# $\Lambda$(1405) in chiral dynamics

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2008, Mar. 18th

# Introduction : (well) known facts on Λ(1405)

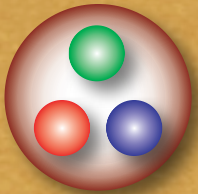
$\Lambda(1405) : J^P = 1/2^-, I = 0$

**Mass : 1406.5 ± 4.0 MeV**

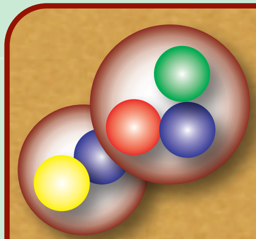
**Width : 50 ± 2 MeV**

**Decay mode :** $\Lambda(1405) \to (\pi\Sigma)_{I=0}$ **100%**

“naive” quark model
: p-wave
~1600 MeV?



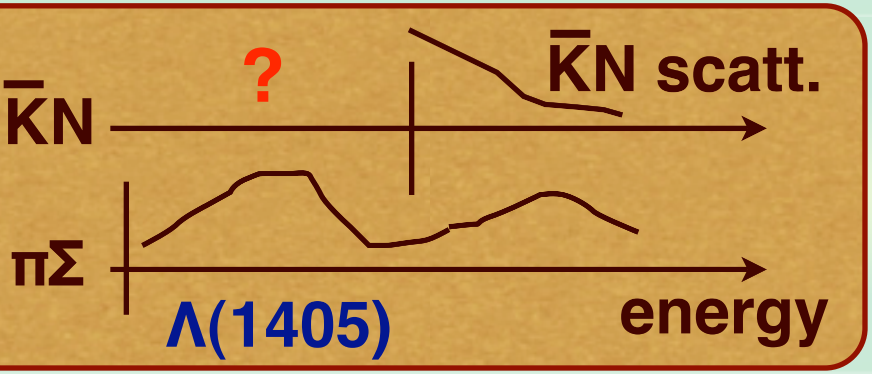
N. Isgur and G. Karl, PRD18, 4187 (1978)



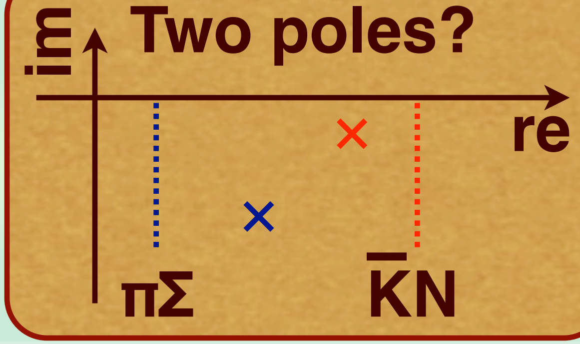
Coupled channel multi-scattering

R.H. Dalitz, T.C. Wong and G. Rajasekaran, PR153, 1617 (1967)

$\bar{K}N$ int. below threshold



Two poles?



# Contents

- Introduction to the chiral unitary approach

## Phenomenology of $\bar{K}N$ interaction

- Construction of local $\bar{K}N$ potential by chiral dynamics

  T. Hyodo, W. Weise, 0712,1613 [nucl-th], Phys. Rev. C, in press.

- Application to three-body $\bar{K}NN$ system

  A. Doté, T. Hyodo, W. Weise, 0802.0238 [nucl-th], Nucl. Phys. A, in press

## Structure of the Λ(1405)

- Nc Behavior and quark structure

  T. Hyodo, D. Jido, L. Roca, 0712.3347 [hep-ph], Phys. Rev. D, in press.

- Dynamical or CDD (genuine quark state) ?

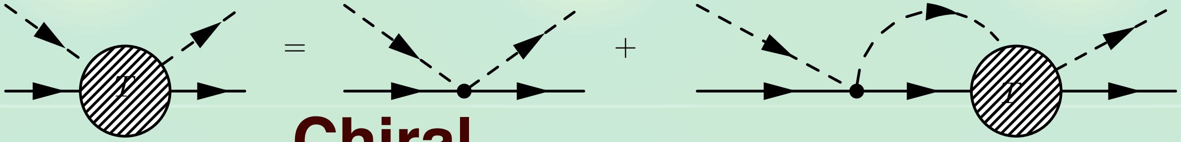
  T. Hyodo, D. Jido, A. Hosaka, 0803.2550 [nucl-th]

# Chiral unitary approach

## S = -1, $\bar{K}N$ s-wave scattering : Λ(1405) in I=0

- ○ Interaction <-- chiral symmetry
- ○ Amplitude <-- unitarity (coupled channel)

$$T = \frac{1}{1 - VG} V$$



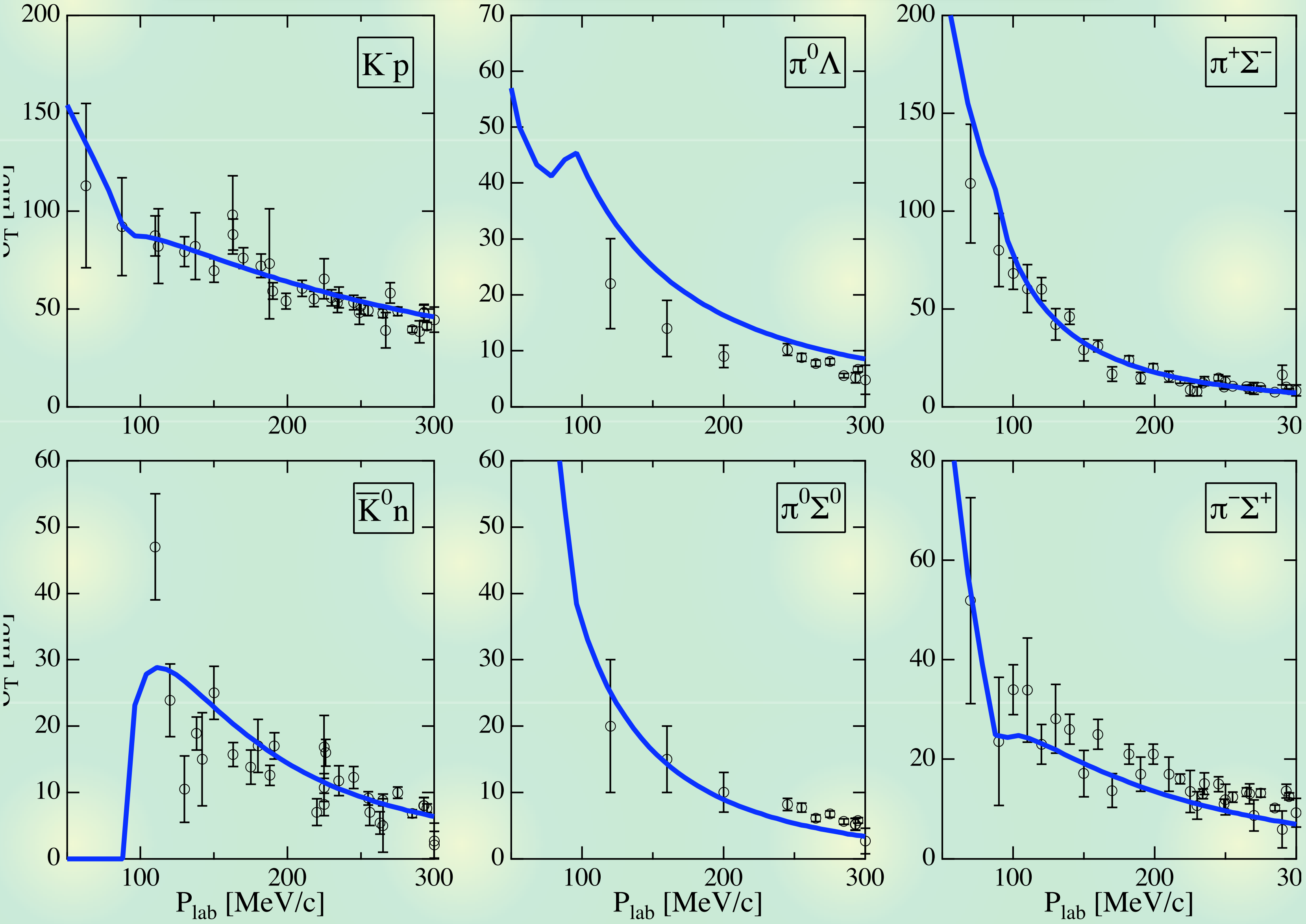
**Chiral (WT interaction)**

cutoff (subtraction constant)

N. Kaiser, P. B. Siegel, W. Weise, Nucl. Phys. A594, 325 (1995)
E. Oset, A. Ramos, Nucl. Phys. A635, 99 (1998)
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001)
M.F.M. Lutz, E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002),
.... many others

strong attraction (<- chiral)
bound state below threshold ➡ **non-perturbative framework**

