

Exotic Hadrons in s-Wave Chiral Dynamics



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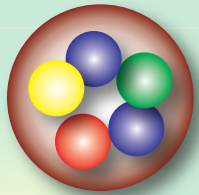
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2006, Dec. 22nd₁

Pentaquark Θ^+

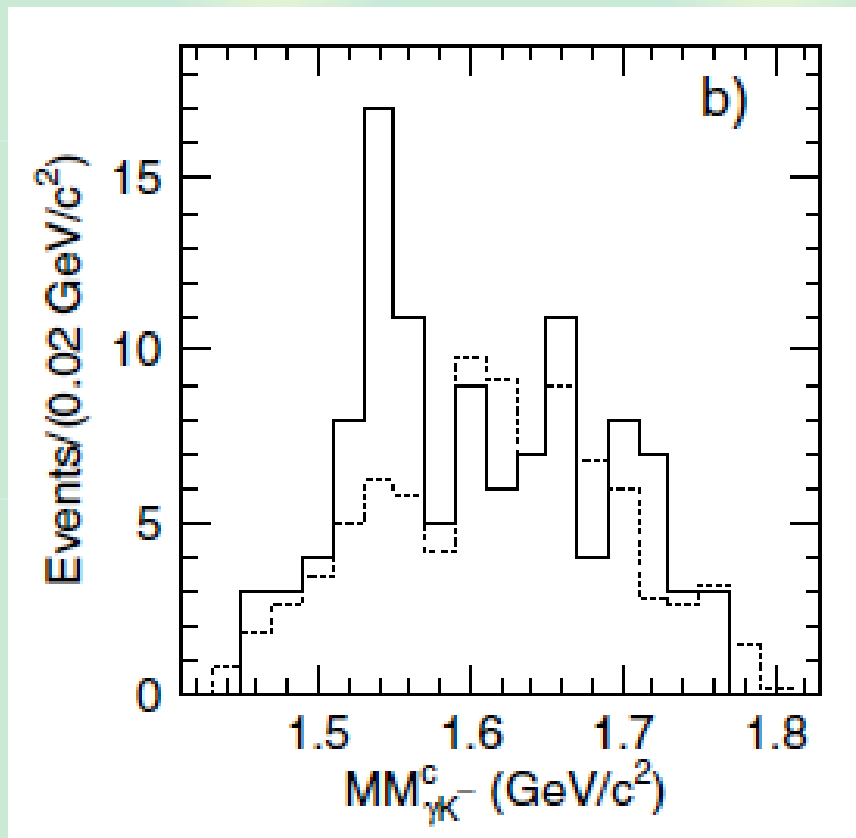
Θ^+ : $uudd\bar{s}$ state



$S = +1$

Candidate for a
manifestly exotic
hadron

$\gamma n \rightarrow K^- K^+ n$ in C

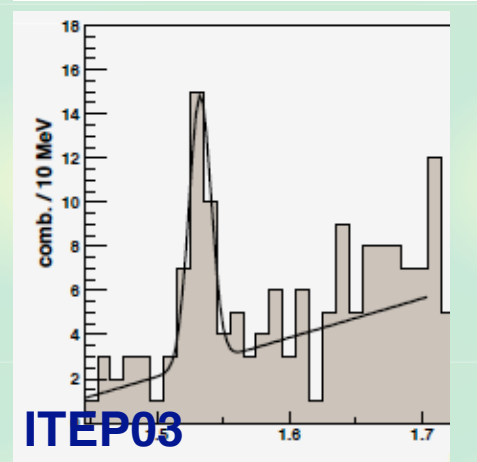
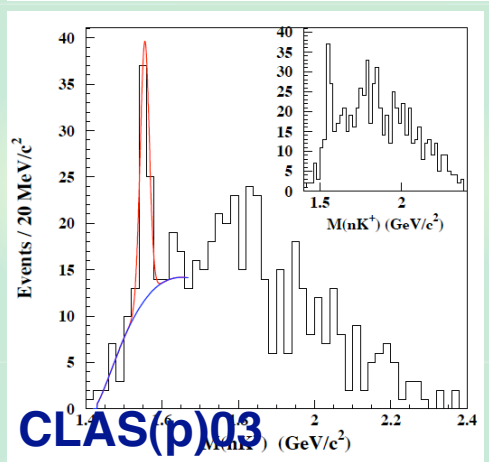
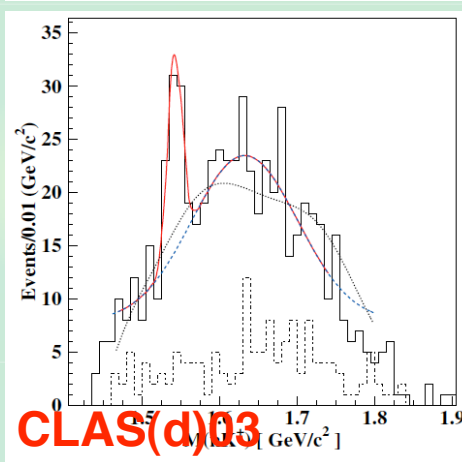
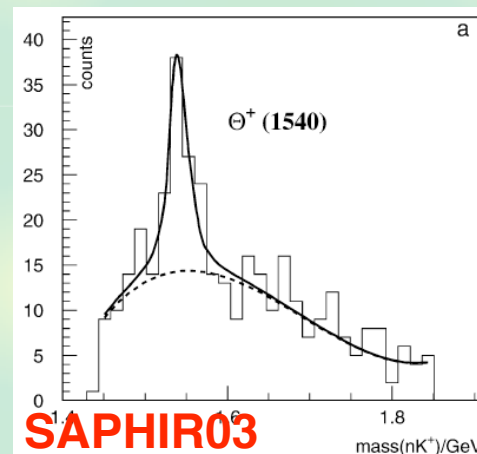
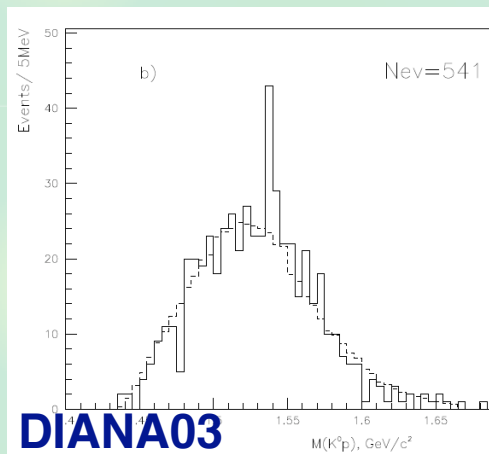
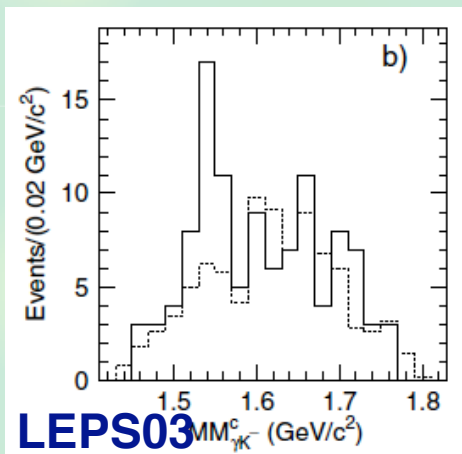


LEPS2003

-> Experimental situation?

