Exotic Hadrons in s-Wave Chiral Dynamics

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Exotic hadrons: states other than $q\bar{q}$, $qqq$. Experimentally, they are exotic. PDG(2006):

- 159 mesons
- 127 baryons
- 1 pentaquark

Theoretically, are they exotic? --> QCD does not forbid exotic states, effective models neither.

We would like to study the existence of exotic hadrons
Motivation 2: Chiral unitary approaches

Hadron excited states ~ \( \pi T \)

- Interaction <-- chiral symmetry
- Amplitude <-- unitarity


Many hadron resonances (\( \Lambda(1405) \), \( N(1535) \), \( \Lambda(1520) \), \( D_s(2317) \), ... ) are well described.

What about exotic hadrons?
-origin of the resonances


Introduction

With SU(3) breaking

\[
\begin{align*}
\text{1390} & \quad \text{(I=0)} \\
\text{1426} & \quad \text{(I=0)} \\
\text{1580} & \quad \text{(I=1)} \\
\text{1680} & \quad \text{(I=0)}
\end{align*}
\]

--- Search for bound states in SU(3) symmetric limit.
Outline

Chiral symmetry

s-wave low energy interaction

\[ V_\alpha = -\frac{\omega}{2f^2}C_{\alpha,T} \quad C_{\text{exotic}} = 1 \]

Scattering theory

Critical strength for a bound state

\[ C_{\text{crit}} = \frac{2f^2}{m(-G(M_T + m))} \]

physical values : \( C_{\text{exotic}} < C_{\text{crit}} \)

No exotic state exists in SU(3) limit.
Low energy s-wave interaction

Scattering of a target (T) with the pion (Ad)

\[ \alpha \left[ \begin{array}{c} \text{Ad}(q) \\ T(p) \end{array} \right] = \frac{1}{f^2} \frac{p \cdot q}{2M_T} \langle F_T \cdot F_{Ad} \rangle_\alpha + \mathcal{O} \left( \left( \frac{m}{M_T} \right)^2 \right) \]

In s-wave,

\[ V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T} \]

- proportional to pion energy
- pion decay constant (No LEC)


\[ C_{\alpha,T} \equiv -\langle 2F_T \cdot F_{Ad} \rangle_\alpha = C_2(T) - C_2(\alpha) + 3 \quad (\text{for } N_f = 3) \]
### Coupling strengths: Examples

Examples of $C_\alpha$: (positive is attractive)

\[ C_{\alpha,T} = C_2(T) - C_2(\alpha) + 3 \]

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>1</th>
<th>8</th>
<th>10</th>
<th>10</th>
<th>27</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T=8$ ($\Lambda, \Sigma, \Xi, \Omega$)</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>$T=10$ ($\Delta, \Sigma^<em>, \Xi^</em>, \Omega$)</td>
<td>6</td>
<td>3</td>
<td></td>
<td></td>
<td>1</td>
<td>-3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\bar{3}$</th>
<th>6</th>
<th>15</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T=\bar{3}$ ($\Lambda_c, \Xi_c$)</td>
<td>3</td>
<td>1</td>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>$T=6$ ($\Sigma_c, \Xi_c^*, \Omega_c$)</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

- **Exotic channels**: mostly repulsive
- **Attractive interaction**: $C = 1$
Coupling strengths : General expression

\[ T = [p, q] \quad \alpha \in [p, q] \otimes [1, 1] \]

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$C_{\alpha,T}$</th>
<th>sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[p + 1, q + 1]$</td>
<td>$-p - q$</td>
<td>repulsive</td>
</tr>
<tr>
<td>$[p + 2, q - 1]$</td>
<td>$1 - p$</td>
<td></td>
</tr>
<tr>
<td>$[p - 1, q + 2]$</td>
<td>$1 - q$</td>
<td></td>
</tr>
<tr>
<td>$[p, q]$</td>
<td>3</td>
<td>attractive</td>
</tr>
<tr>
<td>$[p, q]$</td>
<td>3</td>
<td>attractive</td>
</tr>
<tr>
<td>$[p + 1, q - 2]$</td>
<td>$3 + q$</td>
<td>attractive</td>
</tr>
<tr>
<td>$[p - 2, q + 1]$</td>
<td>$3 + p$</td>
<td>attractive</td>
</tr>
<tr>
<td>$[p - 1, q - 1]$</td>
<td>$4 + p + q$</td>
<td>attractive</td>
</tr>
</tbody>
</table>

- $C$ should be integer.
- Sign is determined for most cases.
Chiral symmetry

Exoticness

Exoticness : minimal number of extra $\bar{q}q$.

