Λ(1405) in chiral dynamics

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Introduction: (well) known facts on $\Lambda(1405)$

$\Lambda(1405)$: $J^P = 1/2^-, I = 0$

Mass : $1406.5 \pm 4.0$ MeV
Width : $50 \pm 2$ MeV
Decay mode : $\Lambda(1405) \rightarrow (\pi \Sigma)_{I=0} \ 100\%$

"naive" quark model:
$p$-wave
$\sim 1600$ MeV?

N. Isgur and G. Karl, PRD18, 4187 (1978)

Coupled channel multi-scattering

R.H. Dalitz, T.C. Wong and G. Rajasekaran, PR153, 1617 (1967)

\[
\sigma(\pi^- \Sigma^+) \propto 1/3 |T^{I=0}|^2 + 1/2 |T^{I=1}|^2 - \frac{2}{\sqrt{6}} \text{Re}(T^{I=0} \cdot T^{I=1})
\]

Spectrum is not in $I=0$, but with the cross term, which may change the shape of the spectrum.
Phenomenology of $\bar{K}N$ interaction

Construction of local $\bar{K}N$ potential by chiral dynamics


Application to three-body $\bar{K}NN$ system


Structure of the $\Lambda(1405)$

Nc Behavior and quark structure


Dynamical or CDD (genuine quark state)?


Electromagnetic properties

Chiral unitary approach and \( \Lambda(1405) \)

\[ T = \frac{1}{1 - VG} V \]

**Strong attraction (\(<-\) chiral)**

**Bound state below threshold**

\( S = -1, \bar{K}N \) s-wave scattering : \( \Lambda(1405) \) in \( I=0 \)

- **Interaction \( <-> \) chiral symmetry**
- **Amplitude \( <-> \) unitarity (coupled channel)**


**Non-perturbative framework**
Total cross sections of $K^-p$ scattering

Chiral unitary approach and $\Lambda(1405)$

Chiral unitary approach and $\Lambda(1405)$

**Description of the resonances**

**Poles of the amplitude: resonance**

$$T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - M_R + i\Gamma_R/2}$$

<table>
<thead>
<tr>
<th>Real part</th>
<th>Mass</th>
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<tbody>
<tr>
<td>Imaginary part</td>
<td>Width/2</td>
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<td>Couplings</td>
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</table>

Successful description of $\overline{K}N$ scattering

**Two poles for the $\Lambda(1405)$**
Motivation

Deeply bound (few-body) kaonic nuclei?

Potential is purely phenomenological. What does chiral dynamics tell us about it?

Construction of the effective interaction

**Effective interaction based on chiral SU(3) dynamics**

Result of chiral dynamics $\rightarrow$ single channel potential

- **Coupled-channel BS** $T_{ij} (\sqrt{s})$
- **+ real interaction** $V_{ij} (\sqrt{s})$

(exact)

- **few-body kaonic nuclei**

- **Single-channel BS** $T^{\text{eff}} (\sqrt{s}) = T_{ii} (\sqrt{s})$
- **+ complex interaction** $V^{\text{eff}} (\sqrt{s})$

(approximate)

- **Schrödinger equation** $f^{\text{eff}} (\sqrt{s}) \sim T^{\text{eff}} (\sqrt{s})$
- **+ local potential** $U^{\text{eff}} (r, \sqrt{s})$

complex, energy-dependent
Construction of the effective interaction

Construction of the single channel interaction

Resummation of the channel to be eliminated

\[ T_{22}^{\text{single}} = V_{22} + V_{22} G_2 T_{22}^{\text{single}} \]

\[ V_{\text{eff}} = V_{11} + V_{12} G_2 V_{21} + V_{12} G_2 T_{22}^{\text{single}} G_2 V_{21} \]

\[ T_{11} = T_{\text{eff}} = V_{\text{eff}} + V_{\text{eff}} G_1 T_{\text{eff}} \]

Equivalent to the coupled-channel equations
Construction of the effective interaction

Single channel $\bar{K}N$ interaction with $\pi\Sigma$ dynamics

Strength: comparable with the WT term

$\sim 1/2$ of phenomenological (Akaishi-Yamazaki) potential

$\pi\Sigma$ resummation: small but pole exists
Construction of the effective interaction