Status of pentaquark Θ^+

2003 ~ 2004 : 6 positive results



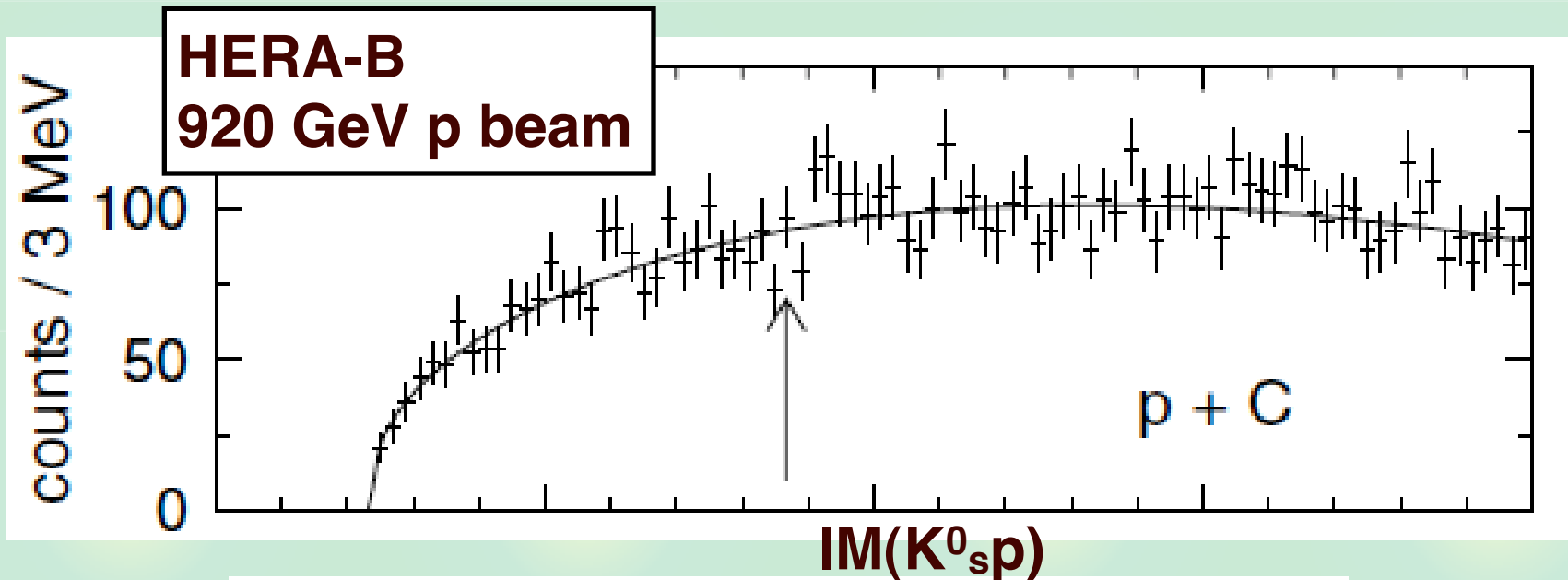
Citation: S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B 592, 1 (2004) (URL: <http://pdg.lbl.gov>)

$\Theta(1540)^+$

$I(J^P) = 0(?^?)$ Status: ***

Status of pentaquark Θ^+

~ 2005 : **12 positive** results,
7 negative (high-energy inclusive).



elman *et al.* (Particle Data Group), Phys. Lett. B 592, 1 (2004) and 2005 partial update for edition 2006 (UR)

$\Theta(1540)^+$

$I(J^P) = 0(?^?)$ Status: **

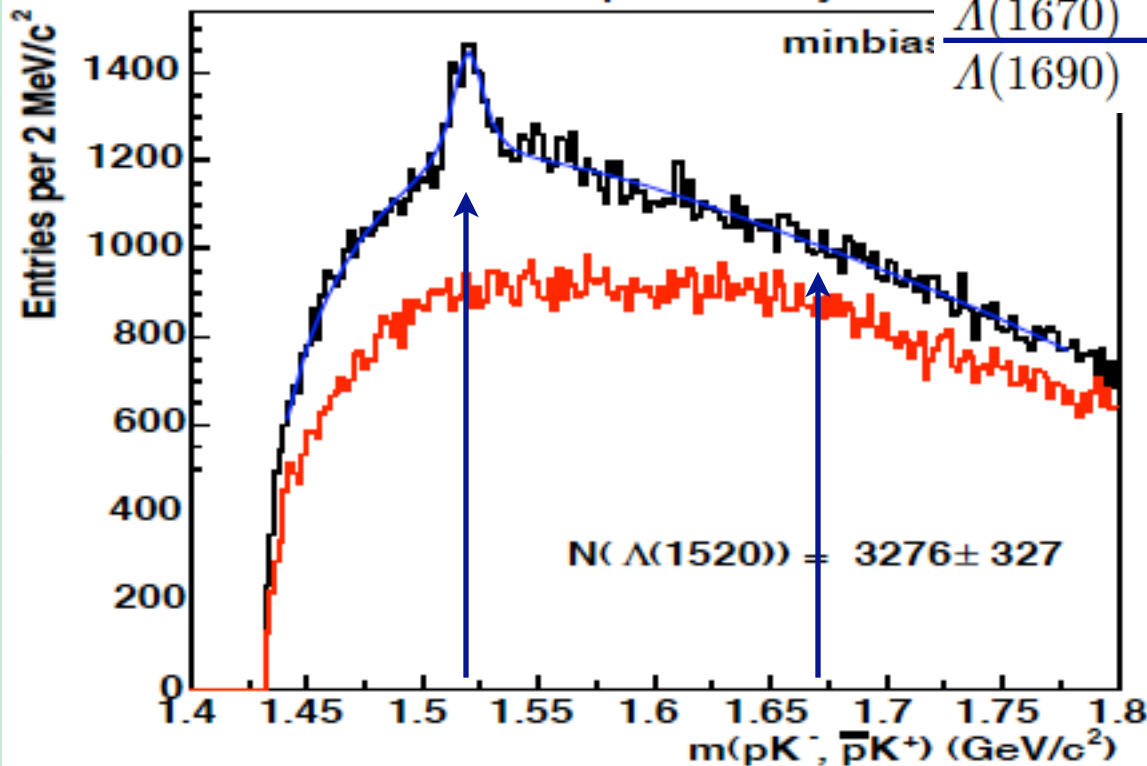
* limited number of resonances are observed in high energy exp.

Status of pentaquark Θ^+

What about $\Lambda(1670)$?

CDF
1.96 TeV $p\bar{p}$

CDF Run II preliminary



Particle	$L_{I,2J}$	Overall status	$N\bar{K}$
$\Lambda(1116)$	P_{01}	****	
$\Lambda(1405)$	S_{01}	****	****
$\Lambda(1520)$	D_{03}	****	****
$\Lambda(1600)$	P_{01}	***	***
$\Lambda(1670)$	S_{01}	****	****
$\Lambda(1690)$	D_{03}	****	****

$\Gamma = 15$ MeV

$\Gamma = 35$ MeV

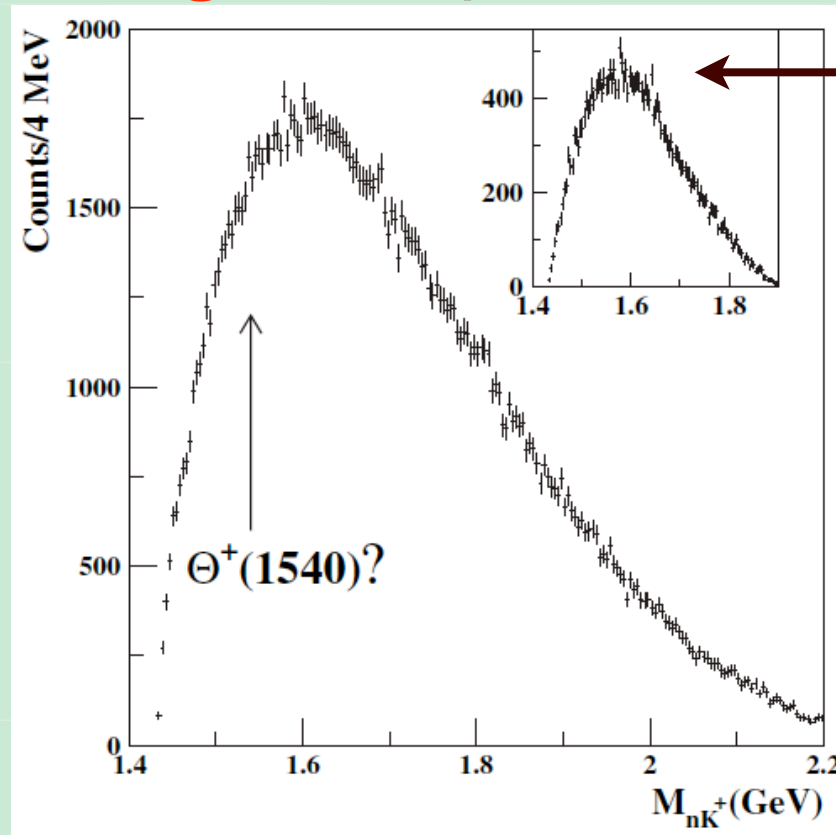
Same argument:
 $\Sigma(1670)$ in K^0p ?
 $\Gamma = 60$ MeV

Status of pentaquark Θ^+

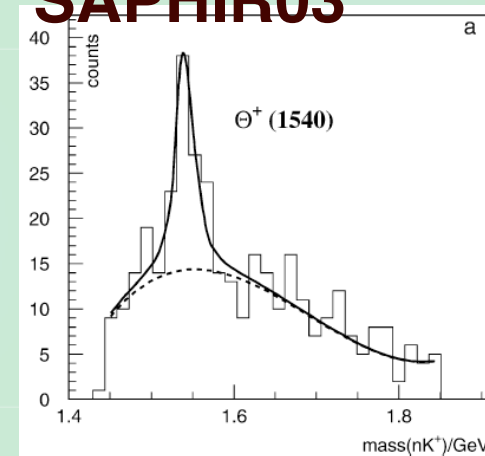
~ present : **14 positive** results,
13 negative (also in low energy)

