

# Exotic hadrons in s-wave chiral dynamics



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# Introduction : QCD at low energy

## Quantum chromodynamics (QCD)

--> strong interaction of quarks and gluons

**At low energies: confinement, chiral symmetry breaking**

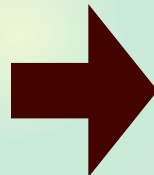
## Chiral perturbation theory (ChPT)

--> Effective Field Theory of mesons and baryons  
current algebra (low energy theorem) : OK  
**systematic** low energy expansion

Ex.)  $\pi N$  scattering length is well reproduced

What happens if the low energy interaction is **strong**?

Ex.)  $\bar{K}N$  scattering  
 $\Lambda(1405)$

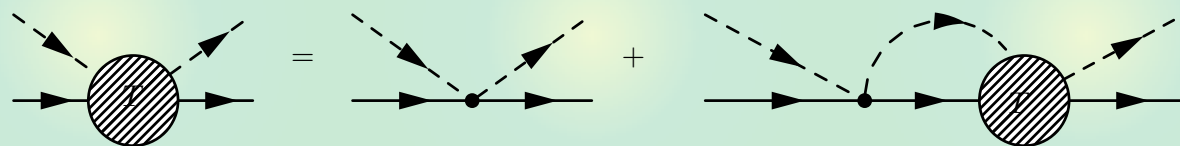


**non-perturbative  
framework**

# Chiral dynamics for non-exotic hadrons

- Interaction  $\leftarrow$  chiral symmetry
- Amplitude  $\leftarrow$  unitarity condition

$$T = \frac{1}{1 - VG} V$$



WT interaction  
model-independent

**cutoff**  
**(subtraction**  
**constant)**

N. Kaiser, P. B. Siegel, W. Weise, Nucl. Phys. A594, 325 (1995)

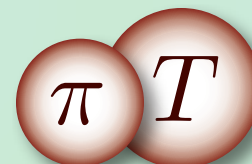
E. Oset, A. Ramos, Nucl. Phys. A635, 99 (1998)

J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001)

M.F.M. Lutz, E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002),

.... many others

**(s-wave) hadron excited states  $\sim$**



# Chiral dynamics for non-exotic hadrons

(s-wave) hadron excited states  $\sim \pi T$

Many hadron resonances are well described.

light baryon	$J^P = 1/2^-$	$\Lambda(1405)$	$\Lambda(1670)$	$\Sigma(1670)$	
		$N(1535)$	$\Xi(1620)$	$\Xi(1690)$	
	$J^P = 3/2^-$	$\Lambda(1520)$	$\Xi(1820)$	$\Sigma(1670)$	
heavy		$\Lambda_c(2880)$	$\Lambda_c(2593)$	$D_s(2317)$	
light meson	$J^P = 1^+$	$b_1(1235)$	$h_1(1170)$	$h_1(1380)$	$a_1(1260)$
		$f_1(1285)$	$K_1(1270)$	$K_1(1440)$	
	$J^P = 0^+$	$\sigma(600)$	$\kappa(900)$	$f_0(980)$	$a_0(980)$

**Chiral interaction is STRONG**





## Exotic hadrons : experiment vs theory

**Exotic hadrons : valence quark-antiquark(s)**

**non-exotic**

$uds, u\bar{d}, udsu\bar{u}, u\bar{d}u\bar{u}, \dots$

**exotic**

$uudd\bar{s}, ud\bar{s}\bar{s}, \dots$

**Experimentally**, they are exotic  $\sim 1/300$ .

**Theoretically**, are they exotic?

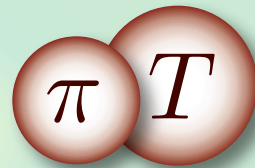
--> There is no simple way to forbid exotic states in QCD, effective models, ...

--> Evidences of multiquark components in non-exotic hadrons.

