Exotic hadrons in s-wave chiral dynamics

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Pentaquark Θ^+

$\Theta(1540) : J^P = ?, I = 0(?)$

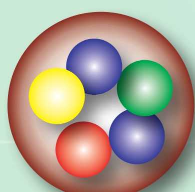
Mass : 1533.6 ± 2.4 MeV

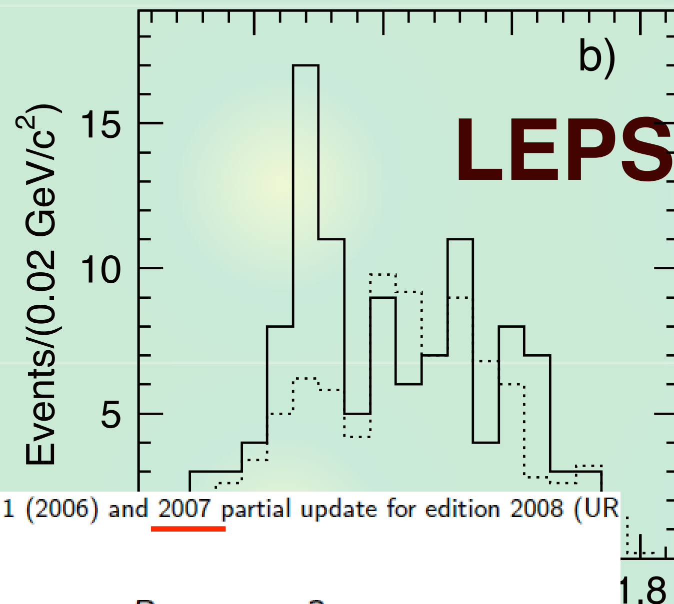
Width : 0.9 ± 0.3 MeV

Decay mode : $\Theta(1540) \rightarrow KN$

S = +1 : uudd$\bar{s}$

-> pentaquark





M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006) and 2007 partial update for edition 2008 (UR

$\Theta(1540)^+$ $I(J^P) = 0(?^?)$

Exotic hadrons in hadron spectrum

Observed hadrons in experiments (PDG06) :

p	P_{11}	****
n	P_{11}	****
$N(1440)$	P_{11}	****
$N(1520)$	D_{13}	****
$N(1535)$	S_{11}	****
$N(1650)$	S_{11}	****
$N(1675)$	D_{15}	****
$N(1680)$	F_{15}	****
$N(1700)$	D_{13}	***
$N(1710)$	P_{11}	***
$N(1720)$	P_{13}	****
$N(1900)$	P_{13}	**
$N(1990)$	F_{17}	**
$N(2000)$	F_{15}	**
$N(2080)$	D_{13}	**
$N(2090)$	S_{11}	*
$N(2100)$	P_{11}	*
$N(2190)$	G_{17}	****
$N(2200)$	D_{15}	**
$N(2220)$	H_{19}	****
$N(2250)$	G_{19}	****
$N(2600)$	$I_{1,11}$	***
$N(2700)$	$K_{1,13}$	**

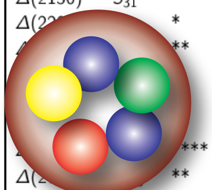
$\Delta(1232)$	P_{33}	****
$\Delta(1600)$	P_{33}	***
$\Delta(1620)$	S_{31}	****
$\Delta(1700)$	D_{33}	****
$\Delta(1750)$	P_{31}	*
$\Delta(1900)$	S_{31}	**
$\Delta(1905)$	F_{35}	****
$\Delta(1910)$	P_{31}	****
$\Delta(1920)$	P_{33}	***
$\Delta(1930)$	D_{35}	***
$\Delta(1940)$	D_{33}	*
$\Delta(1950)$	F_{37}	****
$\Delta(2000)$	F_{35}	**
$\Delta(2150)$	S_{31}	*
$\Delta(22$[illegible]	[illegible]	*
[illegible]	[illegible]	**
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	***
$\Delta(2$[illegible]	[illegible]	**
$\Delta(2950)$	$K_{3,15}$	**
$\Theta(1540)^+$		*

Λ	P_{01}	****
$\Lambda(1405)$	S_{01}	****
$\Lambda(1520)$	D_{03}	****
$\Lambda(1600)$	P_{01}	***
$\Lambda(1670)$	S_{01}	****
$\Lambda(1690)$	D_{03}	****
$\Lambda(1800)$	S_{01}	***
$\Lambda(1810)$	P_{01}	***
$\Lambda(1820)$	F_{05}	****
$\Lambda(1830)$	D_{05}	****
$\Lambda(1890)$	P_{03}	****
$\Lambda(2000)$		*
$\Lambda(2020)$	F_{07}	*
$\Lambda(2100)$	G_{07}	****
$\Lambda(2110)$	F_{05}	***
$\Lambda(2325)$	D_{03}	*
$\Lambda(2350)$	H_{09}	***
$\Lambda(2585)$		**

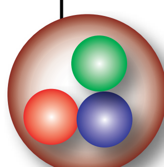
Σ^+	P_{11}	****
Σ^0	P_{11}	****
Σ^-	P_{11}	****
$\Sigma(1385)$	P_{13}	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	D_{13}	*
$\Sigma(1620)$	S_{11}	**
$\Sigma(1660)$	P_{11}	***
$\Sigma(1670)$	D_{13}	****
$\Sigma(1690)$		**
$\Sigma(1750)$	S_{11}	***
$\Sigma(1770)$	P_{11}	*
$\Sigma(1775)$	D_{15}	****
$\Sigma(1840)$	P_{13}	*
$\Sigma(1880)$	P_{11}	**
$\Sigma(1915)$	F_{15}	****
$\Sigma(1940)$	D_{13}	***
$\Sigma(2000)$	S_{11}	*
$\Sigma(2030)$	F_{17}	****
$\Sigma(2070)$	F_{15}	*
$\Sigma(2080)$	P_{13}	**
$\Sigma(2100)$	G_{17}	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	P_{11}	****
Ξ^-	P_{11}	****
$\Xi(1530)$	P_{13}	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	D_{13}	***
$\Xi(1950)$		***
$\Xi(2030)$		***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-		****
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**
Λ_c^+		****
$\Lambda_c(2593)^+$		***
$\Lambda_c(2625)^+$		***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2880)^+$		**
$\Sigma_c(2455)$		****
$\Sigma_c(2520)$		***
$\Sigma_c(2800)$		***
Ξ_c^+		***
Ξ_c^0		***
$\Xi_c'^+$		***
$\Xi_c'^0$		***
$\Xi_c(2645)$		***
$\Xi_c(2790)$		***
$\Xi_c(2815)$		***
Ω_c^0		***
Ξ_{cc}^+		*
Λ_b^0		***

$\sim \dfrac{1}{300}$!