**Scattering amplitude in $\bar{K}N$ and $\pi\Sigma$**

Resonance in $\bar{K}N$: around 1420 MeV

$\leftrightarrow$ two-pole structure (coupled-channel)

Binding energy: $B = 15$ MeV $\leftrightarrow 30$ MeV

$\sim 1420$ MeV

$\sim 1405$ MeV

Experiment
Construction of the effective interaction

**Origin of the two-pole structure**

**Chiral interaction**

\[ V_{ij} = -C_{ij} \frac{\omega_i + \omega_j}{4f^2} \]

\[ C_{ij} = \begin{pmatrix} \overline{KN} & \pi\Sigma \\ 3 & -\sqrt{\frac{3}{2}} \\ -\sqrt{\frac{3}{2}} & 4 \end{pmatrix} \]

\[ \omega_i \sim m_i, \quad 3.3m_\pi \sim m_K \]

Very strong attraction in \( \overline{KN} \) (higher energy) --- bound state

Strong attraction in \( \pi\Sigma \) (lower energy) --- resonance

Two poles : natural consequence of chiral interaction

higher order correction? --> theoretical uncertainty (later)

**Construction of the effective interaction**

**Comparison with phenomenological potential**

### Chiral interaction

$$V_{ij} = -C_{ij} \frac{(\omega_i + \omega_j)}{4f^2}$$

$$C_{ij} = \begin{pmatrix}
3 & -\sqrt{\frac{3}{2}} \\
-\sqrt{\frac{3}{2}} & 4
\end{pmatrix}$$

### Phenomenological

$$v_{ij}(r) \sim -\begin{pmatrix}
436 & 412 \\
412 & 0
\end{pmatrix} g(r)$$

**Absence of $\pi\Sigma$ diagonal coupling**

--> absence of $\pi\Sigma$ dynamics, resonance

--> strong ($\times2$) attractive interaction in $\bar{K}N$

**$\pi\Sigma$ -> $\pi\Sigma$ attraction** : flavor SU(3) symmetry

**Energy dependence** : derivative coupling
Constrution of the effective interaction

\[ U(r, \sqrt{s}) = \frac{M_N V^{\text{eff}}(\sqrt{s})}{2 s^{1/2}} \frac{\sqrt{s}}{\tilde{\omega}(\sqrt{s})} g(r) \]

\[ g(r) = \frac{e^{-r^2/2b^2}}{\pi^{3/2} b^3} \]

\[ b = 0.47 \text{ fm} \] : to reproduce the resonance agreement around threshold : OK

Deviation at lower energy : BS eq. \( <--> \) local potential + Schrödinger eq.
Correction of the strength of the potential

Construction of the effective interaction
We derive the single-channel local potential based on chiral SU(3) dynamics.

Resonance structure in $\bar{K}N$ appears at around 1420 MeV $\leftarrow$ two-pole $\Lambda(1405)$. The strength of the $\bar{K}N$ interaction is comparable with the WT term.

Two poles are the consequence of two attractive interactions in $\bar{K}N$ and $\pi\Sigma$.

Local (non-rel) potential overestimates amplitude at lower energy.

Application to the few-body anti-K system

Application to three-body K-pp system

Hamiltonian: Realistic interactions

\[ \hat{H} = \hat{T} + \hat{V}_{NN} + \text{Re} \hat{V}_{KN}(\sqrt{s}) - \hat{T}_{CM} \]

Realistic NN potential (Av18)

\( \bar{K}N \) potential based on chiral SU(3) dynamics (real part) dispersive effect from imaginary part
\(~ 3-4 \text{ MeV in two-body } \bar{K}N \text{ system} \)

Self-consistency of kaon energy and \( \bar{K}N \) interaction

Variational calculation

Model wave function: \( J^P = 0^-, T = 1/2, T_3 = 1/2 \)

\[ |\Psi\rangle = \mathcal{N}^{-1} [ |\Phi_+\rangle + C |\Phi_-\rangle ] \]

\( T_N = 0 \)

\( T_N = 1, \) dominant, used in Faddeev
Application to the few-body anti-K system

Theoretical uncertainties

Different models of chiral dynamics

Energy dependence of $\bar{K}\Lambda$ interaction

Define antikaon “binding energy”

$$-B_K \equiv \langle \Psi | \hat{H} | \Psi \rangle - \langle \Psi | \hat{H}_N | \Psi \rangle$$

Two options for two-body energy

Type I : $\sqrt{s} = M_N + m_K - B_K$

Type II : $\sqrt{s} = M_N + m_K - B_K/2$
We study the $\bar{K}NN$ system with chiral SU(3) potentials in a variational approach.