**Why aren't the exotics observed??**

## Outline

**Possibility of exotic hadron as a bound state of a target hadron and the NG boson**

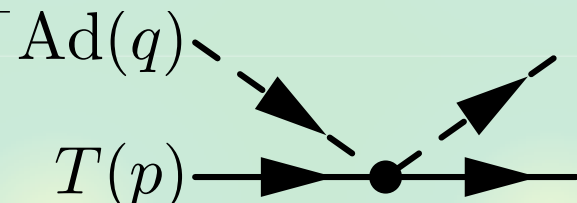


**We need to check**

- **s-wave low energy interaction?**
- **Exotic channel?**
- **Is the interaction strong?**

## Low energy s-wave interaction

### Scattering of a target (T) with the pion ( $\text{Ad}$ )

$$\alpha \left[ \begin{array}{c} \text{Ad}(q) \\ T(p) \end{array} \right] = \frac{1}{f^2} \frac{p \cdot q}{2M_T} \langle \mathbf{F}_T \cdot \mathbf{F}_{\text{Ad}} \rangle_\alpha + \mathcal{O} \left( \left( \frac{m}{M_T} \right)^2 \right)$$


### s-wave : Weinberg-Tomozawa term

Y. Tomozawa, *Nuovo Cim.* 46A, 707 (1966); S. Weinberg, *Phys. Rev. Lett.* 17, 616 (1966)

$$V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}$$

$$C_{\alpha,T} \equiv -\langle 2\mathbf{F}_T \cdot \mathbf{F}_{\text{Ad}} \rangle_\alpha = C_2(T) - C_2(\alpha) + 3 \quad (\text{for } N_f = 3)$$

**model-independent** interaction at low energy

Coupling strength has **linear  $N_c$  dependence**

T. Hyodo, D. Jido, L. Roca, 0712.3347 [hep-ph], *Phys. Rev. D*, in press.



## Coupling strengths : Examples

### Coupling strengths : (positive is attractive)

$$C_{\alpha,T} = C_2(T) - C_2(\alpha) + 3$$

$\alpha$	1	8	10	$\overline{10}$	27	35
$T = \mathbf{8}(N, \Lambda, \Sigma, \Xi)$	6	3	0	0	-2	
$T = \mathbf{10}(\Delta, \Sigma^*, \Xi^*, \Omega)$		6	3		1	-3

$\alpha$	$\overline{3}$	6	$\overline{15}$	24
$T = \overline{\mathbf{3}}(\Lambda_c, \Xi_c)$	3	1	-1	
$T = \mathbf{6}(\Sigma_c, \Xi_c^*, \Omega_c)$	5	3	1	-2

- **Exotic channels** : mostly repulsive
- **Attractive interaction** : **C = 1**

# Coupling strengths : General expression

For a general target  $T = [p, q]$

$\alpha \in [p, q] \otimes [1, 1]$	$C_{\alpha, T}$	sign
$[p + 1, q + 1]$	$-p - q$	<b>repulsive</b>
$[p + 2, q - 1]$	$1 - p$	
$[p - 1, q + 2]$	$1 - q$	
$[p, q]$	3	<b>attractive</b>
$[p, q]$	3	<b>attractive</b>
$[p + 1, q - 2]$	$3 + q$	<b>attractive</b>
$[p - 2, q + 1]$	$3 + p$	<b>attractive</b>
$[p - 1, q - 1]$	$4 + p + q$	<b>attractive</b>

- **Strength should be integer.**
- **Sign is determined for most cases.**

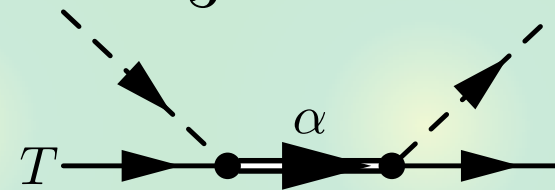
## Exotic channels

**Exoticness : minimal number of extra  $\bar{q}q$ .**

$$E = \epsilon\theta(\epsilon) + \nu\theta(\nu) \quad \epsilon \equiv \frac{p+2q}{3} - B, \quad \nu \equiv \frac{p-q}{3} - B$$