~130 baryons



LIGHT UNFLAVORED ($S = C = B = 0$)	$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(600)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$
• $f_2(1270)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$
• $\pi(1300)$	$1^-(0^{-+})$
• $a_2(1320)$	$1^-(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$
• $\pi_1(1400)$	$1^-(1^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$
• $f_1(1420)$	$0^+(1^{++})$
• $\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$
• $f_0(1500)$	$0^+(0^{++})$
$f_1(1510)$	$0^+(1^{++})$
• $f_2'(1525)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$
$h_1(1595)$	$0^-(1^{+-})$
• $\pi_1(1600)$	$1^-(1^{-+})$
$a_1(1640)$	$1^-(1^{++})$
$f_2(1640)$	$0^+(2^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$
• $\omega(1650)$	$0^-(1^{--})$
• $\omega_3(1670)$	$0^-(3^{--})$

	$I^G(J^{PC})$
• $\pi_2(1670)$	$1^-(2^{-+})$
• $\phi(1680)$	$0^-(1^{--})$
• $\rho_3(1690)$	$1^+(3^{--})$
• $\rho(1700)$	$1^+(1^{--})$
$a_2(1700)$	$1^-(2^{++})$
• $f_0(1710)$	$0^+(0^{++})$
$\eta(1760)$	$0^+(0^{-+})$
• $\pi(1800)$	$1^-(0^{-+})$
$f_2(1810)$	$0^+(2^{++})$
$X(1835)$	$?^?(?^{-+})$
• $\phi_3(1850)$	$0^-(3^{--})$
$\eta_2(1870)$	$0^+(2^{-+})$
$\rho(1900)$	$1^+(1^{--})$
$f_2(1910)$	$0^+(2^{++})$
• $f_2(1950)$	$0^+(2^{++})$
$\rho_3(1990)$	$1^+(3^{--})$
• $f_2(2010)$	$0^+(2^{++})$
$f_0(2020)$	$0^+(0^{++})$
• $a_4(2040)$	$1^-(4^{++})$
• $f_4(2050)$	$0^+(4^{++})$
$\pi_2(2100)$	$1^-(2^{-+})$
$f_0(2100)$	$0^+(0^{++})$
$f_2(2150)$	$0^+(2^{++})$
$\rho(2150)$	$1^+(1^{--})$
$f_0(2200)$	$0^+(0^{++})$
$f_J(2220)$	$0^+(2 \text{ or } 4^{++})$
$\eta(2225)$	$0^+(0^{-+})$
$\rho_3(2250)$	$1^+(3^{--})$
• $f_2(2300)$	$0^+(2^{++})$
$f_4(2300)$	$0^+(4^{++})$
• $f_2(2340)$	$0^+(2^{++})$
$\rho_5(2350)$	$1^+(5^{--})$
$a_6(2450)$	$1^-(6^{++})$
$f_6(2510)$	$0^+(6^{++})$
OTH[illegible]	
Further [illegible]	

STRANGE ($S = \pm1$, $C = B = 0$)	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
$K_0^*(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)	
• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
$D_0^*(2400)^0$	$1/2(0^+)$
$D_0^*(2400)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D^*(2640)^\pm$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)	
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$

BOTTOM ($B = \pm1$)	$I^G(J^{PC})$
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B^0_s/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
$B_J^*(5732)$	$?(?^?)$

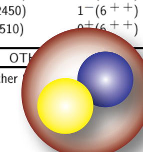
BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)	
• B^0_s	$0(0^-)$
B^*_s	$0(1^-)$
$B^*_{sJ}(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)	
• $B_c^\pm$	$0(0^-)$

$c\bar{c}$	
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
$h_c(1P)$	$?^?(?^{??})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
• $X(3872)$	$0^?(?^{?+})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$Y(3940)$	$?^?(?^{??})$
• $\psi(4040)$	$0^-(1^{--})$
• $\psi(4160)$	$0^-(1^{--})$
$Y(4260)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$

$b\bar{b}$	
$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
• $\Upsilon(2S)$	$0^-(1^{--})$
$\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

NON-$q\bar{q}$ CANDIDATES



~160 mesons

Exotic hadrons are indeed exotic !!

Exotic hadrons : experiment vs theory

Exotic hadrons : valence quark-antiquark(s)

non-exotic $uds, \quad u\bar{d}, \quad uds\, u\bar{u}, \quad u\bar{d}\, u\bar{u}, \ldots$

exotic $uudd\bar{s}, \quad ud\bar{s}\bar{s}, \ldots$

Experimentally, they are exotic ~ 1/300.

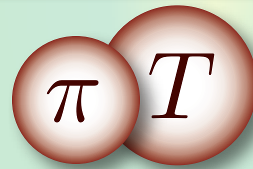
Theoretically, are they exotic?

--> There is no simple way to forbid exotic states in QCD, effective models, ...

--> Evidences of multiquark components in non-exotic hadrons.

Why aren't the exotics observed??

Chiral dynamics for non-exotic hadrons

Hadron excited states ~ 

- Interaction <-- chiral symmetry
- Amplitude <-- unitarity (coupled channel)

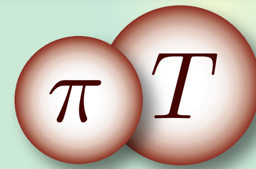
With phenomenological vector meson exchange interaction

R.H. Dalitz, and S.F. Tuan, Ann. Phys. (N.Y.) 10, 307 (1960)
J.H.W. Wyld, Phys. Rev. 155, 1649 (1967)

Chiral perturbation theory for interaction

N. Kaiser, P. B. Siegel and W. Weise, Nucl. Phys. A594, 325 (1995)
E. Oset and A. Ramos, Nucl. Phys. A635, 99 (1998)
J. A. Oller and U. G. Meissner, Phys. Lett. B500, 263 (2001)
M.F.M. Lutz and E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002)

Chiral dynamics for non-exotic hadrons

Hadron excited states ~ 

Many hadron resonances are well described.

