ΛcN interaction from lattice QCD and Λc nuclei

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Various (hyper-) nuclei are found in experiments.

- Are there any **charmed hyper-nuclei**?
- What is the difference between hyper-nuclei and **charmed hyper-nuclei**?

To discuss possibility of **charmed hyper-nuclei**, the interaction between a charmed baryon and a nucleon is a key.

**YcN interactions**

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*Strangeness and charm in hadrons and dense matter 2017 @ YITP, 24 May. 2017*
There are several theoretical studies for charmed baryon interactions

- One-Boson-Exchange (OBE) potential model extended to flavor SU(4) + Infinite hard core at short range (~ 0.5 fm)

These studies show that \(\Lambda c\)-N interactions are attractive.

Various \(\Lambda c\)-nuclei were predicted
Introduction

There are several theoretical studies for charmed baryon interactions

- OBEP model based on the heavy quark effective theory
  - Coupled-channel ($\Lambda cN - \Sigma cN - \Sigma c^*N$) effects are taken into account


Channel coupling has important effects for the $\Lambda cN$ bound states

The possibility of bound $\Lambda cN$ was claimed

The results are sensitive to the phenomenological cutoff parameter, which encodes the information of the short-range interaction.

This feature indicates that the binding energy is sensitive to the short-range interaction.

The determination of interactions from QCD is desirable

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Introduction

Lattice QCD

Our strategy

HAL QCD method

hadron-hadron potential

- Faithful to QCD S-matrix
- No experimental data are needed
- Extending to charmed baryon is easy
Outline

(1) Motivation

(2) HAL QCD method

(3) Simulation setup

(4) Numerical results of $\Lambda cN$ and $\Lambda N$ interactions

(5) Folding potential analysis for $\Lambda c$-nuclei

(6) Summary and conclusion
HAL QCD method

- Nambu-Bethe-Salpeter (NBS) wave functions

\[
\psi_{\Lambda_c N}^{(W_n)}(\vec{r}) = \sum_{\vec{x}} \langle 0|\Lambda_c(\vec{r} + \vec{x}, t) N(\vec{x}, t)|\Lambda_c N, W_n \rangle e^{-W_n t}
\]

\[
W_n = \sqrt{k_n^2 + m_{\Lambda_c}^2} + \sqrt{k_n^2 + m_N^2}
\]

\[
(k_n^2 + \nabla^2)\psi_{\Lambda_c N}^{(W_n)}(\vec{r}) = 0
\]

\[
\psi_{\Lambda_c N}^{(W_n)}(\vec{r}) \propto \sin \left( \frac{k_n r - l \pi}{2} + \delta_l(k_n) \right)
\]

- At large \( r \)
- Outside interactions

Define the **energy-independent non-local potentials** through the **Schrödinger-type equation**

\[
(E_n - H_0) \psi_{\Lambda_c N}^{(W_n)}(\vec{r}) = \int d^3 \vec{r}' U_{\Lambda_c N}(\vec{r}, \vec{r}') \psi_{\Lambda_c N}^{(W_n)}(\vec{r}')
\]

- Potentials **faithful to S-matrix** by construction
- **All 2PI contributions are included** in potentials
- Potentials are energy-independent until a new channel opens

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HAL QCD method

To extract “energy-independent” potentials, we employ time-dependent HAL QCD method

\[
R_{\Lambda^c N}(\vec{r}, t) = \frac{G_{\Lambda^c N}(\vec{r}, t)}{e^{-m_{\Lambda^c} t} e^{-m_N t}}
\]

Normalized 4pt-correlation function (R-correlator)

\[
(E_0 - H_0) \psi_0(\vec{r}) = \int d^3 r' U(\vec{r}, \vec{r}') \psi_0(\vec{r}')
\]

\[
(E_1 - H_0) \psi_1(\vec{r}) = \int d^3 r' U(\vec{r}, \vec{r}') \psi_1(\vec{r}')
\]

\[
(E_2 - H_0) \psi_2(\vec{r}) = \int d^3 r' U(\vec{r}, \vec{r}') \psi_2(\vec{r}')
\]

All equations are combined into one t-dep. eq.

\[
\left(-\frac{\partial}{\partial t} + \left[1 + \frac{\delta^2}{8\mu}\right] \frac{\partial^2}{\partial t^2} - H_0\right) R_{\Lambda^c N}(\vec{r}, t) = \int d^3 r' U_{\Lambda^c N}(\vec{r}, \vec{r}') R_{\Lambda^c N}(\vec{r}', t)
\]