CLAS06

$$\gamma p \rightarrow \bar{K}^0 K^+ n$$



Same
kinematical
cuts with
SAPHIRO3



Citation: W.-M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006) (URL: <http://pdg.lbl.gov>)

$\Theta(1540)^+$

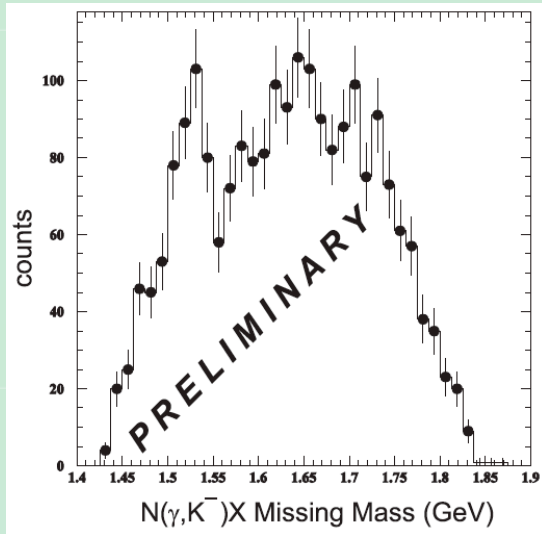
$I(J^P) = 0(??)$ Status: *

Status of pentaquark Θ^+

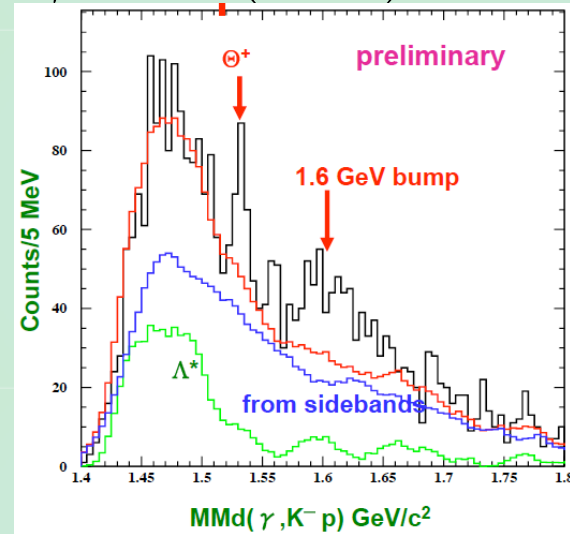
New positive(?) results:

LEPS

$$\gamma n \rightarrow K^- K^+ n \text{ in } d$$

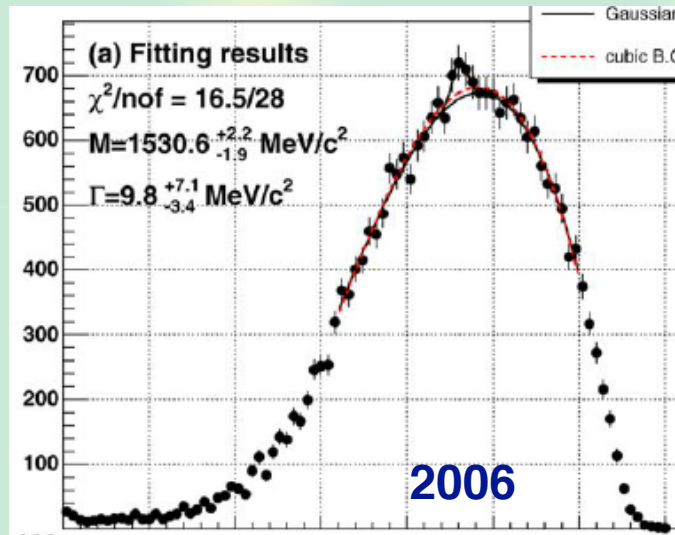


$$\gamma d \rightarrow \Lambda(1520) K N$$



KEK

$$\pi^- p \rightarrow K^- X$$



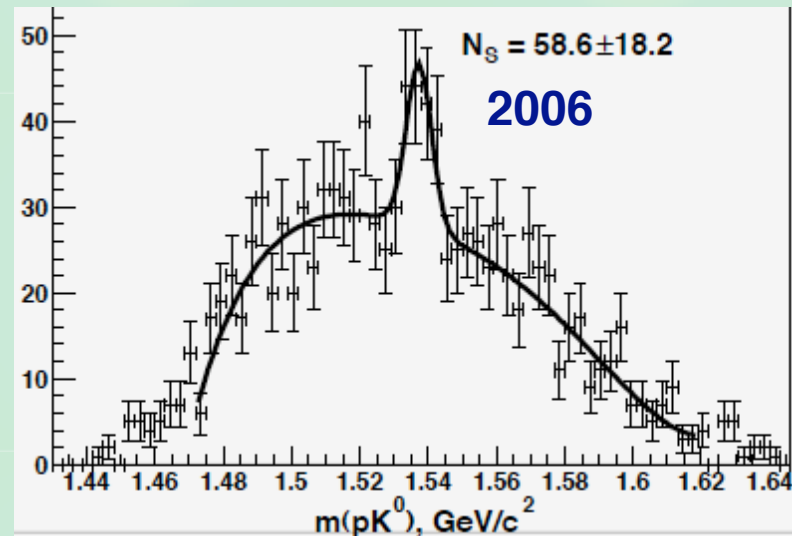
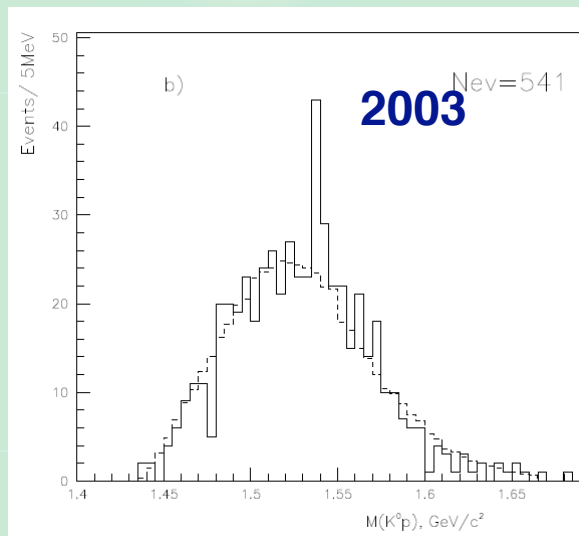
See Titov et al, PRC2006

Status of pentaquark Θ^+

Updated positive results:

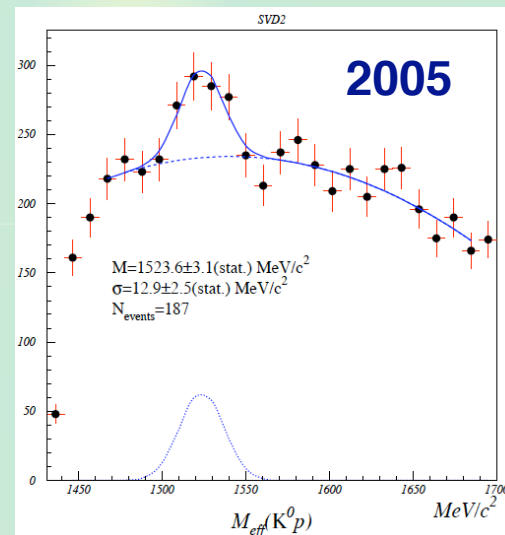
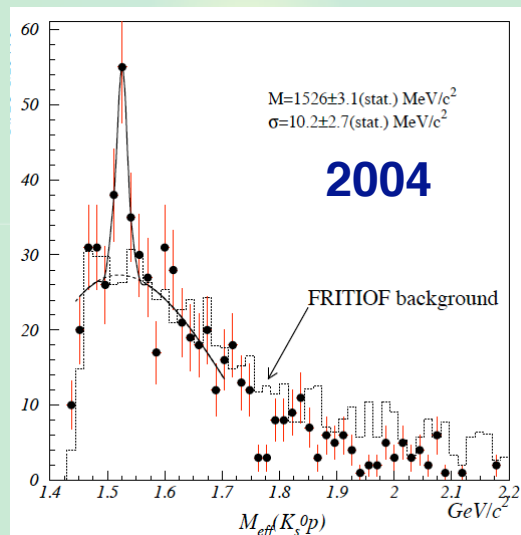
DIANA

$$K^+ n \rightarrow K^0 p \text{ in } Xe$$



SVD

$$pA \rightarrow pK_s^0 + X$$

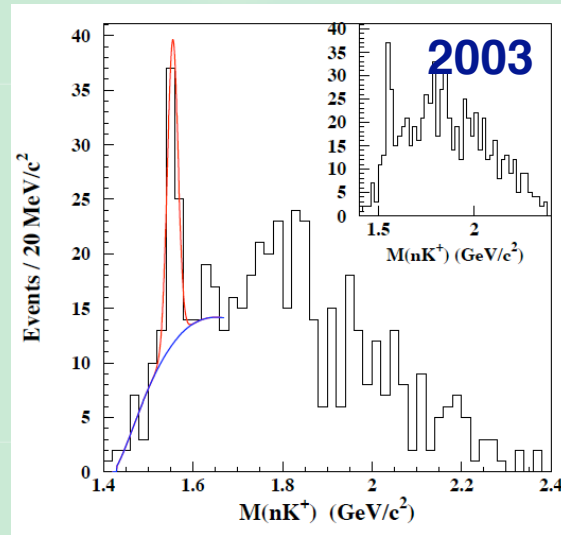


Status of pentaquark Θ^+

Remaining (not yet denied) positive results:

