Origin of resonances in chiral dynamics

Tetsuo Hyodo\textsuperscript{a},

Daisuke Jido\textsuperscript{b}, and Atsushi Hosaka\textsuperscript{c}

\textit{Tokyo Institute of Technology}\textsuperscript{a} \quad \textit{YITP, Kyoto}\textsuperscript{b} \quad \textit{RCNP}\textsuperscript{c}

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Dynamical state and CDD pole

Chiral unitary approach

Natural renormalization scheme

Effective interaction: origin of resonance

Application: Λ(1405) and N(1535)

Classification of resonances

Resonances in two-body scattering

- Knowledge of interaction (potential)
- Experimental data (cross section, ...)

**Dynamical state**: two-body molecule, quasi-bound state, ...

\[ \begin{array}{ccc}
\vdots & + & \vdots \\
\longrightarrow & + & \longrightarrow \\
& + & \ldots
\end{array} \]

- e.g.) Deuteron in NN, positronium in $e^+e^-$, ($\sigma$ in $\pi\pi$), ...

**CDD(≠CCD!) pole**: elementary particle, independent state, ...


- e.g.) J/$\Psi$ in $e^+e^-$, ($\rho$ in $\pi\pi$), ...
Baryon resonances

Meson-baryon molecule (MB) ~ dynamical state
3-quark state (qqq) ~ (representative of) CDD pole

Difficulties: Both arise from QCD
No clear separation of energy scale (c.f. J/ψ)

Find out the (dominant) origin for each resonance.

N. Isgur, G. Karl, Phys. Rev. D18, 4187 (1978)
Model space of scattering and dynamical/CDD

Notion of dynamical/CDD depends on the scattering particles under consideration. It is not an inherent property of the resonance state.

\( e.g. \) \( J/\Psi \) : CDD in \( e^+e^- \), dynamical in \( c\bar{c} \)

Quark structure (for baryon resonances)

For hadron resonances, dynamical/CDD is not directly related to quark structure.

Mixing of dynamical and CDD

When both exist in one system, relative weight is relevant.
Chiral unitary approach

Description of meson-baryon scattering, s-wave resonances

- Interaction \(<--\) chiral symmetry
- Amplitude \(<--\) unitarity (coupled channel)

\[ T = \frac{1}{V^{-1} - G} \]

Chiral (WT interaction) cutoff (subtraction constant)


By construction, generated resonances are all dynamical? Not always...
Scattering theory: N/D method

Single-channel scattering, masses: \( M_T \) and \( m \)

G.F. Chew, S. Mandelstam, Phys. Rev. 119, 467 (1960)

\[
\begin{align*}
\text{unphysical cut(s)} & \quad s^- = (M_T - m)^2 \\
\text{unitarity cut} & \quad s^+ = (M_T + m)^2
\end{align*}
\]

Divide \( T \) into \( N(\text{umerator}) \) and \( D(\text{inominator}) \)

\[
T(s) = \frac{N(s)}{D(s)}
\]

Phase space (optical theorem)

\[
\begin{align*}
\text{Im} D(s) &= \text{Im}[T^{-1}(s)]N(s) = \rho(s)N(s)/2 \quad \text{for } s > s^+ \\
\text{Im} N(s) &= \text{Im}[T(s)]D(s) \quad \text{for } s < s^-
\end{align*}
\]

Dispersion relation for N and D

\[
\text{---> set of integral equations, input : } \text{Im}[T(s)] \quad \text{for } s < s^-
\]
General form of the (s-wave) amplitude

Neglect unphysical cut (crossed diagrams), set N=1


\[
T^{-1}(\sqrt{s}) = \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)}
\]

subtraction constant, not determined

- pole (and zero) of the amplitude


\[
\text{unphysical cut(s)} \quad s^- = (M_T - m)^2 \quad \bigcirc \quad s^+ = (M_T + m)^2 \quad \times \quad \text{unitarity cut}
\]

