Origin of resonances in chiral dynamics

Tetsuo Hyodo[a],

Daisuke Jido[b], **and Atsushi Hosaka**[c]

Tokyo Institute of Technology[a] *YITP, Kyoto*[b] *RCNP*[c]

2009, Jan 15th

Contents

- Dynamical state and CDD pole
- Chiral unitary approach
- Natural renormalization scheme
- Effective interaction: origin of resonance
- Application: Λ(1405) and N(1535)

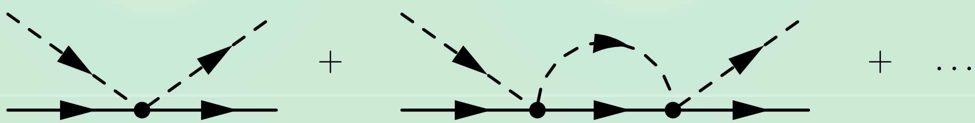
T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

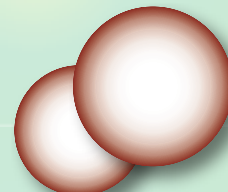
Classification of resonances

Resonances in two-body scattering

- Knowledge of interaction (potential)
- Experimental data (cross section, ...)

Dynamical state: two-body molecule, quasi-bound state, ...

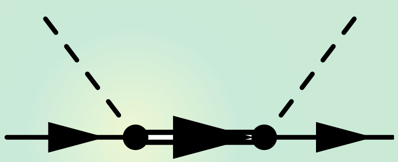


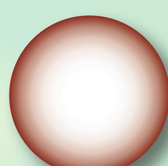


e.g.) Deuteron in NN, positronium in e^+e^-, (σ in π π), ...

CDD(≠CCD!) pole: elementary particle, independent state, ...

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)





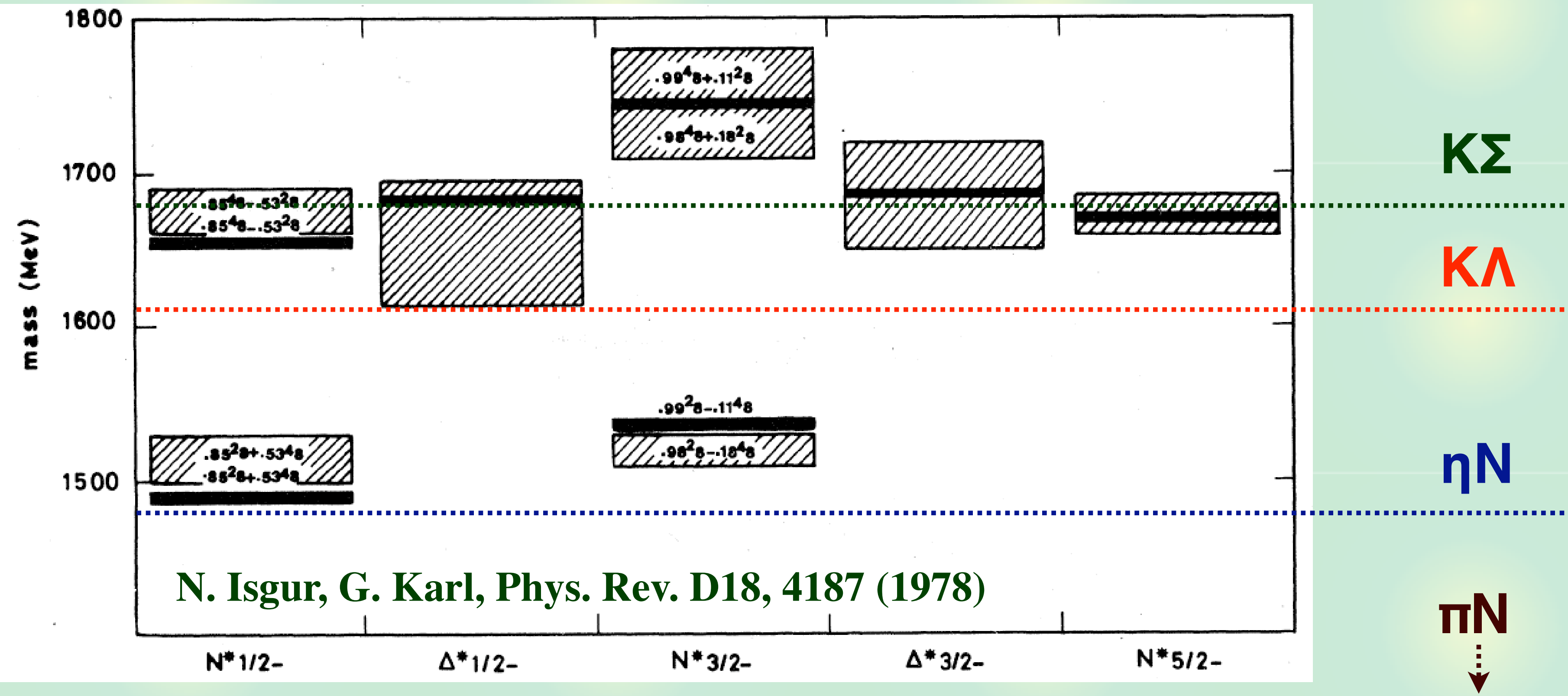
e.g.) J/Ψ in e^+e^-, (ρ in π π), ...