# Total cross sections of K$^-$p scattering



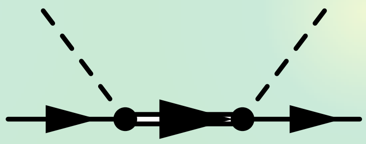
T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Phys. Rev. C68, 018201 (2003)

# Description of the resonances

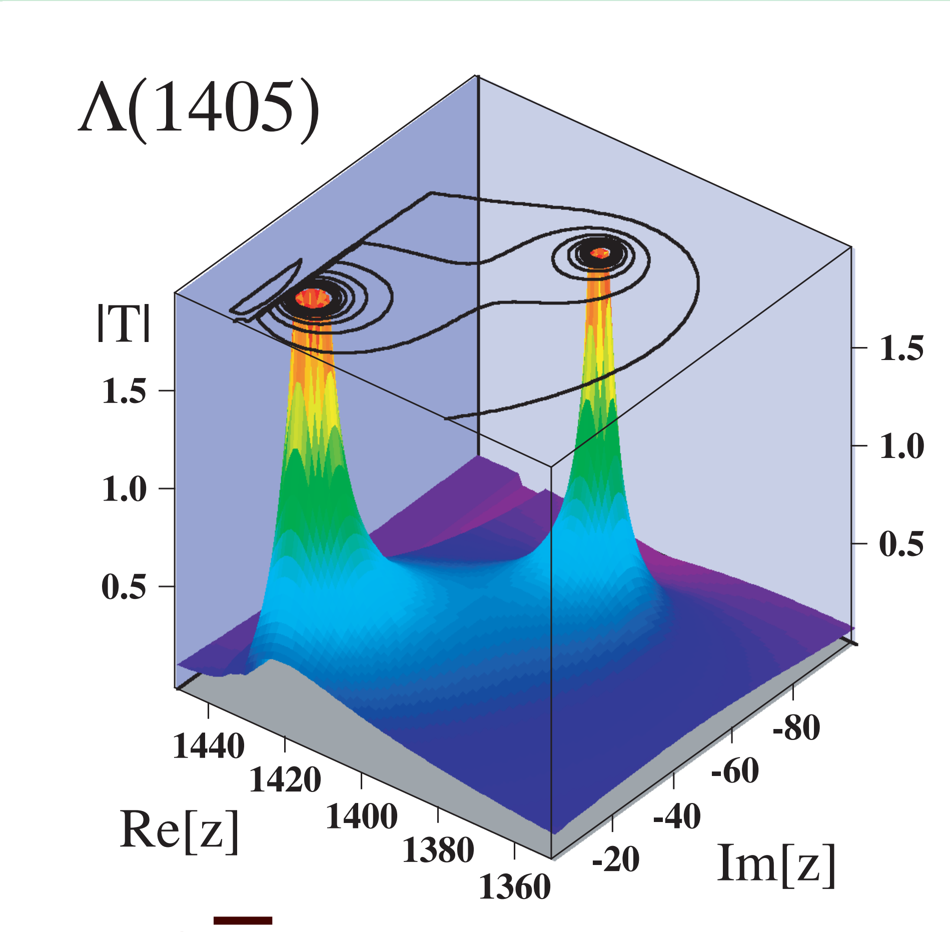
## Poles of the amplitude : resonance

$$T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - M_R + i\Gamma_R/2}$$

$\sim$



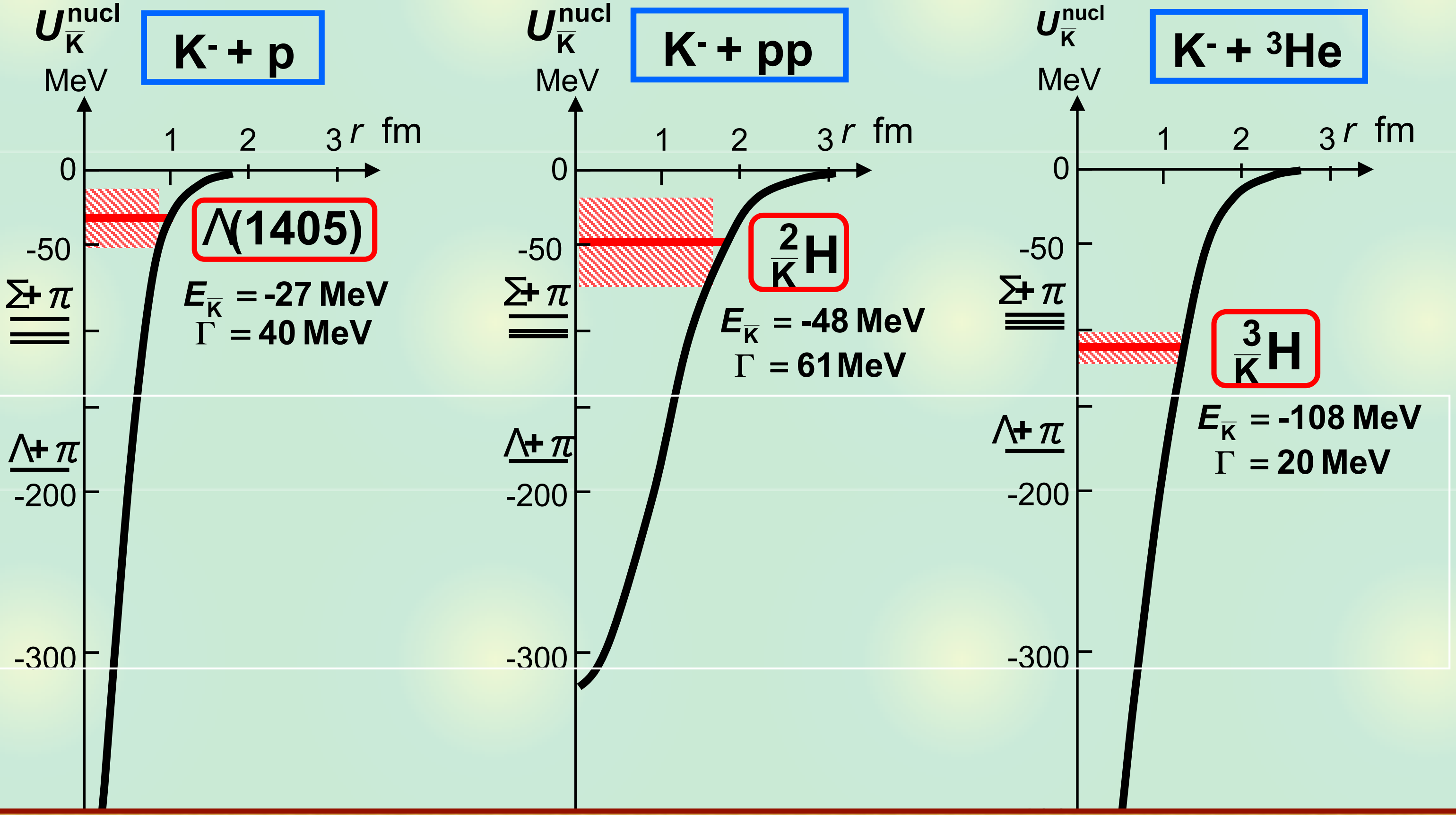
|  |  |
|---|---|
| **Real part** | **Mass** |
| **Imaginary part** | **Width/2** |
| **Residues** | **Couplings** |



◆ **Successful description of $\bar{K}N$ scattering**

◆ **Two poles for the Λ(1405)**

# Deeply bound (few-body) kaonic nuclei?



**Potential is purely phenomenological.**
**What does chiral dynamics tell us about it?**

Y. Akaishi & T. Yamazaki, Phys. Rev. C 65 (2002) 044005
T. Yamazaki & Y. Akaishi, Phys. Lett. B 535 (2002) 70

# Effective interaction based on chiral SU(3) dynamics

Result of chiral dynamics --> single channel potential

**Coupled-channel BS** $T_{ij}(\sqrt{s})$

**+ real interaction** $V_{ij}(\sqrt{s})$

(exact)

**Single-channel BS** $T^{\text{eff}}(\sqrt{s}) = T_{ii}(\sqrt{s})$

**+ complex interaction** $V^{\text{eff}}(\sqrt{s})$

(approximate)

