Use of NN interactions parametrized by ChEFT is now standard for ab initio calculations of nuclear structures.

- Low-energy effective theory of spontaneously broken chiral symmetry of QCD.
- 3NFs are introduced systematically and consistently with 2NFs.

Progress also in strangeness sector; at present NLO.

- Experimental data of elementary processes is insufficient.

It is important to carry out hyper nuclear physics on the basis of YN and YNN interactions of ChEFT.

- Role of YNN interactions. (3NFs are decisively important in nuclear physics.)
- Reason of the existence or non-existence of hyperons in neutron star matter.
  - Negative information from recent observation of $2m_{\odot}$ neutron stars.

Investigate properties of present NLO YN and YNN interactions developed by Bonn-Jülich-München group, by calculating hyperon properties in nuclear matter.
Nuclear physics on the basis of ChEFT

- 3NF is introduced systematically and consistently with 2NF
  - Previous 3NF is largely phenomenological.
  - 3NFs in ChEFT consist of the components determined by the parameters in 2NFs and the new contact terms.

- Basic properties of nuclei are reproduced by adjusting two parameters ($c_D$ and $c_E$) without relying on much phenomenology.
  - Few-body systems and saturation properties (saturation curve)
  - Strength of one-body spin-orbit field essential for nuclear shell structure
  - Enhancement of tensor force (explain the neutron drip-limit of O isotopes)

- ChEFT NN and 3NF are standard for use in ab initio studies of nuclear structures: (CCM, no-core shell model, Monte-Carlo, ...)
  - In place of modern NN potentials such as AV18, CD-Bonn, Nijmegen

- Studies in strangeness sector are in progress (Bonn-Jülich-München group)
Baryon-baryon interactions in chiral effective field theory

- Starting from general Lagrangian written in terms nucleons and pions which satisfies chiral symmetry (of QCD), and expanding it with respect to momentum (power counting) (low energy effective theory)
  - Construction of NN potential (elimination of pions by unitary-transformation or calculation of Feynman diagrams)
    - Coupling constants are determined by $\pi \pi$, $\pi N$, and $NN$ scattering data and few nucleon systems.

- Actual scattering and bound states cannot be treated in perturbation: Lippmann-Schwinger or Schroedinger eq.

- Renormalization in Feynman diagrams and regularization in L-S equation
  - LS eq. $\mathcal{D}$ regulator function $f(\Lambda) = \exp\left(-\frac{p^4 + p'^4}{\Lambda^4}\right)$
    - cutoff scale $\Lambda = 400 - 600$ MeV

- NLO diagrams ($\pi$, $K$, and $\eta$ exchanges in SU(3) )

\[ \text{Diagram images} \]
LOBT calculations with NN+”3NF” of ChEFT

- Calculated saturation curves with 3 choices of cutoff $\Lambda$.
  - Results of $c_D = 0$ and $c_E = 0$.
  - Pauli effects are sizable.
  - Tune $c_D$ and $c_E$.

- It is not easy to explain nuclear matter properties with parameters fitted in few-body systems. There are attempts to readjust parameters to fit simultaneously finite nuclei and nuclear matter at the NNLO level [NNLO$_{\text{sat}}$, Ekström et al., Phys.Rev. C91 (2015) 051301].
NNLO 3NFs in Ch-EFT

\[ V_{3N}^{(2\pi)} = \sum_{i \neq j \neq k} \frac{g_A^2}{8 f_{\pi}^4} \frac{(\sigma_i \cdot q_i)(\sigma_j \cdot q_j)}{(q_i^2 + m_{\pi}^2)(q_j^2 + m_{\pi}^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta \]

\[ F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta} \left[ -4c_1 m_{\pi}^2 + 2c_3 q_i \cdot q_j \right] + c_4 \epsilon^{\alpha\beta\gamma} \tau_k^\gamma \sigma_k \cdot (q_i \times q_j) \]

\[ V_{3N}^{(1\pi)} = -\sum_{i \neq j \neq k} \frac{g_A c_D}{8 f_{\pi}^4 \Lambda_{\chi}} \frac{\sigma_j \cdot q_j}{(q_j^2 + m_{\pi}^2)} \sigma_i \cdot q_j \tau_i \cdot \tau_j \]

\[ V_{3N}^{(ct)} = \sum_{i \neq j \neq k} \frac{c_E}{2 f_{\pi}^4 \Lambda_{\chi}} \tau_i \cdot \tau_j \]

\[ q_i = p_i - p_i, \quad \Lambda_{\chi} = 700 \text{ MeV} \]

\[ c_1 = -0.81 \text{ GeV}^{-1}, \quad c_3 = -3.4 \text{ GeV}^{-1}, \quad \text{and} \quad c_4 = 3.4 \text{ GeV}^{-1} \]

are fixed in NN sector. \( c_D \) and \( c_E \) are to be determined in many-body systems.
Reduction of 3NF $v_{123}$ to density dependent NN $v_{12(3)}$