Baryon resonances

Meson-baryon molecule (MB) ~ dynamical state

3-quark state (qqq) ~ (representative of) CDD pole

Difficulties: Both arise from QCD
No clear separation of energy scale (c.f. J/Ψ)



Find out the (dominant) origin for each resonance.

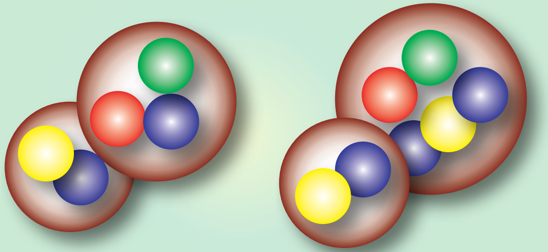
Dynamical state and CDD pole (comments)

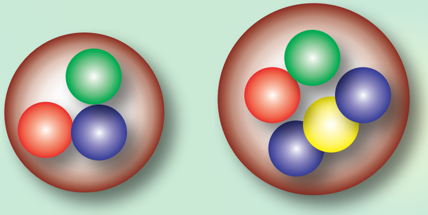
Model space of scattering and dynamical/CDD

Notion of dynamical/CDD **depends** on the scattering particles under consideration. It is **not an inherent property** of the resonance state.

e.g.) J/Ψ : CDD in e^+e^-, dynamical in $c\bar{c}$

Quark structure (for baryon resonances)

dynamical ~ 

CDD ~ 

For hadron resonances, dynamical/CDD is **not directly related to quark structure**.

Mixing of dynamical and CDD

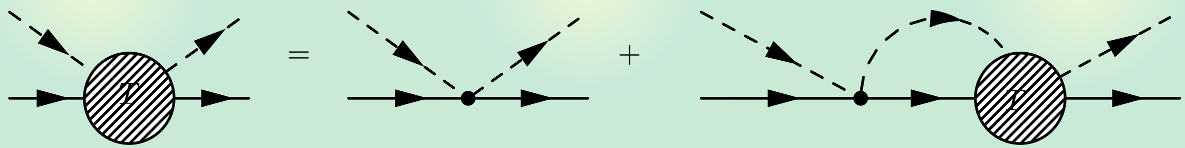
When both exist in one system, **relative weight** is relevant.

Chiral unitary approach

Description of meson-baryon scattering, s-wave resonances

- Interaction <-- chiral symmetry
- Amplitude <-- unitarity (coupled channel)

$$T = \frac{1}{V^{-1} - G}$$



Chiral (WT interaction)

cutoff (subtraction constant)

N. Kaiser, P. B. Siegel, W. Weise, Nucl. Phys. A594, 325 (1995)
E. Oset, A. Ramos, Nucl. Phys. A635, 99 (1998)
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001)
M.F.M. Lutz, E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002),
.... many others

By construction, generated resonances are all dynamical? → **Not always...**

Scattering theory : N/D method

Single-channel scattering, masses: M_T and m

G.F. Chew, S. Mandelstam, Phys. Rev. 119, 467 (1960)

$s = W^2$

Divide T into N(umerator) and D(inominator)

unitarity cut --> D, unphysical cut(s) --> N

$$T(s) = N(s)/D(s)$$

phase space (optical theorem)

$$\mathrm{Im}D(s) = \mathrm{Im}[T^{-1}(s)]N(s) = \boxed{\rho(s)} N(s)/2 \quad \text{for } s > s^+$$

$$\mathrm{Im}N(s) = \mathrm{Im}[T(s)]D(s) \quad \text{for } s < s^-$$

Dispersion relation for N and D

--> set of integral equations, input : $\mathrm{Im}[T(s)] \quad \text{for } s < s^-$