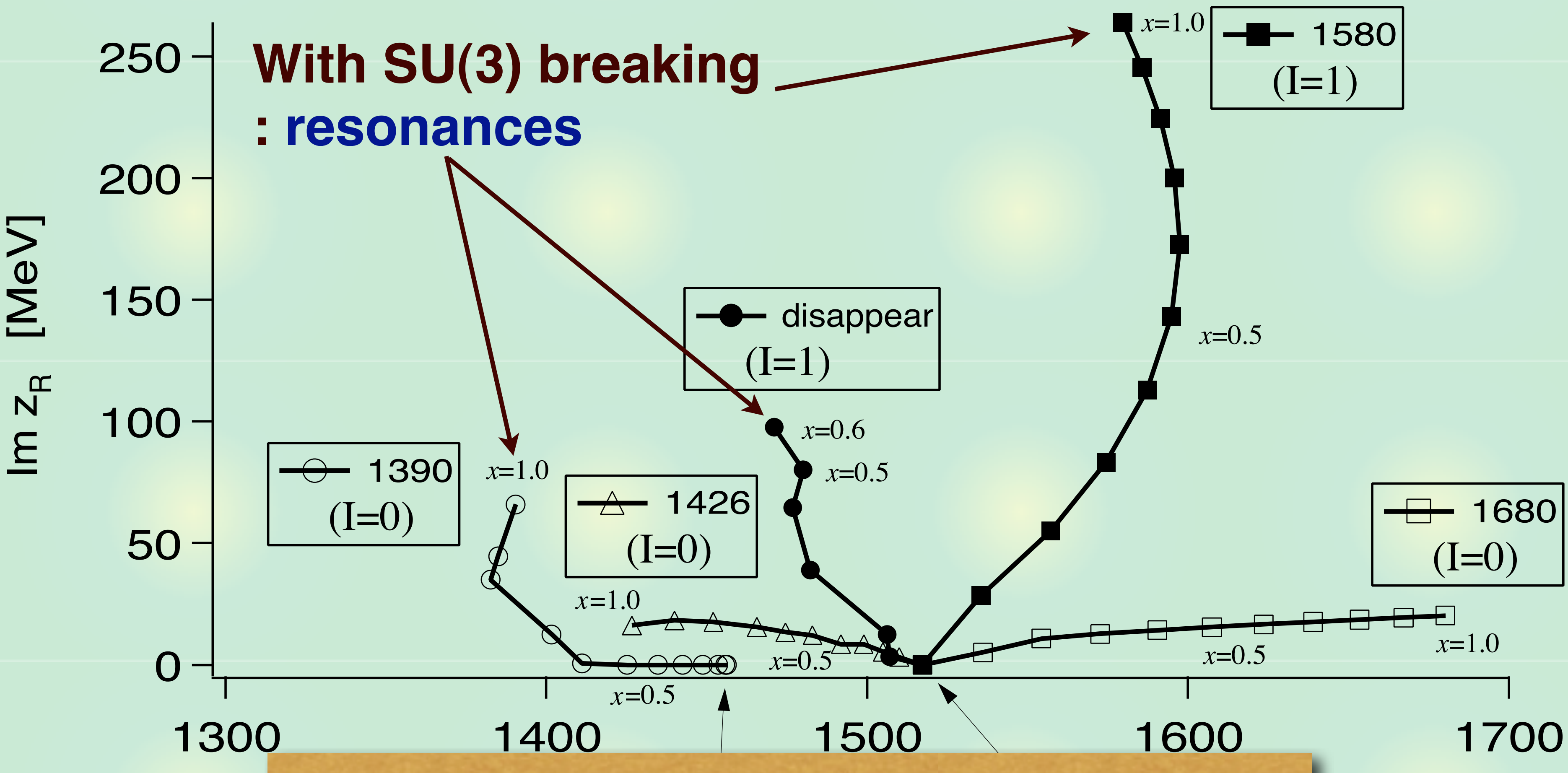
light baryon	$J^P = 1/2^-$	$\Lambda(1405)$ $\Lambda(1670)$ $\Sigma(1670)$ $N(1535)$ $\Xi(1620)$ $\Xi(1690)$
	$J^P = 3/2^-$	$\Lambda(1520)$ $\Xi(1820)$ $\Sigma(1670)$
heavy		$\Lambda_c(2880)$ $\Lambda_c(2593)$ $D_s(2317)$
light meson	$J^P = 1^+$	$b_1(1235)$ $h_1(1170)$ $h_1(1380)$ $a_1(1260)$ $f_1(1285)$ $K_1(1270)$ $K_1(1440)$
	$J^P = 0^+$	$\sigma(600)$ $\kappa(900)$ $f_0(980)$ $a_0(980)$

What about exotic hadrons?

Origin of the resonances

Trajectory of poles

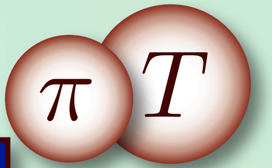
D. Jido, *et al.*, Nucl. Phys. A 723, 205 (2003)



--> Search for **bound states** in **SU(3) symmetric limit.**

Outline

Hadron-NG boson bound state



Chiral Symmetry

s-wave low energy interaction

$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T} \qquad C_{\text{exotic}} = 1$$

Scattering theory

Critical strength for a bound state

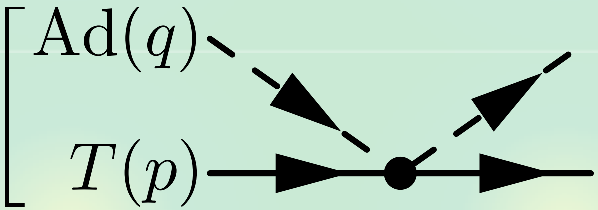
$$C_{\text{crit}} = \frac{2f^2}{m[-G(M_T + m)]}$$

physical values : $C_{\text{exotic}} < C_{\text{crit}}$

→ **No exotic state exists in SU(3) limit.**

Low energy s-wave interaction

Scattering of a target (T) with the pion (Ad)

α 

$$= \frac{1}{f^2}\frac{p\cdot q}{2M_T}\langle \boldsymbol{F}_T\cdot \boldsymbol{F}_{\text{Ad}}\rangle_\alpha + \mathcal{O}\left(\left(\frac{m}{M_T}\right)^2\right)$$

s-wave : Weinberg-Tomozawa term

$$V_\alpha = -\frac{\omega}{2f^2}C_{\alpha,T}$$

$$C_{\alpha,T} \equiv -\langle 2\boldsymbol{F}_T\cdot \boldsymbol{F}_{\text{Ad}}\rangle_\alpha = C_2(T) - C_2(\alpha) + 3 \quad (\text{for } N_f = 3)$$

Coupling : pion decay constant

model-independent interaction at low energy

Y. Tomozawa, Nuovo Cim. 46A, 707 (1966)

S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

Coupling strengths : Examples

Coupling strengths : (positive is attractive)

$$C_{\alpha,T} = C_2(T) - C_2(\alpha) + 3$$

α	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{10}$	$\overline{\mathbf{10}}$	$\mathbf{27}$	$\mathbf{35}$
$T = \mathbf{8}(N, \Lambda, \Sigma, \Xi)$	6	3	0	0	-2	
$T = \mathbf{10}(\Delta, \Sigma^*, \Xi^*, \Omega)$		6	3		1	-3

