S-wave resonances in meson-baryon scattering induced by Weinberg-Tomozawa interaction

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Introduction: QCD at low energy

Quantum chromodynamics (QCD): strong interaction of quarks and gluons

At low energies...

Color confinement
Chiral symmetry breaking

Mesons, baryons (Hadrons)

“exotics”

: elementary excitations of QCD vacuum
Introduction: Exotics and hadron dynamics

\[
|B\rangle = |qqq\rangle + |qqq(q\bar{q})\rangle + \cdots
\]

Hadron structure ↔ meson-baryon dynamics

\[
|P\rangle = |qqqq\bar{q}\rangle + |qqqq\bar{q}(q\bar{q})\rangle + \cdots
\]

meson-baryon molecule
Introduction

Chiral unitary model and Λ(1405)

- Formulation of the model
- Application: estimation of coupling
- Two-pole structure of Λ(1405)
- Experimental verification

Bound states in SU(3) limit

- Weinberg-Tomozawa interaction
- Large Nc limit
- Bound states in exotic channels
Chiral unitary model and two-pole structure of the $\Lambda(1405)$

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Chiral unitary model and Λ(1405)

**Flavor SU(3) meson-baryon scatterings (s-wave)**

- **Chiral symmetry**
  - Low energy behavior

- **Unitarity of S-matrix**
  - Non-perturbative resummation

**Scattering amplitude**

\[ J^P = 1/2^- \] resonances

- Theoretical foundation based on chiral symmetry
- Analytically solvable

---> information of the complex energy plane
Chiral unitary model and $\Lambda(1405)$

Framework of the chiral unitary model: Interaction

Chiral perturbation theory

\[ \mathcal{L}_{WT} = \frac{1}{4f^2} \text{Tr}(\bar{B}i\gamma^{\mu}[(\Phi\partial_{\mu}\Phi - \partial_{\mu}\Phi\Phi), B]) \]

Chiral symmetry

\[ \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \]

SU(3) symmetry

\[ B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix} \]

Ex.)

\[ V^{(WT)}(\bar{K}N \rightarrow \bar{K}N, I = 0) = -\frac{3}{4f^2}(2\sqrt{s} - M_N - M_N) \sqrt{\frac{E_N + M_N}{2M_N}} \sqrt{\frac{E_N + M_N}{2M_N}} \]

\[ V^{(WT)}(\bar{K}N \rightarrow \pi\Sigma, I = 0) = \sqrt{\frac{3}{2}}\frac{1}{4f^2}(2\sqrt{s} - M_N - M_{\Sigma}) \sqrt{\frac{E_N + M_N}{2M_N}} \sqrt{\frac{E_{\Sigma} + M_{\Sigma}}{2M_{\Sigma}}} \]
Framework of the chiral unitary model: Unitarization

\[ T_{ij}^{-1}(\sqrt{s}) = \delta_{ij} \left( \tilde{a}_i(s_0) + \frac{s - s_0}{2\pi} \int_{s_i^+}^{\infty} ds' \frac{\rho_i(s')}{(s' - s)(s' - s_0)} \right) + T_{ij}^{-1} \]

\[ -G_i(\sqrt{s}) = -i \int \frac{d^4q}{(2\pi)^4} \frac{2M_i}{(P - q)^2 - M_i^2 + i\epsilon} \]

\[ T^{-1} = -G + \left( V^{(WT)} \right)^{-1} \]

\[ T = V^{(WT)} + V^{(WT)}GT \]

N/D method: general form of amplitude

physical masses
regularization of loop

Chiral unitary model and \( \Lambda(1405) \)
Chiral unitary model and Λ(1405)

Total cross sections of K⁻-p scattering

Resonance state

If there is a sufficient attraction, resonances can be dynamically generated.

\[ T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - M_R + i \Gamma_R / 2} \]

Position of the pole, Residues

\[ \Lambda(1670) \]

\[ z = 1690 - 22i \]

\[ |T| \]

\[ \text{Re}[z] \]

\[ \text{Im}[z] \]

\[ \sim \]

\[ \sim \]

Position of the pole, Residues

\[ \rightarrow \text{Mass, Width, Coupling strengths} \]
Application: Evaluation of coupling constant