CLAS(p)

$$\gamma p \rightarrow \pi^+ K^- K^+ n$$

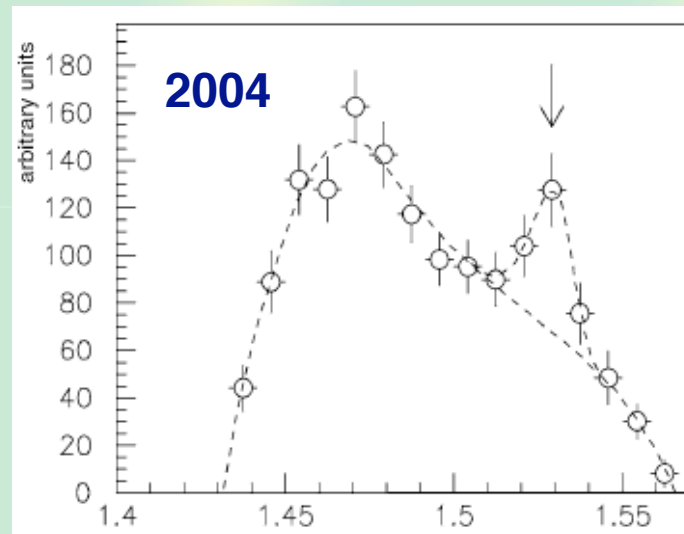


7 sigma!!

The experiment will be run in 2007

COSY-TOF

$$pp \rightarrow pK^0 \Sigma^+$$



4-6 sigma

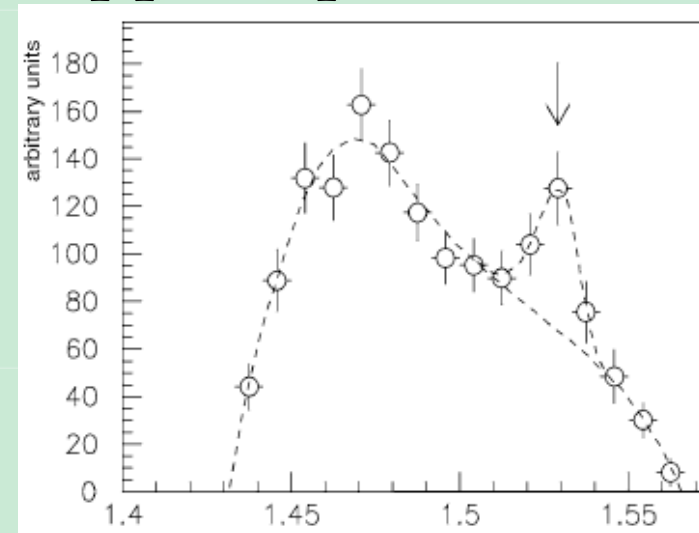
Large no. of events!!

YESTERDAY new data came up...

Status of pentaquark Θ^+

COSY new (hep-ex/0612048)

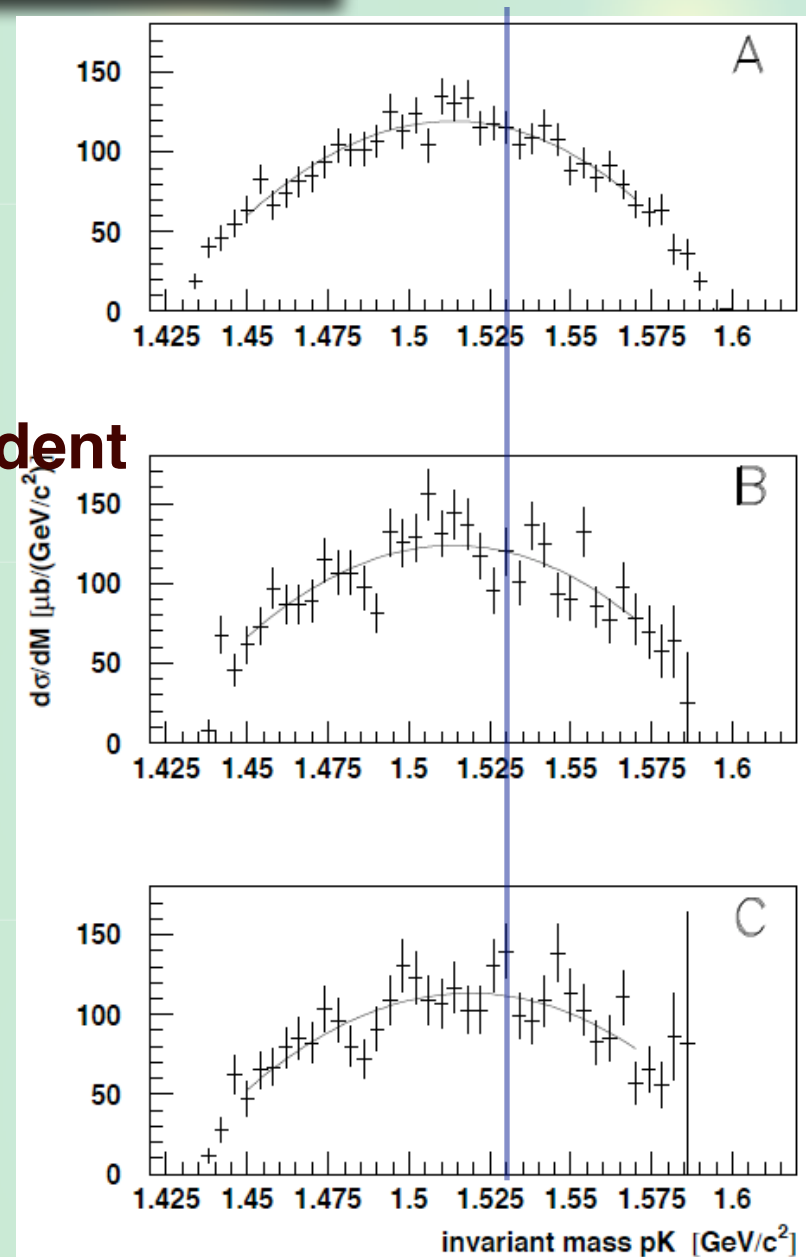
$$pp \rightarrow pK^0\Sigma^+$$



**new data
3 independent
analysis**

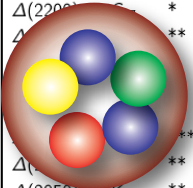
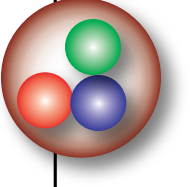
“Analysis does not confirm Θ^+ .”
cross section $\sigma < 0.3 \mu\text{b}$

**Beam energy slightly different
“A” does not tag Σ^+ decay
previous $\sigma = 0.4 \pm 0.1 \pm 0.1 \mu\text{b}$**



Exotic hadrons

Observed hadrons in experiments (PDG06) :