α	$\overline{\mathbf{3}}$	$\mathbf{6}$	$\overline{\mathbf{15}}$	$\mathbf{24}$
$T = \overline{\mathbf{3}}(\Lambda_c, \Xi_c)$	3	1	-1	
$T = \mathbf{6}(\Sigma_c, \Xi_c^*, \Omega_c)$	5	3	1	-2

- ○ Exotic channels : mostly repulsive
- ○ Attractive interaction : C = 1

Coupling strengths : General expression

For a general target $T = [p, q]$

$\alpha \in [p, q] \otimes [1, 1]$	$C_{\alpha,T}$	sign
$[p+1, q+1]$	$-p-q$	repulsive
$[p+2, q-1]$	$1-p$	
$[p-1, q+2]$	$1-q$	
$[p, q]$	3	attractive
$[p, q]$	3	attractive
$[p+1, q-2]$	$3+q$	attractive
$[p-2, q+1]$	$3+p$	attractive
$[p-1, q-1]$	$4+p+q$	attractive

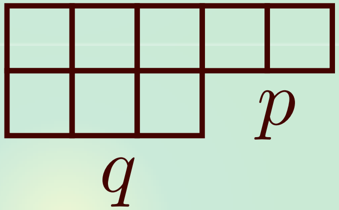
- ○ **Strength should be integer.**
- ○ **Sign is determined for most cases.**

Exoticness

Exoticness : minimal number of extra $\bar{q}q$.

$$E = \epsilon\theta(\epsilon) + \nu\theta(\nu)$$

$$\epsilon = \frac{p+2q}{3} - B \qquad \nu = \frac{p-q}{3} - B$$

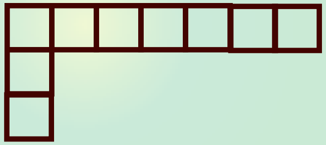


B : baryon number carried by light quarks

V. Kopeliovich, Phys. Lett. B259, 234 (1991)
D. Diakonov and V. Petrov, Phys. Rev. D 69, 056002 (2004)

but... $[p,q] = [6,0] = \mathbf{28}, \quad B = 1$

$E = 2, \quad \epsilon = 1$

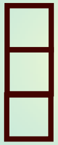


uuu $u\bar{d}$ $u\bar{d}$

E. Jenkins and A.V. Manohar, Phys. Rev. Lett. 93, 022001 (2004)

but... $[p,q] = [0,0] = \mathbf{1}, \quad B = 1$

$E = 0, \quad \epsilon = -1, \quad \nu = -1$



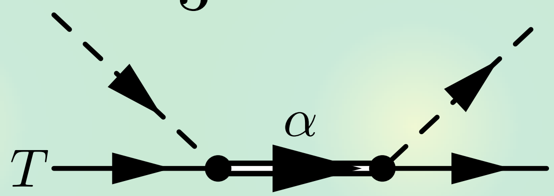
uds

Exotic channels

Exoticness : minimal number of extra $\bar{q}q$.

$$E = \epsilon\theta(\epsilon) + \nu\theta(\nu) \qquad \epsilon \equiv \frac{p+2q}{3} - B, \quad \nu \equiv \frac{p-q}{3} - B$$

$\Delta E = E_\alpha - E_T = +1$ is realized when



○ $\alpha = [p+1, q+1] : C_{\alpha,T} = -p-q$

repulsive

○ $\alpha = [p+2, q-1] : C_{\alpha,T} = 1-p$

attraction : $p = 0$ then $\nu_T \geq 0 \rightarrow B \geq -q/3$

not considered here

○ $\alpha = [p-1, q+2] : C_{\alpha,T} = 1-q$

attraction : $q = 0$ then $\nu_T \leq 0 \rightarrow B \geq p/3$ **OK!**

Universal attraction for more "exotic" channel

$$C_{\text{exotic}} = 1 \quad \text{for} \quad T = [p, 0], \quad \alpha = [p-1, 2]$$

Unitarization : N/D method

Unitarity cut --> N, unphysical cut --> D

$$T(s) = N(s)/D(s)$$

$$\mathrm{Im}D(s) = \mathrm{Im}[T^{-1}(s)]N(s) = -\rho(s)N(s) \quad \text{for } s > s_+$$

$$\mathrm{Im}N(s) = \mathrm{Im}[T(s)]D(s) \quad \text{for } s < s_-$$

Neglect unphysical cut, set N=1

$$T^{-1}(s) = \left(a(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s'-s)(s'-s_0)} \right) + \mathcal{T}^{-1}(s)$$

loop function $\rightarrow G(s)$

Interaction (tree level)

subtraction constant (regularization parameter of the loop)

Renormalization and bound states

Identifying the interaction as $V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}$

$$T_\alpha(\sqrt{s}) = \frac{1}{1 - V_\alpha(\sqrt{s}) G(\sqrt{s})} V_\alpha(\sqrt{s})$$

Renormalization parameter : condition

$$G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T$$

K. Igi, and K. Hikasa, Phys. Rev. D59, 034005 (1999)

M.F.M. Lutz, and E. Kolomeitsev, Nucl. Phys. A700, 193-308 (2002)

Scale at which ChPT works.

Matching with the u-channel amplitude : OK

Bound state:

