Origin of resonances in chiral dynamics

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- Effective interaction: origin of resonance
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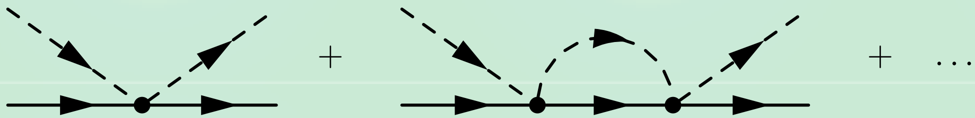
T. Hyodo, D. Jido, A. Hosaka, arXiv:0803.2550 [nucl-th]

Dynamical state and CDD pole

Resonances in two-body scattering

- Knowledge of interaction (potential)
- Experimental data (phase shift, cross section)

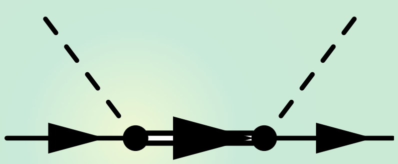
Dynamical state: molecule, quasi-bound, ...



e.g.) Deuteron in NN, positronium in e^+e^-, (σ in $\pi\,\pi$), ...

CDD pole: elementary, independent, ...

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)



e.g.) J/Ψ in e^+e^-, (ρ in $\pi\,\pi$), ...

Dynamical state and CDD pole (notes)

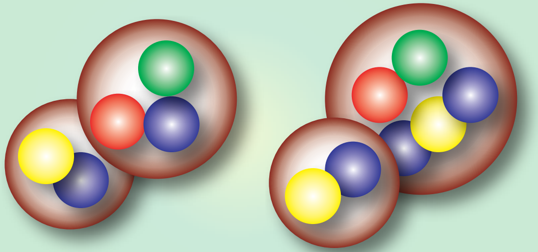
Model space and dynamical/CDD

Notion of dynamical/CDD depends on the scattering particles under consideration. It is not an inherent property of the resonance state.

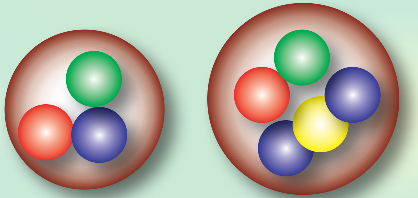
e.g.) J/Ψ : CDD in e^+e^-, dynamical in $c\bar{c}$

Quark structure (for baryon resonances)

dynamical ~



CDD ~



For hadron resonances, dynamical/CDD is not directly related to quark structure.

Mixing of dynamical and CDD

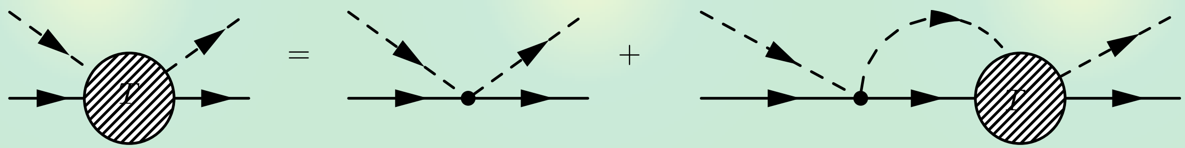
When both exist in one system, relative weight is important.

Chiral unitary approach

S = -1, $\bar{K}N$ s-wave scattering : Λ(1405) in I=0

- **Interaction <-- chiral symmetry**
- **Amplitude <-- unitarity (coupled channel)**

$$T = \frac{1}{V^{-1} - G}$$



Chiral (WT interaction)

cutoff (subtraction constant)

N. Kaiser, P. B. Siegel, W. Weise, Nucl. Phys. A594, 325 (1995)
E. Oset, A. Ramos, Nucl. Phys. A635, 99 (1998)
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001)
M.F.M. Lutz, E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002),
.... many others

