

Exotic hadrons in s-wave chiral dynamics



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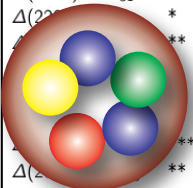
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2007, Mar. 1st

Exotic hadrons

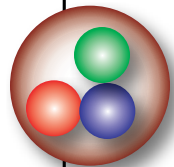
Observed hadrons in experiments (PDG06) :

| LIGHT UNFLAVORED (S = C = B = 0) | | | | | | | | | | | | STRANGE (S = ±1, C = B = 0) | | | BOTTOM (B = ±1) | | | |
|-------------------------------------|------------|------|------------------|------------|------|-----------------|----------|------|----------------|----------|------|--------------------------------|----------|------|--------------------|------------|---|------------|
| $\rho^0 (J^{PC})$ | | | | | | | | | | | | $\rho^0 (J^{PC})$ | | | $\rho^0 (J^{PC})$ | | | |
| ρ | P_{11} | **** | $\Delta(1232)$ | P_{33} | **** | Λ | P_{01} | **** | Σ^+ | P_{11} | **** | Ξ^0 | P_{11} | **** | K^+ | $1/2(0^-)$ | B^+ | $1/2(0^-)$ |
| n | P_{11} | **** | $\Delta(1600)$ | P_{33} | *** | $\Lambda(1405)$ | S_{01} | **** | Σ^0 | P_{11} | **** | Ξ^- | P_{11} | **** | K^0 | $1/2(0^-)$ | B^0 | $1/2(0^-)$ |
| $N(1440)$ | P_{11} | **** | $\Delta(1620)$ | S_{31} | **** | $\Lambda(1520)$ | D_{03} | **** | Σ^- | P_{11} | **** | $\Xi(1530)$ | P_{13} | **** | K_S^0 | $1/2(0^-)$ | B_S^0 | $1/2(0^-)$ |
| $N(1520)$ | D_{13} | **** | $\Delta(1700)$ | D_{33} | **** | $\Lambda(1600)$ | P_{01} | *** | $\Sigma(1385)$ | P_{13} | **** | $\Xi(1620)$ | * | * | K_L^0 | $1/2(0^-)$ | B^+ / B^0 ADMIXTURE | $1/2(0^-)$ |
| $N(1535)$ | S_{11} | **** | $\Delta(1750)$ | P_{31} | * | $\Lambda(1670)$ | S_{01} | **** | $\Sigma(1480)$ | * | * | $\Xi(1690)$ | *** | * | K_2^0 | $1/2(0^-)$ | $B^+ / B^0 / B_S^0 / b$ -baryon ADMIXTURE | $1/2(0^-)$ |
| $N(1650)$ | S_{11} | **** | $\Delta(1900)$ | S_{31} | ** | $\Lambda(1690)$ | D_{03} | **** | $\Sigma(1560)$ | ** | ** | $\Xi(1820)$ | D_{13} | *** | $K_1^0(800)$ | $1/2(0^+)$ | V_{ub} and V_{cb} CKM Matrix Elements | $1/2(1^-)$ |
| $N(1675)$ | D_{15} | **** | $\Delta(1905)$ | F_{35} | **** | $\Lambda(1800)$ | S_{01} | *** | $\Sigma(1580)$ | D_{13} | * | $\Xi(1950)$ | *** | *** | $K^*(892)$ | $1/2(1^-)$ | B^* | $1/2(1^-)$ |