$$1 - V(M_b) G(M_b) = 0 \qquad M_T < M_b < M_T + m$$

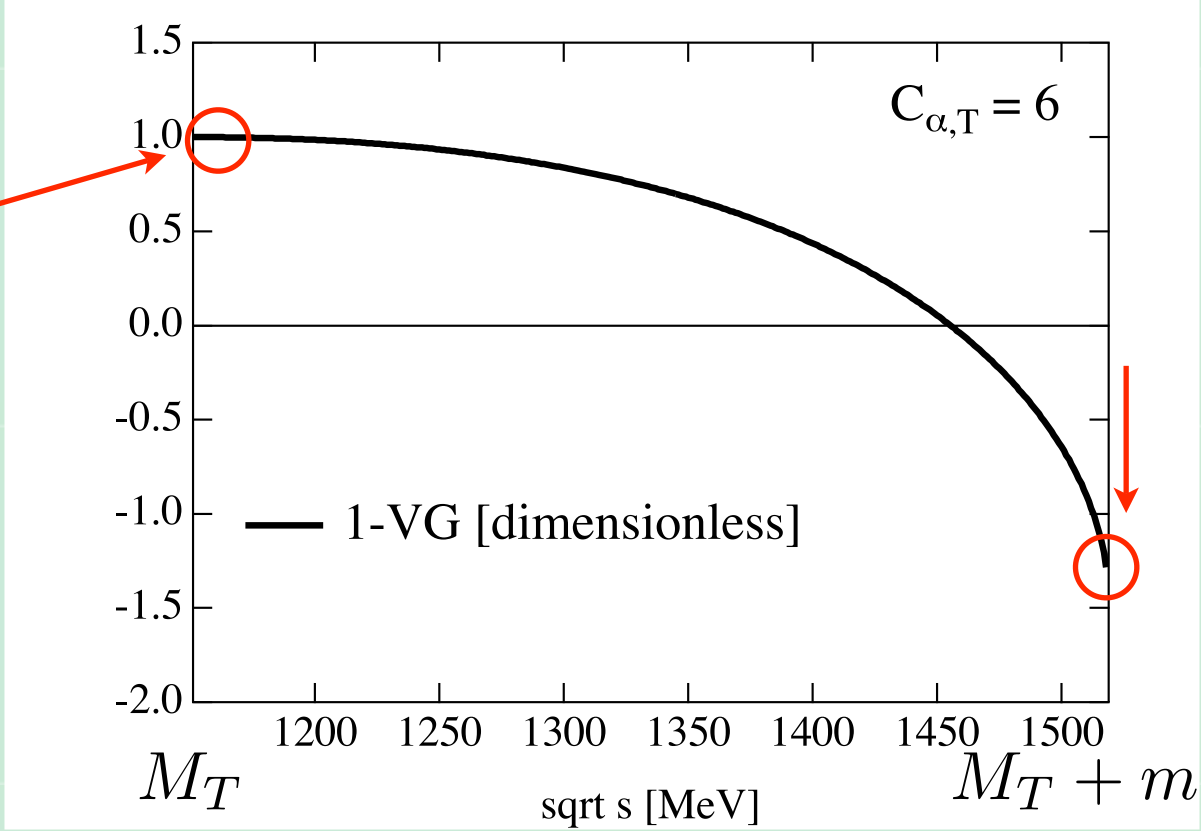
Critical attraction

$1 - V(\sqrt{s})G(\sqrt{s})$: **monotonically decreasing.**

Fixed

$G(M_T) = 0$

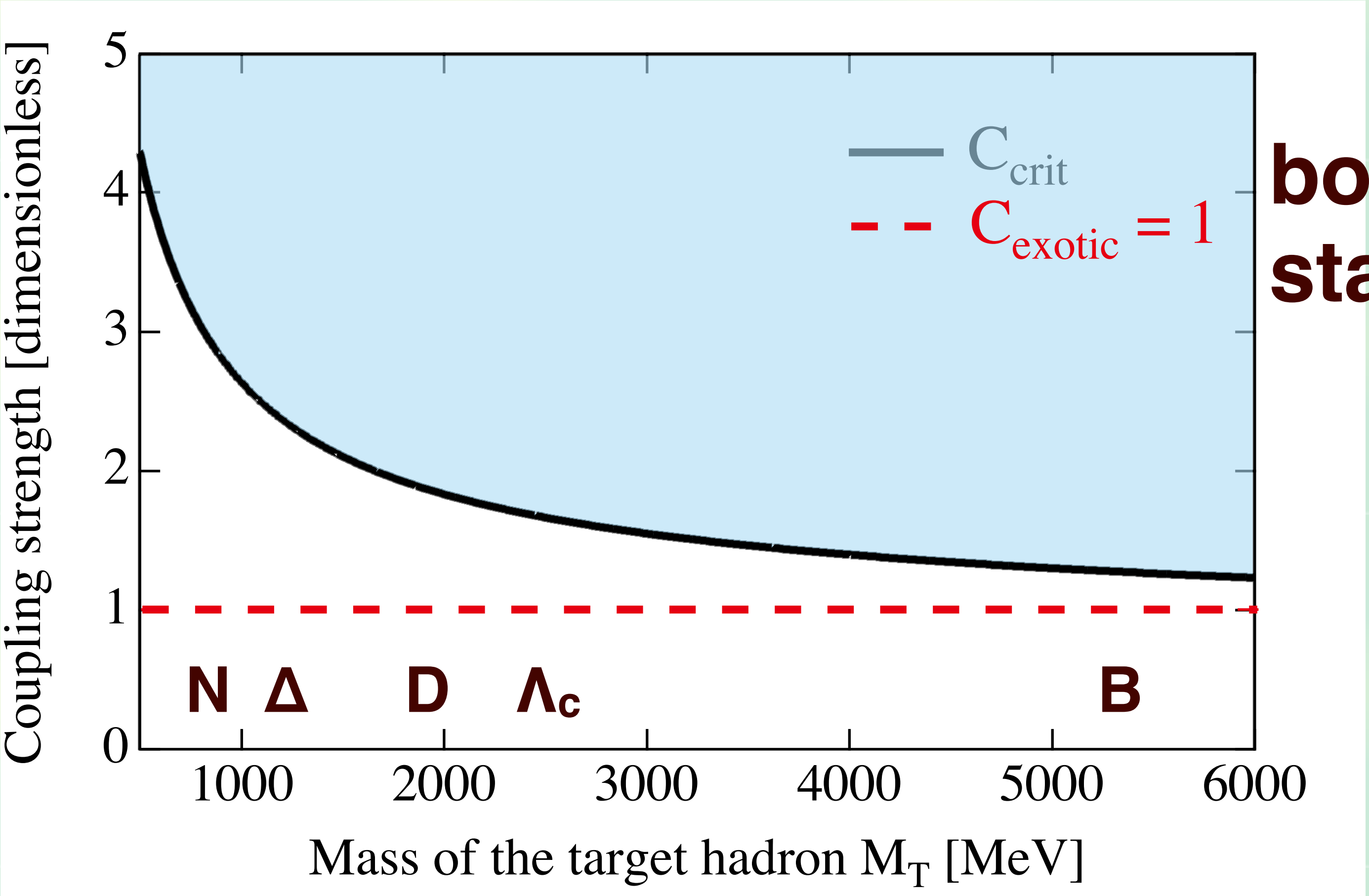
$1 - VG = 1$



Critical attraction : $1 - VG = 0$ at $\sqrt{s} = M_T + m$

$$\Rightarrow \quad C_{\rm crit} = \frac{2f^2}{m[-G(M_T + m)]}$$

Critical attraction and exotic channel



bound state

$$m = 368 \text{ MeV and } f = 93 \text{ MeV}$$

→ **Strength is not enough.**

Summary 1 : SU(3) limit

We study the exotic bound states in s-wave chiral dynamics in flavor SU(3) limit.