Within the approximation up to O(k^2)

Non-local potentials \(\rightarrow\) local potentials

Derivative (velocity) expansion

\[
U(\vec{r}, \vec{r}') = V(\vec{r}, \vec{\nabla}) \delta^3 (\vec{r} - \vec{r}')
\]

\[
V(\vec{r}, \vec{\nabla}) = V_0(\vec{r}) + V_\sigma(\vec{r}) (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T(\vec{r}) S_{12} + O(\vec{\nabla})
\]

In the low energy state, LO term of the potentials is significant.

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(2) HAL QCD method

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Lattice QCD setup

Nf=2+1 full QCD configurations generated by PACS-CS Coll

PACS-CS Collaboration:

- Iwasaki gauge action
- O(a) improved Wilson-clover quark action
- $a \sim 0.09$ fm, $L \sim 3$ fm ($32^3 \times 64$)

$m_\pi \sim 700, 570, 410$ MeV

For charm quark, we use
Relativistic Heavy Quark (RHQ) action
to remove the leading $O((ma)^n)$ discretization error

For more details, see,
Namekawa, et al.,
Phys. Rev. D84 (2011) 074505

BG/Q @ KEK

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S-wave $\Lambda$-c-N effective central potentials

$\Lambda$-c-N potential = repulsive core + attractive pocket

As $m_\Omega$ decreasing,

- repulsive core becomes larger
- attractive pocket shifts outward

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S-wave $\Lambda (c)\cdot N$

effective central potentials

$J^P=0^+$

$J^P=1^+$

$\Lambda N$ potentials

$\Lambda \pi \sim 700$ MeV
$\Lambda \pi \sim 570$ MeV
$\Lambda \pi \sim 410$ MeV
Comparison of $\Lambda N$ and $\Lambda cN$ potential
($m_\pi = 570$ MeV)

Both attractive pocket and repulsive core of $\Lambda cN$ potentials are smaller than those of $\Lambda N$ potentials.
Comparison of $\Lambda N$ and $\Lambda cN$ potential
($m_\pi = 570$ MeV)

Phase shifts of $\Lambda (c)N$ scattering

The net attraction of $\Lambda cN$ is weaker than $\Lambda N$

Scattering length:

$$a_{\Lambda N} = 0.56(30) \text{ fm}$$
$$a_{\Lambda cN} = 0.18(12) \text{ fm}$$

$$a_{\Lambda N} = 0.87(45) \text{ fm}$$
$$a_{\Lambda cN} = 0.21(15) \text{ fm}$$
Comparison of $\Lambda N$ and $\Lambda cN$ potential
$(m_\pi = 570$ MeV$)$

We observed that the scattering lengths of $\Lambda cN$ ($J^P=0^+$) state and $\Lambda cN$ ($J^P=1^+$) state are approximately equal.

To investigate the origin of this observation, we decompose the effective central potentials into spin dep. term and spin indep. term.

Scattering length:

$\quad a_{\Lambda N} = 0.56(30)$ fm
$\quad a_{\Lambda cN} = 0.18(12)$ fm

$\quad a_{\Lambda N} = 0.87(45)$ fm
$\quad a_{\Lambda cN} = 0.21(15)$ fm
Leading order of $\Lambda cN$ potential in velocity expansion

$$V_{\Lambda cN}(\vec{r}) = V_0(\vec{r}) + V_\sigma(\vec{r}) (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T(\vec{r}) S_{12}$$

These three potentials can be obtained from NBS wave functions in $J^P=0^+$ and $1^+$

We found that both spin-spin force and tensor force are weak
Leading order of $\Lambda cN$ potential in velocity expansion

$$V_{\Lambda cN}(\vec{r}) = V_0(\vec{r}) + V_\sigma(\vec{r}) (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T(\vec{r}) S_{12}$$

![Graphs showing potential components](image)

- $V_0(\vec{r})$
- $V_\sigma(\vec{r})$
- $V_T(\vec{r})$

$M_{\pi} \sim 700$ MeV
$M_{\pi} \sim 570$ MeV
$M_{\pi} \sim 410$ MeV

Weak spin-spin force of $\Lambda cN$

$\rightarrow$ It could be explained from the HQ spin symmetry

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Leading order of $\Lambda cN$ potential in velocity expansion

\[
V_{\Lambda cN}(\vec{r}) = V_0(\vec{r}) + V_\sigma(\vec{r}) (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T(\vec{r}) S_{12}
\]