**Schrödinger equation** $f^{\text{eff}}(\sqrt{s}) \sim T^{\text{eff}}(\sqrt{s})$

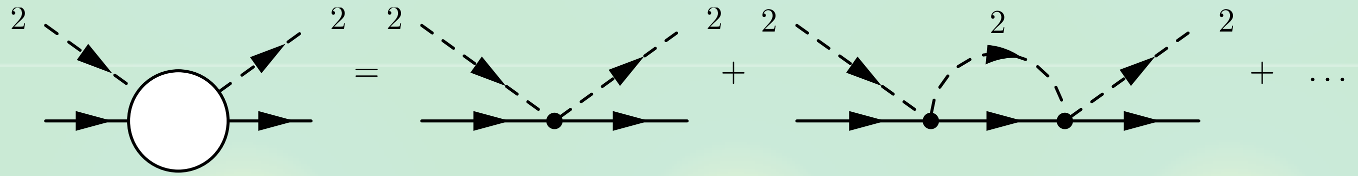
**+ local potential**

**complex, energy-dependent** $U^{\text{eff}}(r, \sqrt{s})$

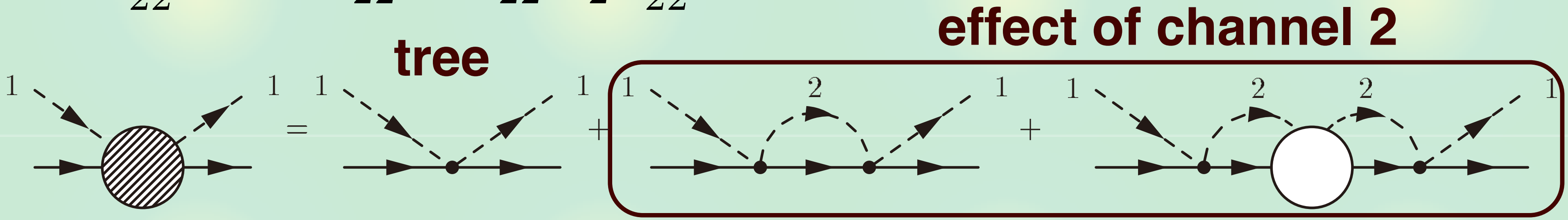
few-body kaonic nuclei

# Construction of the single channel interaction

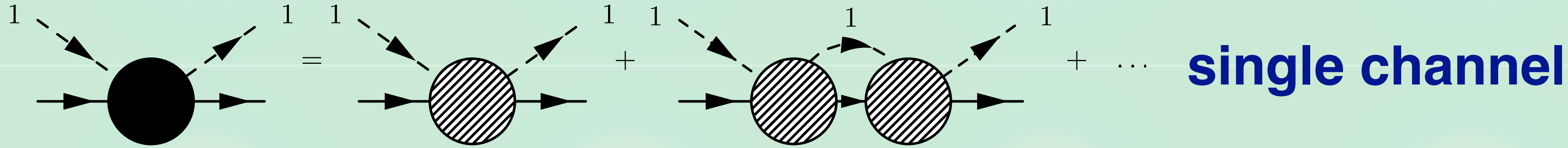
## Channels 1 and 2 --> effective int. in 1



$$T_{22}^{\text{single}} = V_{22} + V_{22}G_2T_{22}^{\text{single}}$$



$$V^{\text{eff}} = V_{11} + V_{12}G_2V_{21} + V_{12}G_2T_{22}^{\text{single}}G_2V_{21}$$

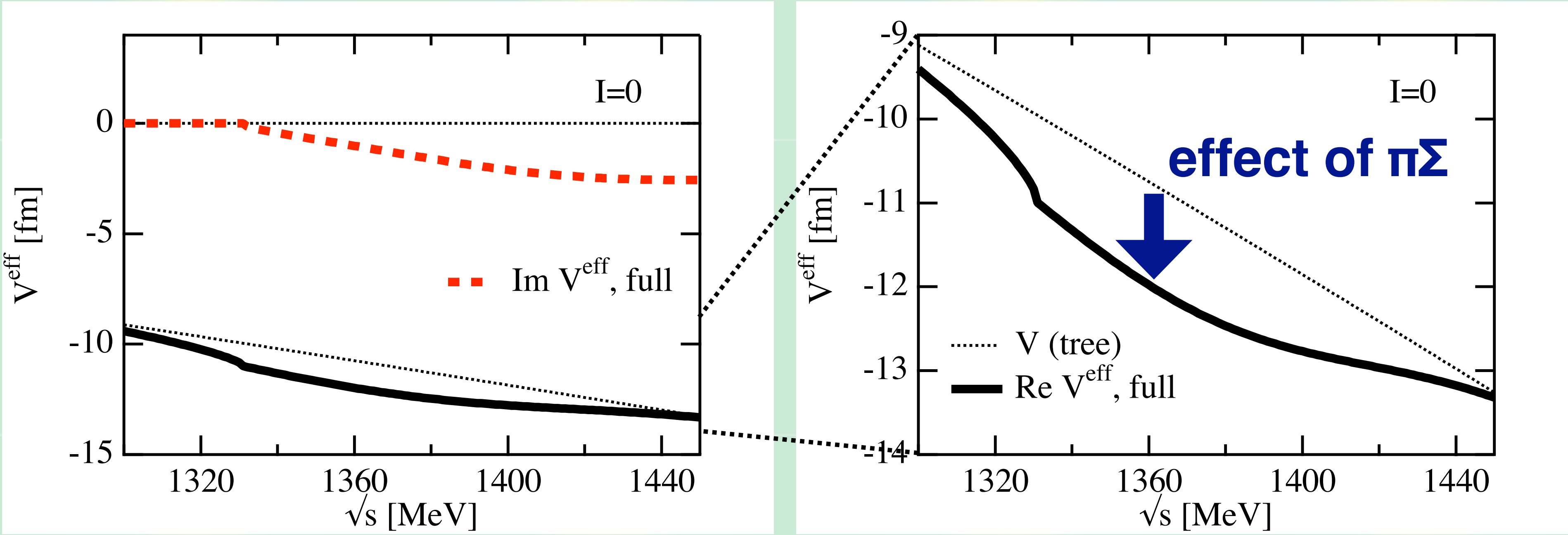


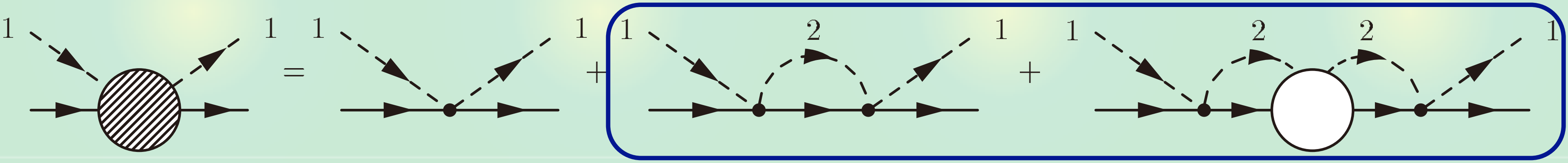
$$T_{11} = T^{\text{eff}} = V^{\text{eff}} + V^{\text{eff}}G_1T^{\text{eff}}$$