- **The interactions in exotic channels are in most cases repulsive.**
- **There are attractive interactions in exotic channels, with universal and the smallest strength :** $C_{\text{exotic}} = 1$
- **The strength is not enough to generate a bound state :** $C_{\text{exotic}} < C_{\text{crit}}$

The result is model independent as far as we respect chiral symmetry.

Summary 2 : Physical world

Caution!

- The exotic hadrons here are the s-wave meson-hadron molecule states ($1/2^-$ for Θ^+).
- We do not exclude the exotics which have other origins (genuine quark state, soliton rotation,...).
- In practice, SU(3) breaking effect, higher order terms,...

In Nature, it is difficult to generate exotic hadrons as in the same way with Λ(1405), Λ(1520),... based on chiral interaction.

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. Lett. 97, 192002 (2006)
T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. D 75, 034002 (2007)

Topics

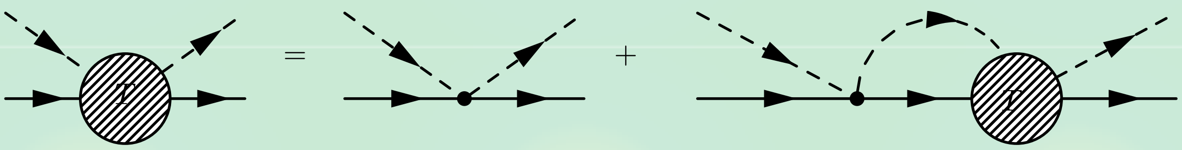
- **Physical meaning of the renormalization condition.**

- **Nc dependence of the interaction and bound states in large Nc limit**

Renormalization condition

Scattering amplitude

$$T = \frac{1}{1 - VG} V$$



$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}$$ **tree (ChPT)**

$$G(\sqrt{s}) = \frac{2M_T}{(4\pi)^2} \left(a(\mu) + \ln \frac{M_T^2}{\mu^2} + \dots \right)$$ **loop**

Subtraction constant $a(\mu)$: condition

$$G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T$$

K. Igi, and K. Hikasa, Phys. Rev. D59, 034005 (1999)

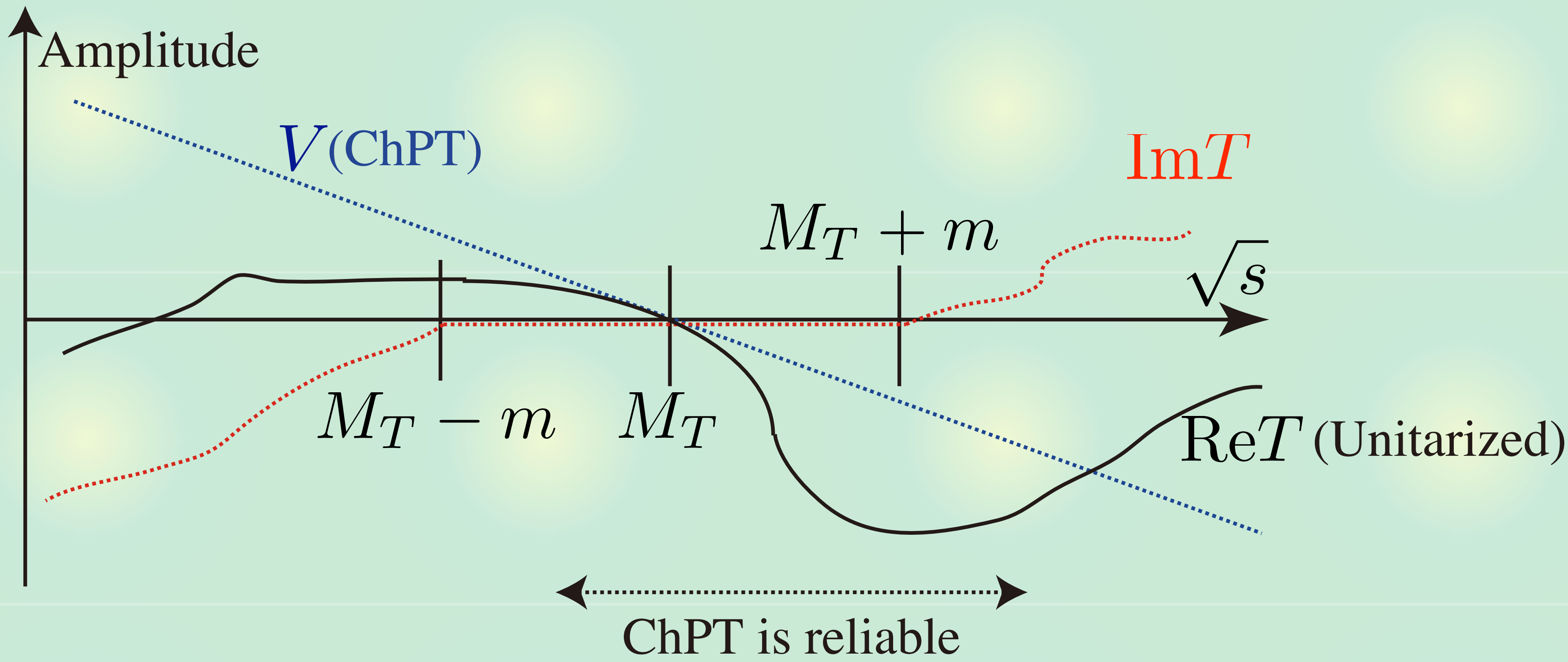
M.F.M. Lutz, and E. Kolomeitsev, Nucl. Phys. A700, 193-308 (2002)

Physical meaning of the condition?

Matching with ChPT

Match the unitarized amplitude with ChPT

$$G(\mu_m) = 0, \quad \Leftrightarrow \quad T(\mu_m) = V(\mu_m)$$



subtraction constant : real

$\Rightarrow \quad M_T - m \leq \mu_m \leq M_T + m$ **natural unitarization**

Effective attraction from the loop

$$T = (V^{-1} - G(a+\tilde{a}))^{-1} = ((V')^{-1} - G(a))^{-1}$$

$$V = -\frac{\omega}{2f^2}C \sim -\frac{\sqrt{s}-M_T}{2f^2}C$$

$$G(a) = \frac{2M_T}{(4\pi)^2}\left(a(\mu) + \ln\frac{M_T^2}{\mu^2} + \dots\right)$$

$$\Rightarrow (f')^2 = \frac{CM\tilde{a}}{16\pi^2}(\sqrt{s}-M) + f^2$$

negative $\tilde{a}$ --> smaller f^2 --> effective attraction

T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Phys. Rev. C. 68, 018201 (2003)
T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Prog. Thor. Phys. 112, 73 (2004)

The condition $\mu_m = M_T$: largest effective attraction in s-channel scattering region

Pole in the effective interaction

$$T = (V^{-1} - G(a+\tilde{a}))^{-1} = ((V')^{-1} - G(a))^{-1}$$

Effective interaction

$$V' = -\frac{8\pi^2}{M\tilde{a}} \frac{\sqrt{s} - M}{\sqrt{s} - M_{\text{eff.}}}$$

pole!

$$M_{\text{eff.}} = M - \frac{16\pi^2 f^2}{CM\tilde{a}}$$

Physically meaningful pole :

$$C > 0, \quad \tilde{a} < 0$$

Origin of dynamical pole?