With theoretical uncertainties,

- $B.E. = 19 \pm 3$ MeV
- $\Gamma(\pi YN) = 40 \sim 70$ MeV

Phenomenological potential

- $B.E. \sim 48$ MeV
- $\Gamma \sim 60$ MeV


Faddeev with chiral interaction

- $B.E. \sim 79$ MeV
- $\Gamma \sim 74$ MeV


No two-nucleon absorption: $\bar{K}NN \rightarrow YN ...$ small?

Structure of dynamically generated resonances

Quark structure of resonances?

\( \text{\textless-- known Nc scaling of } q\bar{q} \text{ meson} \)

\[ m \sim \mathcal{O}(1), \quad \Gamma \sim \mathcal{O}(1/N_c), \]

\text{can be used to distinguish } q\bar{q} \text{ from others}

c.f. \( \rho \) meson in \( \pi\pi \) scattering

\( \text{\textlesssim originate from the contracted resonance propagator} \)

\text{in higher order terms}


\text{analysis of Nc scaling } \rightarrow \rho \sim q\bar{q}


Baryon resonances?

\( \text{\textgreater-- analysis of Nc scaling} \)
Introduce the $N_c$ scaling into the model and study the behavior of resonance.

$$m \sim \mathcal{O}(1), \quad M \sim \mathcal{O}(N_c), \quad f \sim \mathcal{O}(\sqrt{N_c})$$

**Leading order** WT interaction has $N_c$ dep.

$$V = -C \frac{\omega}{2f^2} \sim \mathcal{O}(1/N_c) \quad (\Leftarrow C \sim \mathcal{O}(1))$$

(for baryon and $N_f > 2$)

$$V = -C \frac{\omega}{2f^2}, \quad C \sim \mathcal{O}(N_c) \quad \Rightarrow \quad V \sim \mathcal{O}(1)$$


c.f. meson-meson scattering : $V_{LO} \sim \mathcal{O}(1/N_c) = $ trivial

Nontrivial $N_c$ dependence of the interaction is in **NLO**.
**S = -1, I = 0 channel in SU(3) basis**

**Coupling strengths with Nc dependence**

\[ V = -C \frac{\omega}{2f^2} \quad f \sim O(\sqrt{N_c}) \]

\[
C^{SU(3)}_{ij}(N_c) = \begin{pmatrix}
1 & 8 & 8 & 27 \\
\frac{9}{2} + \frac{N_c}{2} & 0 & 0 & 0 \\
3 & 0 & 0 & 0 \\
-\frac{1}{2} - \frac{N_c}{2} & 3 & 0 & 0
\end{pmatrix}
\]

**C \propto Nc : finite interaction at Nc \rightarrow \infty**

**Attractive interaction in singlet channel**
Coupling strengths with Nc dependence

\[ C^I_{ij}(N_c) = \begin{pmatrix}
\frac{1}{2}(3 + N_c) & -\frac{\sqrt{3}}{2} \sqrt{-1 + N_c} & \frac{\sqrt{3}}{2} \sqrt{3 + N_c} & 0 \\
\frac{\sqrt{3}}{2} \sqrt{-1 + N_c} & 4 & 0 & \frac{\sqrt{3 + N_c}}{2} \\
\frac{\sqrt{3}}{2} \sqrt{3 + N_c} & 0 & 0 & -\frac{3}{2} \sqrt{-1 + N_c} \\
0 & \frac{\sqrt{3 + N_c}}{2} & -\frac{3}{2} \sqrt{-1 + N_c} & \frac{1}{2}(9 - N_c)
\end{pmatrix} \]

Off-diagonal couplings vanish at Nc -> \( \infty \)

--> single-channel problem @ large Nc limit

Attractive interaction in \( \bar{K}N \) -> \( \bar{K}N \)

\( \bar{K}\Xi \) -> \( \bar{K}\Xi \) : attractive -> repulsive for Nc > 9
In the large $N_c$ limit