| $N(1680)$ | F_{15} | **** | $\Delta(1910)$ | P_{31} | **** | $\Lambda(1810)$ | P_{01} | *** | $\Sigma(1620)$ | S_{11} | ** | $\Xi(2030)$ | *** | *** | $K_1(1270)$ | $1/2(1^+)$ | $B_S^+(5732)$ | $?(?)$ |
| $N(1700)$ | D_{13} | *** | $\Delta(1920)$ | P_{33} | *** | $\Lambda(1820)$ | F_{05} | **** | $\Sigma(1660)$ | P_{11} | *** | $\Xi(2120)$ | * | * | $K_1(1400)$ | $1/2(1^+)$ | | |
| $N(1710)$ | P_{11} | *** | $\Delta(1930)$ | D_{35} | *** | $\Lambda(1830)$ | F_{05} | **** | $\Sigma(1670)$ | D_{13} | **** | $\Xi(2250)$ | ** | ** | $K^*(1410)$ | $1/2(1^-)$ | | |
| $N(1720)$ | P_{13} | **** | $\Delta(1940)$ | D_{33} | * | $\Lambda(1890)$ | P_{03} | **** | $\Sigma(1690)$ | * | * | $\Xi(2370)$ | ** | ** | $K_S^*(1430)$ | $1/2(2^+)$ | | |
| $N(1900)$ | P_{13} | ** | $\Delta(1950)$ | F_{37} | **** | $\Lambda(2000)$ | * | * | $\Sigma(1750)$ | S_{11} | * | $\Xi(2500)$ | * | * | $K_S^*(1430)$ | $1/2(2^+)$ | | |
| $N(1990)$ | F_{17} | ** | $\Delta(2000)$ | F_{35} | ** | $\Lambda(2020)$ | F_{07} | * | $\Sigma(1770)$ | P_{11} | ** | Ω^- | **** | **** | $K(1460)$ | $1/2(0^-)$ | | |
| $N(2000)$ | F_{15} | ** | $\Delta(2150)$ | S_{31} | * | $\Lambda(2100)$ | G_{07} | **** | $\Sigma(1775)$ | D_{15} | **** | $\Omega(2250)^-$ | *** | *** | $K_2(1580)$ | $1/2(2^-)$ | B_c^+ | $0(0^-)$ |
| $N(2080)$ | D_{13} | ** | $\Delta(2200)$ | * | * | $\Lambda(2110)$ | F_{05} | *** | $\Sigma(1840)$ | P_{13} | * | $\Omega(2380)^-$ | ** | ** | $K_2(1630)$ | $1/2(2^?)$ | B_c^0 | $0(1^-)$ |
| $N(2090)$ | S_{11} | * | $\Delta(2300)$ | * | * | $\Lambda(2325)$ | D_{03} | * | $\Sigma(1880)$ | P_{11} | ** | $\Omega(2470)^-$ | ** | ** | $K(1630)$ | $1/2(2^?)$ | $B_{S,2}^+(5850)$ | $?(?)$ |
| $N(2100)$ | P_{11} | * | $\Delta(2350)$ | * | * | $\Lambda(2350)$ | H_{09} | *** | $\Sigma(1915)$ | F_{15} | **** | | * | * | $K_1(1650)$ | $1/2(2^?)$ | | |
| $N(2190)$ | G_{17} | **** | $\Delta(2550)$ | * | * | $\Lambda(2585)$ | ** | ** | $\Sigma(1940)$ | D_{13} | **** | | * | * | $K_2(1820)$ | $1/2(2^-)$ | | |