CDD pole(s), \( R_i, W_i \) : not known in advance

\[
T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s - \sqrt{s_i}}} + \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)}
\]

CDD pole contribution \( \rightarrow \) independent particle

Order by order matching with ChPT

Identify loop function $G$, the rest contribution $\Rightarrow V^{-1}$

\[
T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s}-\sqrt{s_i}} + \tilde{a}(s_0) + \frac{s-s_0}{2\pi} \int_{s^+}^{\infty} ds' \rho(s') \frac{\rho(s')}{(s'-s)(s'-s_0)}
\]

\[
-\quad = -i \int \frac{d^4 q}{(2\pi)^4} \frac{2M_T}{(P-q)^2 - M_T^2 + i\epsilon} \frac{1}{q^2 - m^2 + i\epsilon} \left|_{\text{dim.reg.}} \right.
\]

\[
= -\frac{2M_T}{(4\pi)^2} \left\{ a + \frac{m^2 - M_T^2 + s}{2s} \ln \frac{m^2}{M_T^2} + \frac{\bar{q}}{\sqrt{s}} \ln \frac{\phi_{++}(s) \phi_{+-}(s)}{\phi_{--}(s) \phi_{-+}(s)} \right\}
\]

\[
= -G(\sqrt{s}; a)
\]

subtraction constant (cutoff)

\[
T(\sqrt{s}) = [V^{-1}(\sqrt{s}) - G(\sqrt{s}; a)]^{-1}
\]

V? chiral expansion of $T$, (conceptual) matching with ChPT


\[
T^{(1)} = V^{(1)}, \quad T^{(2)} = V^{(2)}, \quad T^{(3)} = V^{(3)} - V^{(1)} G V^{(1)}, \ldots
\]
Summary of chiral unitary approach

Scattering amplitude $T$

$$T(\sqrt{s}) = \frac{1}{V^{-1}(\sqrt{s}) - G(\sqrt{s}; a)}$$

- $V(\sqrt{s})$ : interaction (ChPT at given order)
- $G(\sqrt{s}; a)$ : loop function
- $a$ : subtraction constant (cutoff parameter)

<table>
<thead>
<tr>
<th></th>
<th>ChPT</th>
<th>ChU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unitarity</td>
<td>perturbative</td>
<td>exact</td>
</tr>
<tr>
<td>Dynamical resonance</td>
<td>$\times$</td>
<td>$\bigcirc$</td>
</tr>
<tr>
<td>Crossing symmetry</td>
<td>exact</td>
<td>(perturbative)</td>
</tr>
<tr>
<td>Chiral counting</td>
<td>$\bigcirc$</td>
<td>$\times$</td>
</tr>
</tbody>
</table>

Nonrenormalizable $\rightarrow$ cutoff theory
CDD pole contribution $\rightarrow$ $V$ (interaction)
Chiral unitary approach

(Known) CDD pole in chiral unitary approach

Explicit resonance field in $V$ (interaction)

\[
\begin{align*}
\begin{array}{c}
\vphantom{\text{Diagram}}
\end{array}
\end{align*}
\]


Contracted resonance propagator in higher order $V$

\[
\begin{align*}
\begin{array}{c}
\vphantom{\text{Diagram}}
\end{array}
\end{align*}
\]


\[
\begin{align*}
\begin{array}{c}
\vphantom{\text{Diagram}}
\end{array}
\end{align*}
\]


Is that all? subtraction constant?
Subtraction constant

Phenomenological (standard) scheme
--> V is given, “a” is determined by data

\[
T = \frac{1}{(V^{(1)})^{-1} - G(a)}
\]

leading order

\[
T = \frac{1}{(V^{(1)} + V^{(2)})^{-1} - G(a')}
\]

next to leading order

↑pole ?

“a” represents the effect which is not included in V.

CDD pole contribution in G?