By construction, generated resonances are all dynamical? ➡ **Not always...**

Scattering theory : N/D method

Single-channel scattering, masses: M_T and m

G.F. Chew, S. Mandelstam, Phys. Rev. 119, 467 (1960)

$s = W^2$

Divide T into N(umerator) and D(inominator)
unitarity cut --> D, unphysical cut --> N

$$T(s) = N(s)/D(s)$$

phase space (optical theorem)

$$\mathrm{Im} D(s) = \mathrm{Im}[T^{-1}(s)] N(s) = \rho(s) N(s)/2 \quad \text{for } s > s^+$$

$$\mathrm{Im} N(s) = \mathrm{Im}[T(s)] D(s) \quad \text{for } s < s^-$$

Dispersion relation for N and D
--> set of integral equations, input : $\mathrm{Im}[T(s)] \quad \text{for } s < s^-$

General form of the (s-wave) amplitude

Neglect unphysical cut (crossed diagrams), set N=1

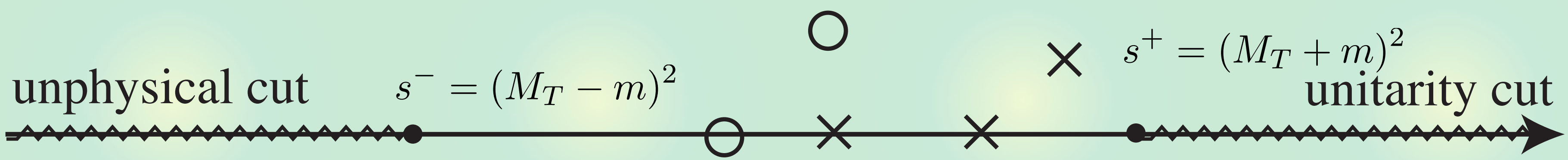
U. G. Meissner, J. A. Oller, Nucl. Phys. A673, 311 (2000)

$$T^{-1}(\sqrt{s}) = \boxed{\tilde{a}(s_0)} + \frac{s-s_0}{2\pi}\int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s'-s)(s'-s_0)}$$

subtraction constant, not determined

- **pole (and zero) of the amplitude**

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)



CDD pole(s), R_i, W_i : not known in advance

$$T^{-1}(\sqrt{s}) = \boxed{\sum_i \frac{R_i}{\sqrt{s}-\sqrt{s_i}}} + \tilde{a}(s_0) + \frac{s-s_0}{2\pi}\int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s'-s)(s'-s_0)}$$

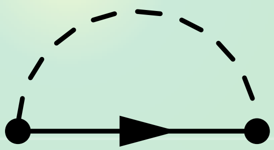
CDD pole contribution --> independent particle

G.F. Chew, S.C. Frautschi, Phys. Rev. 124, 264 (1961)

Order by order matching with ChPT

Identify loop function G, the rest contribution --> V^{-1}

$$T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s}-\sqrt{s_i}} + \tilde{a}(s_0) + \frac{s-s_0}{2\pi}\int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s'-s)(s'-s_0)}$$

$-$ 

$$= -i\int \frac{d^4q}{(2\pi)^4}\frac{2M_T}{(P-q)^2-M_T^2+i\epsilon}\frac{1}{q^2-m^2+i\epsilon}\Bigg|_{\text{dim.reg.}}$$

$$= -\frac{2M_T}{(4\pi)^2}\left\{a + \frac{m^2-M_T^2+s}{2s}\ln\frac{m^2}{M_T^2} + \frac{\bar{q}}{\sqrt{s}}\ln\frac{\phi_{++}(s)\,\phi_{+-}(s)}{\phi_{-+}(s)\,\phi_{--}(s)}\right\}$$

$$= -G(\sqrt{s};a)$$

subtraction constant (cutoff)