Large Nc limit : introduction

1/Nc : a possible expansion parameter

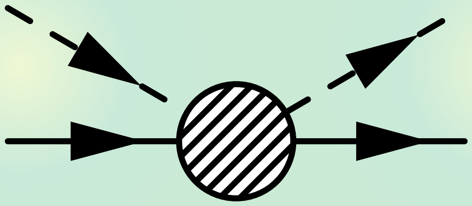
G. 't Hooft, Nucl. Phys. B72, 461 (1974)
E. Witten, Nucl. Phys. B160, 57 (1979)

Scaling of the physical quantities <- Nc^2 gluons and Nc quarks.

Meson mass : $m \sim \mathcal{O}(1)$

Baryon mass : $M \sim \mathcal{O}(N_c)$

Decay constant : $f \sim \mathcal{O}(\sqrt{N_c})$

MB scattering :  $\sim \mathcal{O}(1)$

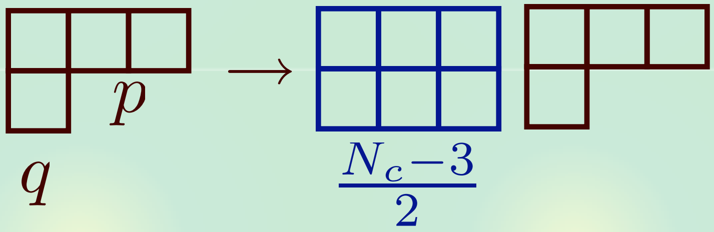
Coupling strengths in large Nc limit

WT interaction in large Nc limit

$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T} \sim \frac{1}{N_c} \times C_{\alpha,T}$$

Flavor representation of baryons

$$[p,q] \to \text{“}[p,q]\text{”} = \left[p, q + \tfrac{N_c - 3}{2}\right]$$



Coupling strength has linear Nc dependence

$$C_{\text{“}\alpha\text{”},\text{“}T\text{”}}(N_c) = C_2(\text{“}T\text{”}) - C_2(\text{“}\alpha\text{”}) + 3$$

$$C\left(\text{“}[p,q]\text{”}\right) = \frac{1}{3}\left(-\frac{9}{4} + p^2 + \frac{3p}{2} + pq + q^2\right) + \frac{1}{3}\left(\frac{p}{2} + q\right) N_c + \frac{N_c^2}{12}$$

Coupling strengths for the general target

For arbitrary Nc, $V \propto -\frac{C}{f^2} \sim -\frac{C(N_c)}{N_c}$

$\alpha \in [p,q] \otimes [1,1]$	$C_{\text{"}\alpha\text{"},\text{"}T\text{"}}(N_c)$	$V(N_c \to \infty)$
$[p+1, q+1]$	$(3-N_c)/2 - p - q$	repulsive
$[p+2, q-1]$	$1-p$	
$[p-1, q+2]$	$(5-N_c)/2 - q$	repulsive
$[p, q]$	3	
$[p, q]$	3	
$[p+1, q-2]$	$(3+N_c)/2 + q$	attractive
$[p-2, q+1]$	$3+p$	
$[p-1, q-1]$	$(5+N_c)/2 + p + q$	attractive

○ **No attraction in exotic channels.**

Coupling strengths : Examples

Coupling strengths with arbitrary Nc

$$C_{``\alpha", ``T"}(N_c) = C_2(``T") - C_2(``\alpha") + 3$$

α	"**1**"	"**8**"	"**10**"	"$\overline{\mathbf{10}}$"	"**27**"	"**35**"
$T =$ "**8**"	$\frac{9+N_c}{2}$	3	0	$\frac{3-N_c}{2}$	$\frac{-2-N_c}{2}$	
$T =$ "**10**"		6	3		$\frac{5-N_c}{2}$	$\frac{-3-N_c}{2}$

α	"$\overline{\mathbf{3}}$"	"**6**"	"$\overline{\mathbf{15}}$"	"**24**"
$T =$ "$\overline{\mathbf{3}}$"	3	1	$\frac{1-N_c}{2}$	
$T =$ "**6**"	5	3	$\frac{5-N_c}{2}$	$\frac{-2-N_c}{2}$

