda</td>
<td>RCNP</td>
<td>(RCNP)</td>
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<td>T. Inoue</td>
<td>Nihon U.</td>
<td>(Nihon U.)</td>
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<td>N. Ishii</td>
<td>RCNP</td>
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<tr>
<td>T. Iritani</td>
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<td>D. Kawai</td>
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<td>T. Miyamoto</td>
<td>YITP</td>
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<td>K. Murano</td>
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<tr>
<td>H. Nemura</td>
<td>U. of Tsukuba</td>
<td>(U. of Tsukuba)</td>
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Kenji Sasaki (YITP, Kyoto University) for HAL QCD Collaboration
Introduction

BB interactions are crucial to investigate the nuclear phenomena

Once we obtain “proper” nuclear potentials, we apply them to the structure of (hyper-) nucleus.

How do we obtain the nuclear force?

Properties of nuclear potential
- State dependence (spin, isospin)
- Long range attraction
- Short range repulsion

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Derivation of hadronic interaction from QCD

Start with the fundamental theory, QCD

Lüscher's finite volume method
M. Lüscher, NPB354(1991)531
1. Measure the discrete energy spectrum, $E$
2. Put the $E$ into the formula which connects $E$ and $\delta$

HAL QCD method
Ishii, Aoki, Hatsuda, PRL99 (2007) 022001
1. Measure the NBS wave function, $\Psi$
2. Calculate potential, $V$, through Schrödinger eq.
3. Calculate observables by scattering theory

Guaranteed to be the same result
T. Kurth et al JHEP 1312 (2013) 015
T. Iritani (HAL QCD) Lattice2015

Scattering phase shift

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**HAL QCD method**

**NBS wave function**

\[
\Psi(E, \vec{r}) e^{-E(t-t_0)} = \sum_{\vec{x}} \langle 0 | B_i(t, \vec{x}+\vec{r}) B_j(t, \vec{x}) | E, t_0 \rangle
\]

- **E**: Total energy of system
- **In asymptotic region**: \((p^2 + \nabla^2)\Psi(E, \vec{r}) = 0\)
- **In interaction region**: \((p^2 + \nabla^2)\Psi(E, \vec{r}) = K(E, \vec{r})\)

\[
\Psi(E, \vec{r}) \approx A \frac{\sin(pr + \delta(E))}{pr}
\]

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

**Modified Schrödinger equation**

\[
\left(-\frac{\partial}{\partial t} + \frac{\nabla^2}{2 \mu}\right) R_{I_{B_1B_2}}^{B_1B_2}(t, \vec{r}) = \int U(\vec{r}, \vec{r}') R_{I_{B_1B_2}}^{B_1B_2}(t, \vec{r}) d^3 r'
\]


**Potential**

\[
V(\vec{r}) = \left(-\frac{\partial}{\partial t} + \frac{\nabla^2}{2 \mu}\right) \frac{R_{I_{B_1B_2}}^{B_1B_2}(t, \vec{r})}{R_{I_{B_1B_2}}^{B_1B_2}(t, \vec{r})}
\]


---

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HAL QCD method (coupled-channel)

NBS wave function

\[ \Psi^\alpha(E_i, \vec{r}) e^{-E_i t} = \langle 0 | (B_1 B_2)^\alpha(\vec{r}) | E_i \rangle \]
\[ \Psi^\beta(E_i, \vec{r}) e^{-E_i t} = \langle 0 | (B_1 B_2)^\beta(\vec{r}) | E_i \rangle \]
\[ \int dr \bar{\Psi}_\beta(E', \vec{r}) \Psi^\gamma(E, \vec{r}) = \delta(E' - E) \delta_\beta^\gamma \]
\[ R^B_{E_1 B_2}(t, \vec{r}) = \Psi_{B_1 B_2}(\vec{r}, E) e^{(-E + m_1 + m_2)t} \]

Leading order of velocity expansion and time-derivative method.

Modified coupled-channel Schrödinger equation

\[
\begin{pmatrix}
    -\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_\alpha} & \frac{\nabla^2}{2\mu_\beta} \\
    -\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_\alpha} & \frac{\nabla^2}{2\mu_\beta}
\end{pmatrix}
\begin{pmatrix}
    V^\alpha_{\alpha}(\vec{r}) & V^\alpha_{\beta}(\vec{r}) \Delta^\gamma_{\beta}(t) \\
    V^\beta_{\alpha}(\vec{r}) \Delta^\alpha_{\alpha}(t) & V^\beta_{\beta}(\vec{r})
\end{pmatrix}
\begin{pmatrix}
    \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_\alpha} & \frac{\nabla^2}{2\mu_\beta} \\
    -\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_\alpha} & \frac{\nabla^2}{2\mu_\beta}
\end{pmatrix}
\begin{pmatrix}
    R^\alpha_{E_0}(t, \vec{r}) & R^\alpha_{E_1}(t, \vec{r}) \\
    R^\beta_{E_0}(t, \vec{r}) & R^\beta_{E_1}(t, \vec{r})
\end{pmatrix}
\]

\[
\Delta^\alpha_{\beta} = \frac{\exp(-(m_\alpha + m_\alpha)t)}{\exp(-(m_\beta + m_\beta)t)}
\]