BARYONS				MESONS													
ρ	P_{11}	****	$\Delta(1232)$	P_{33}	****	Λ	P_{01}	****	Σ^+	P_{11}	****	Ξ^0	P_{11}	****	LIGHT UNFLAVORED	STRANGE	BOTTOM
n	P_{11}	****	$\Delta(1600)$	P_{33}	***	$\Lambda(1405)$	S_{01}	****	Σ^0	P_{11}	****	Ξ^-	P_{11}	****	$(S=C \neq B=0)$	$(S \neq \pm 1, C=B=0)$	$(B = \pm 1)$
$N(1440)$	P_{11}	****	$\Delta(1620)$	S_{31}	****	$\Lambda(1520)$	D_{03}	****	Σ^-	P_{11}	****	$\Xi(1530)$	P_{13}	****	$\rho^0(J^PC)$	$\rho^0(J^PC)$	$\rho^0(J^PC)$
$N(1520)$	D_{13}	****	$\Delta(1700)$	D_{33}	****	$\Lambda(1600)$	P_{01}	***	$\Sigma(1385)$	P_{13}	****	$\Xi(1620)$	*	*	$\pi^+(J^PC)$	$\pi^+(J^PC)$	$\pi^+(J^PC)$
$N(1535)$	S_{11}	****	$\Delta(1750)$	P_{31}	*	$\Lambda(1670)$	S_{01}	****	$\Sigma(1480)$	*	*	$\Xi(1690)$	***	*	$\pi^-(J^PC)$	$\pi^-(J^PC)$	$\pi^-(J^PC)$
$N(1650)$	S_{11}	****	$\Delta(1900)$	S_{31}	**	$\Lambda(1690)$	D_{03}	****	$\Sigma(1560)$	*	*	$\Xi(1820)$	D_{13}	***	$\eta(J^PC)$	$\eta(J^PC)$	$\eta(J^PC)$
$N(1675)$	D_{15}	****	$\Delta(1905)$	F_{35}	****	$\Lambda(1800)$	S_{01}	***	$\Sigma(1580)$	D_{13}	*	$\Xi(1950)$	*	***	$\omega(782)$	$\omega(782)$	$\omega(782)$
$N(1680)$	F_{15}	****	$\Delta(1910)$	P_{31}	****	$\Lambda(1810)$	P_{01}	***	$\Sigma(1620)$	S_{11}	**	$\Xi(2030)$	***	***	$\rho(770)$	$\rho(770)$	$\rho(770)$
$N(1700)$	D_{13}	***	$\Delta(1920)$	P_{33}	***	$\Lambda(1820)$	F_{05}	****	$\Sigma(1660)$	P_{11}	***	$\Xi(2120)$	*	*	$\rho(1700)$	$\rho(1700)$	$\rho(1700)$
$N(1710)$	P_{11}	***	$\Delta(1930)$	D_{35}	***	$\Lambda(1830)$	D_{05}	****	$\Sigma(1670)$	D_{13}	****	$\Xi(2250)$	**	**	$\omega(1720)$	$\omega(1720)$	$\omega(1720)$
$N(1720)$	P_{13}	****	$\Delta(1940)$	D_{33}	*	$\Lambda(1890)$	P_{03}	****	$\Sigma(1690)$	*	*	$\Xi(2370)$	**	**	$\eta(1770)$	$\eta(1770)$	$\eta(1770)$
$N(1900)$	F_{17}	**	$\Delta(2000)$	F_{37}	****	$\Lambda(2000)$	*	*	$\Sigma(1750)$	S_{11}	***	$\Xi(2500)$	*	*	$\eta(1295)$	$\eta(1295)$	$\eta(1295)$
$N(1990)$	F_{17}	**	$\Delta(2000)$	F_{35}	**	$\Lambda(2020)$	F_{07}	*	$\Sigma(1770)$	P_{11}	*	Ω^-	****	****	$\pi(1300)$	$\pi(1300)$	$\pi(1300)$
$N(2000)$	F_{15}	**	$\Delta(2150)$	S_{31}	*	$\Lambda(2100)$	G_{07}	****	$\Sigma(1775)$	D_{15}	****	$\Omega(2250)^-$	***	***	$\pi(1420)$	$\pi(1420)$	$\pi(1420)$
$N(2080)$	D_{13}	**	$\Delta(2200)$	*	*	$\Lambda(2110)$	F_{05}	***	$\Sigma(1840)$	P_{13}	*	$\Omega(2380)^-$	**	**	$\eta(1295)$	$\eta(1295)$	$\eta(1295)$
$N(2090)$	S_{11}	*	$\Delta(2200)$	**	**	$\Lambda(2325)$	D_{03}	***	$\Sigma(1880)$	P_{11}	**	$\Omega(2470)^-$	**	**	$\pi(1300)$	$\pi(1300)$	$\pi(1300)$
$N(2100)$	P_{11}	*	$\Delta(2200)$	**	**	$\Lambda(2350)$	H_{09}	***	$\Sigma(1915)$	F_{15}	****	Λ_c^+	****	****	$\pi(1450)$	$\pi(1450)$	$\pi(1450)$
$N(2190)$	G_{17}	****	$\Delta(2200)$	**	**	$\Lambda(2585)$	**	**	$\Sigma(1940)$	D_{13}	***	$\Lambda_c(2593)^+$	***	***	$\eta(1405)$	$\eta(1405)$	$\eta(1405)$
$N(2200)$	D_{15}	**	$\Delta(2200)$	**	**				$\Sigma(2000)$	S_{11}	*	$\Lambda_c(2625)^+$	***	***	$\eta(1420)$	$\eta(1420)$	$\eta(1420)$
$N(2220)$	H_{19}	****	$\Delta(2200)$	**	**				$\Sigma(2030)$	F_{17}	****	$\Lambda_c(2765)^+$	***	***	$f_1(1285)$	$f_1(1285)$	$f_1(1285)$
$N(2250)$	G_{19}	****	$\Delta(2200)$	**	**				$\Sigma(2070)$	F_{15}	***	$\Lambda_c(2880)^+$	**	**	$f_2(1270)$	$f_2(1270)$	$f_2(1270)$
$N(2600)$	$h_{1,11}$	***	$\Delta(2950)$	$K_{3,15}$	**				$\Sigma(2080)$	P_{13}	**	$\Sigma_c(2455)$	****	****	$\omega(1420)$	$\omega(1420)$	$\omega(1420)$
$N(2700)$	$K_{1,13}$	**	$\Theta(1540)^+$	*	*				$\Sigma(2100)$	G_{17}	*	$\Sigma_c(2520)$	***	***	$a_0(1450)$	$a_0(1450)$	$a_0(1450)$
<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>1</p> <p>286</p> <p>127 baryons</p> </div> <div style="text-align: center;">  <p>!</p> <p>159 mesons</p> </div> </div>																	

Exotic hadrons are indeed exotic !!

Motivation 1 : Exotic hadrons

Exotic hadrons : states other than $q\bar{q}$, qqq .
Experimentally, they are **exotic**.

PDG(2006) :

159 mesons 

127 baryons 

1 pentaquark  with *

Theoretically, are they exotic?

--> QCD does not forbid exotic states,
 effective models neither.

How exotic are they??

Motivation 2 : Chiral unitary approaches

Hadron excited states \sim 

- Interaction \leftarrow chiral symmetry
- Amplitude \leftarrow unitarity

R.H. Dalitz, and S.F. Tuan, *Ann. Phys. (N.Y.)* 10, 307 (1960)

J.H.W. Wyld, *Phys. Rev.* 155, 1649 (1967)

