Hyperons in nuclear matter with YN and YNN interactions of ChEFT

Michio Kohno (RCNP, Osaka University)

- Use of NN interactions parametrized by ChEFT is now standard for ab initio calculations of nuclear structures.
 - > Low-energy effective theory of spontaneously broken chiral symmetry of QCD.
 - > 3NFs are introduced systematically and consistently with 2NFs.
- Progress also in strangeness sector; at present NLO.
 - > Experimental data of elementary processes is insufficient.
- It is important to carry out hyper nuclear physics on the basis of YN and YNN interactions of ChEFT.
 - > Role of YNN interactions. (3NFs are decisively important in nuclear physics.)
 - > Reason of the existence or non-existence of hyperons in neutron star matter.
 - $\succ\,$ Negative information from recent observation of $2m_{\odot}$ neutron stars.
- Investigate properties of present NLO YN and YNN interactions developed by Bonn-Jülich-München group, by calculating hyperon properties in nuclear matter.

Nuclear physics on the basis of ChEFT

- 3NF is introduced systematically and consistently with 2NF
 - Previous 3NF is largely phenomenological.
 - 3NFs in ChEFT consist of the components determined by the parameters in 2NFs and the new contact terms.
- Basic properties of nuclei are reproduced by adjusting two parameters (c_D and c_E) without relying on much phenomenology.
 - Few-body systems and saturation properties (saturation curve)
 - Strength of one-body spin-orbit field essential for nuclear shell structure
 - Enhancement of tensor force (explain the neutron drip-limit of O isotopes)
- ChEFT NN and 3NF are standard for use in ab initio studies of nuclear structures: (CCM, no-core shell model, Monte-Carlo, ...)
 - In place of modern NN potentials such as AV18, CD-Bonn, Nijmegen
- Studies in strangeness sector are in progress (Bonn-Jülich-München group)

Baryon-baryon interactions in chiral effective field theory

- Starting from general Lagrangian written in terms nucleons and pions which satisfies chiral symmetry (of QCD), and expanding it with respect to momentum (power counting) (low energy effective theory)
- Construction of NN potential (elimination of pions by unitary-transformation or calculation of Feynman diagrams)
 - Coupling constants are determined by $\pi\pi$, πN , and NN scattering data and few nucleon systems.
 - E. Epelbaum, H.-W. Hammer and U.-G. Meißner, Rev. Mod. Phys.81 1773 (2009)
 - R. Machleid and D.R. Entem, Phys. Rep. 503,1 (2011)
- Actual scattering and bound states cannot be treated in perturbation:
 Lippmann-Schwinger or Schroedinger eq.
- Renormalization in Feynman diagrams and reguralization in L-S equation
 - LS eq. \mathcal{O} regulator function $f(\Lambda) = \exp(-({p'}^4 + p^4)/\Lambda^4)$

cutoff scale $\Lambda = 400 - 600$ MeV

NLO diagrams (π , K, and η exchanges in SU(3))



LOBT calculations with NN+"3NF" of ChEFT

Tune c_D and c_E .

 C_D

- Calculated saturation curves with 3 choices of cutoff Λ .
 - Results of $c_D = 0$ and $c_E = 0$.
 - Pauli effects are sizable.



It is not easy to explain nuclear matter properties with parameters fitted in few-body systems. There are attempts to readjust parameters to fit simultaneously finite nuclei and nuclear matter at the NNLO level [NNLO_{sat}, Ekström et al., Phys.Rev. C91 (2015) 051301.].





- Expand them into partial waves, add them to NN and carry out G matrix calculation.
- > (*) A factor of $\frac{1}{3}$ is necessary for the calculation of energy at the HF level.

Effective two-body interaction and partial wave expansion

Integrating over the third nucleon in nuclear matter $(|\mathbf{k}_3| \le k_F, \sigma_3, \tau_3)$. Example: c_1 term of the Ch-EFT NNLO 3NF.