Considering two different energy eigen states

\[
\begin{pmatrix}
    V^\alpha_{\alpha}(\vec{r}) & V^\alpha_{\beta}(\vec{r}) \Delta^\ddagger_{\beta}(t) \\
    V^\beta_{\alpha}(\vec{r}) \Delta^\ddagger_{\alpha}(t) & V^\beta_{\beta}(\vec{r})
\end{pmatrix}
\begin{pmatrix}
    \frac{\nabla^2}{2\mu_\alpha} - \frac{\partial}{\partial t} & \frac{\nabla^2}{2\mu_\beta} - \frac{\partial}{\partial t} \\
    \frac{\nabla^2}{2\mu_\alpha} - \frac{\partial}{\partial t} & \frac{\nabla^2}{2\mu_\beta} - \frac{\partial}{\partial t}
\end{pmatrix}
\begin{pmatrix}
    \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_\alpha} & \frac{\nabla^2}{2\mu_\beta} \\
    -\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_\alpha} & \frac{\nabla^2}{2\mu_\beta}
\end{pmatrix}
\begin{pmatrix}
    R^\alpha_{E_0}(t, \vec{r}) & R^\alpha_{E_1}(t, \vec{r}) \\
    R^\beta_{E_0}(t, \vec{r}) & R^\beta_{E_1}(t, \vec{r})
\end{pmatrix}
\]

K. Sasaki et al [HAL QCD Collab.] PTEP no 11 (2015) 113B01

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Introduction

BB interactions are crucial to investigate (hyper-)nuclear structures

- Advantages for more strange quarks
- Signals getting worse as increasing the number of light quarks.
- Complementary role to experiment.

Lattice QCD simulation

Main topics of $S=-2$ multi baryon system

- H-dibaryon
  - R.L. Jaffe, PRL 38 (1977) 195
- Double-$\Lambda$ hypernuclei
  - K.Nakazawa et al, KEK-E176 Collaboration
- $\Xi$-hypernuclei
  - K.Nakazawa et al, KEK-E373 Collaboration

Experiment
**Baryon-baryon system with S=-2**

### Spin singlet states

<table>
<thead>
<tr>
<th>Isospin</th>
<th>BB channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>I=0</td>
<td>$\Lambda\Lambda$</td>
</tr>
<tr>
<td>I=1</td>
<td>$N\Xi$</td>
</tr>
<tr>
<td>I=2</td>
<td>$\Sigma\Sigma$</td>
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</table>

### Spin triplet states

<table>
<thead>
<tr>
<th>Isospin</th>
<th>BB channels</th>
</tr>
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<tbody>
<tr>
<td>I=0</td>
<td>$N\Xi$</td>
</tr>
<tr>
<td>I=1</td>
<td>$N\Xi$</td>
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</table>

### Relations between BB channels and SU(3) irreducible representations

$$8 \times 8 = 27 + 8_s + 1 + 10 + 10 + 8_a$$

#### $J^p=0^+, I=0$

$$\begin{pmatrix} \Lambda\Lambda \\ N\Xi \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -\sqrt{5} & -\sqrt{8} & \sqrt{27} \\ \sqrt{20} & \sqrt{8} & \sqrt{12} \\ \sqrt{15} & -\sqrt{24} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix}$$

#### $J^p=0^+, I=1$

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{5} \begin{pmatrix} \sqrt{2} & -\sqrt{3} & \sqrt{2} \\ \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & \sqrt{4} \end{pmatrix} \begin{pmatrix} 27 \\ 8 \end{pmatrix}$$

#### $J^p=0^+, I=2$

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{2} & -\sqrt{2} & \sqrt{2} \\ \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & \sqrt{4} \end{pmatrix} \begin{pmatrix} 8 \\ 10 \end{pmatrix}$$

### Features of flavor singlet interaction is integrated into the $S=-2$ $J^p=0^+, I=0$ system.
**Keys to understand H-dibaryon**

A strongly bound state predicted by Jaffe in 1977 using MIT bag model.

H-dibaryon state is
- SU(3) flavor singlet [uuddss], strangeness S=-2.
- Spin and isospin equals to zero, and $J^P = 0^+$

Strongly attractive interaction is expected in flavor singlet channel.
- Short range one-gluon exchange contributions
  - Strongly attractive Color Magnetic Interaction
- Symmetry of two-baryon system (Pauli principle)
  - Flavor singlet channel is free from Pauli blocking effect

<table>
<thead>
<tr>
<th>Pauli</th>
<th>27</th>
<th>8</th>
<th>1</th>
<th>10</th>
<th>10</th>
<th>8</th>
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<tbody>
<tr>
<td>mixed</td>
<td>forbidden</td>
<td>allowed</td>
<td>mixed</td>
<td>forbidden</td>
<td>mixed</td>
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</tr>
</tbody>
</table>

| CMI   | repulsive | repulsive | attractive | repulsive | repulsive | repulsive |

Oka, Shimizu and Yazaki NPA464 (1987)
Hunting for H-dibaryon in SU(3) limit

Strongly attractive interaction is expected in flavor singlet channel.