N. Kaiser, P. B. Siegel and W. Weise, *Nucl. Phys.* A594, 325 (1995)

E. Oset and A. Ramos, *Nucl. Phys.* A635, 99 (1998)

J. A. Oller and U. G. Meissner, *Phys. Lett.* B500, 263 (2001)

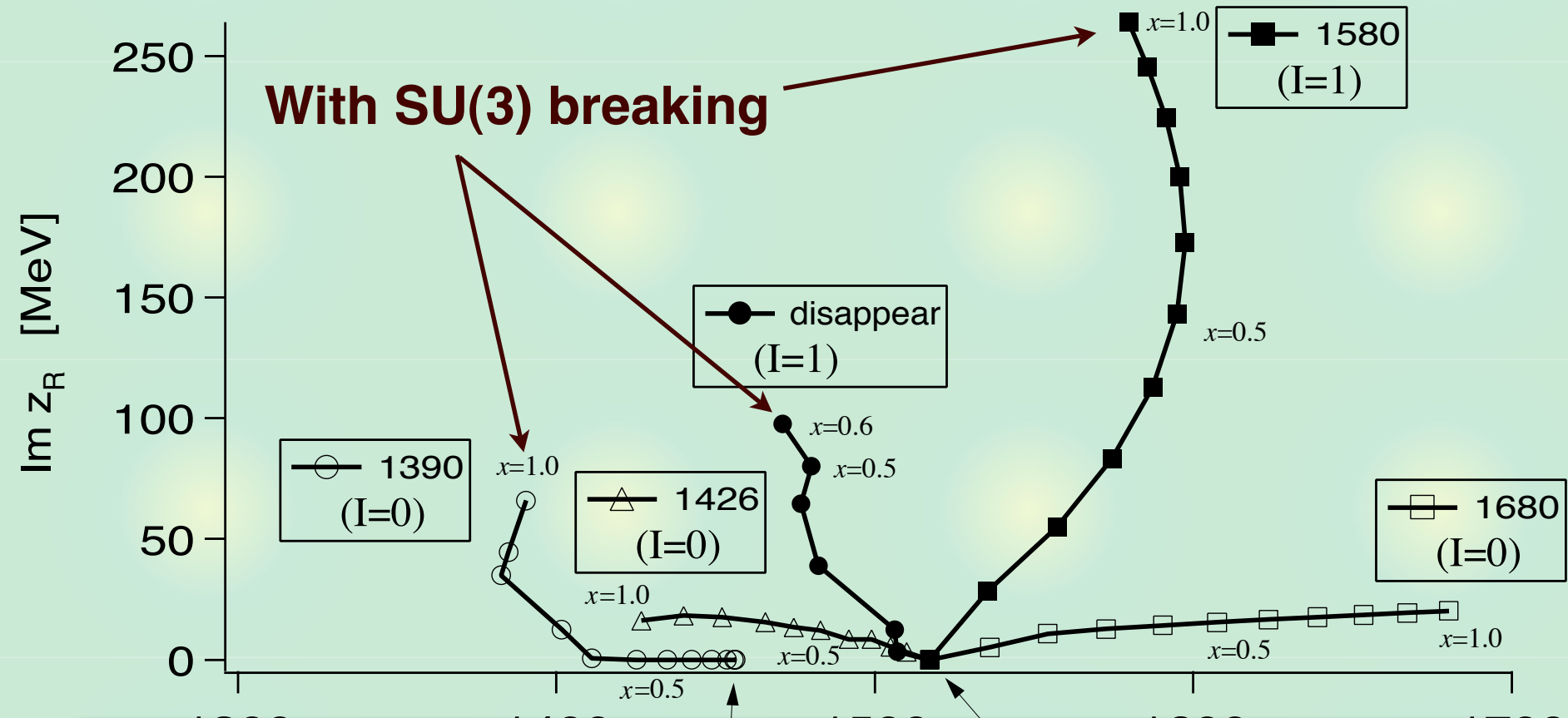
M.F.M. Lutz and E. E. Kolomeitsev, *Nucl. Phys.* A700, 193 (2002)

Many hadron resonances ($\Lambda(1405)$, $N(1535)$, $\Lambda(1520)$, $D_s(2317)$,...) are well described.

What about exotic hadrons?

Origin of the resonances

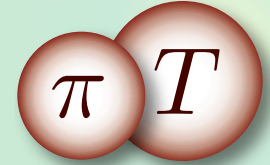
D. Jido, *et al.*, Nucl. Phys. A 723, 205 (2003)



--> Search for bound states in SU(3) symmetric limit.

Outline

Hadron-NG boson bound state



Chiral symmetry

s-wave low energy interaction

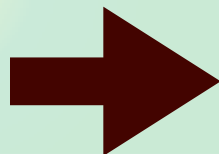
$$V_{\alpha} = -\frac{\omega}{2f^2} C_{\alpha,T} \quad C_{\text{exotic}} = 1$$

Scattering theory

Critical strength for a bound state

$$C_{\text{crit}} = \frac{2f^2}{m(-G(M_T + m))}$$

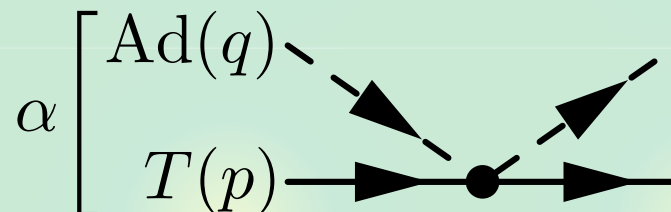
physical values : $C_{\text{exotic}} < C_{\text{crit}}$



No exotic state exists.

Low energy s-wave interaction

Scattering of a target (T) with the pion (Ad)

$$\alpha \left[\begin{array}{c} \text{Ad}(q) \\ T(p) \end{array} \right] = \frac{1}{f^2} \frac{p \cdot q}{2M_T} \langle \mathbf{F}_T \cdot \mathbf{F}_{\text{Ad}} \rangle_\alpha + \mathcal{O}((m/M_T)^2)$$


In s-wave,

$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}$$

- **proportional to pion energy**
- **pion decay constant (No LEC)**

Y. Tomozawa, *Nuovo Cim.* **46A**, 707 (1966)

S. Weinberg, *Phys. Rev. Lett.* **17**, 616 (1966)

$$C_{\alpha,T} \equiv -\langle 2\mathbf{F}_T \cdot \mathbf{F}_{\text{Ad}} \rangle_\alpha = C_2(T) - C_2(\alpha) + 3 \quad (\text{for } N_f = 3)$$

Coupling strengths : Examples

Examples of C_α : (positive is attractive)

$$C_{\alpha,T} = C_2(T) - C_2(\alpha) + 3$$

α	1	8	10	$\bar{10}$	27	35
T=8 (N, Λ, Σ, Ξ)	6	3	0	0	-2	
T=10(Δ, Σ^*, Ξ^*, Ω)		6	3		1	-3

α	$\bar{3}$	6	$\bar{15}$	24
T=$\bar{3}$ (Λ_c, Ξ_c)	3	1	-1	-2
T=6 (Σ_c, Ξ_c^*, Ω_c)	5	3	1	

- **Exotic channels** : mostly repulsive
- **Attractive interaction** : **C = 1**

Coupling strengths : General expression

$$T = [p, q] \quad \alpha \in [p, q] \otimes [1, 1]$$

α	$C_{\alpha, T}$	sign
$[p + 1, q + 1]$	$-p - q$	repulsive
$[p + 2, q - 1]$	$1 - p$	
$[p - 1, q + 2]$	$1 - q$	
$[p, q]$	3	attractive
$[p, q]$	3	attractive
$[p + 1, q - 2]$	$3 + q$	attractive
$[p - 2, q + 1]$	$3 + p$	attractive
$[p - 1, q - 1]$	$4 + p + q$	attractive

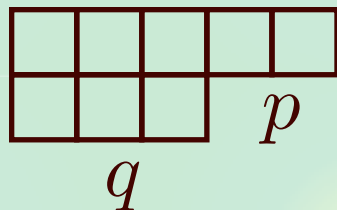
- **C should be integer.**
- **Sign is determined for most cases.**

Exoticness

Exoticness : minimal number of extra $\bar{q}q$.

For $[p, q]$ and baryon number B ,


$$E = \epsilon\theta(\epsilon) + \nu\theta(\nu)$$




$$\epsilon \equiv \frac{p + 2q}{3} - B, \quad \nu \equiv \frac{p - q}{3} - B$$

V. Kopeliovich, Phys. Lett. B259, 234 (1991)

D. Diakonov and V. Petrov, Phys. Rev. D 69, 056002 (2004)

but... $[p, q] = [6, 0] = \mathbf{28}, \quad B = 1$  $uuu \bar{u}\bar{d} \bar{u}\bar{d}$
 $E = 2, \quad \epsilon = 1$

E. Jenkins and A.V. Manohar, Phys. Rev. Lett. 93, 022001 (2004)

but... $[p, q] = [0, 0] = \mathbf{1}, \quad B = 1$  uds
 $E = 0, \quad \epsilon = -1, \quad \nu = -1$

Exotic channels

Consider α is more “exotic” than T

For $[p, q]$ and baryon number B ,

$$E = \epsilon\theta(\epsilon) + \nu\theta(\nu) \quad \epsilon \equiv \frac{p+2q}{3} - B, \quad \nu \equiv \frac{p-q}{3} - B$$

$\Delta E = E_\alpha - E_T = +1$ is realized when

○ $\Delta\epsilon = 1, \Delta\nu = 0, \epsilon_T \geq 0,$

$\alpha = [p+1, q+1] : C_{\alpha,T} = -p - q$ **repulsive**

○ $\Delta\epsilon = 0, \Delta\nu = 1, \nu_T \geq 0,$

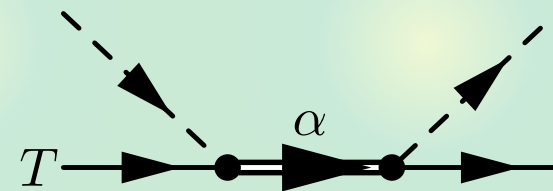
$\alpha = [p+2, q-1] : C_{\alpha,T} = 1 - p$

attraction : $p = 0$ then $\nu_T \geq 0 \rightarrow B \leq -q/3$ **not considered here**

○ $\Delta\epsilon = 1, \Delta\nu = -1, \nu_T \leq 0,$

$\alpha = [p-1, q+2] : C_{\alpha,T} = 1 - q$

attraction : $q = 0$ then $\nu_T \leq 0 \rightarrow B \geq p/3$ **OK!**



Universal attraction for more “exotic” channel

$$C_{\text{exotic}} = 1 \quad \text{for} \quad T = [p, 0], \quad \alpha = [p-1, 2]_{20}$$