$$\begin{split} \left\langle \mathbf{k}' \sigma_{1}' \tau_{1}', -\mathbf{k}' \sigma_{2}' \tau_{2}' \Big| V_{12(3)}^{c_{1}} \Big| \mathbf{k} \sigma_{1} \tau_{1}, -\mathbf{k} \sigma_{2} \tau_{2} \right\rangle \\ &= - \frac{c_{1} g_{A}^{2} m_{\pi}^{2}}{f_{\pi}^{4}} \sum_{\mathbf{k}_{3}, \sigma_{3}, \tau_{3}} \langle \mathbf{k}' \sigma_{1}' \tau_{1}', -\mathbf{k}' \sigma_{2}' \tau_{2}', \mathbf{k}_{3} \sigma_{3} \tau_{3} \Big| \left\{ \frac{(\sigma_{1} \cdot \mathbf{q}_{1})(\sigma_{2} \cdot \mathbf{q}_{2})}{(\mathbf{q}_{1}^{2} + m_{\pi}^{2})(\mathbf{q}_{2}^{2} + m_{\pi}^{2})} (\tau_{1} \cdot \tau_{2}) \right. \\ &+ \frac{(\sigma_{1} \cdot \mathbf{q}_{1})(\sigma_{3} \cdot \mathbf{q}_{3})}{(\mathbf{q}_{1}^{2} + m_{\pi}^{2})(\mathbf{q}_{2}^{2} + m_{\pi}^{2})} (\tau_{1} \cdot \tau_{3}) + \frac{(\sigma_{2} \cdot \mathbf{q}_{2})(\sigma_{3} \cdot \mathbf{q}_{3})}{(\mathbf{q}_{2}^{2} + m_{\pi}^{2})(\mathbf{q}_{2}^{2} + m_{\pi}^{2})} (\tau_{2} \cdot \tau_{3}) \right\} \\ &+ \left[\left[\mathbf{k} \sigma_{1} \tau_{1}, -\mathbf{k} \sigma_{2} \tau_{2} \right]_{a}, \mathbf{k}_{3} \sigma_{3} \tau_{3} + \left[-\mathbf{k} \sigma_{2} \tau_{2} \right]_{a}, \mathbf{k}_{3} \sigma_{3} \tau_{3}, \left[\mathbf{k} \sigma_{1} \tau_{1} \right]_{a} + \mathbf{k}_{3} \sigma_{3} \tau_{3}, \left[\mathbf{k} \sigma_{1} \tau_{1}, -\mathbf{k} \sigma_{2} \tau_{2} \right]_{a} \right\rangle \end{split}$$

Spin and isospin summations

two-body central, spin-orbit, and tensor components

- **k**₃-integration for general case of $k' \neq k$
- expand the result into partial waves

• Then, form factor in the form of
$$\exp\left\{-\left(\frac{k'}{\Lambda}\right)^6 - \left(\frac{k}{\Lambda}\right)^6\right\}$$
 is multiplied

Neutron matter

- EoS of neutron matter: basic to theoretical studies of neutron star.
 - EoS of APR, including phenomenological 3NFs, has been standard.
 - Necessity of the repulsive contributions from 3NF.



- Dependence on different twobody NN interactions is small, because of the absence of tensor effects in the ³E state.
- The contribution of ChEFT 3NFs (no c_D and c_E terms) is similar to the standard phenomenological one by APR.
 - ChEFT is not applicable to the high-density region of $\rho > 2\rho_0$.

APR: Akmal, Pandharipande, and Ravenhall, PRC58, 1804 (1998)



 To construct instantaneous interaction between two nucleons, meson degrees of freedom is eliminated by some unitary transformation. [Okubo, PTP64 (1958)]

π

π

- The unitary tf. $e^{s_{12}}$ should satisfy a decoupling condition $\langle Q | \tilde{H} | P \rangle = \langle P | \tilde{H} | Q \rangle = 0$ in two-body space.
- (Induced) Many-body forces appear
 in many-nucleon space. Typical example: Fujita-Miyazawa type.

Equivalent interaction in restricted (low-mom.) space



- Apply some unitary transformation e^{S₁₂} to H to obtain an equivalent Hamiltonian H̃ in a restricted (P) space
 [Suzuki and Lee, PTP64 (1980)]
- The unitary tf. $e^{s_{12}}$ should satisfy a decoupling condition $\langle Q | \tilde{H} | P \rangle = \langle P | \tilde{H} | Q \rangle = 0$ in two-body space (block diagonal).

Singular high-momentum components are eliminated.

Eigenvalues, namely on-shell properties, in the restricted (P) space do not change.

> Off-shell properties naturally change.

Induced many-body forces appear in many-body space.

Example of induced 3NF in SRG method

- Red curve: calculation with low-momentum interaction by SRG method
- When induced 3NF is included, the exact energy is recovered (black curve).
 - > λ represents the scale of low-momentum space.
 - > To reproduce experimental values, genuine 3NF is needed (attractive).