Natural renormalization scheme
--> fix “a” first, then determine V

exclude CDD pole contribution from G, based on theoretical argument.
Natural renormalization scheme

Loop function below threshold

Below threshold, $G$ is real and NEGATIVE
($\sim$ assume no states below threshold)

\[ G(\sqrt{s}) = \leq 0 \quad \text{(for } \sqrt{s} \leq M_T + m) \]

It is automatically satisfied in 3d cutoff. However, ...

\[ G(\sqrt{s}; a) = \frac{2 M_T}{(4\pi)^2} \left\{ a + \frac{m^2 - M_T^2 + s}{2s} \ln \frac{m^2}{M_T^2} + \frac{q}{\sqrt{s}} \ln \frac{\phi_{++}(s) \phi_{+-}(s)}{\phi_{++}(s) \phi_{--}(s)} \right\} \]

Large (positive) "a" can make $G$ positive.
Avoid this for s-channel region ($> M_T$),

\[ a \leq a_{\max}(M_T, m) \]

or equivalently

(G: decreasing),

\[ G(\sqrt{s} = M_T) \leq 0 \]
V is given by ChPT.
At a “low energy”, T should be matched with V:

\[ G(\sqrt{s} = \mu m) = 0, \quad \Leftrightarrow \quad T(\mu m) = V(\mu m) \]

matching in s-channel region, subtraction constant is real

\[ \Rightarrow \quad M_T \leq \mu m \leq M_T + m \]

consistent with “low energy” requirement

\[ \sqrt{s} = M_T + m \quad \Rightarrow \quad p = 0, \quad \sqrt{s} = M_T \quad \Rightarrow \quad \omega \sim 0 \]
Natural renormalization condition

- Loop function should be negative below threshold
- $T$ matches with $V$ at low energy scale

"a" is uniquely determined such that

$$G(\sqrt{s} = M_T) = 0, \quad \Leftrightarrow \quad T(M_T) = V(M_T)$$

matching with low energy interaction


crossing symmetry (matching with u-channel amplitude)


We regard this condition as the exclusion of the CDD pole contribution from $G$
Two renormalization schemes

**Phenomenological** scheme

V is given by ChPT (for instance, leading order term), fit cutoff in G to data

**Natural** renormalization scheme

determine G to exclude CDD pole contribution, V is to be determined

Same physics (scattering amplitude $T$)

$$T = \frac{1}{V_{\text{ChPT}}^{-1} - G(a_{\text{pheno}})} = \frac{1}{(V_{\text{natural}})^{-1} - G(a_{\text{natural}})}$$

↑Effective interaction

Origin of the resonance
Pole in the effective interaction

Leading order V : Weinberg-Tomozawa term

\[ V_{WT} = - \frac{C}{2 f^2} (\sqrt{s} - M_T) \quad \text{C/f}^2 : \text{coupling constant} \]

no s-wave resonance

\[ T^{-1} = V_{WT}^{-1} - G(a_{\text{pheno}}) = (V_{\text{natural}})^{-1} - G(a_{\text{natural}}) \]

↑ChPT ↑data fit ↑given

Effective interaction in natural scheme

\[ V_{\text{natural}} = - \frac{C}{2 f^2} (\sqrt{s} - M_T) + \frac{C}{2 f^2} \frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\text{eff}}} \]

\[ M_{\text{eff}} = M_T - \frac{16 \pi^2 f^2}{C M_T \Delta a}, \quad \Delta a = a_{\text{pheno}} - a_{\text{natural}} \]

Pole!

Physically meaningful pole : \( C > 0, \quad \Delta a < 0 \)

There is always a pole for \( a_{\text{pheno}} \neq a_{\text{natural}} \)

---> energy scale of the effective pole is relevant.
S=-1 and S=0 meson-baryon scatterings

Models for the Meson-baryon scattering:


K-p total cross sections

<table>
<thead>
<tr>
<th>γ</th>
<th>R_c</th>
<th>R_n</th>
</tr>
</thead>
<tbody>
<tr>
<td>exp.</td>
<td>2.36</td>
<td>0.664</td>
</tr>
<tr>
<td>theo.</td>
<td>1.80</td>
<td>0.624</td>
</tr>
</tbody>
</table>