## Equivalent to the coupled-channel equations

**Construction of the effective interaction**

# Single channel $\bar{K}N$ interaction with $\pi\Sigma$ dynamics



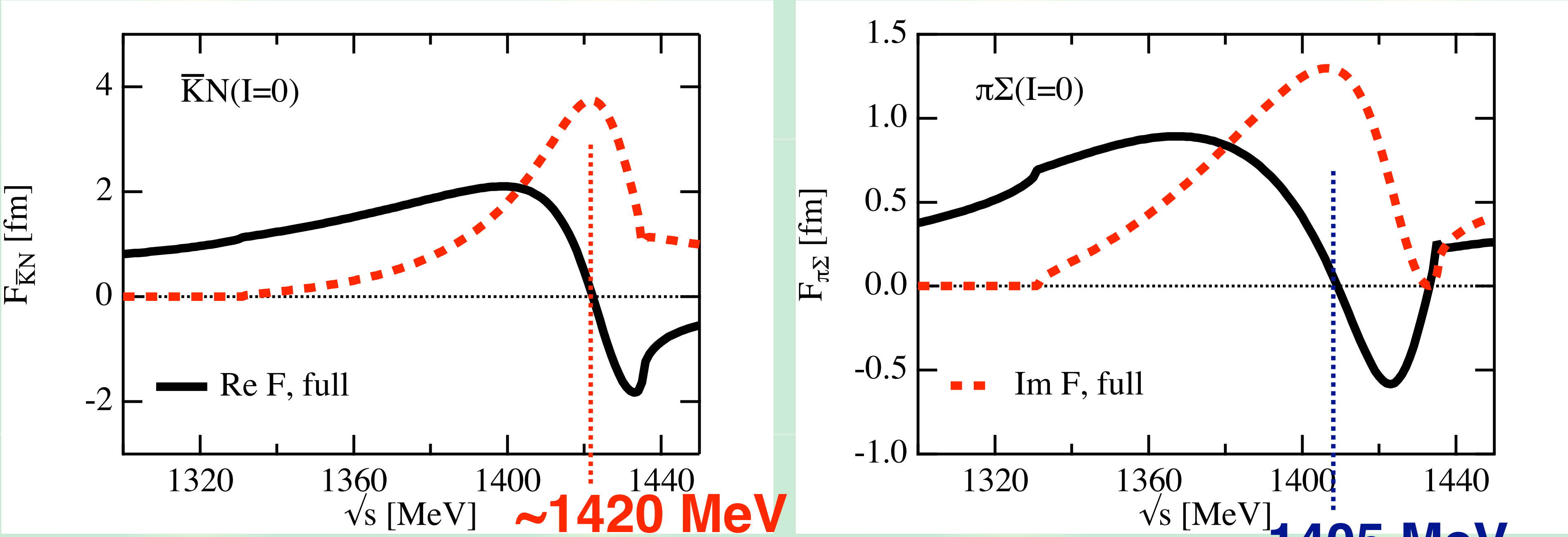


## Strength : comparable with the WT term

~ 1/2 of phenomenological (Akaishi-Yamazaki) potential

effect of πΣ resummation in $\bar{K}N$ channel is not large

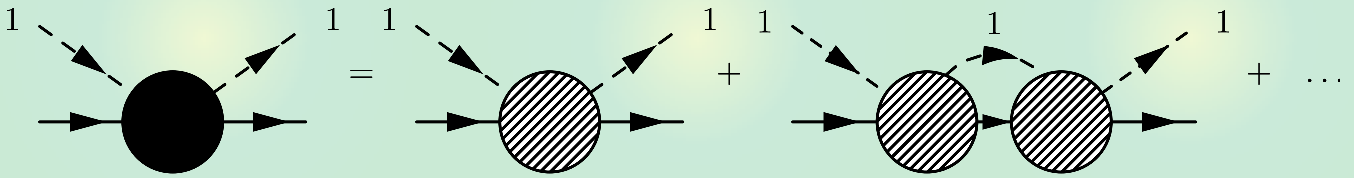
# Scattering amplitude in $\bar{K}N$ and $\pi\Sigma$



~1420 MeV

~1405 MeV

Experiment



**Resonance in $\bar{K}$N : around 1420 MeV**

**<-- two-pole structure (coupled-channel)**

**Binding energy : B = 15 MeV <--> 30 MeV**

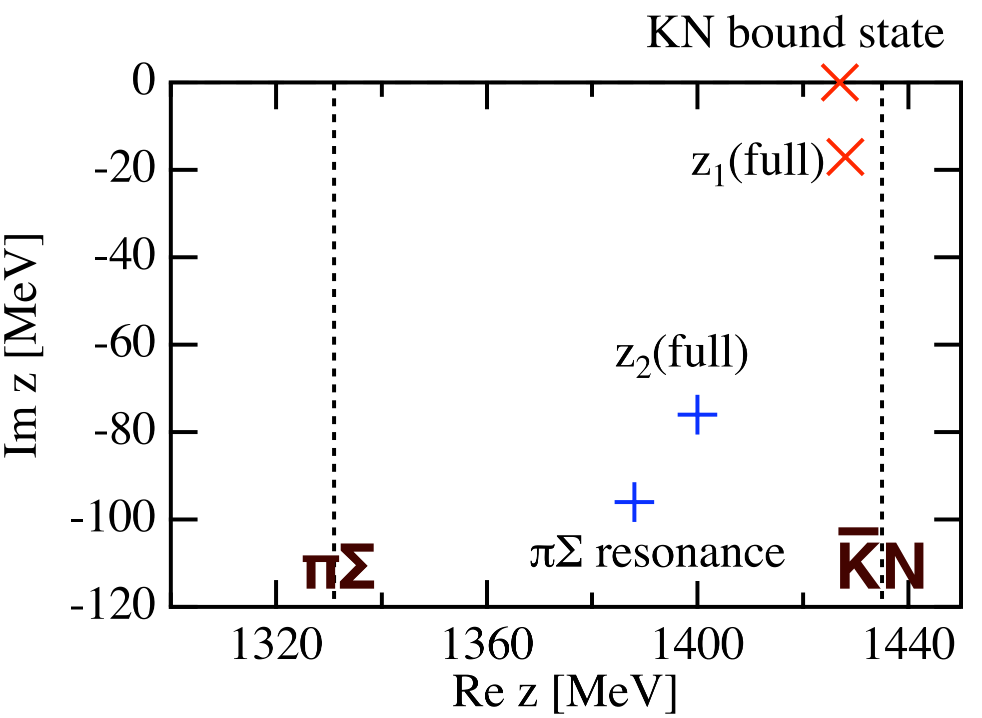
# Origin of the two-pole structure

## Chiral interaction

$$V_{ij} = -C_{ij}\frac{\omega_i + \omega_j}{4f^2}$$

$$C_{ij} = \begin{array}{cc} \bar{K}N & \pi\Sigma \\ \left(\begin{array}{c} 3 \\ -\sqrt{\frac{3}{2}} \end{array}\right. & \left.\begin{array}{c} -\sqrt{\frac{3}{2}} \\ 4 \end{array}\right) \end{array}$$

$$\omega_i \sim m_i, \quad 3.3m_\pi \sim m_K$$



**Very strong attraction in $\bar{K}N$ (higher energy) --> bound state**

**Strong attraction in $\pi\Sigma$ (lower energy) --> resonance**

**Two poles : natural consequence of chiral interaction**

**higher order correction? --> theoretical uncertainty (later)**

**B. Borasoy, R. Nissler, W. Weise, Eur. Phys. J. A25, 79-96 (2005)**

# Comparison with phenomenological potential

## Chiral interaction

$$V_{ij} = -C_{ij}\frac{\boxed{\omega_i + \omega_j}}{4f^2}$$

$$\begin{array}{cc} & \begin{array}{cc} \bar{K}N & \pi\Sigma \end{array} \\ C_{ij} = & \begin{pmatrix} 3 & -\sqrt{\frac{3}{2}} \\ -\sqrt{\frac{3}{2}} & \boxed{4} \end{pmatrix} \end{array}$$

## phenomenological

T. Yamazaki, Y. Akaishi,
Phys. Rev. C76, 045201 (2007)

$$\begin{array}{cc} & \begin{array}{cc} \bar{K}N & \pi\Sigma \end{array} \\ v_{ij}(r) \sim - & \begin{pmatrix} 436 & 412 \\ 412 & \boxed{0} \end{pmatrix} g(r) \end{array}$$

**Absence of πΣ diagonal coupling**

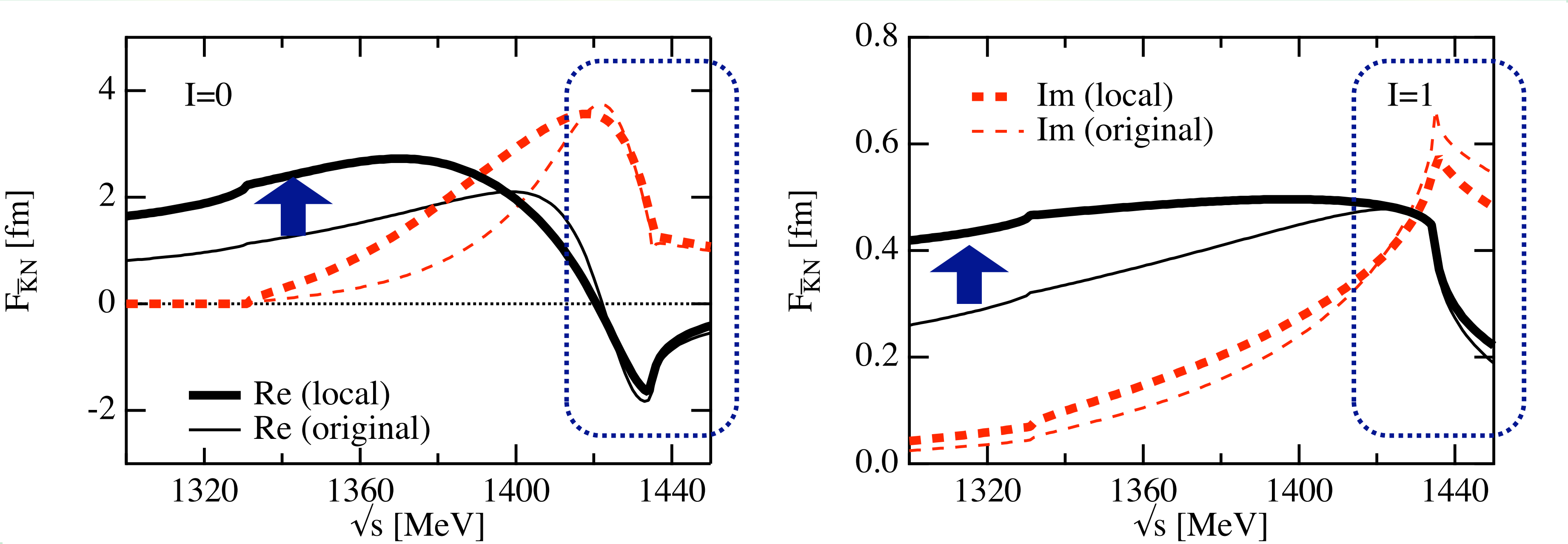
**--> absence of πΣ dynamics, resonance**