Renormalization and bound states

Solve the scattering problem with $V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}$

$$T = \frac{1}{1 - VG} V$$

Unitarity : OK

Renormalization parameter : condition

$$G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T$$

K. Igi, and K. Hikasa, Phys. Rev. D59, 034005 (1999)

M.F.M. Lutz, and E. Kolomeitsev, Nucl. Phys. A700, 193-308 (2002)

Matching with the u-channel amplitude : OK

Bound state:

$$1 - V(M_b)G(M_b) = 0 \quad M_T < M_b < M_T + m_{21}$$

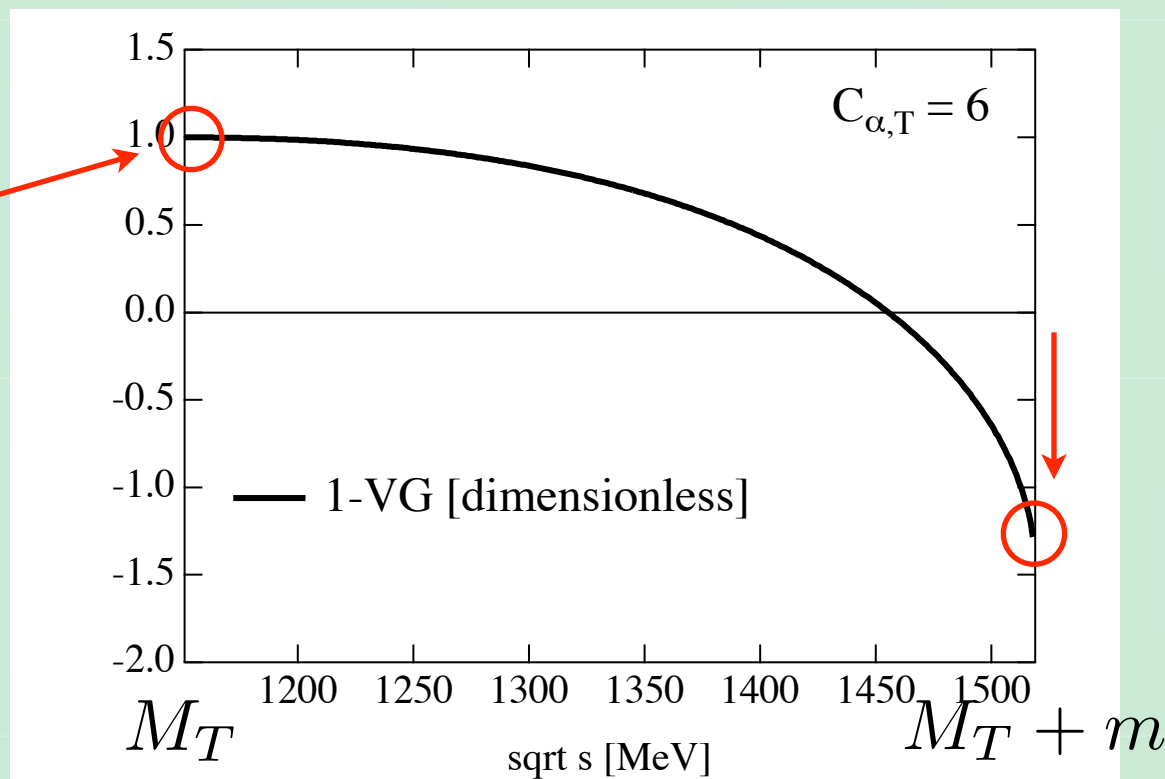
Critical attraction

$1 - V(\sqrt{s})G(\sqrt{s})$: monotonically decreasing.