$$T(\sqrt{s}) = [V^{-1}(\sqrt{s}) - G(\sqrt{s};a)]^{-1}$$

V? chiral expansion of T, (conceptual) matching with ChPT

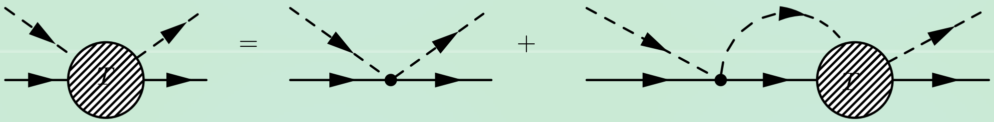
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001)

$$T^{(1)} = V^{(1)}, \quad T^{(2)} = V^{(2)}, \quad T^{(3)} = V^{(3)} - V^{(1)}GV^{(1)}, \ldots$$

Summary of chiral unitary appraoch

Scattering amplitude T

$$T(\sqrt{s}) = \frac{1}{V^{-1}(\sqrt{s}) - G(\sqrt{s};a)}$$



$V(\sqrt{s})$: interaction (ChPT at given order)

$G(\sqrt{s};a)$: loop function

a : subtraction constant (cutoff parameter)

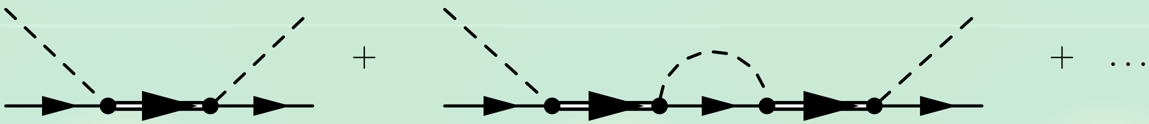
	ChPT	ChU
Unitarity	perturbative	exact
Dynamical resonance	×	○
Crossing symmetry	exact	(perturbative)
Chiral counting	○	×

Nonrenormalizable --> cutoff theory

CDD pole contribution --> V (interaction)

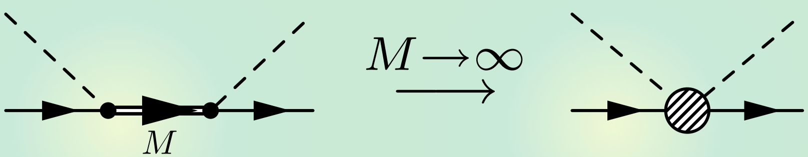
(Known) CDD pole in chiral unitary approach

Explicit resonance field in V (interaction)

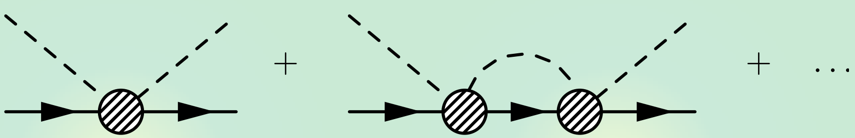


U.G. Meissner, J.A. Oller, Nucl. Phys. A673, 311 (2000)
D. Jido, E. Oset, A. Ramos, Phys. Rev. C66, 055203 (2002)

Contracted resonance propagator in V



G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B321, 311 (1989)
V. Bernard, N. Kaiser, U.G. Meissner, Nucl. Phys. A615, 483 (1997)



J.A. Oller, E. Oset, J.R. Pelaez, Phys. Rev. D59, 074001 (1999)

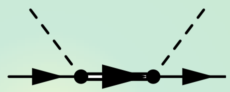
Is that all? subtraction constant?

Subtraction constant

Phenomenological (standard) scheme
--> V is given, "a" is determined by data

$$T = \frac{1}{(V^{(1)})^{-1} - G(\underline{a})}$$ leading order

$$T = \frac{1}{(V^{(1)} + V^{(2)})^{-1} - G(\underline{a'})}$$ next to leading order

↑pole  ?

"a" represents the effect which is not included in V.

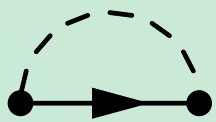
The CDD pole contribution in G?