- $V_0(\vec{r})$
- $V_\sigma(\vec{r})$
- $V_T(\vec{r})$

- $\Sigma^*N$
- $\Sigma c^*N$
- $\Sigma cN$
- $\Lambda cN$

Weak tensor force of $\Lambda cN$

$\rightarrow$ It could be explained from large difference of $\Lambda cN - \Sigma cN$ threshold
Comparison of $\Lambda N$ potential and $\Lambda cN$ potential

$$V_{\Lambda cN}(\vec{r}) = V_0(\vec{r}) + V_\sigma(\vec{r}) (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T(\vec{r}) S_{12}$$

Spin independent central potential is significant in $\Lambda cN$ interaction

We use this potential for the investigation of $\Lambda c$-nuclei
Λc-nuclei

(Single-) folding potential

\[ V_F(r) = \int d^3r' \rho_A(r') V_{\Lambda c N}(r - r') \]

density distributions for nuclear matter

\[ \rho_A(r) = \rho_0 \left[ 1 + \exp \left( \frac{r - c}{a} \right) \right]^{-1} \]

two-parameter Fermi (FM) form

\[ \left( \int d^3r \rho_A(r) = A \right) \]

- Parameters for several stable nuclei

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$^{12}$C</th>
<th>$^{28}$Si</th>
<th>$^{40}$Ca</th>
<th>$^{58}$Ni</th>
<th>$^{90}$Zr</th>
<th>$^{208}$Pb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_0$ (fm$^{-3}$)</td>
<td>0.207</td>
<td>0.175</td>
<td>0.169</td>
<td>0.172</td>
<td>0.165</td>
<td>0.150</td>
</tr>
<tr>
<td>$c$ (fm)</td>
<td>2.1545</td>
<td>3.15</td>
<td>3.60</td>
<td>4.094</td>
<td>4.90</td>
<td>6.80</td>
</tr>
<tr>
<td>$\alpha$ (fm)</td>
<td>0.425</td>
<td>0.475</td>
<td>0.523</td>
<td>0.54</td>
<td>0.515</td>
<td>0.515</td>
</tr>
</tbody>
</table>

Since the spin dep. force is weak, we use only the spin indep. force for the folding potentials.

Ref: M. El-Azab Farid a, M.A. Hassanain, Nuclear Physics A 678 (2000) 39–75
Λc-nuclei

(Single-) folding potential

\[ V_F(r) = \int d^3 r' \rho_A(r') V_{ΛcN}(r - r') \]

HAL QCD potential (spin-indep. force)

- Binding energies

\[ V_F(r) \]

(Λc-\(^{208}\)Pb)

\[ M_{\pi} \sim 700 \text{ MeV} \]
\[ M_{\pi} \sim 570 \text{ MeV} \]
\[ M_{\pi} \sim 410 \text{ MeV} \]

w/o Coulomb force
Λc-nuclei

(Single-) folding potential

\[ V_F(r) = \int d^3 r' \rho_A(r') V_{ΛcN}(r - r') \]

HAL QCD potential (spin-indep. force)

Binding energies increase as pion mass approaches the physical point

- Binding energies

\[ V_{F(r)} \text{[MeV]} \]

\[ r \text{[fm]} \]

\[ (Λc-^{208}\text{Pb}) \]

\[ \text{Mpi} \sim 700 \text{ MeV} \]
\[ \text{Mpi} \sim 570 \text{ MeV} \]
\[ \text{Mpi} \sim 410 \text{ MeV} \]

w/o Coulomb force
Conclusions

- We investigate $\Lambda cN$ interactions by the HAL QCD method

Our results show that $\Lambda cN$ interactions are attractive

The feature of $\Lambda cN$ potentials is spin-independent. The spin-spin force and the tensor force are almost negligible

The analysis of the folding potentials with LQCD potentials shows that $\Lambda c$ could be bound with heavy nuclei.

Prospects:
- We will investigate $\Lambda c$ - light nuclei by using few/many-body calculation (GEM, AMD, ⋯)
- We will also investigate $\Sigma c$-nuclei
- Physical point calculation is the next step