$\Delta E = E_\alpha - E_T = +1$  is realized when

○  $\alpha = [p+1, q+1] : C_{\alpha,T} = -p - q$   
repulsive



○  $\alpha = [p+2, q-1] : C_{\alpha,T} = 1 - p$

attraction :  $p = 0$  then  $\nu_T \geq 0 \rightarrow B \geq -q/3$   
not considered here

○  $\alpha = [p-1, q+2] : C_{\alpha,T} = 1 - q$

attraction :  $q = 0$  then  $\nu_T \leq 0 \rightarrow B \geq p/3$  OK!

**Universal attraction for more “exotic” channel**

$$C_{\text{exotic}} = 1 \quad \text{for} \quad T = [p, 0], \quad \alpha = [p-1, 2]$$

# Unitarization : N/D method

**Unitarity cut --> N, unphysical cut --> D**

$$T(s) = N(s)/D(s)$$

$$\text{Im}D(s) = \text{Im}[T^{-1}(s)]N(s) = -\rho(s)N(s) \quad \text{for } s > s_+$$

$$\text{Im}N(s) = \text{Im}[T(s)]D(s) \quad \text{for } s < s_-$$

**Neglect unphysical cut, set N=1**

$$T^{-1}(s) = \left( a(s_0) + \frac{s - s_0}{2\pi} \int_{s_+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)} \right) + \mathcal{T}^{-1}(s)$$

**loop function**

$$\rightarrow G(s)$$

**Interaction  
(tree level)**

**subtraction constant**

**(regularization parameter of the loop)**

# Renormalization and bound states

Identifying the interaction as  $V_\alpha = -\frac{\omega}{2f^2}C_{\alpha,T}$

$$T_\alpha(\sqrt{s}) = \frac{1}{1 - V_\alpha(\sqrt{s})G(\sqrt{s})}V_\alpha(\sqrt{s})$$

