Exotic hadrons
in s-wave chiral dynamics

Tetsuo Hyodo\textsuperscript{a},
Daisuke Jido\textsuperscript{a}, and Atsushi Hosaka\textsuperscript{b}
\textit{YITP, Kyoto}\textsuperscript{a} \textit{RCNP, Osaka}\textsuperscript{b}

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Exotic hadrons

Observed hadrons in experiments (PDG06):

~130 baryons

~160 mesons

Exotic hadrons are indeed exotic!!
Motivation 1: Exotic hadrons

Exotic hadrons: more than 4 valence quarks

- non-exotic
  \[ uds, \quad ud\bar{d}, \quad udsu\bar{u}, \quad ud\bar{u}u\bar{u}, \ldots \]

- exotic (in this talk)
  \[ uu\bar{u}dd\bar{s}, \quad ud\bar{d}s\bar{s}, \ldots \]

- not considered
  \[ uu\bar{u}ddss, \quad c\bar{c}g, \quad \bar{u}\bar{u}\bar{d}\bar{d}s, \ldots \]

Experimentally, they are exotic \( \sim 1/300 \).

Theoretically, are they exotic?

- There is no simple way to forbid exotic states in QCD, effective models, ...

- Evidences of multiquark components in non-exotic hadrons.

Why aren’t the exotics observed??
Motivation 2 : Chiral unitary approaches

Hadron excited states ~ \( \pi T \)

- Interaction <-- chiral symmetry
- Amplitude <-- unitarity


Many hadron resonances (\( \Lambda(1405), N(1535), \Lambda(1520), D_s(2317), \ldots \)) are well described.

What about exotic hadrons?
Origin of the resonances


Trajectory of poles

With SU(3) breaking:
- resonances

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Search for bound states in SU(3) symmetric limit.
s-wave low energy interaction

\[ V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T} \]

Critical strength for a bound state

\[ C_{\text{crit}} = \frac{2f^2}{m[-G(M_T + m)]} \]

physical values: \( C_{\text{exotic}} < C_{\text{crit}} \)

No exotic state exists.
Chiral symmetry

Low energy s-wave interaction

Scattering of a target (T) with the pion (Ad)

\[ \alpha \left[ \begin{array}{c} \text{Ad}(q) \\ \text{T}(p) \end{array} \right] = \frac{1}{f^2} \frac{p \cdot q}{2M_T} \langle F_T \cdot F_{Ad} \rangle_\alpha + O \left( \left( \frac{m}{M_T} \right)^2 \right) \]

s-wave : Weinberg-Tomozawa term

\[ V_\alpha = -\frac{\omega}{2f^2 C_{\alpha,T}} \]

\[ C_{\alpha,T} \equiv -\langle 2F_T \cdot F_{Ad} \rangle_\alpha = C_2(T) - C_2(\alpha) + 3 \quad \text{(for } N_f = 3) \]

Coupling : pion decay constant

model-independent interaction at low energy

Chiral symmetry

Coupling strengths : Examples

Coupling strengths : (positive is attractive)

\[ C_{\alpha,T} = C_2(T) - C_2(\alpha) + 3 \]

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>1</th>
<th>8</th>
<th>10</th>
<th>10</th>
<th>27</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T = 8(N, \Lambda, \Sigma, \Xi) )</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>−2</td>
<td></td>
</tr>
<tr>
<td>( T = 10(\Delta, \Sigma^<em>, \Xi^</em>, \Omega) )</td>
<td>6</td>
<td>3</td>
<td></td>
<td>1</td>
<td>−3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \bar{3} )</th>
<th>6</th>
<th>15</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T = \bar{3}(\Lambda_c, \Xi_c) )</td>
<td>3</td>
<td>1</td>
<td>−1</td>
<td></td>
</tr>
<tr>
<td>( T = 6(\Sigma_c, \Xi^*_c, \Omega_c) )</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>−2</td>
</tr>
</tbody>
</table>

- Exotic channels : mostly repulsive
- Attractive interaction : \( C = 1 \)
**Coupling strengths : General expression**

For a general target \( T = [p, q] \)