| $N(2200)$ | D_{15} | ** | $\Delta(2600)$ | * | * | | | | $\Sigma(2000)$ | S_{11} | * | Λ_c^+ | **** | **** | $K(1830)$ | $1/2(0^+)$ | $c\bar{c}$ | |
| $N(2220)$ | H_{19} | **** | $\Delta(2700)$ | * | * | | | | $\Sigma(2030)$ | F_{17} | **** | $\Lambda_c(2593)^+$ | *** | *** | $K_2^*(1950)$ | $1/2(0^+)$ | $\eta_c(1S)$ | $0^+(0^-)$ |
| $N(2250)$ | G_{19} | **** | $\Delta(2800)$ | * | * | | | | $\Sigma(2070)$ | F_{15} | * | $\Lambda_c(2625)^+$ | *** | *** | $K_2^*(1980)$ | $1/2(0^+)$ | $J/\psi(1S)$ | $0^-(1^-)$ |
| $N(2600)$ | $h_{1,11}$ | *** | $\Delta(2950)$ | $K_{3,15}$ | ** | | | | $\Sigma(2080)$ | P_{13} | ** | $\Lambda_c(2765)^+$ | * | * | $K_2^*(2045)$ | $1/2(4^+)$ | $\chi_{c0}(1P)$ | $0^+(0^+)$ |
| $N(2700)$ | $K_{1,13}$ | ** | $\Theta(1540)^+$ | * | * | | | | $\Sigma(2100)$ | G_{17} | ** | $\Lambda_c(2880)^+$ | ** | ** | $K_2(2250)$ | $1/2(2^-)$ | $\chi_{c1}(1P)$ | $0^+(1^+)$ |
| | | | | | | | | | $\Sigma(2250)$ | | *** | $\Sigma_c(2455)$ | **** | **** | $K_2(2320)$ | $1/2(3^+)$ | $\chi_{c2}(1P)$ | $0^+(2^+)$ |
| | | | | | | | | | $\Sigma(2455)$ | | ** | $\Sigma_c(2520)$ | *** | *** | $K_1(2380)$ | $1/2(5^-)$ | $\eta_c(2S)$ | $0^+(0^+)$ |
| | | | | | | | | | $\Sigma(2620)$ | | ** | $\Sigma_c(2800)$ | *** | *** | $K(2500)$ | $1/2(4^-)$ | $\psi(2S)$ | $0^-(1^-)$ |
| | | | | | | | | | $\Sigma(3000)$ | | * | | *** | *** | $K(3100)$ | $?(?)$ | $\psi(3770)$ | $0^-(1^-)$ |
| | | | | | | | | | $\Sigma(3170)$ | | * | | *** | *** | | | $\chi(3870)$ | $0^2(2^+)$ |
| | | | | | | | | | | | | Ξ_c^+ | **** | **** | | | $\chi(3940)$ | $?(?)$ |
| | | | | | | | | | | | | Ξ_c^0 | **** | **** | | | $\psi(4040)$ | $0^-(1^-)$ |
| | | | | | | | | | | | | Ξ_c^- | **** | **** | | | $\psi(4160)$ | $0^-(1^-)$ |
| | | | | | | | | | | | | Ξ_c' | **** | **** | | | $\chi(4260)$ | $?(?)$ |
| | | | | | | | | | | | | Ω_c^+ | **** | **** | | | $\psi(4415)$ | $0^-(1^-)$ |
| | | | | | | | | | | | | Ω_c^0 | **** | **** | | | | |
| | | | | | | | | | | | | Ω_c^- | **** | **** | | | | |
| | | | | | | | | | | | | Λ_b^0 | **** | **** | | | | |