## Renormalization parameter : condition

$$G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T$$

**K. Igi and K. Hikasa, Phys. Rev. D59, 034005 (1999)**

**M.F.M. Lutz and E. Kolomeitsev, Nucl. Phys. A700, 193-308 (2002)**

## Exclude the genuine quark states in G

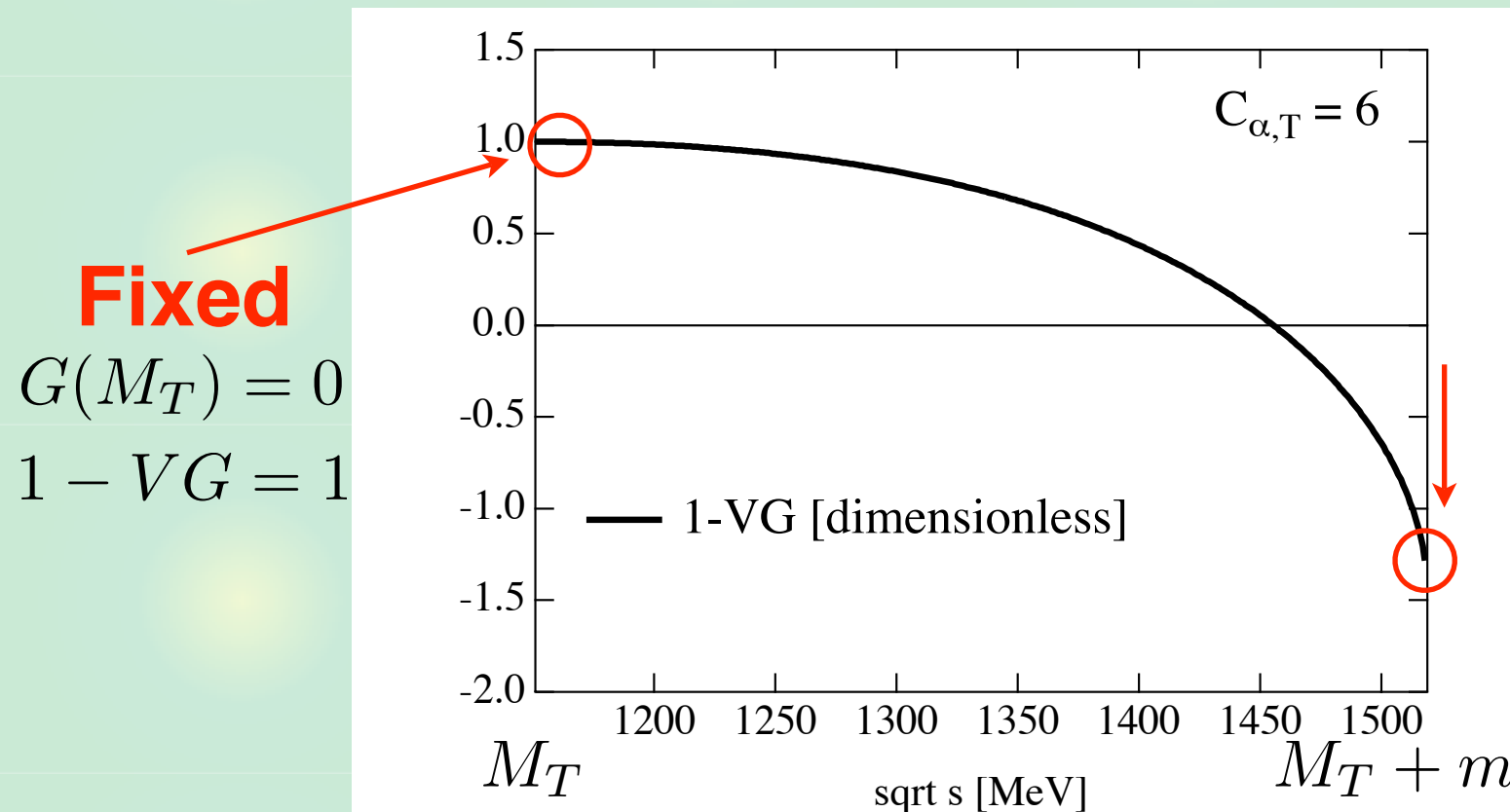
**T. Hyodo, D. Jido and A. Hosaka, arXiv: 0803.2550 [nucl-th]**

## Bound state:

$$1 - V(M_b)G(M_b) = 0 \quad M_T < M_b < M_T + m$$

# Critical attraction

$1 - V(\sqrt{s})G(\sqrt{s})$  : monotonically decreasing.

