Exotic hadrons in s-wave chiral dynamics

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Exotic hadrons

Observed hadrons in experiments (PDG06):

- ~160 mesons
- ~130 baryons

Exotic hadrons are indeed exotic!!
Motivation 1: Exotic hadrons

Exotic hadrons: more than 4 valence quarks
- non-exotic: $uds, \ u\bar{d}, \ uds\ u\bar{u}, \ u\bar{d}\ u\bar{u}, \ldots$
- exotic (in this talk): $uudd\bar{s}, \ ud\bar{s}\bar{s}, \ldots$
- not considered: $uuddss, \ c\bar{c}g, \ \bar{u}\bar{u}\bar{d}\bar{d}s, \ldots$

Experimentally, they are exotic $\sim 1/300$.

Theoretically, are they exotic?
- There is no simple way to forbid exotic states in QCD, effective models, ...
- Evidences of multiquark components in non-exotic hadrons.

Why aren’t the exotics observed??
**Motivation 2: Chiral unitary approaches**

**Introduction**

Hadron excited states ~ \( \pi T \)

- Interaction <-- chiral symmetry
- Amplitude <-- unitarity


Many hadron resonances (\( \Lambda(1405) \), \( N(1535) \), \( \Lambda(1520) \), \( D_s(2317) \),...) are well described.

What about exotic hadrons?
Trajectory of poles

With SU(3) breaking: resonances

--> Search for bound states in SU(3) symmetric limit.
Outline

Hadron-NG boson bound state

Chiral Symmetry

s-wave low energy interaction

\[ V_\alpha = -\frac{\omega}{2f^2}C_{\alpha,T} \quad C_{\text{exotic}} = 1 \]

Scattering theory

Critical strength for a bound state

\[ C_{\text{crit}} = \frac{2f^2}{m[-G(M_T + m)]} \]

physical values: \[ C_{\text{exotic}} < C_{\text{crit}} \]

No exotic state exists.
Chiral symmetry

Low energy s-wave interaction

Scattering of a target (T) with the pion (Ad)

\[
\alpha \left[ \begin{array}{c}
\text{Ad}(q) \\
T(p) 
\end{array} \right] = \frac{1}{f^2} \frac{p \cdot q}{2M_T} \langle F_T \cdot F_{Ad} \rangle_\alpha + O \left( \left( \frac{m}{M_T} \right)^2 \right)
\]

s-wave : Weinberg-Tomozawa term

\[
V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}
\]

\[
C_{\alpha,T} \equiv -\langle 2F_T \cdot F_{Ad} \rangle_\alpha = C_2(T) - C_2(\alpha) + 3 \quad (\text{for } N_f = 3)
\]

Coupling : pion decay constant

model-independent interaction at low energy

Coupling strengths : Examples

Coupling strengths : (positive is attractive)

\[ C_{\alpha,T} = C_2(T) - C_2(\alpha) + 3 \]

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>1</th>
<th>8</th>
<th>10</th>
<th>10</th>
<th>27</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T = 8(N, \Lambda, \Sigma, \Xi) )</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>( T = 10(\Delta, \Sigma^<em>, \Xi^</em>, \Omega) )</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>-3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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<th>( \alpha )</th>
<th>3</th>
<th>6</th>
<th>15</th>
<th>24</th>
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<tbody>
<tr>
<td>( T = 3(\Lambda_c, \Xi_c) )</td>
<td>3</td>
<td>1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>( T = 6(\Sigma_c, \Xi^*_c, \Omega_c) )</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>

- Exotic channels : mostly repulsive
- Attractive interaction : \( C = 1 \)
## Chiral symmetry