Normal-ordered 2NF from 3NF with respect to nuclear matter:

$$\langle ab|v_{12(3)}|cd\rangle_A \equiv \sum_h \langle abh|v_{123}|cdh\rangle_A$$

Diagrammatical representation by Holt et al.

- Contributions from the two left diagrams tend to cancel.
- This diagram partly corresponds to the Pauli blocking of the isobar $\Delta$ excitation in a conventional picture.
- Expand them into partial waves, add them to NN and carry out G matrix calculation.

(*) A factor of $\frac{1}{3}$ is necessary for the calculation of energy at the HF level.
Integrating over the third nucleon in nuclear matter ($|\mathbf{k}_3| \leq k_F, \sigma_3, \tau_3$).

Example: $c_1$ term of the Ch-EFT NNLO 3NF.

$$\left\langle k' \sigma'_1 \tau'_1, -k' \sigma'_2 \tau'_2 \left| V_{12(3)}^{c_1} \right| k \sigma_1 \tau_1, -k \sigma_2 \tau_2 \right\rangle$$

$$= -\frac{c_1 g_A^2 m^2}{f^4} \sum_{k_3, \sigma_3, \tau_3} \left\langle k' \sigma'_1 \tau'_1, -k' \sigma'_2 \tau'_2, k_3 \sigma_3 \tau_3 \left| \frac{(\sigma_1 \cdot q_1)(\sigma_2 \cdot q_2)}{(q_1^2 + m_\pi^2)(q_2^2 + m_\pi^2)} (\tau_1 \cdot \tau_2) + \frac{(\sigma_2 \cdot q_2)(\sigma_3 \cdot q_3)}{(q_2^2 + m_\pi^2)(q_3^2 + m_\pi^2)} (\tau_2 \cdot \tau_3) \right\rangle$$

$$\left| [k \sigma_1 \tau_1, -k \sigma_2 \tau_2]_a, k_3 \sigma_3 \tau_3 + [-k \sigma_2 \tau_2]_a, k_3 \sigma_3 \tau_3, [k \sigma_1 \tau_1]_a + k_3 \sigma_3 \tau_3, [k \sigma_1 \tau_1, -k \sigma_2 \tau_2]_a \right\rangle$$

- Spin and isospin summations
  - two-body central, spin-orbit, and tensor components

- $k_3$-integration for general case of $k' \neq k$

- expand the result into partial waves

- Then, form factor in the form of $\exp \left\{ - \left( \frac{k'}{\Lambda} \right)^6 - \left( \frac{k}{\Lambda} \right)^6 \right\}$ is multiplied
Neutron matter

- EoS of neutron matter: basic to theoretical studies of neutron star.
  - EoS of APR, including phenomenological 3NFs, has been standard.
    - Necessity of the repulsive contributions from 3NF.

- Dependence on different two-body NN interactions is small, because of the absence of tensor effects in the $^3$E state.
- The contribution of ChEFT 3NFs (no $c_D$ and $c_E$ terms) is similar to the standard phenomenological one by APR.
  - ChEFT is not applicable to the high-density region of $\rho > 2\rho_0$.

To construct instantaneous interaction between two nucleons, meson degrees of freedom is eliminated by some unitary transformation. [Okubo, PTP64 (1958)]

The unitary tf. $e^{S_{12}}$ should satisfy a decoupling condition $\langle Q | \tilde{H} | P \rangle = \langle P | \tilde{H} | Q \rangle = 0$ in two-body space.

(Induced) Many-body forces appear in many-nucleon space. Typical example: Fujita-Miyazawa type.
The unitary tf. $e^{S_{12}}$ should satisfy a decoupling condition $\langle Q | \tilde{H} | P \rangle = \langle P | \tilde{H} | Q \rangle = 0$ in two-body space (block diagonal).

- Singular high-momentum components are eliminated.

- Eigenvalues, namely on-shell properties, in the restricted (P) space do not change.
  - Off-shell properties naturally change.

- Induced many-body forces appear in many-body space.
Example of induced 3NF in SRG method

- Red curve: calculation with low-momentum interaction by SRG method
- When induced 3NF is included, the exact energy is recovered (black curve).
  - $\lambda$ represents the scale of low-momentum space.
  - To reproduce experimental values, genuine 3NF is needed (attractive).