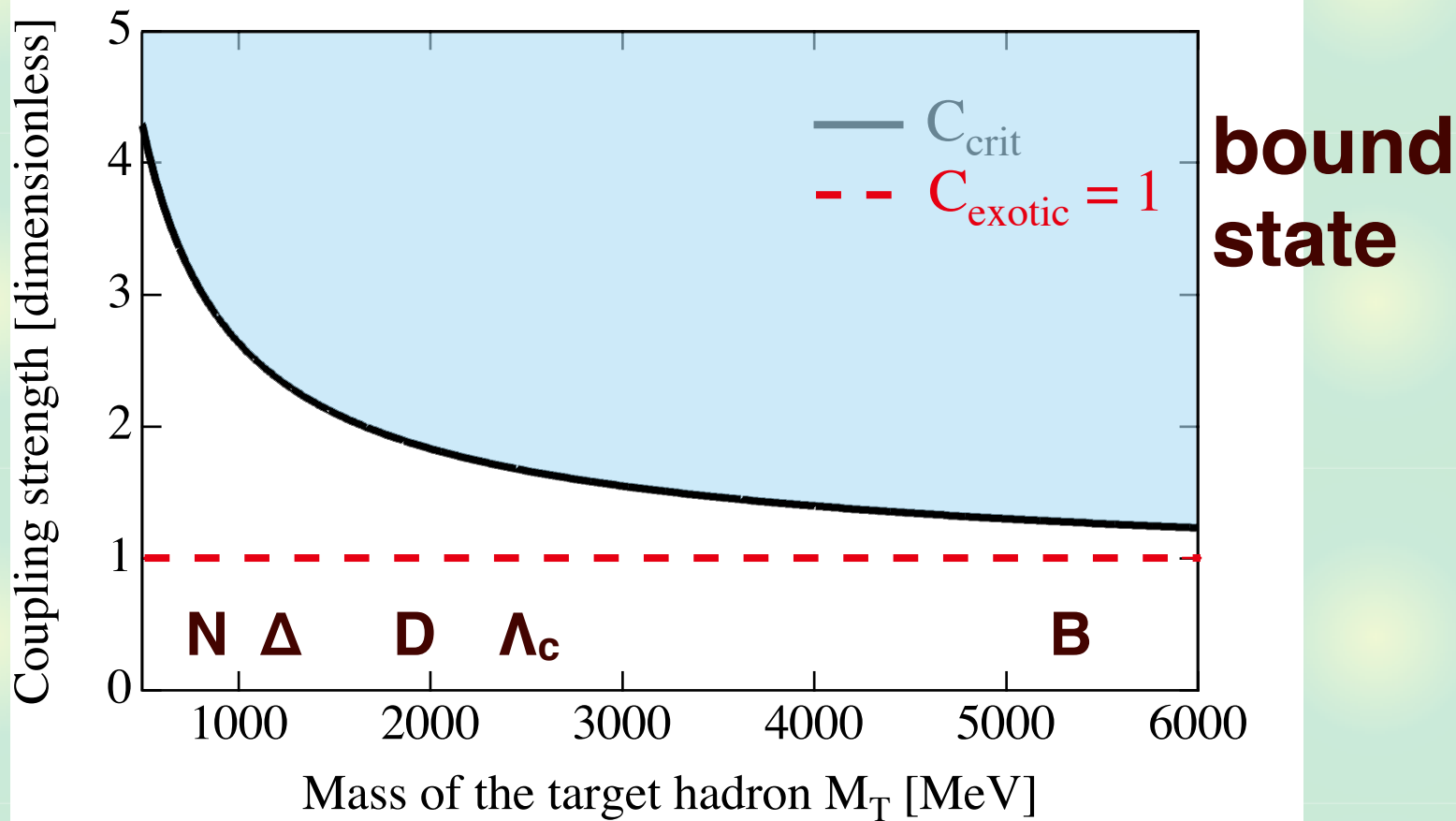


**Critical attraction** :  $1 - VG = 0$  at  $\sqrt{s} = M_T + m$

$$\longrightarrow C_{\text{crit}} = \frac{2f^2}{m[-G(M_T + m)]}$$



# Critical attraction and exotic channel



$$m = 368 \text{ MeV and } f = 93 \text{ MeV}$$

**➔ Strength is not enough.**

## Summary 1 : SU(3) limit




We study the exotic bound states in s-wave chiral dynamics in flavor SU(3) limit.

- The interactions in exotic channels are in most cases **repulsive**.
- There are **attractive interactions** in exotic channels, with **universal** and the smallest strength :  $C_{\text{exotic}} = 1$
- The strength is **not enough** to generate a bound state :  $C_{\text{exotic}} < C_{\text{crit}}$

The result is **model independent** as far as we respect chiral symmetry.

## Summary 2 : Physical world

### Caution!

-  The exotic hadrons here are the **s-wave** meson-hadron molecule states ( $1/2^-$  for  $\Theta^+$ ).
-  We do not exclude the exotics which have **other origins** (genuine quark state, soliton rotation,...).
-  In practice, **SU(3) breaking** effect, **higher order** terms,...

In Nature, it is **difficult** to generate exotic hadrons as in the same way with  $\Lambda(1405)$ ,  $\Lambda(1520)$ ,... based on chiral interaction.

[T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. Lett. 97, 192002 \(2006\)](#)

[T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. D 75, 034002 \(2007\)](#)