Natural renormalization scheme
--> fix "a" first, then determine V

exclude CDD pole contribution from G,
based on theoretical argument.

Loop function below threshold

Below threshold, G is real and NEGATIVE (~ assume no states below threshold)

$G(\sqrt{s}) =$  $\leq 0 \quad (\text{for } \sqrt{s} \leq M_T + m)$

It is automatically satisfied in 3d cutoff. However, ...

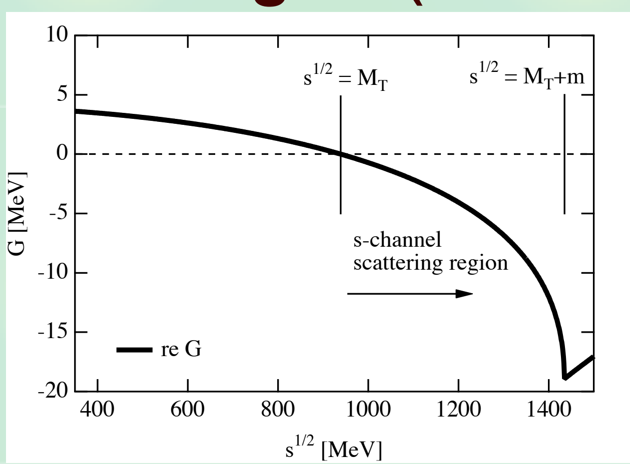
$$G(\sqrt{s}; a) = \frac{2M_T}{(4\pi)^2}\left\{a + \frac{m^2 - M_T^2 + s}{2s}\ln\frac{m^2}{M_T^2} + \frac{\bar{q}}{\sqrt{s}}\ln\frac{\phi_{++}(s)\,\phi_{+-}(s)}{\phi_{-+}(s)\,\phi_{--}(s)}\right\}$$

Large (positive) “a” can make G positive.
Avoid this for s-channel region (above M_T),