- **Exotic attraction -> repulsive**
- **Two poles of Λ(1405)?**

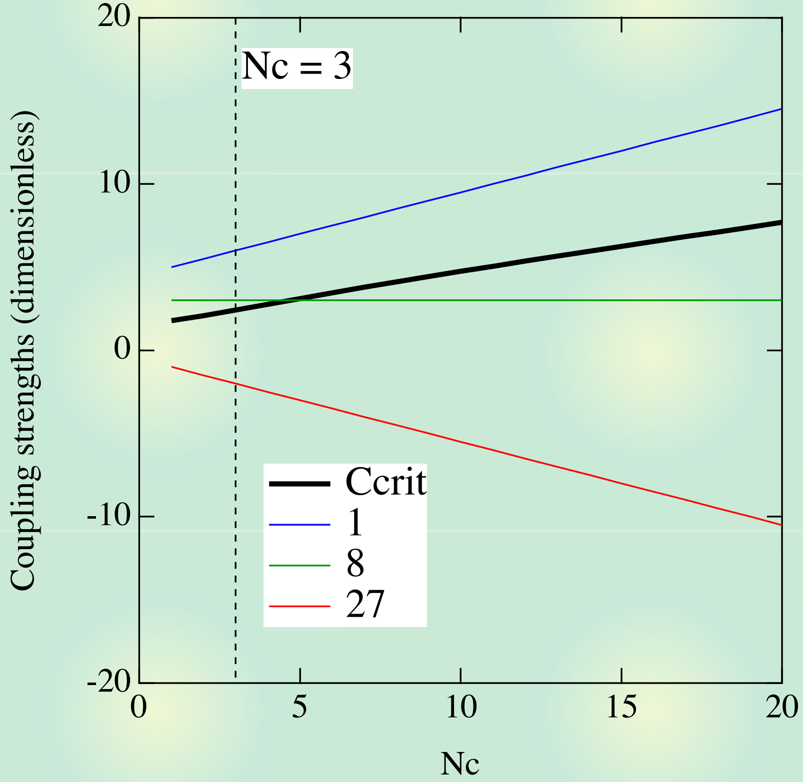
S = -1 I = 0 channel in SU(3) basis

α	"1"	"8_s"	"8_a"	"27"
$T =$ "8"	$\frac{9+N_c}{2}$	3	3	$\frac{-2-N_c}{2}$

$$C_{\rm crit}(N_c) = \frac{2[f(N_c)]^2}{m[-G(M_T(N_c)+m)]}$$

$$M_T(N_c) = M_0 \times \frac{N_c}{3}$$

$$f(N_c) = f_0 \times \sqrt{\frac{N_c}{3}}$$



○ **Bound state in "1" in the large Nc limit.**

S = -1 I = 0 channel in Isospin basis

Basis transformation via CG Coef. with Nc

$$
\begin{array}{ccccc}
 & \bar{K}N & \pi\Sigma & \eta\Lambda & K\Xi
\end{array}
$$

$$
C_{ij}(N_c) = \begin{pmatrix}
\frac{1}{2}(3+N_c) & -\frac{\sqrt{3}}{2}\sqrt{-1+N_c} & \frac{\sqrt{3}}{2}\sqrt{3+N_c} & 0 \\
-\frac{\sqrt{3}}{2}\sqrt{-1+N_c} & 4 & 0 & \frac{\sqrt{3+N_c}}{2} \\
\frac{\sqrt{3}}{2}\sqrt{3+N_c} & 0 & 0 & -\frac{\sqrt{3}}{2}\sqrt{-1+N_c} \\
0 & \frac{\sqrt{3+N_c}}{2} & -\frac{\sqrt{3}}{2}\sqrt{-1+N_c} & \frac{1}{2}(9-N_c)
\end{pmatrix}
$$

Combining with the 1/Nc factor of $1/f^2$,

- $\bar{K}N \to \bar{K}N$: attractive at large Nc
- $\bar{K}N \to \pi\Sigma$: $\mathcal{O}(1/\sqrt{N_c})$
- $\pi\Sigma \to \pi\Sigma$: $\mathcal{O}(1/N_c)$

Summary 2 : Topics

- **Physical meaning of the renormalization condition.**

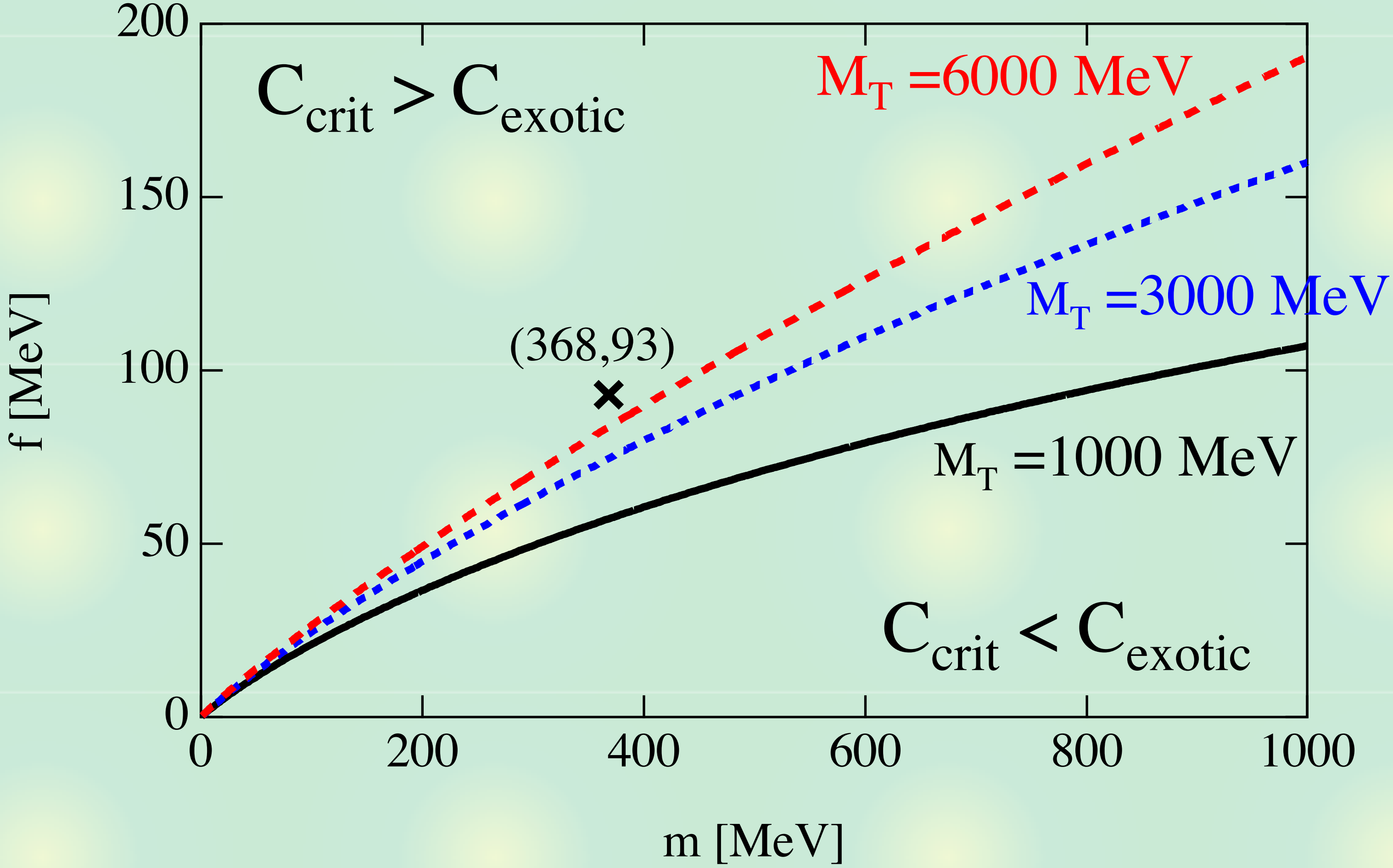
  The renomlization condition provides the largest strength of the effective attraction in the s-channel region.

- **Nc dependence of the interaction and bound states in large Nc limit**

  Attraction in “1” and $\bar{K}N$ channels is strong enough to provide a bound state in the large Nc limit.

Discussion : Dependence on the parameters

Lines for $C_{crit} = 1$ in (m, f) plane



○ C_{crit} becomes smaller for M_T↗, m↗ and f↘.