πΣ spectrum

πN scattering amplitude
Application: $\Lambda(1405)$ and $N(1535)$

**Comparison of pole positions**

**Pole of the full amplitude: physical state**

$z_1^{\Lambda^*} = 1429 - 14i$ MeV, $z_2^{\Lambda^*} = 1397 - 73i$ MeV

$z^{N^*} = 1493 - 31i$ MeV

**Pole of the $V_{WT}$ + natural: pure dynamical**

$z_1^{\Lambda^*} = 1417 - 19i$ MeV, $z_2^{\Lambda^*} = 1402 - 72i$ MeV

$z^{N^*} = 1582 - 61i$ MeV

Two poles for $\Lambda(1405)$

$\implies \Lambda(1405)$ is mostly dynamical state
Application: $\Lambda$(1405) and N(1535)

**Pole in the effective interaction**

\[
T^{-1} = V_{WT}^{-1} - G(a_{\text{pheno}}) = (V_{\text{natural}})^{-1} - G(a_{\text{natural}})
\]

Pole of the effective interaction (Meff) : pure CDD pole

\[z_{\text{eff}}^\Lambda \sim 7.9 \text{ GeV}\]

\[z_{\text{eff}}^N = 1693 \pm 37i \text{ MeV}\]

**Difference of interactions** \(\Delta V \equiv V_{\text{natural}} - V_{WT}\)

\[\Delta V \equiv \begin{array}{cccc}
\Delta V_{11} & \Lambda(1405) \\
\Delta V_{22} & \\
\Delta V_{33} & \\
\Delta V_{44} & \\
\end{array}\]

\[\Delta V \equiv \begin{array}{cccc}
\Delta V_{11} & N(1535) \\
\Delta V_{22} & \\
\Delta V_{33} & \\
\Delta V_{44} & \\
\end{array}\]

\[\Rightarrow \text{Important CDD pole contribution in N}(1535)\]
Application: $\Lambda(1405)$ and N(1535)

**N(1535) coupling strengths**

Residues of the pole --> coupling strengths

$$T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s - M_R} + i\Gamma_R/2}$$

<table>
<thead>
<tr>
<th>pole in</th>
<th>property</th>
<th>$\pi N$</th>
<th>$\eta N$</th>
<th>$K\Lambda$</th>
<th>$K\Sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>full T</td>
<td>physical</td>
<td>0.949</td>
<td>1.64</td>
<td>1.45</td>
<td>2.96</td>
</tr>
<tr>
<td>$V_{\text{natural}}$</td>
<td>CDD</td>
<td>4.67</td>
<td>2.15</td>
<td>5.71</td>
<td>7.44</td>
</tr>
<tr>
<td>WT+natural</td>
<td>Dynamical</td>
<td>0.353</td>
<td>2.11</td>
<td>1.71</td>
<td>2.93</td>
</tr>
</tbody>
</table>

Coupling properties of the physical pole is similar with those of dynamical pole.

Dynamical nature (on top of CDD pole) is also important?
Summary

We study the origin (dynamical/CDD) of the resonances in the chiral unitary approach.

Natural renormalization scheme

Exclude CDD pole contribution from the loop function, consistent with N/D.

Comparison with phenomenology

--> Pole in the effective interaction

We extract the CDD pole contribution hidden in the subtraction constant into effective interaction $V$.

The origin of the $\Lambda(1405)$ is dominated by dynamical component.

**Nc scaling analysis**


$\rightarrow$ $\Lambda(1405)$ is non-qqq dominant

**Electromagnetic property**


$\rightarrow$ relatively large charge radius

= consistent with present analysis
Summary: application2 N(1535)

The N(1535) consists of both CDD pole and dynamical component.

Comparison of pole position
  --> large effect of the CDD pole
  --> 3-quark state?
  Chiral partner of the nucleon?

Residues (coupling strengths)
  --> important role of the dynamical component