$$a \leq a_{\max}(M_T, m)$$

or equivalently (G: decreasing),

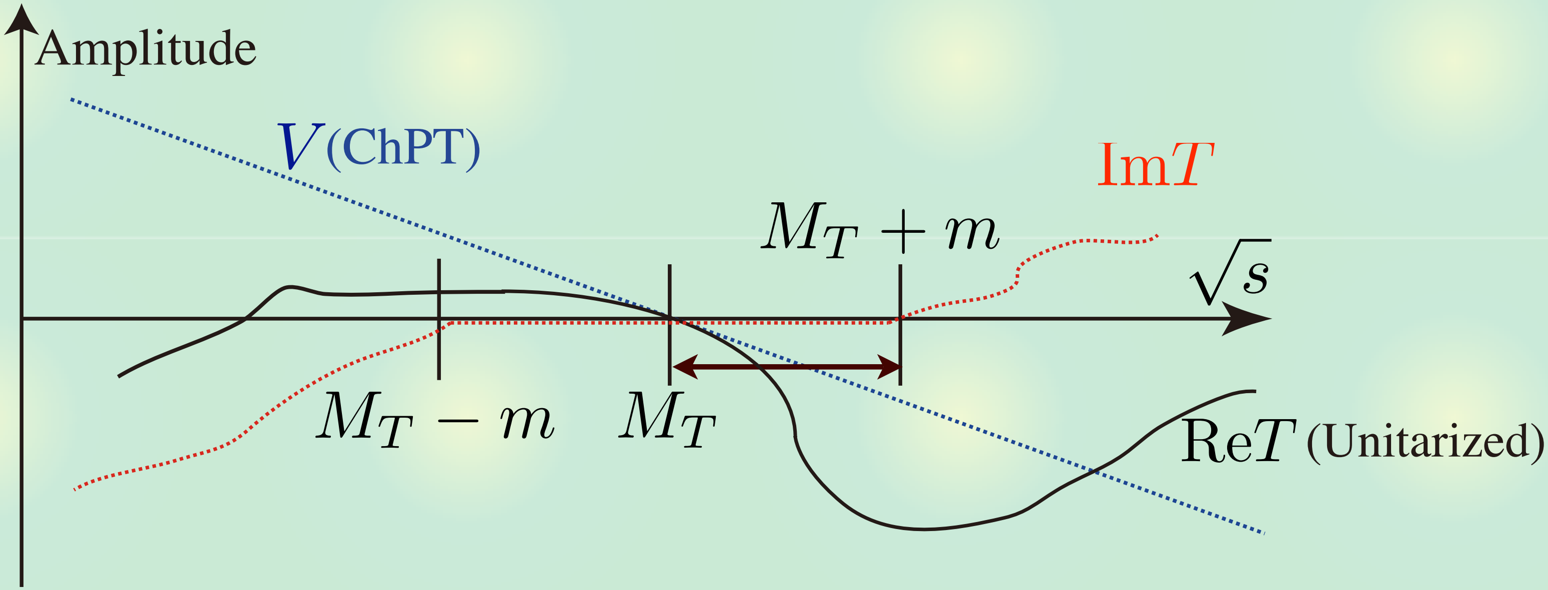
$$G(\sqrt{s} = M_T) \leq 0$$



(Explicit) matching with ChPT

V is given by ChPT.
At a "low energy", T should be matched with V:

$$G(\sqrt{s}=\mu_m)=0, \quad \Leftrightarrow \quad T(\mu_m)=V(\mu_m)$$



subtraction constant : real

$$\Rightarrow \quad M_T \le \mu_m \le M_T + m$$

consistent with "low energy" requirement

$$\sqrt{s}=M_T+m \ \Rightarrow \ \boldsymbol{p}=0, \quad \sqrt{s}=M_T \ \Rightarrow \ \omega \sim 0$$

Natural renormalization condition : summary

Natural renormalization condition

- Loop function should be negative below threshold
- T matches with V at low energy scale

"a" is uniquely determined such that

$$G(\sqrt{s} = M_T) = 0, \quad \Leftrightarrow \quad T(M_T) = V(M_T)$$

matching with low energy interaction

K. Igi, K. Hikasa, Phys. Rev. D59, 034005 (1999)
U.G. Meissner, J.A. Oller, Nucl. Phys. A673, 311 (2000)

crossing symmetry (matching with u-channel amplitude)

M.F.M. Lutz, E. Kolomeitsev, Nucl. Phys. A700, 193 (2002)

We regard this condition as the exclusion of the CDD pole contribution from G

Two renormalization schemes

Phenomenological scheme

V is given by ChPT (for instance, leading order term), fit cutoff in G to data

Natural renormalization scheme

determine G to exclude CDD pole contribution, V is to be determined

Same physics (scattering amplitude T)

$$T = \frac{1}{V_{\mathrm{ChPT}}^{-1} - G(a_{\mathrm{pheno}})} = \frac{1}{(V_{\mathrm{natural}})^{-1} - G(a_{\mathrm{natural}})}$$

↑Effective interaction
Origin of the resonance

Pole in the effective interaction

Leading order V : Weinberg-Tomozawa term

$$V_{\mathrm{WT}} = -\frac{C}{2f^2}(\sqrt{s} - M_T)$$

C/f² : coupling constant
no s-wave resonance

$$T^{-1} = V_{\mathrm{WT}}^{-1} - G(a_{\mathrm{pheno}}) = (V_{\mathrm{natural}})^{-1} - G(a_{\mathrm{natural}})$$

↑ChPT ↑data fit ↑given

Effective interaction in natural scheme

$$V_{\mathrm{natural}} = -\frac{C}{2f^2}(\sqrt{s} - M_T) + \frac{C}{2f^2}\frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\mathrm{eff}}}$$

pole!

$$M_{\mathrm{eff}} = M_T - \frac{16\pi^2 f^2}{C M_T \Delta a}, \qquad \Delta a = a_{\mathrm{pheno}} - a_{\mathrm{natural}}$$

Physically meaningful pole : $C > 0, \quad \Delta a < 0$

There is always a pole for $a_{\mathrm{pheno}} \neq a_{\mathrm{natural}}$
--> energy scale of the effective pole is relevant.