General form of the (s-wave) amplitude

Neglect unphysical cut (crossed diagrams), set N=1

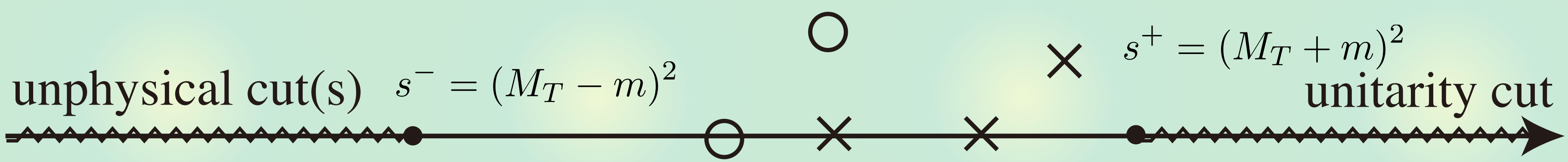
U. G. Meissner, J. A. Oller, Nucl. Phys. A673, 311 (2000)

$$T^{-1}(\sqrt{s}) = \boxed{\tilde{a}(s_0)} + \frac{s - s_0}{2\pi}\int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s'-s)(s'-s_0)}$$

subtraction constant, not determined

- **pole (and zero) of the amplitude**

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)



CDD pole(s), R_i, W_i : not known in advance

$$T^{-1}(\sqrt{s}) = \boxed{\sum_i \frac{R_i}{\sqrt{s} - \sqrt{s_i}}} + \tilde{a}(s_0) + \frac{s - s_0}{2\pi}\int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s'-s)(s'-s_0)}$$

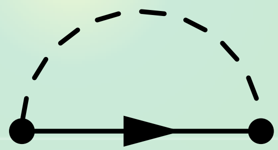
CDD pole contribution --> independent particle

G.F. Chew, S.C. Frautschi, Phys. Rev. 124, 264 (1961)

Order by order matching with ChPT

Identify loop function G, the rest contribution --> V^{-1}

$$T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s}-\sqrt{s_i}} + \tilde{a}(s_0) + \frac{s-s_0}{2\pi}\int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s'-s)(s'-s_0)}$$

$-$ 

$$= -i\int \frac{d^4q}{(2\pi)^4}\frac{2M_T}{(P-q)^2-M_T^2+i\epsilon}\frac{1}{q^2-m^2+i\epsilon}\Big|_{\mathrm{dim.reg.}}$$

$$= -\frac{2M_T}{(4\pi)^2}\left\{a + \frac{m^2-M_T^2+s}{2s}\ln\frac{m^2}{M_T^2} + \frac{\bar{q}}{\sqrt{s}}\ln\frac{\phi_{++}(s)\,\phi_{+-}(s)}{\phi_{-+}(s)\,\phi_{--}(s)}\right\}$$

$$= -G(\sqrt{s};a)$$ **subtraction constant (cutoff)**

$$T(\sqrt{s}) = [V^{-1}(\sqrt{s}) - G(\sqrt{s};a)]^{-1}$$

V? chiral expansion of T, (conceptual) matching with ChPT

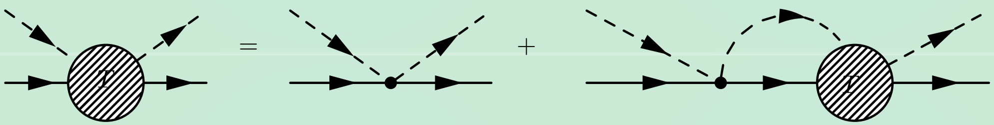
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001)

$$T^{(1)} = V^{(1)}, \quad T^{(2)} = V^{(2)}, \quad T^{(3)} = V^{(3)} - V^{(1)}GV^{(1)}, \dots$$

Summary of chiral unitary appraoch

Scattering amplitude T

$$T(\sqrt{s}) = \frac{1}{V^{-1}(\sqrt{s}) - G(\sqrt{s}; a)}$$



$V(\sqrt{s})$: interaction (ChPT at given order)

$G(\sqrt{s}; a)$: loop function

a : subtraction constant (cutoff parameter)