E.D. Jurgenson, P. Navratil, and R.J. Furnstahl, Phys. Rev. C83, 034301 (2011) "Evolving nuclear many-body forces with the similarity renormalization group"

Summary of effects of 3NF in chiral effective field theory

- Phenomenological adjustment is minimal.
- 3NFs have to be expected as induced effective interaction when other degrees of freedom than nucleons are eliminated in many-nucleon space.
- The repulsive contribution is essential, at least semi-quantitatively, to explain basic saturation properties.
- In addition:
 - Enhancement of spin-orbit strength of the nuclear mean field.
 - Enhancement of tensor component: due to the suppression of the reduction of the tensor force from NNLO 2-π exchange.
 - > E.g., the neutron drip limit is reproduced in oxygen isotopes.
- Subjects to be studied: 3BF contributions in the strangeness sector; hyper nuclei and neutron star matter.

Strangeness sector: YN and YNN interaction in ChEFT

- Parametrization by Bonn-Jülich-München
- Lowest order:
 - Polinder, Haidenbauer, and Meißner,
 Nucl. Phys. A779, 244 (2006)
 - > Parameters: $f_{NN\pi} = \frac{2f_{\pi}}{g_A}$ and $\alpha = \frac{F}{F+D}$, also five low-energy constants:

 $C_{1S0}^{\Lambda\Lambda}, C_{3S1}^{\Lambda\Lambda}, C_{1S0}^{\Sigma\Sigma}, C_{3S1}^{\Sigma\Sigma}, C_{3S1}^{\Lambda\Sigma}$

- Next-to-Leading order
 - Haidenbauer, Petschauer, Kaiser, Meißner, Nogga, and Weise, Nucl. Phys. A915, 24 (2013)
- Leading three-baryon forces
 - Petschauer, Kaiser, Haidenbauer, Meißner, and Weise, Phys. Rev. C93, 014001 (2016)
 - > Estimation of 1π and contact LEC: decouplet dominance



Hyperon s.p. potentials in nuclear matter

- First, hyperon (Λ , Σ and Ξ) single-particle potentials are calculated in nuclear matter using Ch-NLO YN interactions only.
 - cutoff scale of 400~600 MeV is still hard to treat the interaction by HF method.

Standard lowest-order Brueckner method to take care of high momentum part with the continuous choice for intermediate spectra.

> Salient features, in comparison with other previous YN interactions.

- Strong ΛN - ΣN coupling, which generates ΛN attraction
 - Charge symmetry breaking in few-body systems may be accounted for.
- Λ s.p. potential does not become deeper at high densities.

Λ and $~\Sigma$ s.p. potentials from G-matrices in symmetric nuclear matter

- The ΛN attraction (experimentally, s.p. potential depth is around 30 MeV) comes from ΛN -ΣN coupling through tensor force.
- The coupling effect is large in ChNLO potential.



Cf: Results of Nijmegen and quak model (fss2) potentials



Calculations at other densities of $k_F = 1.07, 1.50$ and 1,60 fm⁻¹

NLO ChEFT The Λ potential stays shallow at higher densities.



Nijmegen



Density-dependence of the potential of the Λ hyperon at rest



- Shallowness of the potential of the Λ hyperon at rest is due to behavior of the ³S₁ contribution.
- Qualitatively same results with those of Haidenbauer, Meißner, Keiser, and Weise [arXiv:1612.03758, EPJA in print].

Comparison with the results from Nijmegen 97f potential



- Qualitative resemblance between ChEFT and NSC97F.
 - Note that NSC97f predicts attractive Σ potential; inconsistent with experimental data.

comment

- The tensor component from one-pion exchange process in the ΛN-ΣN coupling at the NLO may be overestimated.
 - In the NN case, it is known that 2π exchange NNLO diagram (a) reduces the strong one-pion exchange tensor force, which corresponds to the role of the ρ meson in the OBEP picture.

This is related to the later observation that the effect of ΛNN- ΣNN
 3BF coupling enhances the tensor component.

$$N \xrightarrow{\Sigma} N \xrightarrow{N} Occupied state (Pauli blocking)$$

Inclusion of YNN interactions

- Next, introducing normal-ordering for YNN force in the 2BF level, LOBT calculations are carried out.
 - Petschauer et al., "Leading three-baryon force from SU(3) chiral effective filed theory", Phys. Rev. C93, 014001 (2016)



 YNN 3BF(a) is reduced to density-dependent 2BF(c) in nuclear matter (normal-ordering). Solve G-matrix equations, with 2BF + DD 2BF, to obtain Λ s.p. potential.