S=-1 and S=0 meson-baryon scatterings

Models for the Meson-baryon scattering :

E. Oset, A. Ramos, C. Bennhold, Phys. Lett. B527, 99 (2002),
T. Inoue, E. Oset, M.J. Vicente Vacas, Phys. Rev. C. 65, 035204 (2002)
T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Phys. Rev. C. 68, 018201 (2003)
T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Prog. Thor. Phys. 112, 73 (2004)

$$T^{-1} = V_{\rm WT}^{-1} - G(a_{\rm pheno}) = (V_{\rm natural})^{-1} - G(a_{\rm natural})$$

Pole of the full amplitude

physical state

Pole of the effective interaction ($M_{\rm eff}$)

pure CDD pole contribution
(can be complex for coupled-channel case)

Pole of the $V_{\rm WT}$ + natural

pure dynamical contribution

Comparison of pole positions

Pole of the full amplitude

physical state

$$z_1^{\Lambda^*} = 1429 - 14i \text{ MeV}, \quad z_2^{\Lambda^*} = 1397 - 73i \text{ MeV}$$

$$z^{N^*} = 1493 - 31i \text{ MeV}$$

Pole of the effective interaction (Meff)

pure CDD pole contribution

$$z_{\text{eff}}^{\Lambda^*} \sim 7.9 \text{ GeV}$$ **irrelevant!**

$$z_{\text{eff}}^{N^*} = 1693 \pm 37i \text{ MeV}$$ **relevant?**

Pole of the V_{WT} + natural

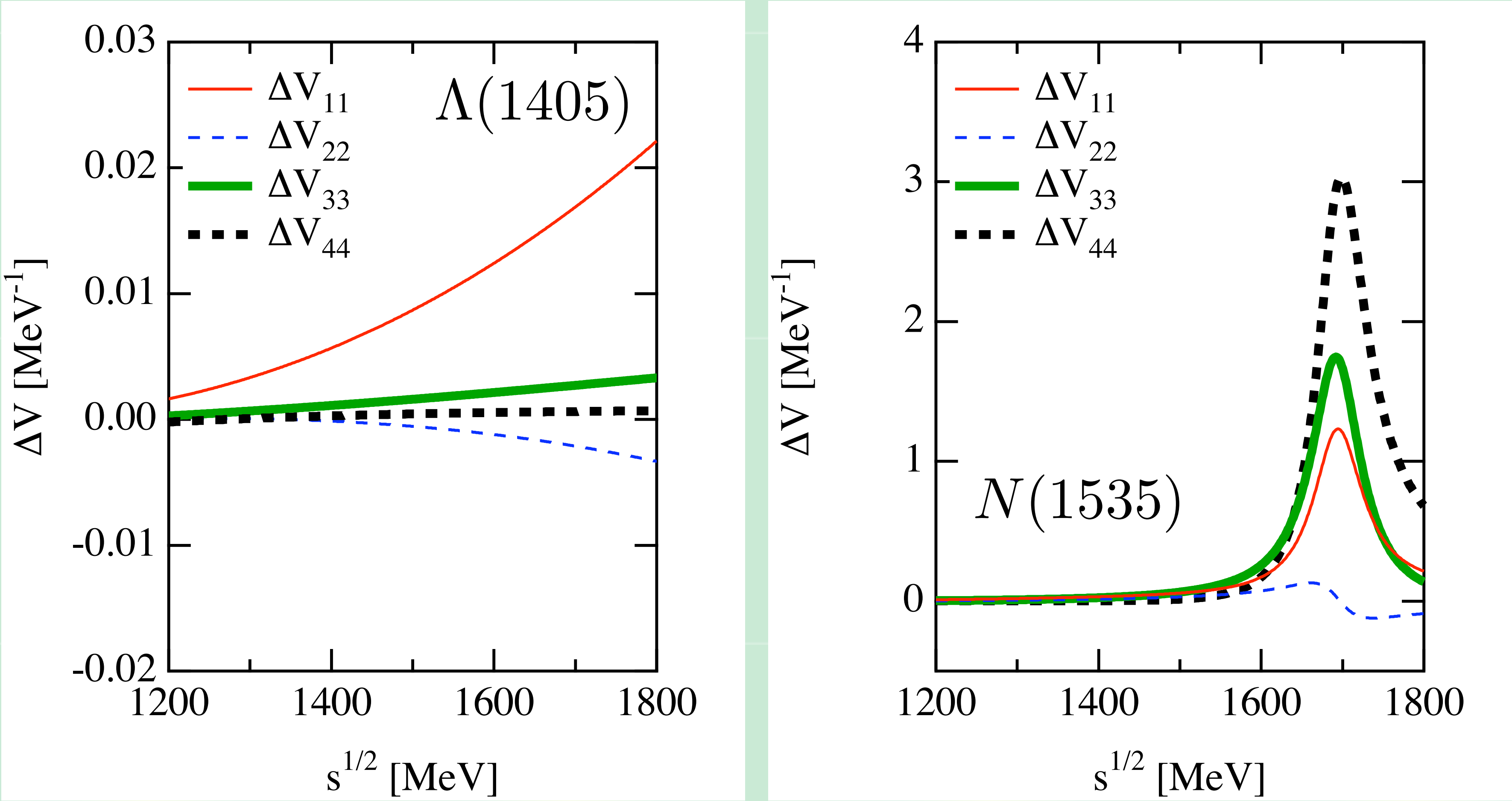
pure dynamical contribution

$$z_1^{\Lambda^*} = 1417 - 19i \text{ MeV}, \quad z_2^{\Lambda^*} = 1402 - 72i \text{ MeV}$$

$$z^{N^*} = 1582 - 61i \text{ MeV}$$

Example : Λ(1405) and N(1535)