<table>
<thead>
<tr>
<th>( \alpha \in [p, q] \otimes [1, 1] )</th>
<th>( C_{\alpha, T} )</th>
<th>Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>([p + 1, q + 1])</td>
<td>(-p - q)</td>
<td>repulsive</td>
</tr>
<tr>
<td>([p + 2, q - 1])</td>
<td>(1 - p)</td>
<td></td>
</tr>
<tr>
<td>([p - 1, q + 2])</td>
<td>(1 - q)</td>
<td></td>
</tr>
<tr>
<td>([p, q])</td>
<td>(3)</td>
<td>attractive</td>
</tr>
<tr>
<td>([p, q])</td>
<td>(3)</td>
<td>attractive</td>
</tr>
<tr>
<td>([p + 1, q - 2])</td>
<td>(3 + q)</td>
<td>attractive</td>
</tr>
<tr>
<td>([p - 2, q + 1])</td>
<td>(3 + p)</td>
<td>attractive</td>
</tr>
<tr>
<td>([p - 1, q - 1])</td>
<td>(4 + p + q)</td>
<td>attractive</td>
</tr>
</tbody>
</table>

- Strength should be integer.
- Sign is determined for most cases.
Chiral symmetry

**Exotic channels**

**Exoticness : minimal number of extra $\bar{q}q$.**

$$E = \epsilon \theta(\epsilon) + \nu \theta(\nu) \quad \epsilon \equiv \frac{p + 2q}{3} - B, \quad \nu \equiv \frac{p - q}{3} - B$$

$$\Delta E = E_\alpha - E_T = +1$$ is realized when

- $\alpha = [p + 1, q + 1] : C_{\alpha,T} = -p - q$
  - Repulsive

- $\alpha = [p + 2, q - 1] : C_{\alpha,T} = 1 - p$
  - Attraction: $p = 0 \text{ then } \nu_T \geq 0 \rightarrow B \geq -q/3$
  - Not considered here

- $\alpha = [p - 1, q + 2] : C_{\alpha,T} = 1 - q$
  - Attraction: $q = 0 \text{ then } \nu_T \leq 0 \rightarrow B \geq p/3$ \textbf{OK!}

**Universal attraction for more “exotic” channel**

$$C_{\text{exotic}} = 1 \quad \text{for} \quad T = [p, 0], \quad \alpha = [p - 1, 2]$$
Renormalization and bound states

Solve the scattering problem with

\[ V_\alpha = -\frac{\omega}{2f^2}C_{\alpha,T} \]

Unitarity: OK

Renormalization parameter: condition

\[ G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T \]


Scale at which ChPT works.
Matching with the u-channel amplitude: OK

Bound state:

\[ 1 - V(M_b)G(M_b) = 0 \quad \text{at} \quad M_T < M_b < M_T + m \]
Critical attraction

\[ 1 - V(\sqrt{s})G(\sqrt{s}) \]: monotonically decreasing.

Fixed \[ G(M_T) = 0 \]

\[ 1 - VG = 1 \]

Critical attraction: \[ 1 - VG = 0 \] at \( \sqrt{s} = M_T + m \)

\[ C_{\text{crit}} = \frac{2f^2}{m[-G(M_T + m)]} \]
Critical attraction and exotic channel

$m = 368$ MeV and $f = 93$ MeV

Strength is not enough.
We study the exotic bound states in s-wave chiral dynamics in flavor SU(3) limit.

The interaction in exotic channels is in most cases repulsive.

There are attractive interactions in exotic channels, with universal and the smallest strength: \( C_{\text{exotic}} = 1 \)

The strength is not enough to generate a bound state: \( C_{\text{exotic}} < C_{\text{crit}} \)

The result is model independent as far as we respect chiral symmetry.
Caution!

The exotic hadrons here are the s-wave meson-hadron molecule states (1/2- for Θ+).

We do not exclude the exotics which have other origins (genuine quark state, soliton rotation,...).

In practice, SU(3) breaking effect, higher order terms,...

In Nature, it is difficult to generate exotic hadrons as in the same way with Λ(1405), Λ(1520),... based on chiral interaction.