For $[p, q]$ and baryon number $B$, 

$$E = \epsilon \theta(\epsilon) + \nu \theta(\nu)$$

$$\epsilon \equiv \frac{p + 2q}{3} - B, \quad \nu \equiv \frac{p - q}{3} - B$$


but... $[p, q] = [6, 0] = 28, \quad B = 1$

$$E = 2, \quad \epsilon = 1$$


but... $[p, q] = [0, 0] = 1, \quad B = 1$

$$E = 0, \quad \epsilon = -1, \quad \nu = -1$$
Exotic channels

Consider $\alpha$ is more “exotic” than $T$

For $[p, q]$ and baryon number $B$,

$$E = \epsilon \theta(\epsilon) + \nu \theta(\nu) \quad \epsilon \equiv \frac{p + 2q}{3} - B, \quad \nu \equiv \frac{p - q}{3} - B$$

$\Delta E = E_\alpha - E_T = +1$ is realized when

- $\Delta \epsilon = 1, \Delta \nu = 0, \epsilon_T \geq 0,$
  $\alpha = [p + 1, q + 1] : C_{\alpha, T} = -p - q$ repulsive

- $\Delta \epsilon = 0, \Delta \nu = 1, \nu_T \geq 0,$
  $\alpha = [p + 2, q - 1] : C_{\alpha, T} = 1 - p$
  attraction: $p = 0$ then $\nu_T \geq 0 \rightarrow B \leq -q/3$ not considered here

- $\Delta \epsilon = 1, \Delta \nu = -1, \nu_T \leq 0,$
  $\alpha = [p - 1, q + 2] : C_{\alpha, T} = 1 - q$
  attraction: $q = 0$ then $\nu_T \leq 0 \rightarrow B \geq p/3$ OK!

Universal attraction for more “exotic” channel

$C_{\text{exotic}} = 1$ for $T = [p, 0], \quad \alpha = [p - 1, 2]$
Solve the scattering problem with \( V_\alpha = -\frac{\omega}{2f^2}C_{\alpha,T} \)

\[
T = \frac{1}{1 - VG} V
\]

Elastic unitarity : OK

Renormalization parameter : condition

\( G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T \)


Matching with the u-channel amplitude : OK

Bound state:

\[
1 - V(M_b)G(M_b) = 0 \quad M_T < M_b < M_T + m
\]
Critical attraction: \( 1 - V (\sqrt{s}) G(\sqrt{s}) \): monotonically decreasing.

\[ C_{\alpha,T} = \frac{2f^2}{m \left( -G(M_T + m) \right)} \]

Critical attraction: \( 1 - VG = 0 \) at \( \sqrt{s} = M_T + m \)
Scattering theory

Critical attraction and exotic channel

\[ m = 368 \text{ MeV} \text{ and } f = 93 \text{ MeV} \]

![Graph showing coupling strength against mass of the target hadron. The graph includes two lines: one for \( C_{\text{crit}} \) and another for \( C_{\text{exotic}} = 1 \). The graph indicates that the strength is not enough.](image)

Strength is not enough.
We study the exotic bound states in s-wave chiral dynamics in flavor SU(3) limit.

The interaction in exotic channels are in most cases repulsive.

There are attractions in exotic channels, with universal and the smallest strength:

\[ C_{\text{exotic}} = 1 \]

This is not enough to generate a bound state:

\[ C_{\text{exotic}} < C_{\text{crit}} \]
The exotic hadrons here are the s-wave meson-hadron molecule states (1/2- for $\Theta^+$).

We do not exclude the exotics which have other origins (genuine quark state, soliton rotation,...)

In practice, SU(3) breaking effect, higher order terms,...

It is difficult to generate exotic hadrons as in the same way with $\Lambda(1405)$, $\Lambda(1520)$,... based on chiral dynamics.

Possible exotic states 1: Genuine quark state

\[ q^2 \overline{q}^2 \]

Coupling of four-quark and meson-molecule

(a) Confined states

(b) Meson-meson

Potential

Channel radius \( r \)

Y. Kanada-En’yo @ YKIS06
Possibility of exotic states 2


**S = +1, l=1, KΔ resonance?**

27 plet in SU(3) : \( C_{\text{exotic}} = 1 \)

Large dependence on the input parameter (subtraction constant)


**S = +1, K*N bound state at 1.7-1.8 GeV**

SU(6) extension of the WT term --- valid for chiral mesons?
Possibility of exotic states 3


$S = 0, I = 3/2, \Delta(1700)$

$\Delta \pi, \Delta \eta, \Sigma^* K : 10, 27, 35$


Couplings of the generated resonance are very different from 10 assignment

--> dominated by 27 plet?

(Attractive) coupled channels in lower energy

--> resonance in exotic channel ??

$K \Theta N \quad K\pi N \quad K\Delta \quad K^* N$