Difference of interactions $\Delta V \equiv V_{\text{natural}} - V_{\text{WT}}$



Meff ~ 8 GeV **Meff ~ 1.7 GeV**

Important CDD pole contribution to N(1535)

N(1535) coupling strengths

Residues of the pole --> coupling strengths

$$T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - M_R + i\Gamma_R/2}$$

pole in	property	πN	ηN	KΛ	KΣ
full T	physical	0.949	1.64	1.45	2.96
$V_{natural}$	CDD	4.67	2.15	5.71	7.44
WT+natural	Dynamical	0.353	2.11	1.71	2.93

Coupling properties of the physical pole is similar with those of dynamical pole.

Dynamical component is also important?

Summary: formulation

We study the origin (dynamical/CDD) of the resonances in the chiral unitary approach

- **Natural renormalization scheme**

  Exclude CDD pole contribution from the loop function, consistent with N/D.

- **Comparison with phenomenology**

  --> Pole in the effective interaction

  We extract the CDD pole contribution hidden in the subtraction constant into effective interaction V.

Summary: application

- Λ(1405) : predominantly dynamical

  consistent with Nc scaling

  T. Hyodo, D. Jido, R. Loca, Phys. Rev. D77, 056010 (2008)
  R. Loca, T. Hyodo, D. Jido, arXiv:0804.1210 [hep-ph]

  --> Λ(1405) is non-qqq dominant

- N(1535) : mixture of both components

  Energy of the pole in the effective interaction --> CDD pole nature

  Analysis of the coupling strengths --> dynamical nature

  T. Hyodo, D. Jido, A. Hosaka, arXiv:0803.2550 [nucl-th]