Attractive interaction in KN(singlet) channels

$C \sim N_c/2$

Critical coupling strength (with $N_c$ dep)

$$C_{\text{crit}}(N_c) = \frac{2[f(N_c)]^2}{m[-G(M_T(N_c) + m)]}$$

$$N_c/2 > C_{\text{crit}}(N_c)$$

Bound state in “1” or KN channels
Nc behavior and quark structure

With SU(3) breaking: Pole trajectories with varying Nc

1 bound state and 1 dissolving resonance

\[ \Gamma_R \neq O(1) \]

\[ \sim \text{non-qqq (i.e. dynamical) structure} \]

Nc scaling of excited qqq baryon

\[ M_R \sim O(N_c), \quad \Gamma_R \sim O(1) \]

Residues in the isospin basis

\[ \frac{|g_i|}{|g_{KN}|} \left\{ \begin{array}{ll}
< 1 : \bar{K}N \text{ dominant} \\
> 1 : \text{non} \bar{K}N \text{ dominant}
\end{array} \right. \]

Bound state \(\bar{K}N\) dominant

Dissolving other components

Nc behavior and quark structure
SU(3) components of the poles

Residues in the SU(3) basis

\[
\frac{|g_i|}{|g_1|} \begin{cases} < 1 : \text{singlet dominant} \\ > 1 : \text{non singlet dominant} \end{cases}
\]

bound state
1 dominant
dissolving
other components
We study the Nc scaling of the Λ(1405)

**Large Nc limit**
- Existence of a **bound state** in “1” or $\bar{K}N$ channel even in the **large Nc limit**

**Behavior around Nc = 3**
- 1 bound state and 1 dissolving pole: signal of the **non-qqq state**.
- Residues of the would-be-bound-state: dominated by “1” or $\bar{K}N$; consistent with large Nc limit.

Structure of dynamically generated resonances

Resonances ~ quasi-bound two-body states

<--> in some case, CDD pole (genuine state).

Renormalization
change of loop function
~ change of interaction kernel

Formulation of the N/D method
and the structure of low energy interaction
Renormalization schemes

Scattering amplitude in N/D method

\[ T = \frac{1}{V^{-1} - G} \]

**V**: interaction  \quad **G**: loop function (cutoff)

**Phenomenological scheme**

: \( V \) is given by ChPT, fit cutoff to data

**N/D method**: CDD pole contribution --> \( V \)

**Natural renormalization scheme**

: exclude CDD pole contribution from \( G \)

\[ G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T \]

Pole in the effective interaction

\[ T = (V^{-1} - G(a + \Delta a))^{-1} = (\left(\frac{1}{V'}\right)^{-1} - G(a))^{-1} \]

↑phenomenological ↑natural

Effective interaction in natural scheme

\[ V' = -\frac{8\pi^2}{M\Delta a} \frac{\sqrt{s} - M}{\sqrt{s} - M_{\text{eff}}} \]

\[ = -\frac{C}{2f^2} (\sqrt{s} - M_T) + \frac{C}{2f^2} \frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\text{eff}}} \]

\[ M_{\text{eff}} = M - \frac{16\pi^2 f^2}{CM\Delta a} \]

Physically meaningful pole:

\[ C > 0, \quad \Delta a < 0 \]

** energy scale of the effective pole **
Example: $\Lambda(1405)$ and $N(1535)$

$$\Delta V \equiv V' - V_{WT}$$

Dynamical or CDD

Origin of dynamical pole?
Summary 4: dynamical or CDD?

We study the origin of the resonances in the chiral unitary approach.

Natural renormalization

Exclude CDD pole contribution from the loop function, consistent with N/D.

Analysis of Λ(1405) and N(1535)

Λ(1405) : CDD pole would be small

N(1535) : appreciable contribution from CDD pole

Large Nc behavior

Summary 5: Structure of $\Lambda(1405)$

Schematic decomposition of $\Lambda(1405)$

$$|\Lambda(1405)\rangle = N_3|qqq\rangle + N_5|qqqq\bar{q}\rangle + N_{MB}|B\rangle|M\rangle + \ldots$$

- Analysis of Nc behavior
  - $N_3 << 1$

- Analysis of natural renormalization
  - $N_{MB}$ dominates

- Both analyses consistently indicate the dominance of $N_{MB}$ component

Not trivial! c.f. rho meson, $N(1535)$, ...