Phenomenological study for the $\Theta^+$ and two-meson coupling

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Contents

★ Introduction
★ Pure antidecuplet case
★ 8–10 mixing case
★ Mass spectra
★ Decay widths
★ Two-meson coupling
★ Coupling constants
★ Meson induced Θ production
★ Summary
Existence of $\Theta^+$ + Flavor SU(3) symmetry

Existence of flavor partners of $\Theta^+$

Assuming the flavor multiplet that $\Theta^+$ belongs to, we examine its properties by symmetry relation, in connection with known baryon resonances.

to determine the $J^P$ of $\Theta^+$

Phenomenological but model independent analysis up to $O(m_s)$
Pure antidecuplet case

Simplest assignment for $\Theta^+$

Test the masses and widths of partners via flavor SU(3) symmetry relations
Pure antidecuplet case

Mass: Gell-Mann—Okubo formula

\[ M(\bar{10}; Y) = M_{10} - aY \]

Two parameters ← Mass of Θ and N*

Width: SU(3) symmetric coupling

\[ g_{\Theta KN} = \sqrt{6} g_{N^* \pi N} \]

\[ \Gamma_R = g_R^2 F_I \frac{p^{2l+1}}{M_R^{2l}} \]

One parameter ← Width of N*
**Mass and width [MeV]**

\[ M(\bar{10}; Y) = M_{10} - \alpha Y, \quad g_{\Theta KN} = \sqrt{6}g_{N^* \pi N}, \quad \Gamma_R = g_R^2 F_I \frac{p^{2l+1}}{M_R^{2l}} \]

<table>
<thead>
<tr>
<th>( J^P )</th>
<th>( M_{\Theta} )</th>
<th>( M_N )</th>
<th>( M_\Sigma )</th>
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<th>( \Gamma_{\Theta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2(^-)</td>
<td>( 1540 )</td>
<td>( 1647 )</td>
<td>( 1753 )</td>
<td>( 1860 )</td>
<td>( 156.1 )</td>
</tr>
<tr>
<td>exp.</td>
<td>( \Theta(1540) )</td>
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<td>1/2(^+)</td>
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Pure antidecuplet case

Mass and width [MeV]

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<td>1900</td>
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<td>10.6</td>
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</tr>
<tr>
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<td>1540 Θ(1540)</td>
<td>1700 N(1700)</td>
<td>1860</td>
<td>2020 Ξ(2030)</td>
<td>1.3</td>
</tr>
</tbody>
</table>

are not reproduced simultaneously.
Octet-antidecuplet mixing

Second simplest assignment for $\Theta^+$

Mixing is induced by the SU(3) breaking in mass term.
Octet-antidecuplet mixing

Mass formulae: GMO + mixing \((N, \Sigma)\)

\[
M_\Theta = M_{10} - 2a \\
M_{\Xi_{10}} = M_{10} + a \\
M_\Lambda = M_8 \\
M_{\Xi_8} = M_8 + b + \frac{1}{2}c \\
M_{N_1} = \left( M_8 - b + \frac{1}{2}c \right) \cos^2 \theta_N + (M_{10} - a) \sin^2 \theta_N - \delta \sin 2\theta_N \\
M_{N_2} = \left( M_8 - b + \frac{1}{2}c \right) \sin^2 \theta_N + (M_{10} - a) \cos^2 \theta_N + \delta \sin 2\theta_N \\
M_{\Sigma_1} = (M_8 + 2c) \cos^2 \theta_\Sigma + M_{10} \sin^2 \theta_\Sigma - \delta \sin 2\theta_\Sigma \\
M_{\Sigma_2} = (M_8 + 2c) \sin^2 \theta_\Sigma + M_{10} \cos^2 \theta_\Sigma + \delta \sin 2\theta_\Sigma \\
\]

8 masses v.s. 6 parameters

\(J^P = 1/2^-\) : too wide width
\(J^P = 3/2^+\) : states are not well established
Mass spectra

1/2^+ predicted

3/2^- predicted

Mass spectrum [MeV]
1/2^+ predicted

3/2^- predicted

Mass spectrum [MeV]

Mass spectra
Decay width of $\Theta$

$g_\Theta = \sqrt{6} \left( g_{N_2} \cos \theta_N - g_{N_1} \sin \theta_N \right)$

$\Gamma_R = g_R^2 F_L \frac{p^{2l+1}}{M_R^{2l}}$

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>$\theta_N$ [deg]</th>
<th>$\Gamma_\Theta$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2^+$</td>
<td>29</td>
<td>29.1</td>
</tr>
<tr>
<td>$3/2^-$</td>
<td>33</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Narrow width
Then, what about two-meson coupling?

