Determining the $\Theta^+$ quantum numbers through $K^+p \rightarrow \pi^+KN$

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Motivation: Spin parity determination


\[ | \Theta \rangle = | uudd\bar{s} \rangle \]

S = +1, manifestly exotic

Spin and parity are not determined.

Find a reaction where qualitatively different results depending on the quantum numbers are observed.

Utilize the polarization of particles
Motivation: Advantage of hadronic process

We propose

\[ K^+ p \rightarrow \pi^+ \Theta^+ \rightarrow \pi^+ K^+ n (K^0 p) \]

at threshold of \( \pi \) and \( \Theta \).

- Low energy (\( k_{\text{in}} \sim 350 \text{ MeV in c.m. frame} \))
- Decay is considered \( \rightarrow \) background, interference, ...
- Hadronic process: clear mechanism
  : large cross section \( \sim 10^2 \text{ \( \mu b \)} \)
Chiral model for the reaction: Background


Threshold production of $\pi$ and $\Theta$

Vertices $\leftarrow$ chiral Lagrangian

Assume the final $\pi^+$ is almost at rest
Chiral model for the reaction: Resonance term

Background (tree level)

Resonance (one loop)
Chiral model for the reaction: Resonance term

Proportional to $\sigma \cdot p_{\pi^+}$ -> vanishes
Spin and parity: \( KN \rightarrow \Theta \rightarrow KN \)

\( 1/2^- \) (KN s-wave resonance)

\( 1/2^+ \), \( 3/2^+ \) (KN p-wave resonance)

\[
t_{K+n(K^0 p)\rightarrow K+n}^{(s)} = \frac{(\pm)g_{K+n}^2}{M_I - M_R + i\Gamma/2}
\]

\[
t_{K+n(K^0 p)\rightarrow K+n}^{(p,1/2)} = \frac{(\pm)\bar{g}_{K+n}^2(\sigma \cdot q')(\sigma \cdot q)}{M_I - M_R + i\Gamma/2}
\]

\[
t_{K+n(K^0 p)\rightarrow K+n}^{(p,3/2)} = \frac{(\pm)\tilde{g}_{K+n}^2(S \cdot q')(S^\dagger \cdot q)}{M_I - M_R + i\Gamma/2}
\]

\[
g_{K+n}^2 = \frac{\pi M_R \Gamma}{M q}, \quad \bar{g}_{K+n}^2 = \frac{\pi M_R \Gamma}{M q^3}, \quad \tilde{g}_{K+n}^2 = \frac{3\pi M_R \Gamma}{M q^3}.
\]

\( M_R = 1540 \text{ MeV} \)

\( \Gamma_R = 20 \text{ MeV} \)
Spin and parity: Resonance amplitude

Resonance term for $K^+ p \rightarrow \pi^+ K^+ n$

$$-i\tilde{t}^{(s)}_i = \frac{g_{K^+n}^2}{M_I - M_R + i\Gamma/2} \left\{ G(M_I)(a_i + c_i) - \frac{1}{3} \tilde{G}(M_I)b_i \right\} \sigma \cdot k_{in} S_I(i)$$

$$-i\tilde{t}^{(p,1/2)}_i = \frac{\tilde{g}_{K^+n}^2}{M_I - M_R + i\Gamma/2} \tilde{G}(M_I) \left\{ \frac{1}{3} b_i k_{in}^2 - a_i + d_i \right\} \sigma \cdot q' S_I(i)$$

$$-i\tilde{t}^{(p,3/2)}_i = \frac{\tilde{g}_{K^+n}^2}{M_I - M_R + i\Gamma/2} \tilde{G}(M_I) \frac{1}{3} b_i \left\{ (k_{in} \cdot q')(\sigma \cdot k_{in}) - \frac{1}{3} k_{in}^2 \sigma \cdot q' \right\} S_I(i)$$
Numerical results: Mass distributions

- \( I, J^P = 0, 1/2^- \)
- \( I, J^P = 0, 1/2^+ \) \( k_{in} \text{(Lab)} = 850 \text{ MeV/c} \)
- \( I, J^P = 0, 3/2^+ \) \( \theta = 0 \text{ deg} \)
Numerical results: Mass distributions

Deviation of the peak from B.W. mass
→ interference effect
size ~ width
Production 1: $K^+ p \rightarrow \pi^+ K^- n$

Numerical results: Angular dependence

I, J$^P$ = 0, 1/2$^-$

I, J$^P$ = 0, 1/2$^+$

I, J$^P$ = 0, 3/2$^+$

- total
- resonance
- background
Numerical results: Polarization test

\[
\langle -1/2 | \sigma \cdot k_{in} | 1/2 \rangle = 0
\]

\[
\langle -1/2 | \sigma \cdot q' | 1/2 \rangle \propto q' \sin \theta
\]

# Same result is obtained for final pK^0
Numerical results: Mass distributions

\[ \frac{d^2 \sigma}{dM_I d\cos \theta} \quad [\text{[\mu b/MeV]}] \]

- \( I,J^P = 0,1/2^- \)
- \( I,J^P = 0,1/2^+ \)
- \( I,J^P = 0,3/2^+ \)

\[ k_{\text{in (Lab)}} = 850 \text{ MeV/c} \]
\[ \theta = 90 \text{ deg} \]

Polarization test

\[ \frac{d^2 \sigma}{dM_I d\cos \theta} \quad [\text{[\mu b/MeV]}] \]

\[ \text{Mass distributions} \]
Numerical results: Angular dependence

\[ \frac{d^2\sigma}{dM d\cos\theta} \left[ \mu^2b/MeV \right] \]

1.0 0.5 0.0 -0.5 -1.0

\( I, J^P = 0, 1/2^- \)

\( I, J^P = 0, 1/2^+ \)

\( I, J^P = 0, 3/2^+ \)

Polarization test
Numerical results: Incomplete polarization

Polarization

\[ k_{in}(Lab) = 850 \text{ MeV/c} \]
\[ \theta = 90 \text{ deg} \]

\[ d^2 \sigma / dM_I d\cos \theta \ [\mu b/\text{MeV}] \]

\[ I,J^P=0,1/2^- \]
\[ I,J^P=0,1/2^+ \]

80%  100%  0%
We calculate the $K^+ p \rightarrow \pi^+ K^+ n$ reaction using a chiral model, assuming the possible quantum numbers of $\Theta^+$ baryon.

If we find the resonance in the polarization test, the quantum numbers of $\Theta^+$ can be determined as $I=0$, $J^P=1/2^+$

Problems

- 0 momentum $\pi$
- polarization of final N

As energy of initial Kaon increases, $\Delta$ contribution becomes dominant.

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