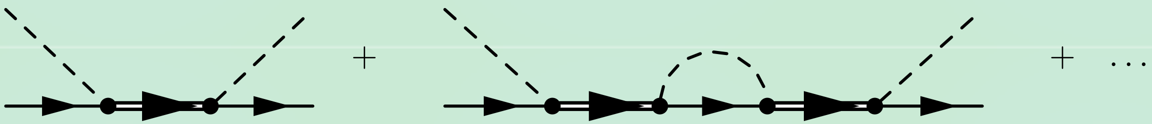
	ChPT	ChU
Unitarity	perturbative	exact
Dynamical resonance	×	○
Crossing symmetry	exact	(perturbative)
Chiral counting	○	×

Nonrenormalizable --> cutoff theory

CDD pole contribution --> V (interaction)

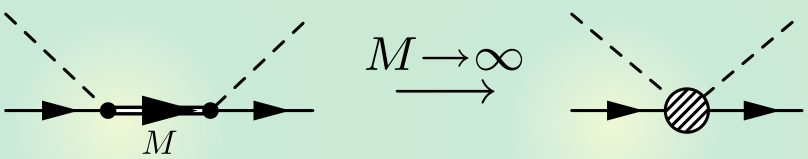
(Known) CDD pole in chiral unitary approach

Explicit resonance field in V (interaction)

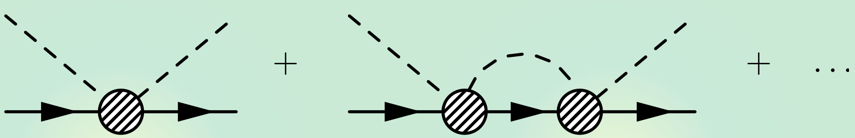


U.G. Meissner, J.A. Oller, Nucl. Phys. A673, 311 (2000)
D. Jido, E. Oset, A. Ramos, Phys. Rev. C66, 055203 (2002)

Contracted resonance propagator in higher order V



G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B321, 311 (1989)
V. Bernard, N. Kaiser, U.G. Meissner, Nucl. Phys. A615, 483 (1997)



J.A. Oller, E. Oset, J.R. Pelaez, Phys. Rev. D59, 074001 (1999)

Is that all? subtraction constant?

Subtraction constant

Phenomenological (standard) scheme

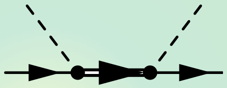
--> V is given, “a” is determined by data

$$T = \frac{1}{(V^{(1)})^{-1} - G(\underline{a})}$$

leading order

$$T = \frac{1}{(V^{(1)} + V^{(2)})^{-1} - G(\underline{a'})}$$

next to leading order

↑pole  ?

“a” represents the effect which is not included in V.

CDD pole contribution in G?

Natural renormalization scheme

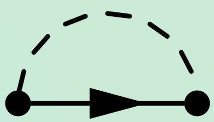
--> fix “a” first, then determine V

exclude CDD pole contribution from G, based on theoretical argument.

Loop function below threshold

Below threshold, G is real and NEGATIVE (~ assume no states below threshold)

$$G(\sqrt{s}) =$$



$$\leq 0 \quad (\text{for } \sqrt{s} \leq M_T + m)$$

It is automatically satisfied in 3d cutoff. However, ...

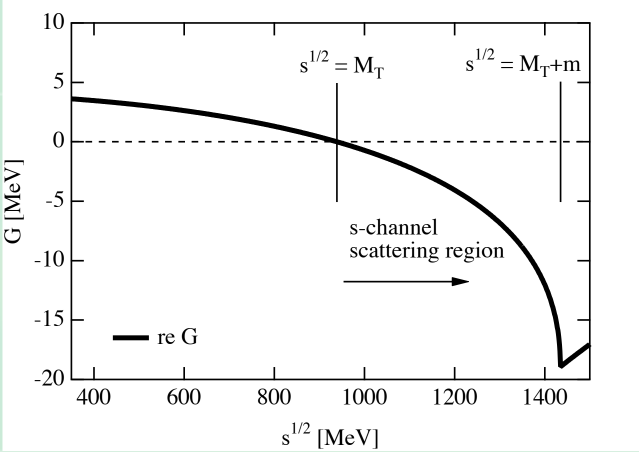
$$G(\sqrt{s}; a) = \frac{2M_T}{(4\pi)^2}\left\{ a + \frac{m^2 - M_T^2 + s}{2s} \ln \frac{m^2}{M_T^2} + \frac{\bar{q}}{\sqrt{s}} \ln \frac{\phi_{++}(s)\,\phi_{+-}(s)}{\phi_{-+}(s)\,\phi_{--}(s)} \right\}$$