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"Evolving nuclear many-body forces with the similarity renormalization group"
Summary of effects of 3NF in chiral effective field theory

- Phenomenological adjustment is minimal.
- 3NFs have to be expected as induced effective interaction when other degrees of freedom than nucleons are eliminated in many-nucleon space.
- The repulsive contribution is essential, at least semi-quantitatively, to explain basic saturation properties.
- In addition:
  - Enhancement of spin-orbit strength of the nuclear mean field.
  - Enhancement of tensor component: due to the suppression of the reduction of the tensor force from NNLO 2-\(\pi\) exchange.
    - E.g., the neutron drip limit is reproduced in oxygen isotopes.
- Subjects to be studied: 3BF contributions in the strangeness sector; hyper nuclei and neutron star matter.
Strangeness sector: YN and YNN interaction in ChEFT

- Parametrization by Bonn-Jülich-München
  - Lowest order:
    - Parameters: $f_{NN\pi} = \frac{2f_\pi}{g_A}$ and $\alpha = \frac{F}{F+D}$, also five low-energy constants:
      $$C_{1S0}^{\Lambda\Lambda}, C_{3S1}^{\Lambda\Lambda}, C_{1S0}^{\Sigma\Sigma}, C_{3S1}^{\Sigma\Sigma}, C_{3S1}^{\Lambda\Sigma}$$
  - Next-to-Leading order
  - Leading three-baryon forces
    - Estimation of $1\pi$ and contact LEC: decouplet dominance
First, hyperon (Λ, Σ and Ξ) single-particle potentials are calculated in nuclear matter using Ch-NLO YN interactions only.

- Cutoff scale of 400~600 MeV is still hard to treat the interaction by HF method.

Standard lowest-order Brueckner method to take care of high momentum part with the continuous choice for intermediate spectra.

\[ G = v + v \frac{Q}{\omega - (t_1 + U_1 + t_2 + U_2)} G \]

Salient features, in comparison with other previous YN interactions.

- Strong ΛN-ΣN coupling, which generates ΛN attraction
- Charge symmetry breaking in few-body systems may be accounted for.
- Λ s.p. potential does not become deeper at high densities.
The ΛN attraction (experimentally, s.p. potential depth is around 30 MeV) comes from ΛN -ΣN coupling through tensor force.

The coupling effect is large in ChNLO potential.
Calculations at other densities of $k_F = 1.07, 1.50$ and $1.60 \text{ fm}^{-1}$

- **NLO ChEFT**  The $\Lambda$ potential stays shallow at higher densities.

- **Nijmegen**
Shallowness of the potential of the $\Lambda$ hyperon at rest is due to behavior of the $^3S_1$ contribution.

Qualitatively same results with those of Haidenbauer, Meißner, Keiser, and Weise [arXiv:1612.03758, EPJA in print].
Comparison with the results from Nijmegen 97f potential

- Qualitative resemblance between ChEFT and NSC97F.
- Note that NSC97f predicts attractive $\Sigma$ potential; inconsistent with experimental data.
The tensor component from one-pion exchange process in the ΛN-ΣN coupling at the NLO may be overestimated.

In the NN case, it is known that $2\pi$ exchange NNLO diagram (a) reduces the strong one-pion exchange tensor force, which corresponds to the role of the $\rho$ meson in the OBEP picture.

This is related to the later observation that the effect of ΛNN-ΣNN 3BF coupling enhances the tensor component.
Next, introducing normal-ordering for YNN force in the 2BF level, LOBT calculations are carried out.


YNN 3BF(a) is reduced to density-dependent 2BF(c) in nuclear matter (normal-ordering). Solve G-matrix equations, with 2BF + DD 2BF, to obtain Λ s.p. potential.
Pauli blocking type contribution (c) suppresses the attraction in free space, namely the repulsive contribution. The suppression becomes larger at higher density, like in the NNN case.

Contributions of $1\pi$ exchange 3BF can be attractive contribution, depending the sign of the coupling constant.