~130 baryons

~ $\frac{1}{300}$!



~160 mesons

Exotic hadrons are indeed exotic !!

Motivation 1 : Exotic hadrons

Exotic hadrons : more than 4 valence quarks

non-exotic

$uds, u\bar{d}, udsu\bar{u}, u\bar{d}u\bar{u}, \dots$

exotic (in this talk)

$uudd\bar{s}, ud\bar{s}\bar{s}, \dots$

not considered

$uuddss, c\bar{c}g, \bar{u}\bar{u}\bar{d}\bar{d}\bar{s}, \dots$

Experimentally, they are exotic $\sim 1/300$.

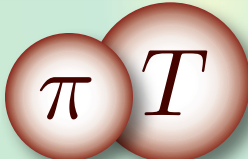
Theoretically, are they exotic?

--> There is no simple way to forbid exotic states in QCD, effective models, ...

--> Evidences of multiquark components in non-exotic hadrons.

Why aren't the exotics observed??

Motivation 2 : Chiral unitary approaches

Hadron excited states \sim 

- Interaction \leftarrow chiral symmetry
- Amplitude \leftarrow unitarity

R.H. Dalitz, and S.F. Tuan, *Ann. Phys. (N.Y.)* 10, 307 (1960)

J.H.W. Wyld, *Phys. Rev.* 155, 1649 (1967)

N. Kaiser, P. B. Siegel and W. Weise, *Nucl. Phys.* A594, 325 (1995)

E. Oset and A. Ramos, *Nucl. Phys.* A635, 99 (1998)

J. A. Oller and U. G. Meissner, *Phys. Lett.* B500, 263 (2001)

M.F.M. Lutz and E. E. Kolomeitsev, *Nucl. Phys.* A700, 193 (2002)

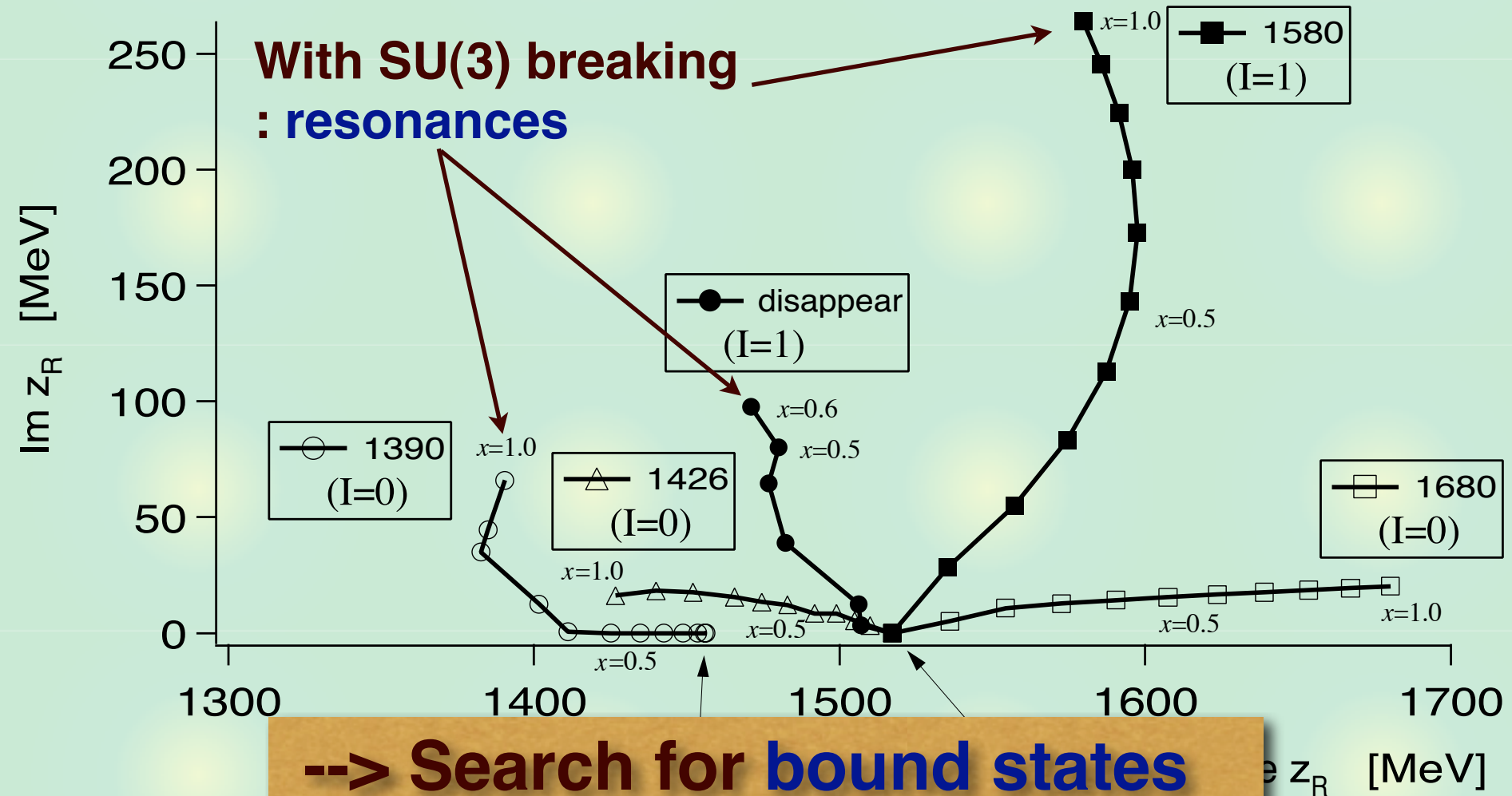
Many hadron resonances ($\Lambda(1405)$, $N(1535)$, $\Lambda(1520)$, $D_s(2317)$,...) are well described.