- Strongly attractive potential was found in the flavor singlet channel.
- Bound state was found in this mass range with SU(3) symmetry.

What happens at the physical point?

- SU(3) breaking effects
  - Threshold separation
  - Changes of interactions

Non-trivial contributions


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Works on H-dibaryon state

**Theoretical status**

Several sort of calculations and results (bag models, NRQM, Quenched LQCD….)

There were no conclusive result.

Chiral extrapolations of recent LQCD data

Unbound or resonance

**Experimental status**

"NAGARA Event"

K.Nakazawa et al KEK-E176 & E373 Coll.
PRL87(2001) 212502

- Deeply bound dibaryon state is ruled out

"$^{12}$C(K~,$K^+\Lambda\Lambda$) reaction"

C.J.Yoon et al KEK-PS E522 Coll.

PRC75(2007) 022201(R)

- Significance of enhancements below 30 MeV.

Larger statistics J-PARC E42

"Y(1S) and Y(2S) decays"

B.H. Kim et al Belle Coll.
PRL110(2013) 222002

- There is no sign of near threshold enhancement.

P. E. Shanahan et al
PRL 107(2011) 092004

Y.Yamaguchi and T.Hyodo
hep-ph:1607.04053

K. Nakazawa et al
KEK-E176 & E373 Coll.

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Numerical setup

- **2+1 flavor** gauge configurations.
  - Iwasaki gauge action & O(a) improved Wilson quark action
  - $a = 0.086 \, [fm]$, $a^{-1} = 2.300 \, GeV$.
  - $96^3 \times 96$ lattice, $L = 8.24 \, [fm]$.
  - 414 confs x 28 sources x 4 rotations.
- **Flat wall source** is considered to produce S-wave B-B state.

<table>
<thead>
<tr>
<th>Mass [MeV]</th>
<th>π</th>
<th>K</th>
<th>$m_{\pi}/m_K$</th>
<th>N</th>
<th>Λ</th>
<th>Σ</th>
<th>Ξ</th>
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<tbody>
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<tr>
<td>K</td>
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<tr>
<td>$m_{\pi}/m_K$</td>
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<tr>
<td>N</td>
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<tr>
<td>Λ</td>
<td>1121±4</td>
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<tr>
<td>Σ</td>
<td>1201±3</td>
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<tr>
<td>Ξ</td>
<td>1328±3</td>
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</tbody>
</table>

Masses in MeV:
- $\pi$: 146 MeV
- $K$: 525 MeV
- $m_{\pi}/m_K$: 0.28
- $N$: 956±12 MeV
- $\Lambda$: 1121±4 MeV
- $\Sigma$: 1201±3 MeV
- $\Xi$: 1328±3 MeV

- $l=0$ channel: 42 MeV
- $l=1$ channel: 118 MeV, 91 MeV, 27 MeV

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$\Lambda \Lambda$, $N \Xi \ (l=0)$ $^1S_0$ potential (2ch calc.)

- $N_f = 2+1$ full QCD with $L = 8\text{fm}$, $m_\pi = 146 \text{ MeV}$

- Potential calculated by only using $\Lambda \Lambda$ and $N \Xi$ channels.
- Long range part of potential is almost stable against the time slice.
- Short range part of $N \Xi$ potential changes as time $t$ goes.
- $\Lambda \Lambda$–$N \Xi$ transition potential is quite small in $r > 0.7\text{fm}$ region

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$\Lambda\Lambda$ and $N\Xi$ phase shift and inelasticity

$N_f = 2+1$ full QCD with $L = 8\text{fm}$, $m_\pi = 146 \text{ MeV}$

- $\Lambda\Lambda$ and $N\Xi$ phase shift is calculated by using 2ch effective potential.
- A sharp resonance is found just below the $N\Xi$ threshold.
- Inelasticity is small.

Preliminary!

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In the vicinity of resonance point,

\[ \delta(E) = \delta_B - \arctan\left( \frac{\Gamma/2}{E - E_r} \right) \]

thus

\[ \frac{d \delta(E)}{dE} = \frac{\Gamma/2}{(E - E_r)^2 + (\Gamma/2)^2} \]
• H-dibaryon state is investigated using 414confs x 28src x 4rot.

• We perform $\Lambda\Lambda - N\Xi$ coupled channel calculation.

• Sharp resonance is found just below the $N\Xi$ threshold.

• Resonance position and width from Breit-Wigner type fit

• We continue to study it by using higher statistical data.