Effects from $\Lambda NN-\Sigma NN$ coupling can be attractive by the enhancement of the tensor component.
Contributions of 2-pion exchange $\Lambda NN$ interaction

- $2\pi$ exchange $\Lambda NN$ 3-body interaction $V_{\Lambda NN}^{TPE}$
  \[
g_A^2 \frac{f_0^2}{3f_0^2} (\tau_2 \cdot \tau_3) \frac{(\sigma_3 \cdot q_{63})(\sigma_2 \cdot q_{52})}{(q_{63}^2 + m_{\pi}^2)(q_{52}^2 + m_{\pi}^2)} \{-A m_{\pi}^2 + B q_{63} \cdot q_{52}\}
\]

  Estimation by Petschauer: $A = 0, B = -3.0$ GeV$^{-1}$

- Normal-ordered 2-body int. in symmetric nuclear matter
  \[
  \langle k' \sigma'_{\Lambda'}, -k' \sigma' \tau' | V_{\Lambda N\langle N \rangle} | k \sigma_{\Lambda}, -k \sigma \tau \rangle \\
  \equiv \frac{1}{2} \sum_{k_h, \sigma_h, \tau_h} \langle k' \sigma'_{\Lambda'}, -k' \sigma' \tau', k_h, \sigma_h, \tau_h | V_{\Lambda NN} | k \sigma_{\Lambda}, -k \sigma \tau, k_h, \sigma_h, \tau_h \rangle_A
\]

  - Density-dependent central, LS, ALS 成分
  - No tensor component
  - Additional statistical factor $\frac{1}{2}$
  - Repulsive character of Pauli blocking type
  - Partial waves expansion $\Rightarrow$ G-matrix calculation
HF contribution of $2\pi$ exchange $\Lambda NN$ force to $\Lambda$ potential

$$V_{\Lambda NN}^{TPE} = \frac{g_A^2}{3f_0^4} (\tau_2 \cdot \tau_3) \frac{(\sigma_3 \cdot q_{63})(\sigma_2 \cdot q_{52})}{(q_{63}^2 + m_\pi^2)(q_{63}^2 + m_\pi^2)}$$

$$\times \{- (3b_0 + b_D)m_\pi^2 + (2b_2 + 3b_4)q_{63} \cdot q_{52}\}$$

$$\Delta U_\Lambda(k_\Lambda) = \frac{1}{2} \sum_{2,3} \langle k_\Lambda k_2 k_3 | V_{\Lambda NN}^{TPE} | k_\Lambda k_2 k_3 \rangle_A$$

$$= \frac{g_A^2}{3f_0^4} \frac{1}{(2\pi)^6} \int_0^{k_F} q^2 dq \frac{64\pi^2}{3} (k_F - q)^2$$

$$\times (2k_F + q) \frac{4q^2}{(4q^2 + m_\pi^2)^2}$$

$$\times \{- (3b_0 + b_D)m_\pi^2 + (2b_2 + 3b_4)q^2\}$$

- ChEFT may not be relevant to $\rho > 2\rho_0$.
- Contributions from contact terms have also to be considered.
HF contribution of $2\pi$ and $1\pi$ exchange $\Lambda$NN forces to $\Lambda$ potential

$$V_{\Lambda NN}^{TPE} = \frac{g_A^2}{3f_0^2}(\mathbf{\tau}_2 \cdot \mathbf{\tau}_3) \frac{(|\mathbf{\sigma}_3 \cdot \mathbf{q}_{63}|)(|\mathbf{\sigma}_2 \cdot \mathbf{q}_{52}|)}{(\mathbf{q}_{63}^2 + m_\pi^2)(\mathbf{q}_{63}^2 + m_\pi^2)} \times \{-3b_0 + b_D) m_\pi^2 + (2b_2 + 3b_4)q_{63} \cdot q_{52}\}$$

$$V_{\Lambda NN}^{\Lambda \pi \pi} = \frac{1}{2f_0^4} \frac{(|\mathbf{\sigma}_2 \cdot \mathbf{q}_{52}|)}{(\mathbf{q}_{52}^2 + m_\pi^2)} \times \{N_1 \mathbf{\sigma}_3 \cdot \mathbf{q}_{52} + N_2 i(\mathbf{\sigma}_1 \times \mathbf{\sigma}_3) \cdot \mathbf{q}_{52}\}$$
Contributions of $2\pi$ exchange $\Lambda NN$ force in G-matrix calculations

- Absolute values of the are reduced by the correlation from the G-matrix equation.
- Contributions below the normal density are not large, though depending on the sign of the 1-pion exchange contact term.
- Effects of the $\Lambda NN$-$\Sigma NN$ coupling are to be included.

\[ V_{TPE}^{\Lambda NN} = \frac{g_A^2}{3f_0^4} (\tau_2 \cdot \tau_3) \frac{(\sigma_3 \cdot q_{63})(\sigma_2 \cdot q_{52})}{(q_{63}^2 + m_\pi^2)(q_{63}^2 + m_\pi^2)} \times \{-3b_0 + b_D\}m_\pi^2 + (2b_2 + 3b_4)q_{63} \cdot q_{52}\]

\[ V_{OPE}^{\Lambda NN} = \frac{1}{2f_0^4} \frac{(\sigma_2 \cdot q_{52})}{(q_{52}^2 + m_\pi^2)} \times \{N_1\sigma_3 \cdot q_{52} + N_2i(\sigma_1 \times \sigma_3) \cdot q_{52}\} \]
Again, normal-ordered 2NF is introduced.