**--> strong (×2) attractive interaction in $\bar{K}N$**

**πΣ -> πΣ attraction : flavor SU(3) symmetry**

**energy dependence : derivative coupling**

# $\bar{\rm K}$N amplitude with local potential



$$U(r,\sqrt{s}) = \frac{M_N V^{\rm eff}(\sqrt{s})}{2\sqrt{s}\tilde{\omega}(\sqrt{s})} g(r) \qquad g(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2} b^3}$$

$b = 0.47$ fm **: to reproduce the resonance**

**agreement around threshold : OK**

**Deviation at lower energy :**

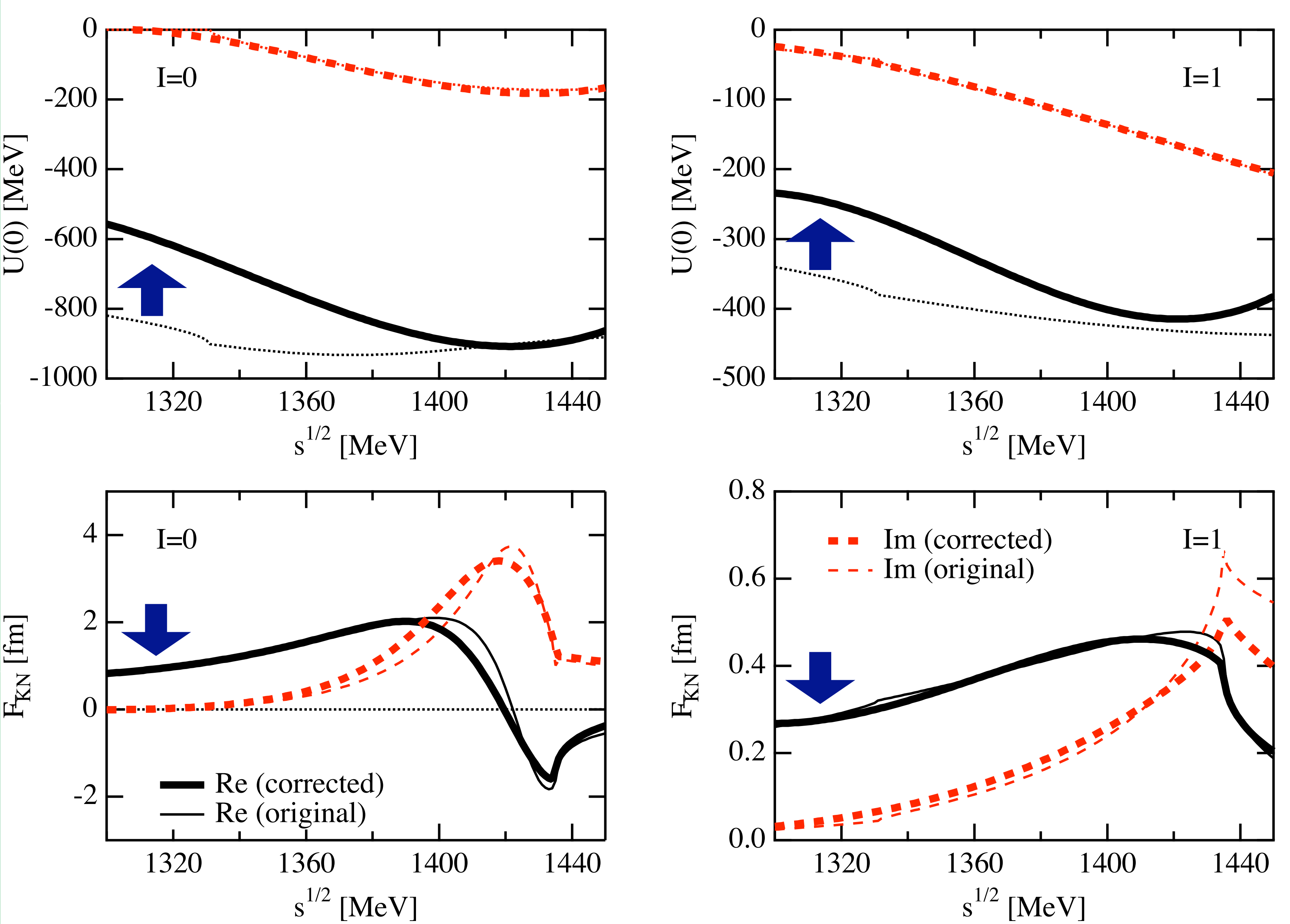
**BS eq. <--> local potential + Schrödinger eq.**

**Construction of the effective interaction**

# Correction of the strength of the potential

**Potential**

**Amplitude**



# Summary 1 : $\bar{K}N$ interaction

**We derive the single-channel local potential based on chiral SU(3) dynamics.**

- **Resonance structure in $\bar{K}N$ appears at around 1420 MeV <-- two-pole Λ(1405). The strength of the $\bar{K}N$ interaction is comparable with the WT term.**
- **Two poles are the consequence of two attractive interactions in $\bar{K}N$ and πΣ.**
- **Local (non-rel) potential overestimates amplitude at lower energy.**

T. Hyodo, W. Weise, 0712,1613 [nucl-th], Phys. Rev. C, in press.

# Application to three-body K-pp system

## Hamiltonian : Realistic interactions

$$\hat{H} = \hat{T} + \boxed{\hat{V}_{NN}} + \boxed{\mathrm{Re}\,\hat{V}_{\bar{K}N}(\sqrt{s})} - \hat{T}_{CM}$$

**Realistic NN potential (Av18)**

**$\bar{K}N$ potential based on chiral SU(3) dynamics (real part)**
**dispersive effect from imaginary part**
**~ 3-4 MeV in two-body $\bar{K}N$ system**

**Self-consistency of kaon energy and $\bar{K}N$ interaction**

## Model wave function

**$J^P = 0^-$ , $T = 1/2$, $T_3 = 1/2$**

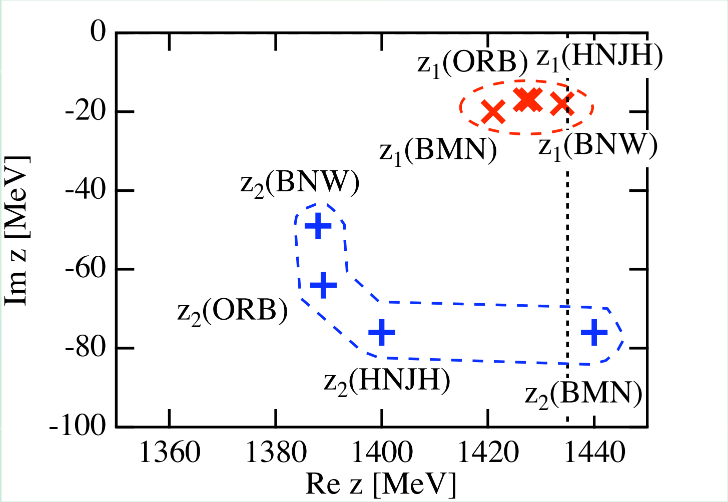
$$|\Psi\rangle = \mathcal{N}^{-1}[\,|\Phi_+\rangle + C\,|\Phi_-\rangle\,]$$

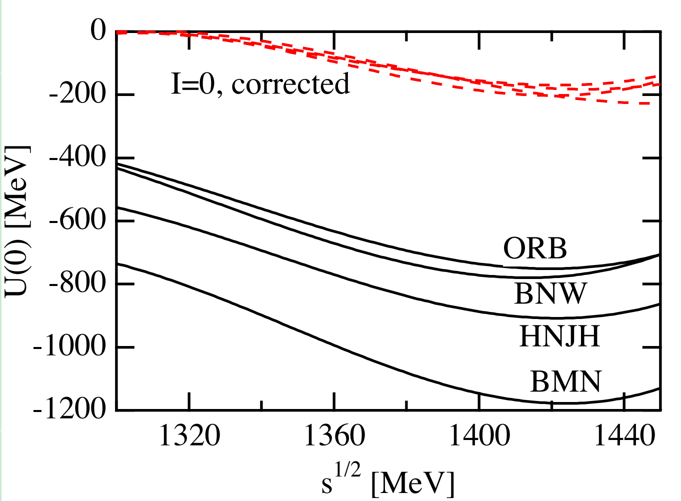
**$T_N = 0$**

**$T_N = 1$, dominant, used in Faddeev**

# Theoretical uncertainties

## Different models of chiral dynamics





## Energy dependence of $\bar{K}N$ interaction

**Define antikaon "binding energy"**

$$-B_K \equiv \langle\Psi|\hat{H}|\Psi\rangle - \langle\Psi|\hat{H}_N|\Psi\rangle$$

**Two options for two-body energy**

$$\text{Type I} \;: \sqrt{s} = M_N + m_K - B_K$$

$$\text{Type II} \;: \sqrt{s} = M_N + m_K - B_K/2$$

# Summary 2 : K-pp system

## We study the K-pp system with chiral SU(3) potentials in a variational approach.

**With theoretical uncertainties,**

B.E. = 19 ± 3 MeV

Γ(πYN) = 40 ~ 70 MeV

| | |
|---|---|
| Phenomenological potential<br>(~ 2 times stronger than ours) | B.E. ~ 48 MeV<br>Γ ~ 60 MeV |

T. Yamazaki, Y. Akaishi, Phys. Rev. C76, 045201 (2007)

| | |
|---|---|
| Faddeev with chiral interaction<br>(separable, non-rel, ...?) | B.E. ~ 79 MeV<br>Γ ~ 74 MeV |

Y. Ikeda, T. Sato, Phys. Rev. C76, 035203 (2007)

No two-nucleon absorption : $\bar{K}$NN -> YN ... small?