Compare two amplitudes for the same process

ChU amplitude

Resonance dominance

\[ \overline{K}^* \pi N \rightarrow \Sigma^* \Sigma^* \]

\[ \Lambda (1520) \]

\[ g_{\Lambda^* \overline{K}^* N} S^\dagger \cdot \epsilon \]

\[ P_0 \sim M_{\Lambda^*} \]

\[ T_{\text{ChU}} \]

\[ \Rightarrow \]

---> Extract the coupling constants

• \( K^* N \Lambda (1520) \) coupling


• Magnetic moments


Hyodo, Nam, Jido, Hosaka, nucl-th/0305023
Two poles of the scattering amplitude were found around nominal Λ(1405) energy region.

- **Cloudy bag model**
- **Chiral unitary model**

Λ(1405) : superposition of two states?
Introduction: Λ(1405)

Λ(1405): $J^P = 1/2^-, I = 0$

- Mass: $1406.5 \pm 4.0$ MeV
- Width: $50 \pm 2$ MeV
- Decay mode: $\Lambda(1405) \rightarrow (\pi \Sigma)_{I=0} 100\%$

Quark model: p-wave, $\sim 1600$ MeV?

- N. Isgur, and G. Karl, PRD 18, 4187 (1978)

Coupled channel multi-scattering

- R.H. Dalitz, T.C. Wong and G. Rajasekaran PR 153, 1617 (1967)

Deeply bound Kaonic nuclei?

Chiral unitary model and $\Lambda(1405)$

$\Lambda(1405)$ in the chiral unitary model

Two poles? -> to be checked experimentally

$1426 - 16i \ (KN)$

$1390 - 66i \ (\pi \Sigma)$

$\pi \Sigma$ mass distribution

Shape of $\pi \Sigma$ spectrum depends on initial state
Chiral unitary model and Λ(1405)

Production reaction for the Λ(1405)

\[
\frac{d\sigma}{dM_I} = C |t_{\pi\Sigma \rightarrow \pi\Sigma}|^2 p_{CM} \quad \rightarrow \quad \frac{d\sigma}{dM_I} = |\sum_i C_i t_{i \rightarrow \pi\Sigma}|^2 p_{CM}
\]

In order to clarify the two-pole structure, we study two reactions.

\[
\pi^- p \rightarrow K^0 \Lambda(1405) \rightarrow K^0 \pi\Sigma
\]

- **Experimental result -> lower energy pole**


\[
\gamma p \rightarrow K^* \Lambda(1405) \rightarrow K^0 \pi^+ \pi\Sigma
\]

- **higher energy pole?**

Photoproduction of $K^*$ and $\Lambda(1405)$

Only $K^-p$ channel appears at the initial stage

$\rightarrow$ higher energy pole

$\Sigma(1385)$ is included $\leftarrow$ background estimation

Chiral unitary model and $\Lambda(1405)$
Isospin decomposition of final states

Since initial state is $\bar{K}N$, no $I=2$ component.

\[ \sigma(\pi^0\Sigma^0) \propto \frac{1}{3} |T^{(0)}|^2 \]

- Pure $I=0$ amplitude $\leftarrow\rightarrow \Lambda(1405)$

\[ \sigma(\pi^0\Lambda) \propto |T^{(1)}|^2 \]

- Pure $I=1$ amplitude $\leftarrow\rightarrow \Sigma(1385)$

\[ \sigma(\pi^{\pm}\Sigma^{\mp}) \propto \frac{1}{3} |T^{(0)}|^2 + \frac{1}{2} |T^{(1)}|^2 \pm \frac{2}{\sqrt{6}} \text{Re}(T^{(0)}T^{(1)*}) \]

- Mixture of $I=0$ and $I=1$
Invariant mass distributions

$P_{lab} = 2.5 \text{ GeV/c}$

$sqrt s \sim 2.35 \text{ GeV}$

$\pi^- p \rightarrow K^0 \pi \Sigma$

Thomas et al., (1973)

- dominance of higher pole -> different shape

Chiral unitary model and $\Lambda(1405)$
Invariant mass distributions 2

\[ P_{\text{lab}} = 2.5 \text{ GeV/c} \]
\[ \sqrt{s} \sim 2.35 \text{ GeV} \]

\[ \pi^- p \rightarrow K^0 \pi \Sigma \]

Thomas et al., (1973)

\[ \Sigma(1385) \text{ contribution is small} \]
In this model, $\Lambda(1405)$ is generated as a quasi-bound state in coupled channel meson-baryon scattering. Two poles of the scattering amplitude are found around the $\Lambda(1405)$ energy region.