Effects of YNN interactions in ChEFT

 Pauli blocking type contribution (c) suppresses the attraction in free space, namely the repulsive contribution. The suppression becomes become larger at higher density, like in the NNN case.



- Contributions of 1πexchange 3BF can be attractive contribution, depending the sign of the coupling constant.
- Effects from ΛNN-ΣNN coupling can be attractive by the enhancement of the tensor component.

Contributions of 2-pion exchange ΛNN interaction

 $2\pi \text{ exchange } \Lambda \text{NN 3-body interaction } V_{\Lambda NN}^{TPE} \qquad \begin{array}{c} & 5 & 4 & 6 \\ & & \\ N & & \frac{g_A^2}{3f_0^2} (\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3) \frac{(\boldsymbol{\sigma}_3 \cdot \boldsymbol{q}_{63})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}_{52})}{(\boldsymbol{q}_{63}^2 + m_\pi^2)(\boldsymbol{q}_{52}^2 + m_\pi^2)} \{-Am_\pi^2 + B\boldsymbol{q}_{63} \cdot \boldsymbol{q}_{52}\} \qquad \begin{array}{c} & & \\ & &$

Estimation by Petschauer: A = 0, $B = -3.0 \text{ GeV}^{-1}$

Normal-ordered 2-body int. in symmetric nuclear matter

$$\langle \mathbf{k}' \sigma_{\Lambda}', -\mathbf{k}' \sigma' \tau' | V_{\Lambda N(N)} | \mathbf{k} \sigma_{\Lambda}, -\mathbf{k} \sigma \tau \rangle$$

$$\equiv \frac{1}{2} \sum_{\mathbf{k}_{h}, \sigma_{h}, \tau_{h}} \langle \mathbf{k}' \sigma_{\Lambda}', -\mathbf{k}' \sigma' \tau', \mathbf{k}_{h}, \sigma_{h}, \tau_{h} | V_{\Lambda NN} | \mathbf{k} \sigma_{\Lambda}, -\mathbf{k} \sigma \tau, \mathbf{k}_{h}, \sigma_{h}, \tau_{h} \rangle_{A}$$

- Density-dependent central, LS, ALS 成分
- No tensor component
- Additional statistical factor $\frac{1}{2}$

- Repulsive character of Pauli blocking type
- Partial waves expansion \rightarrow G-matrix calculation

HF contribution of 2π exchange ANN force to A potential



- ChEFT may not be relevant to $\rho > 2\rho_0$.
- Contributions from contact terms have also to be considered.

$$V_{\Lambda NN}^{TPE} = \frac{g_A^2}{3f_0^4} (\mathbf{\tau}_2 \cdot \mathbf{\tau}_3) \frac{(\mathbf{\sigma}_3 \cdot \mathbf{q}_{63})(\mathbf{\sigma}_2 \cdot \mathbf{q}_{52})}{(\mathbf{q}_{63}^2 + m_\pi^2)(\mathbf{q}_{63}^2 + m_\pi^2)} \times \{-(3b_0 + b_D)m_\pi^2 + (2b_2 + 3b_4)\mathbf{q}_{63} \cdot \mathbf{q}_{52}\}$$

$$N = \frac{1}{2} \sum_{k=1}^{N} \langle \mathbf{k}_{\Lambda} \mathbf{k}_{2k} \mathbf{k}_{3} | V_{\Lambda NN}^{TPE} | \mathbf{k}_{\Lambda} \mathbf{k}_{2k} \mathbf{k}_{3} \rangle_{A}$$

$$= \frac{g_A^2}{3f_0^4} \frac{1}{(2\pi)^6} \int_0^{k_F} q^2 dq \frac{64\pi^2}{3} (k_F - q)^2 \times (2k_F + q) \frac{4q^2}{(4q^2 + m_\pi^2)^2} \times \{-(3b_0 + b_D)m_\pi^2 + (2b_2 + 3b_4)q^2\}$$

HF contribution of 2π and 1π exchange ANN forces to A potential



$$V_{\Lambda NN}^{TPE} = \frac{g_A^2}{3f_0^4} (\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3) \frac{(\boldsymbol{\sigma}_3 \cdot \boldsymbol{q}_{63})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}_{52})}{(\boldsymbol{q}_{63}^2 + m_\pi^2)(\boldsymbol{q}_{63}^2 + m_\pi^2)} \times \{-(3b_0 + b_D)m_\pi^2 + (2b_2 + 3b_4)\boldsymbol{q}_{63} \cdot \boldsymbol{q}_{52}\}$$