As in the case of 3NFs

- Contributions of (d) and (e) diagrams cancel each other.
- The diagram (c) enhances the tensor component.
Λ s.p. potential in symmetric nuclear matter including 3BFs

- $k_F = 1.07, 1.35$ and $1.60$ fm$^{-1}$ ($\rho = \frac{1}{2}\rho_0$, $\rho_0$, and $1.66\rho_0$, respectively)
  
  2$\pi$-exchange $\Lambda$NN interaction
  2$\pi$-exchange $\Lambda$NN-$\Sigma$NN interaction

![Graphs showing $U_\Lambda(k)$ in SNM for $k_F = 1.07$ fm$^{-1}$, $1.35$ fm$^{-1}$, and $1.60$ fm$^{-1}$.]
Λ s.p. potential in symmetric nuclear matter including 3BFs

- \( k_F = 1.07, 1.35 \) and \( 1.60 \text{ fm}^{-1} \) (\( \rho = \frac{1}{2} \rho_0, \ \rho_0, \) and \( 1.66 \rho_0 \), respectively)
  - 2\( \pi \)-exchange \( \Lambda NN \) interaction
  - 2\( \pi \)-exchange \( \Lambda NN\)-\( \Sigma NN \) interaction
  - Contact + attractive 1\( \pi \)-exchange \( \Lambda NN \) interaction

![Graphs showing the \( U(k) \) in SNM for different values of \( k_F \) and including 3BF effects.](image)
A naive condition for the $\Lambda$ hyperon to appear in pure neutron matter.

$$U_\Lambda(0) < \frac{\hbar^2}{2m_n} k_{F_n}^2 + m_n - m_\Lambda + U_n(k_{F_n})$$

- $\Lambda$ s.p. potential predicted by NLO YN potential does not become deep at high densities.
  - $U_\Lambda(0) > -30$ MeV
  - The attractive effect from $\Lambda$NN-$\Sigma$NN coupling does not change this feature qualitatively.

- $U_\Lambda(0)$ is always above $\frac{\hbar^2}{2m_n} k_{F_n}^2 + m_n - m_\Lambda + U_n(k_{F_n})$; namely, $\Lambda$ hyperon does not appear in the high density medium.

(Note: Actual calculations in neutron matter are in progress.)
Coupling constants in $S = -2$ sector are largely uncertain.

- Constraint from the non-existence of $H$ and bound $\Xi N$ state.

Various baryon-channel coupling ($\Xi N-\Lambda \Lambda -\Sigma \Sigma (T=0)$, $\Xi N-\Lambda \Sigma -\Sigma \Sigma (T=1)$) state and density dependences

$\Xi$-nucleus potential from ChEFT NLO interaction may be attractive but shallow at the surface region.

Recent experimental data suggest some weakly bound $\Xi$-nucleus states.

- Kiso event: $\Xi^- - ^{14}N$ (Nakazawa et al.)
- $^{12}C(K^-,K^+)X$ spectra at 1.8 GeV/c (Nagae et al.), which showed some strength below the threshold, though not conclusive until the future experiment.

- Effects from 3BFs should be included.
General remarks for 3-body interaction

LOBT G-matrix calculations to obtain hyperon single-particle potentials in (symmetric) nuclear matter, using Ch-EFT NLO interactions in the strangeness sector developed by the Bonn-Jülich-München group.

- Strong ΛN-ΣN coupling
- Λ s.p. potential does not become deeper at higher densities.
  - Λ is energetically unfavorable in high density matter. Does hyperon puzzle dissolve?
- Contributions of 3BF do not change the situation.
- ΛNN 3BF gives repulsive contribution, while the effect from ΛNN- ΣNN is attractive.

Subjects to be studied in the future

- NNLO
- New experiments to reduce uncertainties of coupling constants and low energy constants.
- Consideration of diagrams including $K$ and $\eta$ exchanges.
- Explicit (ab initio) calculations of finite hypernuclei.

Summary