### Coupling strengths : General expression

#### For a general target

$$T = [p, q]$$

<table>
<thead>
<tr>
<th>$\alpha \in [p, q] \otimes [1, 1]$</th>
<th>$C_{\alpha, T}$</th>
<th>sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[p + 1, q + 1]$</td>
<td>$-p - q$</td>
<td>repulsive</td>
</tr>
<tr>
<td>$[p + 2, q - 1]$</td>
<td>$1 - p$</td>
<td></td>
</tr>
<tr>
<td>$[p - 1, q + 2]$</td>
<td>$1 - q$</td>
<td></td>
</tr>
<tr>
<td>$[p, q]$</td>
<td>$3$</td>
<td>attractive</td>
</tr>
<tr>
<td>$[p, q]$</td>
<td>$3$</td>
<td>attractive</td>
</tr>
<tr>
<td>$[p + 1, q - 2]$</td>
<td>$3 + q$</td>
<td>attractive</td>
</tr>
<tr>
<td>$[p - 2, q + 1]$</td>
<td>$3 + p$</td>
<td>attractive</td>
</tr>
<tr>
<td>$[p - 1, q - 1]$</td>
<td>$4 + p + q$</td>
<td>attractive</td>
</tr>
</tbody>
</table>

- **Strength should be integer.**
- **Sign is determined for most cases.**
Exotic channels

Exoticness: minimal number of extra $\bar{q}q$.

$$E = \epsilon \theta(\epsilon) + \nu \theta(\nu)$$

$$\epsilon \equiv \frac{p + 2q}{3} - B, \quad \nu \equiv \frac{p - q}{3} - B$$

$$\Delta E = E_\alpha - E_T = +1$$ is realized when

- $\alpha = [p + 1, q + 1] : C_{\alpha,T} = -p - q$
  - repulsive

- $\alpha = [p + 2, q - 1] : C_{\alpha,T} = 1 - p$
  - attraction: $p = 0$ then $\nu_T \geq 0 \rightarrow B \geq -q/3$

- $\alpha = [p - 1, q + 2] : C_{\alpha,T} = 1 - q$
  - attraction: $q = 0$ then $\nu_T \leq 0 \rightarrow B \geq p/3$ \textbf{OK!}

Universal attraction for more "exotic" channel

$$C_{\text{exotic}} = 1 \quad \text{for} \quad T = [p, 0], \quad \alpha = [p - 1, 2]$$
Scattering theory

Renormalization and bound states

Solve the scattering problem with $V_\alpha = -\frac{\omega}{2f^2}C_{\alpha,T}$

$$T = \frac{1}{1 - VG}V$$

Unitarity : OK

Renormalization parameter : condition

$$G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T$$


Scale at which ChPT works.
Matching with the u-channel amplitude : OK

Bound state:

$$1 - V(M_b)G(M_b) = 0 \quad \text{at} \quad M_T < M_b < M_T + m$$
Critical attraction

$1 - V(\sqrt{s})G(\sqrt{s})$ : monotonically decreasing.

Critical attraction:

$1 - VG = 0$ at $\sqrt{s} = M_T + m$

$C_{\text{crit}} = \frac{2f^2}{m \left[ -G(M_T + m) \right]}$
Critical attraction and exotic channel

Mass of the target hadron $M_T$ [MeV]

$C_{\text{crit}}$

$C_{\text{exotic}} = 1$

$m = 368$ MeV and $f = 93$ MeV

Strength is not enough.
Summary 1 : SU(3) limit

We study the exotic bound states in s-wave chiral dynamics in flavor SU(3) limit.

The interaction in exotic channels is in most cases **repulsive**.

There are **attractive interactions** in exotic channels, with **universal** and the smallest strength : \( C_{\text{exotic}} = 1 \)

The strength is **not enough** to generate a bound state : \( C_{\text{exotic}} < C_{\text{crit}} \)

The result is **model independent** as far as we respect chiral symmetry.
Caution!

The exotic hadrons here are the s-wave meson-hadron molecule states (1/2⁻ for Θ⁺).

We do not exclude the exotics which have other origins (genuine quark state, soliton rotation,...).

In practice, SU(3) breaking effect, higher order terms,...

In Nature, it is difficult to generate exotic hadrons as in the same way with Λ(1405), Λ(1520),... based on chiral interaction.