**Conclusion for part I**

We study the **structure of $\Lambda(1405)$ using the chiral unitary model.**

- **Pole 1 (1426-16i)**: strongly couples to $\bar{K}N$ state
- **Pole 2 (1390-66i)**: strongly couples to $\pi\Sigma$ state
We propose the reaction $\gamma p \rightarrow K^* \Lambda(1405)$, which provides a different shape of spectrum from the nominal one. Observation of this feature give a support of the two-pole structure.

We estimate the effect of $\Sigma(1385)$ in $I=1$ channel, which is found to be small for the $\pi\Sigma$ spectrum.
s-wave exotic bound states in the SU(3) symmetric limit

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\textit{in preparation}...
Bound states in SU(3) limit

Pentaquark $\Theta$


$$|\Theta\rangle = |uudds\rangle$$

$S = +1$, manifestly exotic

light mass 1540 MeV

$$M_\Theta \sim 4m_{ud} + m_s \sim 1700 \text{ MeV}$$

narrow width < 15 MeV (1 MeV)

$$\Gamma_{B^*} \sim 100 \text{ MeV}$$

Experimental situation is still controversial

Γ $\sim$ 100 MeV
QCD does not forbid exotic states.

Effective models, lattice simulations, ...

Experimentally, (almost?) completely absent
--> highly non-trivial fact

We need theoretical explanation.

Does exotic states exist in the chiral unitary model?

--> In the simplest case: flavor SU(3) limit.
Bound states in SU(3) limit

**Weinberg-Tomozawa interaction**

**Coupling structure : chiral symmetry**

\[
V^{(WT)}_{\alpha\beta} = -\frac{1}{2f^2} C_\alpha (\sqrt{s} - M_\alpha) \frac{E_\alpha + M_\alpha}{2M_\alpha} \delta_{\alpha\beta}
\]

**Coupling strength : SU(3) symmetry**

\[
C_\alpha = -2 \langle [MT]_\alpha | F_M^\alpha F_T^\alpha | [MT]_\alpha \rangle = - [C(\alpha) - C(M) - C(T)]
\]

--> universal for any target particles

In SU(3) basis, channel coupling disappears.
Bound states in SU(3) limit

**Examples of C\(\alpha\): (positive is attractive)**

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>1</th>
<th>8</th>
<th>10</th>
<th>10</th>
<th>27</th>
<th>35</th>
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<tbody>
<tr>
<td>T=8</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>−2</td>
<td></td>
</tr>
<tr>
<td>T=10</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>−3</td>
<td></td>
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</tbody>
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<table>
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<th>6</th>
<th>15</th>
<th>24</th>
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</thead>
<tbody>
<tr>
<td>T=3</td>
<td>3</td>
<td>1</td>
<td>−1</td>
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<tr>
<td>T=6</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>−2</td>
</tr>
</tbody>
</table>
Bound states in SU(3) limit

**Coupling strengths in large Nc limit**

**Take large Nc limit**

\[ V_{\alpha\beta}^{(WT)} \sim -\frac{1}{2f^2} C_{\alpha} \omega_\alpha \delta_{\alpha\beta} \sim \frac{1}{N_c} \times C_{\alpha} \]

**Flavor representation**

\[ [p, q] \rightarrow [p, q + \frac{3 - N_c}{2}] \]

\[ C_\alpha = -2\langle [MT]_\alpha | F_M^a F_T^a | [MT]_\alpha \rangle = - [C(\alpha) - C(M) - C(T)] \]

\[ C([p, q + \frac{3 - N_c}{2}]) = \frac{1}{3} \left( \frac{-9}{4} + p^2 + \frac{3q}{2} + pq + q^2 \right) + \frac{1}{3} \left( p + \frac{q}{2} \right) N_c + \frac{N_c^2}{12} \]