What about exotic hadrons?

Origin of the resonances

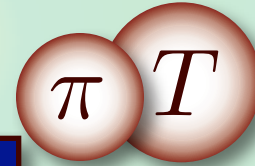
Trajectory of poles

D. Jido, *et al.*, Nucl. Phys. A 723, 205 (2003)



Outline

Hadron-NG boson bound state



Chiral Symmetry

s-wave low energy interaction

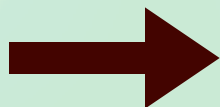
$$V_{\alpha} = -\frac{\omega}{2f^2} C_{\alpha,T} \quad C_{\text{exotic}} = 1$$

Scattering theory

Critical strength for a bound state

$$C_{\text{crit}} = \frac{2f^2}{m[-G(M_T + m)]}$$

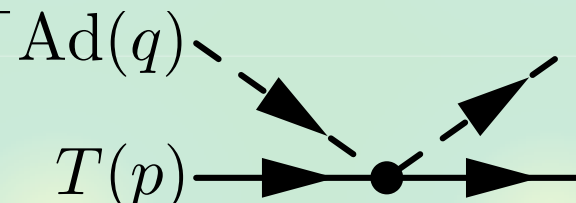
physical values : $C_{\text{exotic}} < C_{\text{crit}}$



No exotic state exists.

Low energy s-wave interaction

Scattering of a target (T) with the pion (Ad)

$$\alpha \left[\begin{array}{c} \text{Ad}(q) \\ T(p) \end{array} \right] = \frac{1}{f^2} \frac{p \cdot q}{2M_T} \langle \mathbf{F}_T \cdot \mathbf{F}_{\text{Ad}} \rangle_\alpha + \mathcal{O}((m/M_T)^2)$$


s-wave : Weinberg-Tomozawa term

$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}$$

$$C_{\alpha,T} \equiv -\langle 2\mathbf{F}_T \cdot \mathbf{F}_{\text{Ad}} \rangle_\alpha = C_2(T) - C_2(\alpha) + 3 \quad (\text{for } N_f = 3)$$

Coupling : pion decay constant model-independent interaction at low energy

Y. Tomozawa, *Nuovo Cim.* 46A, 707 (1966)

S. Weinberg, *Phys. Rev. Lett.* 17, 616 (1966)

Coupling strengths : Examples

Coupling strengths : (positive is attractive)

$$C_{\alpha,T} = C_2(T) - C_2(\alpha) + 3$$

| | | | | | | |
|--|---|---|----|-----------------|----|----|
| α | 1 | 8 | 10 | $\overline{10}$ | 27 | 35 |
| $T = \mathbf{8}(N, \Lambda, \Sigma, \Xi)$ | 6 | 3 | 0 | 0 | -2 | |
| $T = \mathbf{10}(\Delta, \Sigma^*, \Xi^*, \Omega)$ | | 6 | 3 | | 1 | -3 |

| | | | | |
|---|----------------|---|-----------------|----|
| α | $\overline{3}$ | 6 | $\overline{15}$ | 24 |
| $T = \overline{\mathbf{3}}(\Lambda_c, \Xi_c)$ | 3 | 1 | -1 | |
| $T = \mathbf{6}(\Sigma_c, \Xi_c^*, \Omega_c)$ | 5 | 3 | 1 | -2 |

- **Exotic channels** : mostly repulsive
- **Attractive interaction** : **C = 1**