Large (positive) "a" can make G positive.
Avoid this for s-channel region (> M_T),

$$a \leq a_{\max}(M_T, m)$$

or equivalently (G: decreasing),

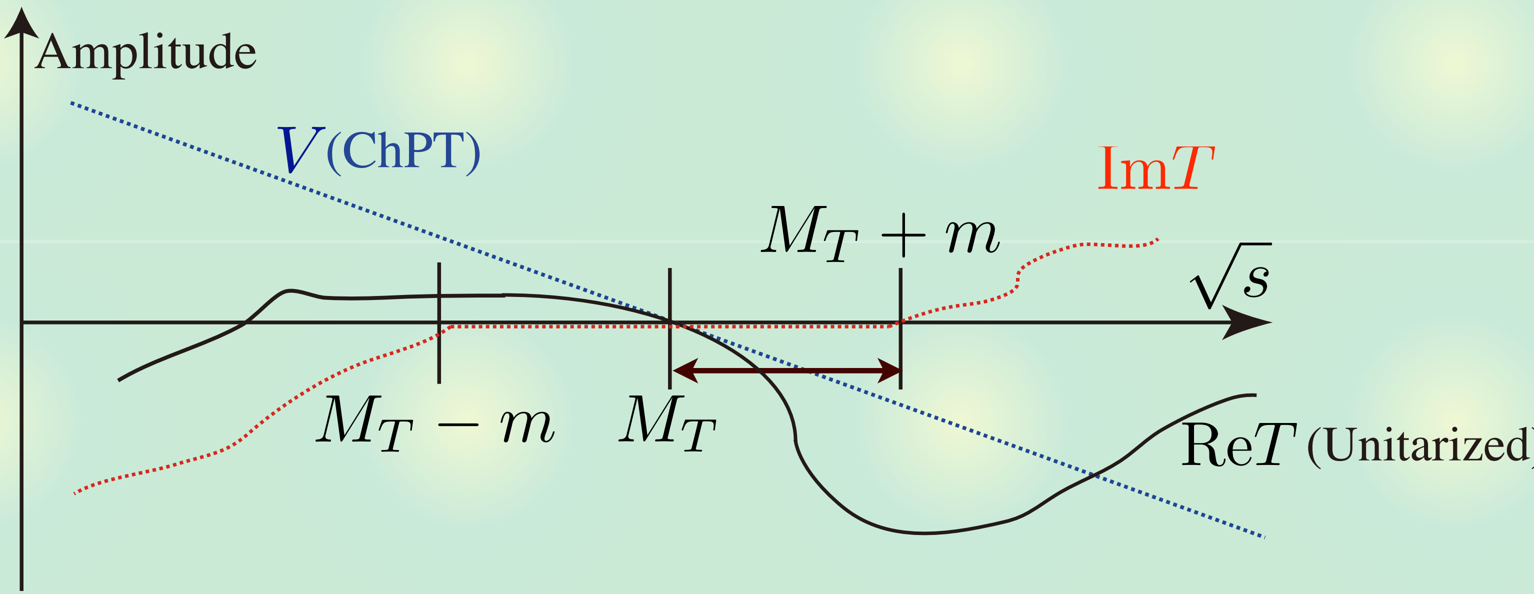
$$G(\sqrt{s} = M_T) \leq 0$$



(Explicit) matching with ChPT

V is given by ChPT.
At a "low energy", T should be matched with V:

$$G(\sqrt{s}=\mu_m)=0, \quad \Leftrightarrow \quad T(\mu_m)=V(\mu_m)$$



matching in s-channel region, subtraction constant is real

$$\Rightarrow \quad M_T \le \mu_m \le M_T+m$$

consistent with "low energy" requirement

$$\sqrt{s}=M_T+m \;\Rightarrow\; \boldsymbol{p}=0, \quad \sqrt{s}=M_T \;\Rightarrow\; \omega\sim 0$$

Natural renormalization condition : summary

Natural renormalization condition

- Loop function should be negative below threshold
- T matches with V at low energy scale