A. Doté, T. Hyodo, W. Weise, 0802.0238 [nucl-th], Nucl. Phys. A, in press

# Structure of dynamically generated resonances

**Quark structure of resonances?**

**<-- known Nc scaling of $q\bar{q}$ meson**

$$m \sim \mathcal{O}(1), \quad \Gamma \sim \mathcal{O}(1/N_c),$$

**can be used to distinguish $q\bar{q}$ from others**

c.f. ρ meson in ππ scattering

<-- originate from the contracted resonance propagator in higher order terms

J.A. Oller, E. Oset and J.R. Pelaez, Phys. Rev. D59, 074001 (1999)

G. Ecker, J. Gasser, A. Pich, and E. de Rafael, Nucl. Phys. B321, 311 (1989)

analysis of Nc scaling --> ρ ~ $q\bar{q}$

J.R. Pelaez, Phys. Rev. Lett. 92, 102001 (2004)

**Baryon resonances?**

**--> analysis of Nc scaling**

# Nc scaling in the model

**Introduce the Nc scaling into the model and study the behavior of resonance.**

$$m \sim \mathcal{O}(1), \quad M \sim \mathcal{O}(N_c), \quad f \sim \mathcal{O}(\sqrt{N_c})$$

**Leading order WT interaction has Nc dep.**

$$V = -C\frac{\omega}{2f^2} \sim \mathcal{O}(1/N_c) \quad (\Leftarrow C \sim \mathcal{O}(1))$$

**(for baryon and Nf > 2)**

$$V = -C\frac{\omega}{2f^2}, \quad \underline{C \sim \mathcal{O}(N_c)} \quad \Rightarrow V \sim \mathcal{O}(1)$$

**T. Hyodo, D. Jido and A. Hosaka, Phys. Rev. Lett. 97, 192002 (2006)**
**T. Hyodo, D. Jido and A. Hosaka, Phys. Rev. D75, 034002 (2007)**

**c.f. meson-meson scattering :** $V_{\mathrm{LO}} \sim \mathcal{O}(1/N_c)$ **= trivial**
**Nontrivial Nc dependence of the interaction is in NLO.**

# S = -1, I = 0 channel in SU(3) basis

## Coupling strengths with Nc dependence

$$V = -C\frac{\omega}{2f^2} \qquad f \sim \mathcal{O}(\sqrt{N_c})$$

$$C^{SU(3)}_{ij}(N_c) = \begin{array}{c} \begin{array}{cccc} \mathbf{1} & \mathbf{8} & \mathbf{8} & \mathbf{27} \end{array} \\ \begin{pmatrix} \boxed{\frac{9}{2}+\frac{N_c}{2}} & 0 & 0 & 0 \\ & 3 & 0 & 0 \\ & & 3 & 0 \\ & & & -\frac{1}{2}-\frac{N_c}{2} \end{pmatrix} \end{array}$$

**C ∝ Nc : finite interaction at Nc -> ∞**

**Attractive interaction in singlet channel**

# S = -1, I = 0 channel in Isospin basis

## Coupling strengths with Nc dependence

$$C^I_{ij}(N_c) = \begin{array}{c} \begin{array}{cccc} \bar{K}N & \pi\Sigma & \eta\Lambda & K\Xi \end{array} \\ \begin{pmatrix} \frac{1}{2}(3+N_c) & -\frac{\sqrt{3}}{2}\sqrt{-1+N_c} & \frac{\sqrt{3}}{2}\sqrt{3+N_c} & 0 \\ & 4 & 0 & \frac{\sqrt{3+N_c}}{2} \\ & & 0 & -\frac{3}{2}\sqrt{-1+N_c} \\ & & & \frac{1}{2}(9-N_c) \end{pmatrix} \end{array}$$

**Off-diagonal couplings vanish at Nc -> ∞**

**--> single-channel problem @ large Nc limit**

**Attractive interaction in $\bar{K}N$ -> $\bar{K}N$**

**KΞ -> KΞ : attractive -> repulsive for Nc > 9**

# In the large Nc limit

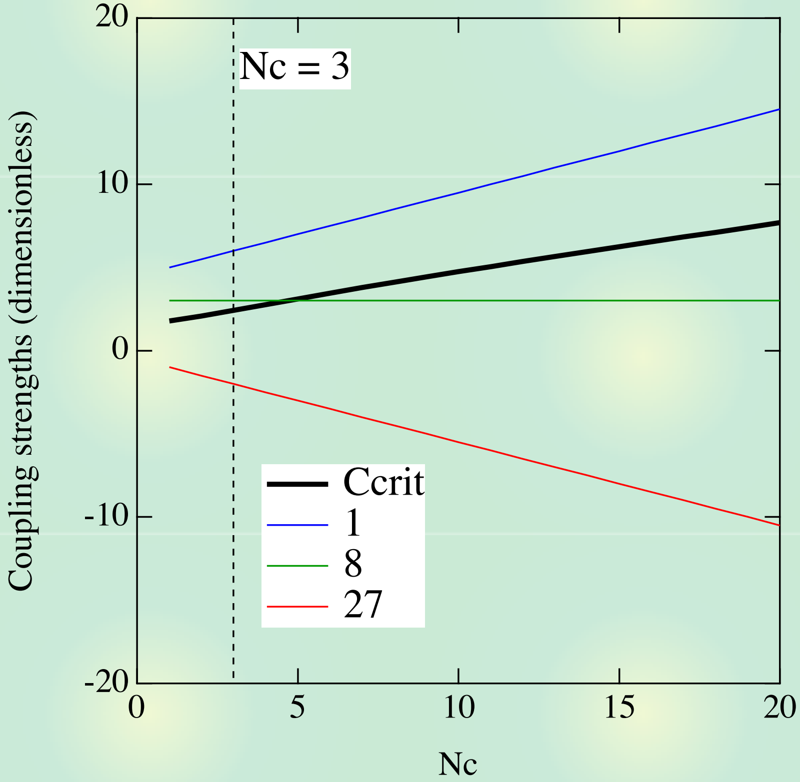
## Attractive interaction in $\bar{K}$N(singlet) channels

$$C \sim N_c/2$$

## Critical coupling strength (with Nc dep)

$$C_{\mathrm{crit}}(N_c) = \frac{2[f(N_c)]^2}{m[-G(M_T(N_c)+m)]}$$

$$N_c/2 > C_{\mathrm{crit}}(N_c)$$



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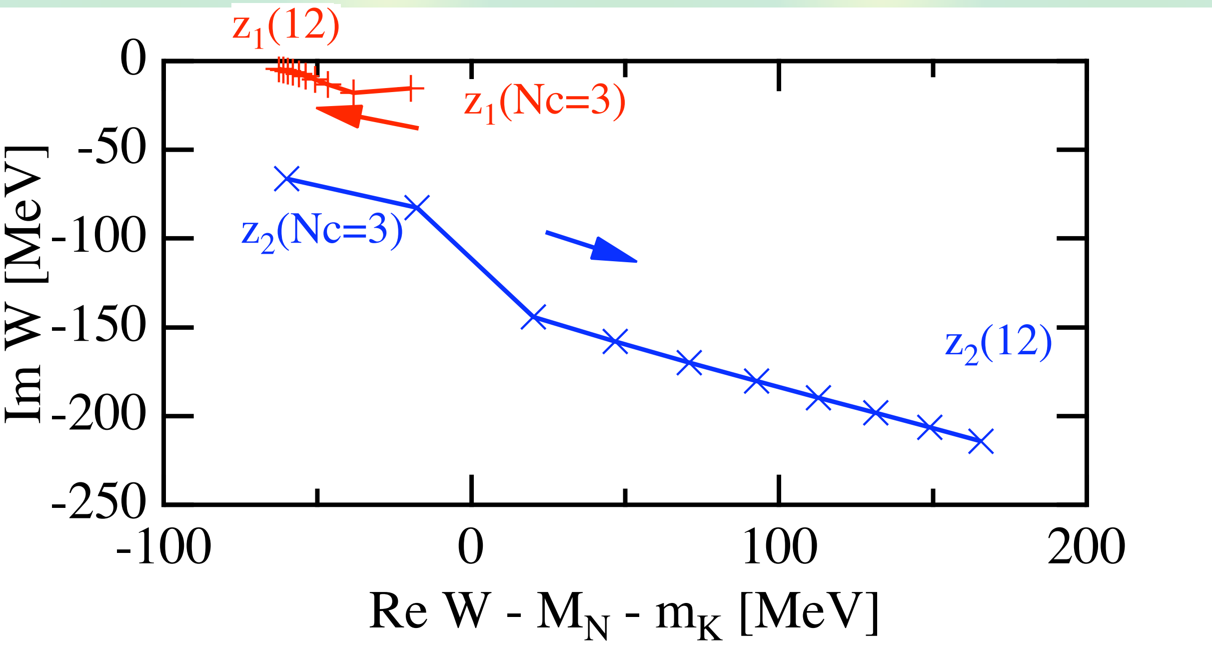
## **Bound state** in “1” or $\bar{K}$N channels