 $V_{OPE}^{\Lambda NN} = \frac{1}{2f_0^4} \frac{(\sigma_2 \cdot q_{52})}{(q_{52}^2 + m_\pi^2)}$ $\times \{N_1 \sigma_3 \cdot q_{52} + N_2 i(\sigma_1 \times \sigma_3) \cdot q_{52}\}$ $5 \quad 4 \quad 6$



Contributions of 2π exchange ANN force in G-matrix calculations





- Absolute values of the are reduced by the correlation from the G-matrix equation.
- Contributions below the normal density are not large, though depending on the sign of the 1-pion exchange contact term.
- = Effects of the $\Lambda NN-\Sigma NN$ coupling are to be included.

Normal-ordered 2body force from ΛNN - ΣNN coupling interaction



- Again, normal-ordered 2NF is introduced.
- As in the case of 3NFs
 - Contributions of (d) and (e) diagrams cancel each other.
 - The diagram (c) enhances the tensor component.

Λ s.p. potential in symmetric nuclear matter including 3BFs

- $k_F = 1.07, 1.35 \text{ and } 1,60 \text{ fm}^{-1}$ ($\rho = \frac{1}{2}\rho_0, \rho_{0}$ and $1.66\rho_0$, respectively)
 - > 2π -exchange Λ NN interaction
 - > 2π -exchange Λ NN- Σ NN interaction



Λ s.p. potential in symmetric nuclear matter including 3BFs

- $k_F = 1.07, 1.35 \text{ and } 1,60 \text{ fm}^{-1}$ ($\rho = \frac{1}{2}\rho_0, \rho_{0}$ and $1.66\rho_0$, respectively)
 - > 2π -exchange Λ NN interaction
 - > 2π -exchange ANN- Σ NN interaction
 - > Contact + attractive 1π -exchange ANN interaction



Possibility to resolve hyperon puzzle

• A naive condition for the Λ hyperon to appear in pure neutron matter. $U_{\Lambda}(0) < \frac{\hbar^2}{2m_n} k_{F_n}^2 + m_n - m_{\Lambda} + U_n(k_{F_n})$



- Λ s.p. potential predicted by NLO YN potential does not become deep at high densities.
 - ► $U_{\Lambda}(0) > -30 \text{ MeV}$
 - The attractive effect from ΛNN-ΣNN coupling does not change this feature qualitatively.

• $U_{\Lambda}(0)$ is always above $\frac{\hbar^2}{2m_n} \times k_{F_n}^2 + m_n - m_{\Lambda} + U_n(k_{F_n})$; namely, Λ hyperon does not appear in the high density medium.

(Note: Actual calculations in neutron matter are in progress.)

Ξ s.p. potential in symmetric nuclear matter



- Coupling constants in S = -2 sector are largely uncertain.
 - Constraint from the non-existence of H and bound EN state.
- Various baryon-channel coupling ($\Xi N-\Lambda\Lambda-\Sigma\Sigma$ (T=0), $\Xi N-\Lambda\Sigma-\Sigma\Sigma$ (T=1)) state and density dependences
- Ξ-nucleus potential from ChEFT NLO interaction may be attractive but shallow at the surface region.
- Recent experimental data suggest some weakly bound Ξ-nucleus states.
 - > Kiso event: $\Xi^{-14}N$ (Nakazawa et al.)
 - ¹²C(K⁻,K⁺)X spectra at 1.8 GeV/c (Nagae et al.), which showed some strength below the threshold, though not conclusive until the future experiment.
- > Effects from 3BFs should be included.

Summary

- General remarks for 3-body interaction
- LOBT G-matrix calculations to obtain hyperon single-particle potentials in (symmetric) nuclear matter, using Ch-EFT NLO interactions in the strangeness sector developed by the Bonn-Jülich-München group.
 - Strong ΛΝ-ΣΝ coupling
 - A s.p. potential does not become deeper at higher densities.
 - > A is energetically unfavorable in high density matter. Does hyperon puzzle dissolve?
 - Contributions of 3BF do not change the situation.
 - > ANN 3BF gives repulsive contribution, while the effect from ANN- ΣNN is attractive.
- Subjects to be studied in the future
 - > NNLO
 - New experiments to reduce uncertainties of coupling constants and low energy constants.
 - > Consideration of diagrams including K and η exchanges.
 - > Explicit (ab initio) calculations of finite hypernuclei.