: large branching ratio of $N^* \rightarrow \pi \pi N$
: $\pi K N$ molecule picture for the $\Theta$

SU(3) relation enable us to calculate

the cross section of

from the decay of $N^* \rightarrow \pi \pi N$
Two-meson coupling

The structure of the two-meson coupling


• The effect of the two-meson coupling was studied by evaluating the self-energy.

• We examined possible structures, and found that two types of the interaction Lagrangians were important.

• These terms provided a sizable contribution.
# Two-meson coupling

## Branching fraction [%]

<table>
<thead>
<tr>
<th>J^P</th>
<th>state</th>
<th>(\pi N)</th>
<th>(\pi\pi N(s))</th>
<th>(\pi\pi N(v))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1/2^+)</td>
<td>N(1440)</td>
<td>65</td>
<td>7.5</td>
<td>&lt;8</td>
</tr>
<tr>
<td></td>
<td>N(1710)</td>
<td>15</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>(3/2^-)</td>
<td>N(1520)</td>
<td>55</td>
<td>25</td>
<td>&lt;85-95</td>
</tr>
<tr>
<td></td>
<td>N(1700)</td>
<td>10</td>
<td>&lt;35</td>
<td></td>
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</table>

Still large uncertainty
Constraints on the coupling

We impose phenomenological constraints.

Self-energy: not too large, but not too small

$\pi^- p \rightarrow K^- \Theta^+$ at KEK: upper limit is $\sim 4.1 \mu b$

$\sim 100 \text{ MeV}$

$\pi^-$

$p$

$K^-$

$\Theta^+$

$< 4.1 \mu b$
Constraints on the coupling

Two structures should be added coherently.

\[ \pi^- K^- + \pi^+ K^+ \]

\[ \Theta^+ \]

\[ \pi^- K^- \]

\[ \Theta^+ \]

\[ \pi^- K^- \]

\[ \Theta^+ \]

\[ p \]

\[ s \]

\[ \Theta^+ \]

\[ \Theta^+ \]

\[ v \]

\[ \rightarrow \text{interference effect among } s \text{ and } v. \]
\[ \pi^- p \rightarrow K^- \Theta^+ \]

\[ K^+ p \rightarrow \pi^+ \Theta^+ \]

**1/2^+**

\[ \sigma(K^+) \approx 50 \]

**3/2^-**

\[ \sigma(\pi^-) \approx 3 \]

**large interference**

**small interference**
Masses of $\Theta(1540)$ and $\Xi(1860)$ are well fitted in the $8$–$10$ mixing scheme with $J^P = 1/2^+$ or $3/2^-$ baryons.

A very narrow width of $\Theta$ can be obtained for the $J^P = 3/2^-$ case.

For both $J^P$, the mixing angle is close to the ideal angle.

Summary 2: Two-meson coupling and $\Theta$ production

Based on the mixing scheme, we evaluate the two-meson coupling of $\Theta$, and calculate the reaction process for $\Theta$ production.

There is an interference effect between two amplitudes, which is prominent for $1/2^+$ case and rather moderate for $3/2^-$ case.

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>$g_s$</th>
<th>$g^\nu$</th>
<th>$\sigma_{K^+}/\sigma_{\pi^-}$</th>
<th>$\text{Re}\Sigma_\Theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2^+$</td>
<td>1.59</td>
<td>-0.27</td>
<td>50</td>
<td>-78 MeV</td>
</tr>
<tr>
<td>$3/2^-$</td>
<td>0.104</td>
<td>0.209</td>
<td>3</td>
<td>-23 MeV</td>
</tr>
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