# With SU(3) breaking : Pole trajectories around Nc = 3



## 1 bound state and 1 dissolving resonance

**Nc scaling of (excited) qqq baryon**

$$M_R \sim \mathcal{O}(N_c), \quad \Gamma_R \sim \mathcal{O}(1)$$

T.D. Cohen, D.C. Dakin, A. Nellore, Phys. Rev. D69, 056001 (2004)

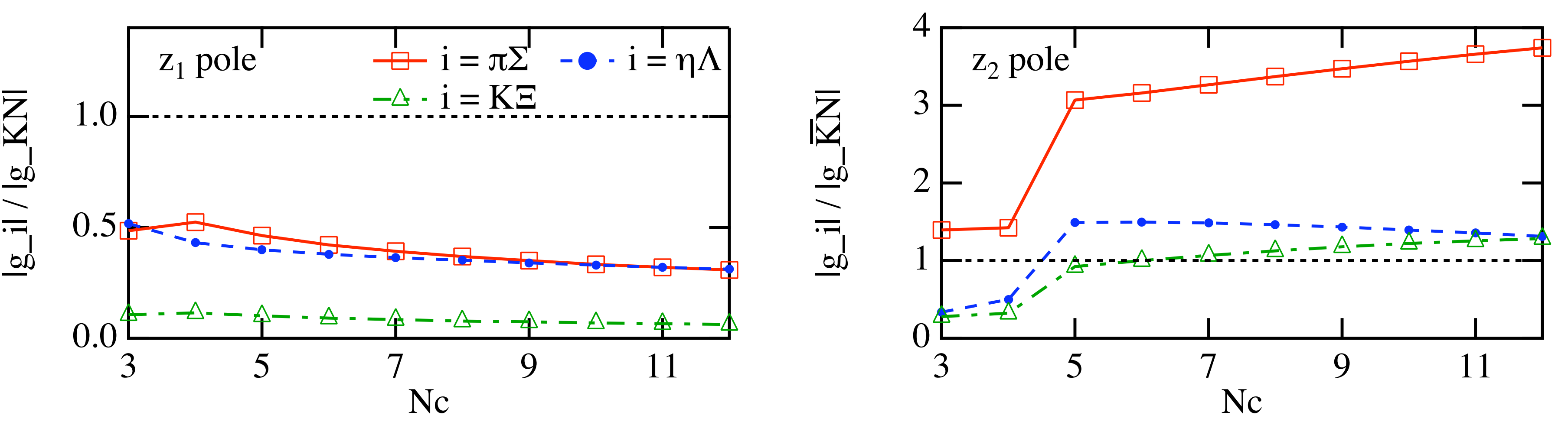
$$\Gamma_R \neq \mathcal{O}(1)$$

## ~ non-qqq (i.e. dynamical) structure

# Isospin conponents of the poles

## Residues in the isospin basis

$$\frac{|g_i|}{|g_{\bar{K}N}|}\begin{cases}<1: \bar{K}N \text{ dominant} \\ >1: \text{non } \bar{K}N \text{ dominant}\end{cases}$$



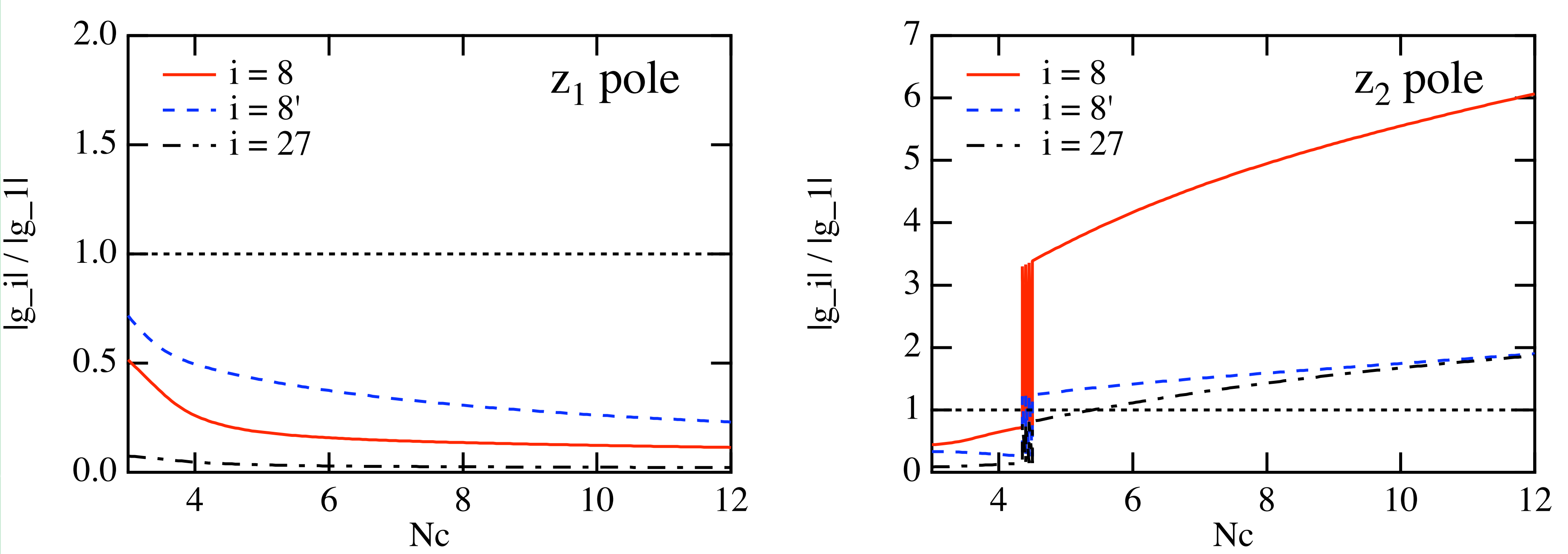
**bound state**
**K̄N dominant**

**dissolving**
**other components**

# SU(3) components of the poles

## Residues in the SU(3) basis

$$\frac{|g_i|}{|g_{\mathbf{1}}|}\begin{cases}<1 : \text{singlet dominant}\\>1 : \text{non singlet dominant}\end{cases}$$



**bound state**
**1 dominant**

**dissolving**
**other components**

# Summary 3 : Nc behavior of Λ(1405)

## We study the Nc scaling of the Λ(1405)

- **Large Nc limit**

  Existence of a bound state in "1" or $\bar{K}N$ channel even in the large Nc limit

- **Behavior around Nc = 3**

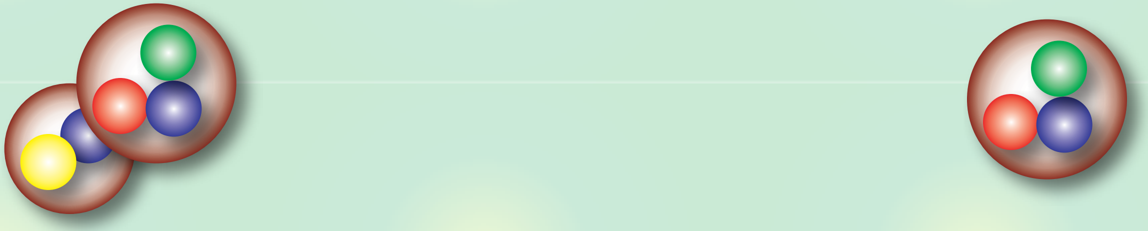
  1 bound state and 1 dissolving pole
  : signal of the non-qqq state.

  Residues of the would-be-bound-state
  : dominated by "1" or $\bar{K}N$
  : consistent with large Nc limit.

T. Hyodo, D. Jido, L. Roca, 0712.3347 [hep-ph], Phys. Rev. D, in press.

# Structure of dynamically generated resonances

Resonances ~ quasi-bound two-body states



<--> in some case, CDD pole (genuine state).