Coupling strengths : General expression

For a general target $T = [p, q]$

| $\alpha \in [p, q] \otimes [1, 1]$ | $C_{\alpha, T}$ | sign |
|------------------------------------|-----------------|-------------------|
| $[p + 1, q + 1]$ | $-p - q$ | repulsive |
| $[p + 2, q - 1]$ | $1 - p$ | |
| $[p - 1, q + 2]$ | $1 - q$ | |
| $[p, q]$ | 3 | attractive |
| $[p, q]$ | 3 | attractive |
| $[p + 1, q - 2]$ | $3 + q$ | attractive |
| $[p - 2, q + 1]$ | $3 + p$ | attractive |
| $[p - 1, q - 1]$ | $4 + p + q$ | attractive |

- **Strength should be integer.**
- **Sign is determined for most cases.**

Exotic channels

Exoticness : minimal number of extra $\bar{q}q$.

$$E = \epsilon\theta(\epsilon) + \nu\theta(\nu) \quad \epsilon \equiv \frac{p + 2q}{3} - B, \quad \nu \equiv \frac{p - q}{3} - B$$

$\Delta E = E_\alpha - E_T = +1$ is realized when

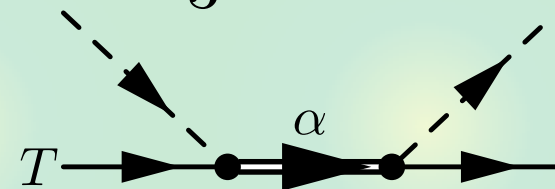
○ $\alpha = [p + 1, q + 1] : C_{\alpha,T} = -p - q$
repulsive

○ $\alpha = [p + 2, q - 1] : C_{\alpha,T} = 1 - p$

attraction : $p = 0$ then $\nu_T \geq 0 \rightarrow B \geq -q/3$
not considered here

○ $\alpha = [p - 1, q + 2] : C_{\alpha,T} = 1 - q$

attraction : $q = 0$ then $\nu_T \leq 0 \rightarrow B \geq p/3$ OK!



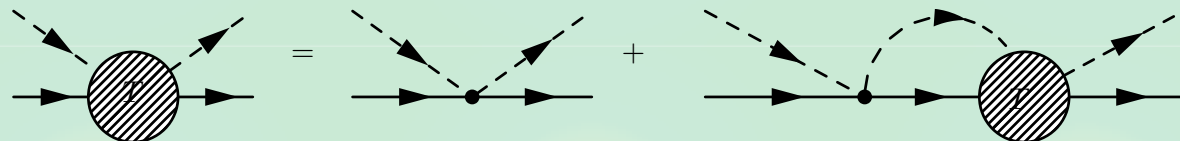
Universal attraction for more “exotic” channel

$$C_{\text{exotic}} = 1 \quad \text{for} \quad T = [p, 0], \quad \alpha = [p - 1, 2]$$

Renormalization and bound states

Solve the scattering problem with $V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}$

$$T = \frac{1}{1 - VG} V$$



Unitarity : OK

Renormalization parameter : condition

$$G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T$$

K. Igi, and K. Hikasa, Phys. Rev. D59, 034005 (1999)

M.F.M. Lutz, and E. Kolomeitsev, Nucl. Phys. A700, 193-308 (2002)

Scale at which ChPT works.

Matching with the u-channel amplitude : OK

Bound state:

$$1 - V(M_b)G(M_b) = 0 \quad M_T < M_b < M_T + m$$

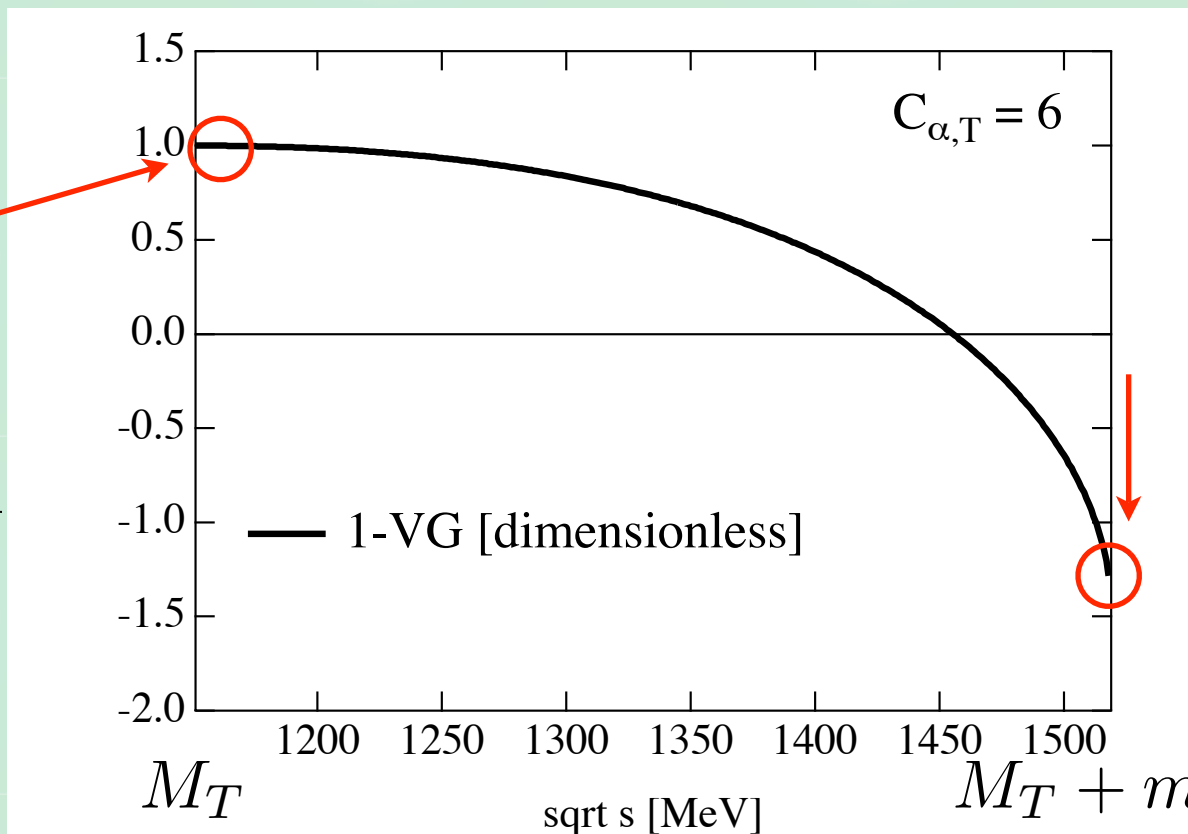
Critical attraction

$1 - V(\sqrt{s})G(\sqrt{s})$: monotonically decreasing.