Fixed

$$G(M_T) = 0$$

$$1 - VG = 1$$

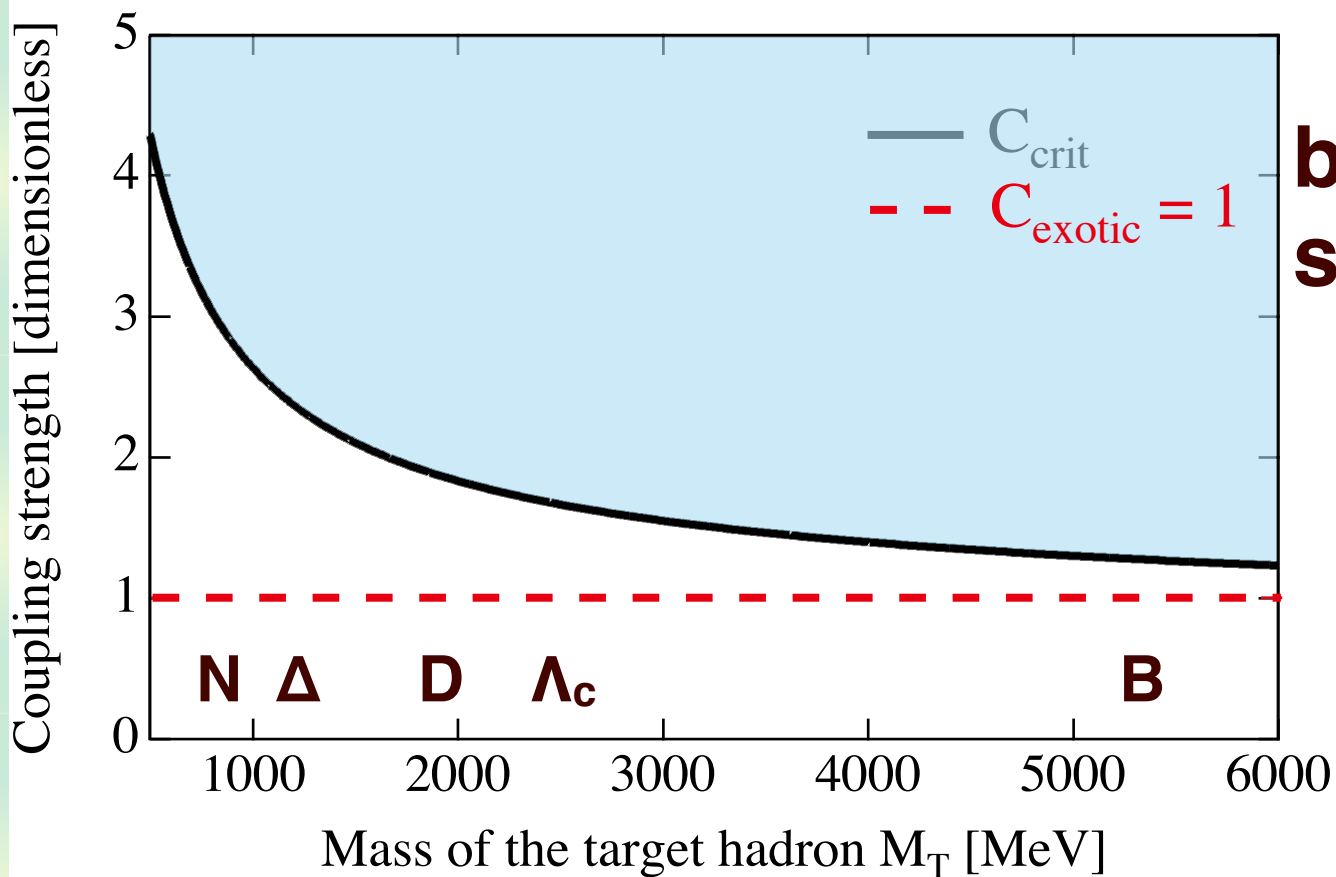


Critical attraction : $1 - VG = 0$ at $\sqrt{s} = M_T + m$

$$C_{\text{crit}} = \frac{2f^2}{m(-G(M_T + m))}$$

Critical attraction and exotic channel

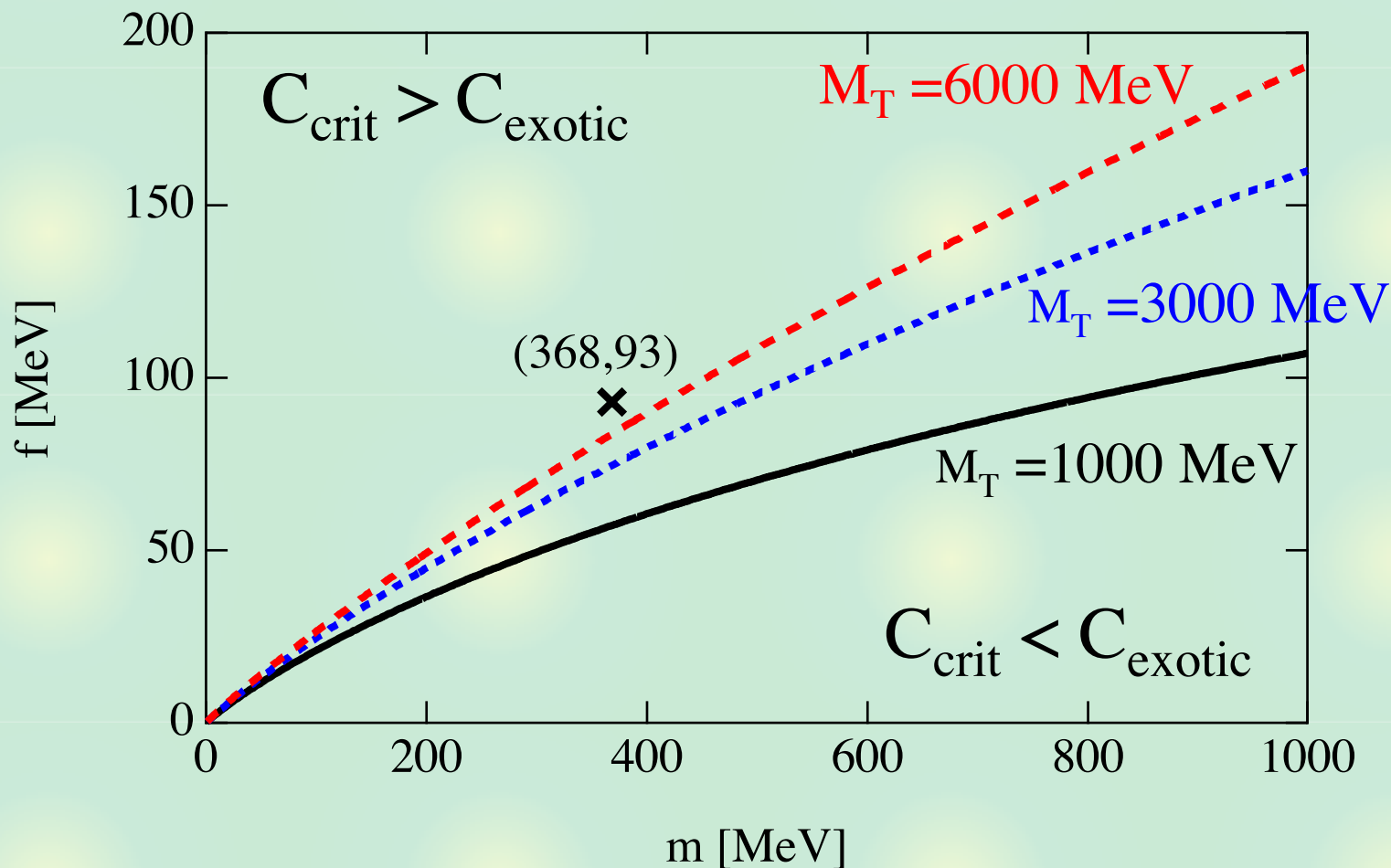
$$m = 368 \text{ MeV} \text{ and } f = 93 \text{ MeV}$$



➔ Strength is not enough.

Discussion : Dependence on the parameters

Lines for $C_{\text{crit}} = 1$ in (m, f) plane



- C_{crit} becomes smaller for $M_T \nearrow$, $m \nearrow$ and $f \searrow$.
- difficult to generate a bound state.

Coupling strengths in large N_c limit

In large N_c limit

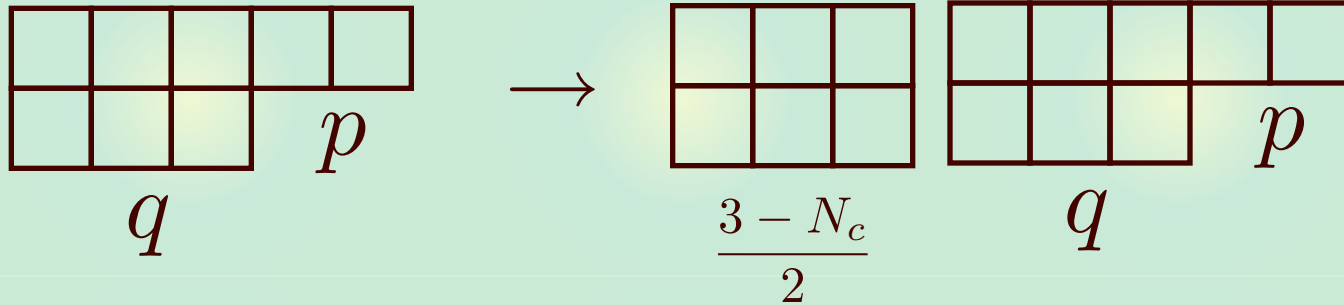
c.f. T.D. Cohen and R.F. Lebed
 Phys. Rev. D74, 056006 (2006)
 Praszalowicz, Talk at YKIS

$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T} \sim \frac{1}{N_c} \times C_{\alpha,T}$$

Flavor representation

$$[p, q] \rightarrow \left[p, q + \frac{3 - N_c}{2} \right]$$

$$C_2(T) - C_2(\alpha) + 3$$



$$C\left(\left[p, q + \frac{3 - N_c}{2}\right]\right) = \frac{1}{3} \left(-\frac{9}{4} + p^2 + \frac{3q}{2} + pq + q^2 \right) + \frac{1}{3} \left(p + \frac{q}{2} \right) N_c + \frac{N_c^2}{12}$$

Non-trivial N_c dependence

Coupling strengths in large N_c limit

C_α with arbitrary N_c : (positive is attractive)

α	“1”	“8”	“10”	“ $\bar{10}$ ”	“27”	“35”
$T=$ “8”	$\frac{9}{2} + \frac{N_c}{2}$	3	0	$\frac{3}{2} - \frac{N_c}{2}$	$-\frac{1}{2} - \frac{N_c}{2}$	
$T=$ “10”	$\Lambda(1405)$ two-pole?		3		$\frac{5}{2} - \frac{N_c}{2}$	$-\frac{1}{2} - \frac{N_c}{2}$

α	“ $\bar{3}$ ”	“6”	“ $\bar{15}$ ”	“24”
$T=$ “ $\bar{3}$ ”	3	1	$-\frac{N_c}{3}$	
$T=$ “6”	5	3	$\frac{5}{2} - \frac{N_c}{2}$	$\frac{1}{2} - \frac{5N_c}{6}$

Exotic attractions --> repulsions

Discussion 1 : large N_c behavior

For arbitrary N_c , $[p, q] \rightarrow \left[p, q + \frac{3 - N_c}{2} \right] \quad V \propto -\frac{1}{f^2} C \sim \frac{1}{N_c} C(N_c)$

α	$C^{\text{"}\alpha\text{"}, \text{"}T\text{"}}(N_c)$	$V(N_c \rightarrow \infty)$	ΔE
$[p + 1, q + 1]$	$(3 - N_c)/2 - p - q$	repulsive	1 or 0
$[p + 2, q - 1]$	$1 - p$	0	1 or 0
$[p - 1, q + 2]$	$(5 - N_c)/2 - q$	repulsive	1 or 0
$[p, q]$	3	0	0
$[p, q]$	3	0	0
$[p + 1, q - 2]$	$(3 + N_c)/2 + q$	attractive	0 or -1
$[p - 2, q + 1]$	$3 + p$	0	0 or -1
$[p - 1, q - 1]$	$(5 + N_c)/2 + p + q$	attractive	0 or -1

- **Exotic attraction --> repulsion**
- **No attraction in exotic channels.**




Summary 1 : SU(3) limit

We study the exotic bound states in s-wave chiral dynamics in flavor SU(3) limit.

- The interaction in exotic channels are in most cases **repulsive**.
- There are **attractions** in exotic channels, with **universal** and the smallest strength : $C_{\text{exotic}} = 1$
- This is **not enough** to generate a bound state : $C_{\text{exotic}} < C_{\text{crit}}$
- **No attractive interaction exists** in exotic channels in the large N_c limit.

Summary 2 : Physical world

Caution!

-  The exotic hadrons here are the **s-wave** meson-hadron molecule states ($1/2^-$ for Θ^+).
-  We do not exclude the exotics which have **other origins** (genuine quark state, soliton rotation,...)
-  In practice, **SU(3) breaking** effect, **higher order** terms,...

It is **difficult** to generate exotic hadrons as in the same way with $\Lambda(1405)$, $\Lambda(1520)$,... based on chiral dynamics.

[T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. Lett. 97, 192002 \(2006\)](#)

[T. Hyodo, D. Jido, A. Hosaka, hep-ph/0611004](#)