Renormalization
change of loop function
~ change of interaction kernel

Formulation of the N/D method
and the structure of low energy interaction

# Renormalization schemes

## Scattering amplitude in N/D method

$$T = \frac{1}{V^{-1} - G}$$

G : unitarity cut

V : other contribution (e.g. CDD pole)

## For meson-baryon scattering

- Identify G as loop function
- Matching with ChPT order by order
- V is given by ChPT (interaction kernel)

--> equivalent to solving BS equation

# Renormalization schemes

## Renormalization procedure

### Phenomenological scheme

: V is given by ChPT, fit cutoff in G to data

### Natural renormalization scheme

: determine G to exclude CDD pole contribution,
V is to be determined

$$G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T$$

c. f. K. Igi, and K. Hikasa, Phys. Rev. D59, 034005 (1999)
M.F.M. Lutz, and E. Kolomeitsev, Nucl. Phys. A700, 193-308 (2002)

## Same physics (scattering amplitude)

$$T = \frac{1}{V_{\mathrm{WT}}^{-1} - G(a_{\mathrm{pheno}})} = \frac{1}{(V_{\mathrm{natural}})^{-1} - G(a_{\mathrm{natural}})}$$

# Pole in the effective interaction

$$T=(V_{\mathrm{WT}}^{-1}-G(a_{\mathrm{pheno}}))^{-1}=((V_{\mathrm{natural}})^{-1}-G(a_{\mathrm{natural}}))^{-1}$$

**↑ChPT ↑data fit ↑given**

## Interaction kernel in natural scheme

$$\begin{aligned}V_{\mathrm{natural}} &= -\frac{8\pi^2}{M\Delta a}\frac{\sqrt{s}-M}{\sqrt{s}-M_{\mathrm{eff}}}\\ &= -\frac{C}{2f^2}(\sqrt{s}-M_T)+\boxed{\frac{C}{2f^2}\frac{(\sqrt{s}-M_T)^2}{\sqrt{s}-M_{\mathrm{eff}}}}\end{aligned}$$

**pole!**

$$M_{\mathrm{eff}}=M-\frac{16\pi^2 f^2}{CM\Delta a},\quad \Delta a=a_{\mathrm{pheno}}-a_{\mathrm{natural}}$$

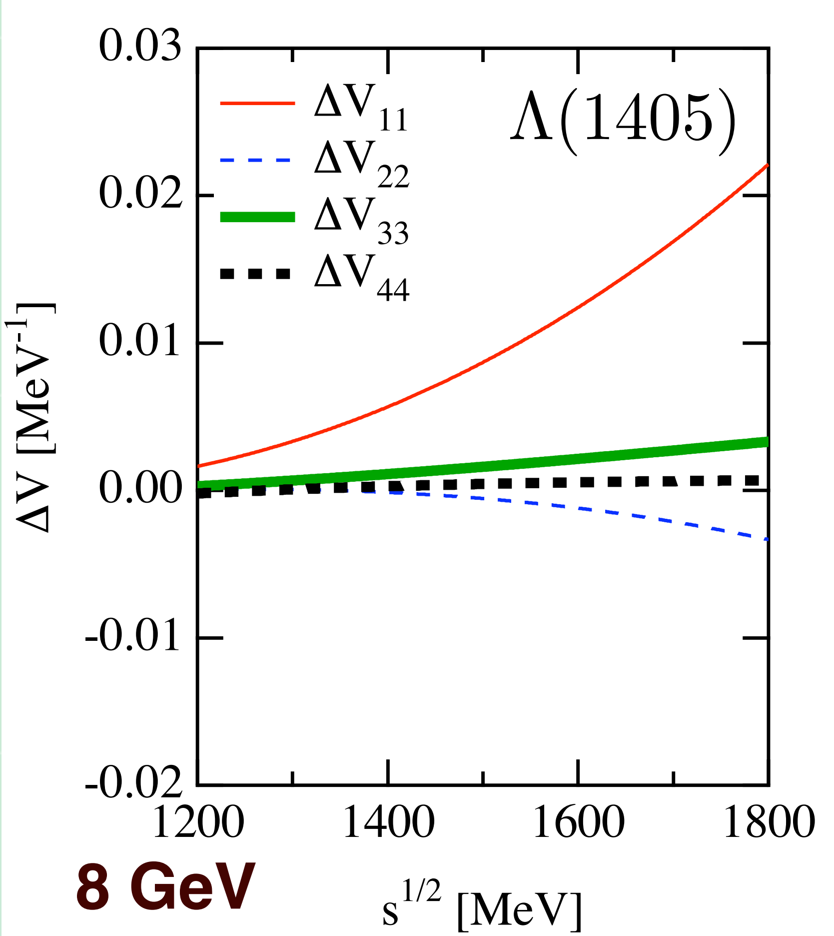
## Physically meaningful pole :

$$C>0,\quad \Delta a<0$$

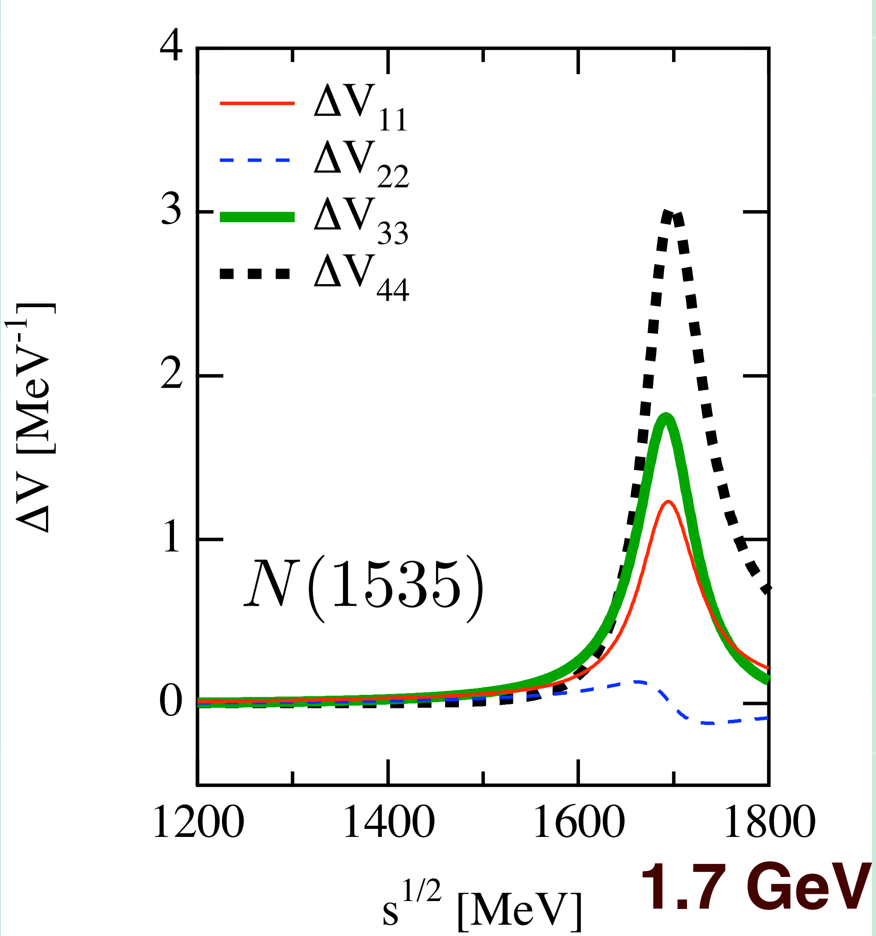
## ** energy scale of the effective pole **

# Example : Λ(1405) and N(1535)

$$\Delta V \equiv V_{\mathrm{natural}} - V_{\mathrm{WT}}$$



**8 GeV**



**1.7 GeV**

**Origin of resonances?**

# Summary 4 : dynamical or CDD?

## We study the origin of the resonances in the chiral unitary approach

- **Natural renormalization**

  Exclude CDD pole contribution from the loop function, consistent with N/D

- **Analysis of Λ(1405) and N(1535)**

  Λ(1405) : CDD pole would be small
  N(1535) : appreciable contribution
  from CDD pole

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Dynamical or CDD

# Summary 5 : Structure of Λ(1405)

## Schematic decomposition of Λ(1405)

$$|\,\Lambda(1405)\,\rangle = N_{MB}|\,B\,\rangle|\,M\,\rangle + N_3|\,qqq\,\rangle + N_5|\,qqqq\bar{q}\,\rangle + \ldots$$

- **Analysis of Nc behavior**

  **$N_3$ << 1**

- **Analysis of natural renormalization**

  **$N_{MB}$ dominates**

➡ **Both analyses consistently indicate the *dominance of $N_{MB}$* component**

**Not trivial ! c.f. rho meson, N(1535), ...**