Fixed

$$G(M_T) = 0$$

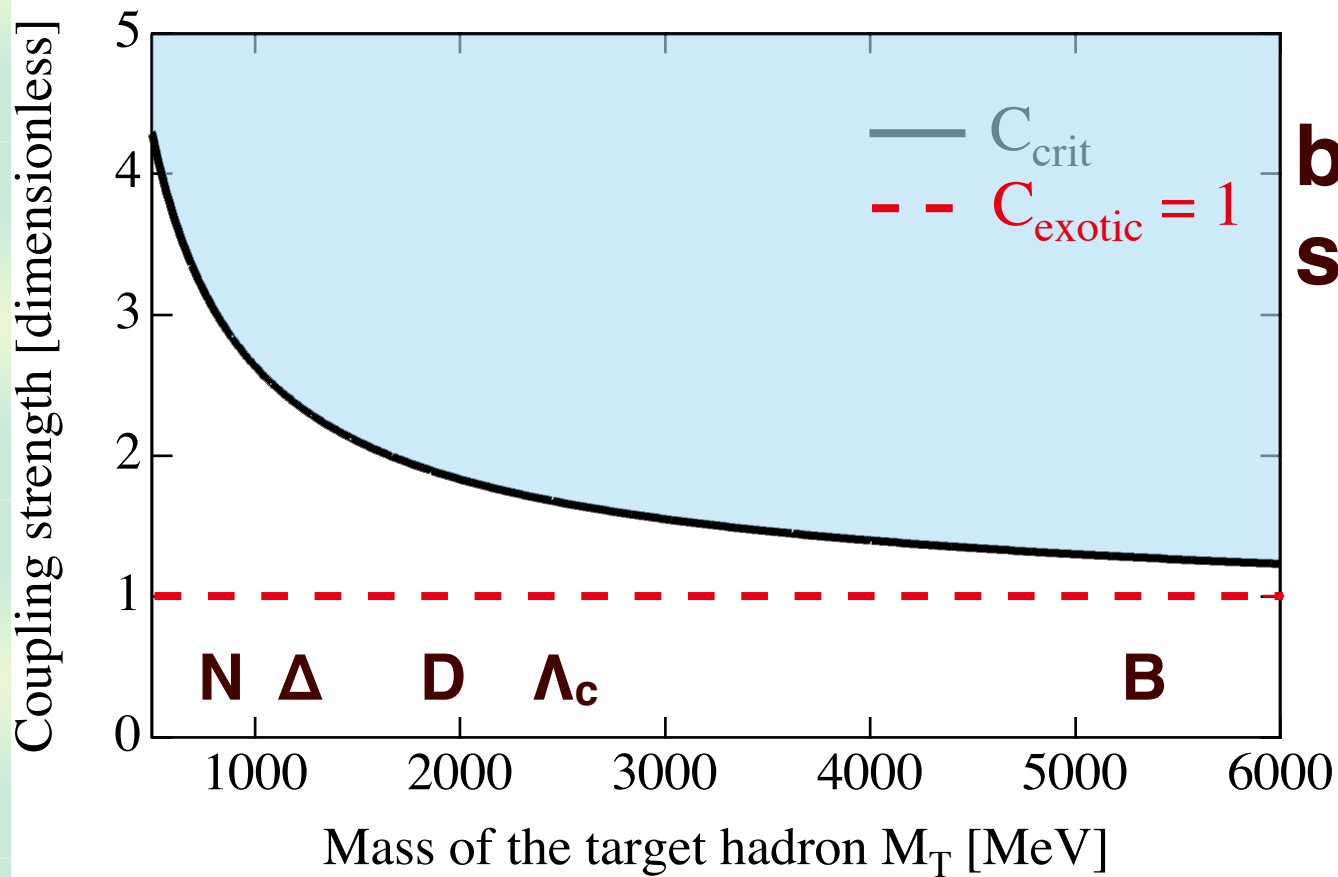
$$1 - VG = 1$$



Critical attraction : $1 - VG = 0$ at $\sqrt{s} = M_T + m$

$$\longrightarrow C_{\text{crit}} = \frac{2f^2}{m[-G(M_T + m)]}$$

Critical attraction and exotic channel



$$m = 368 \text{ MeV and } f = 93 \text{ MeV}$$

➔ Strength is not enough.

Summary 1 : SU(3) limit




We study the exotic bound states in s-wave chiral dynamics in flavor SU(3) limit.

- The interaction in exotic channels is in most cases **repulsive**.
- There are **attractive interactions** in exotic channels, with **universal** and the smallest strength : $C_{\text{exotic}} = 1$
- The strength is **not enough** to generate a bound state : $C_{\text{exotic}} < C_{\text{crit}}$

The result is **model independent** as far as we respect chiral symmetry.

Summary 2 : Physical world

Caution!

-  The exotic hadrons here are the **s-wave** meson-hadron molecule states ($1/2^-$ for Θ^+).
-  We do not exclude the exotics which have **other origins** (genuine quark state, soliton rotation,...).
-  In practice, **SU(3) breaking** effect, **higher order** terms,...

In Nature, it is **difficult** to generate exotic hadrons as in the same way with $\Lambda(1405)$, $\Lambda(1520)$,... based on chiral interaction.

[T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. Lett. 97, 192002 \(2006\)](#)

[T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. D 75, 034002 \(2007\)](#)