Non-trivial Nc dependence
Bound states in SU(3) limit

**Coupling strengths in large Nc limit**

$C_\alpha$ in large Nc : (positive is attractive)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>1</th>
<th>8</th>
<th>10</th>
<th>10</th>
<th>27</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>T=8</td>
<td>$\frac{9}{2} + \frac{N_c}{2}$</td>
<td>3</td>
<td>0</td>
<td>$\frac{3}{2} - \frac{N_c}{2}$</td>
<td>$-\frac{1}{2} - \frac{N_c}{2}$</td>
<td></td>
</tr>
<tr>
<td>T=10</td>
<td>6</td>
<td>3</td>
<td>$\frac{5}{2} - \frac{N_c}{2}$</td>
<td>$-\frac{1}{2} - \frac{N_c}{2}$</td>
<td></td>
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</tr>
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<tr>
<th>$\alpha$</th>
<th>$\bar{3}$</th>
<th>6</th>
<th>$\bar{15}$</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>T=$\bar{3}$</td>
<td>3</td>
<td>1</td>
<td>$-\frac{N_c}{3}$</td>
<td></td>
</tr>
<tr>
<td>T=6</td>
<td>5</td>
<td>3</td>
<td>$\frac{5}{2} - \frac{N_c}{2}$</td>
<td>$\frac{1}{2} - \frac{5N_c}{6}$</td>
</tr>
</tbody>
</table>

Exotic attractions --> repulsions
Renormalization and bound states

\[ T = \frac{1}{1 - V G} V \]

Renormalization condition:

\[ \mathcal{G}(\mu) = 0, \quad \iff \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M \]


Condition is fixed by the mass of target.
It almost agrees with the natural value of cutoff.

Bound state:

\[ \Rightarrow \quad 1 - V(M_b)G(M_b) = 0 \quad M < M_b < M + m \]
Parameters for numerical analysis

Bound states in SU(3) limit

Mass of the target

<table>
<thead>
<tr>
<th>Target</th>
<th>Irrep.</th>
<th>M [MeV]</th>
</tr>
</thead>
<tbody>
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<td>light baryon</td>
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<tr>
<td></td>
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<td>1382</td>
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<td></td>
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<td>2534</td>
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<tr>
<td>D meson</td>
<td>3</td>
<td>1900</td>
</tr>
<tr>
<td>B meson</td>
<td>3</td>
<td>5309</td>
</tr>
</tbody>
</table>

Mass of NG boson : $m=368$ MeV
Meson decay constant : $f=93$ MeV
Bound states in SU(3) limit

**Numerical result for 1-VG**

\[ 1 - V(M_b)G(M_b) = 0 \]

No bound state for exotic channel:
Strength of the attraction in exotic channel is not enough to generate a bound state.
Critical attraction and applicability of the model

Since \( G(M) = 0 \) by renormalization, \( 1 - V(M)G(M) = 1 \) \( 1 - VG \) is monotonically decreasing.

--> Critical attraction : \( 1 - VG = 0 \) at \( M + m \)

\[
C_{\text{crit}} = -\frac{2f^2}{mG(M + m)}
\]

No appearance of artificial pole

--> Maximal attraction : \( 1 - VG = 0 \) at \( M - m \)

\[
C_{\text{Max}} = \frac{2f^2}{mG(M - m)}
\]

Physically meaningful bound state appears when

\[
C_{\text{crit}} < C < C_{\text{Max}}
\]
On the fate of the bound state

Plot of critical attraction.

Critical coupling vs. Baryon mass [MeV] for $m = 368$ MeV.
Bound states in SU(3) limit

On the fate of the bound state

It turns into virtual state.

Another state $V' \Leftarrow$ due to $V(M)=0$ ?
We study the exotics bound states in chiral unitary model in flavor SU(3) limit.

We give the general formula of coupling strength of WT interaction.

There are attractions in exotic channels, though the strength is week.

In large $N_c$ limit, these attractions turns into repulsive.
We give the critical and maximal attractions which determine the region of coupling constant for physically meaningful bound states. Attraction in exotic channel is beyond this region.

For the less attraction than the critical value, bound state becomes virtual state.