"a" is uniquely determined such that

$$G(\sqrt{s} = M_T) = 0, \quad \Leftrightarrow \quad T(M_T) = V(M_T)$$

matching with low energy interaction

K. Igi, K. Hikasa, Phys. Rev. D59, 034005 (1999)
U.G. Meissner, J.A. Oller, Nucl. Phys. A673, 311 (2000)

crossing symmetry (matching with u-channel amplitude)

M.F.M. Lutz, E. Kolomeitsev, Nucl. Phys. A700, 193 (2002)

We regard this condition as the **exclusion of the CDD pole contribution from G**

Two renormalization schemes

Phenomenological scheme

V is given by ChPT (for instance, leading order term),
fit cutoff in G to data

Natural renormalization scheme

determine G to exclude CDD pole contribution,
V is to be determined

Same physics (scattering amplitude T)

$$T = \frac{1}{V_{\mathrm{ChPT}}^{-1} - G(a_{\mathrm{pheno}})} = \frac{1}{(V_{\mathrm{natural}})^{-1} - G(a_{\mathrm{natural}})}$$

↑Effective interaction
Origin of the resonance

Pole in the effective interaction

Leading order V : Weinberg-Tomozawa term

$$V_{\mathrm{WT}} = -\frac{C}{2f^2}(\sqrt{s} - M_T)$$

C/f² : coupling constant
no s-wave resonance

$$T^{-1} = V_{\mathrm{WT}}^{-1} - G(a_{\mathrm{pheno}}) = (V_{\mathrm{natural}})^{-1} - G(a_{\mathrm{natural}})$$

↑ChPT ↑data fit ↑given

Effective interaction in natural scheme

$$V_{\mathrm{natural}} = -\frac{C}{2f^2}(\sqrt{s} - M_T) + \boxed{\frac{C}{2f^2}\frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\mathrm{eff}}}}$$

pole!

$$M_{\mathrm{eff}} = M_T - \frac{16\pi^2 f^2}{C M_T \Delta a}, \quad \Delta a = a_{\mathrm{pheno}} - a_{\mathrm{natural}}$$

Physically meaningful pole : $C > 0, \quad \Delta a < 0$

There is always a pole for $a_{\mathrm{pheno}} \neq a_{\mathrm{natural}}$
--> energy scale of the effective pole is relevant.

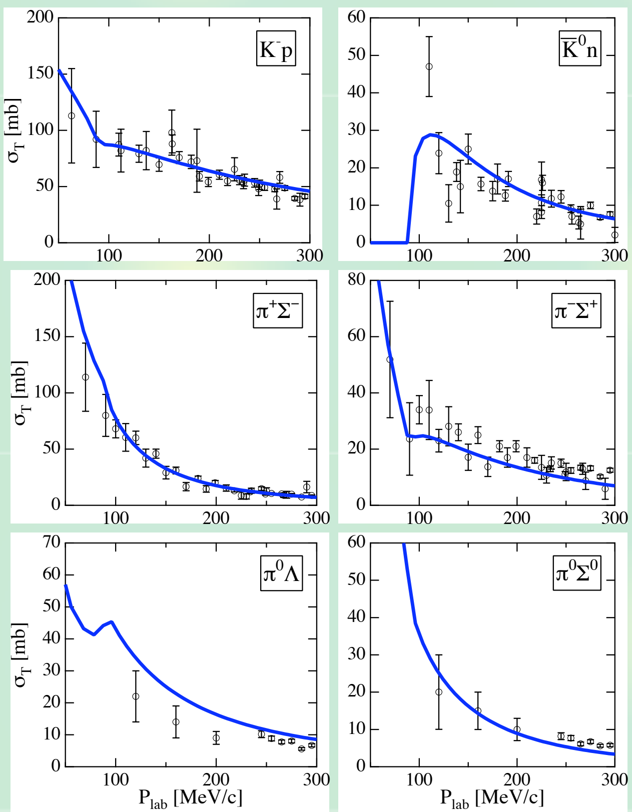
Application: Λ(1405) and N(1535)

S=-1 and S=0 meson-baryon scatterings

Models for the Meson-baryon scattering :

E. Oset, A. Ramos, C. Bennhold, Phys. Lett. B527, 99 (2002),
T. Inoue, E. Oset, M.J. Vicente Vacas, Phys. Rev. C. 65, 035204 (2002)
T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Phys. Rev. C. 68, 018201 (2003)
T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Prog. Thor. Phys. 112, 73 (2004)

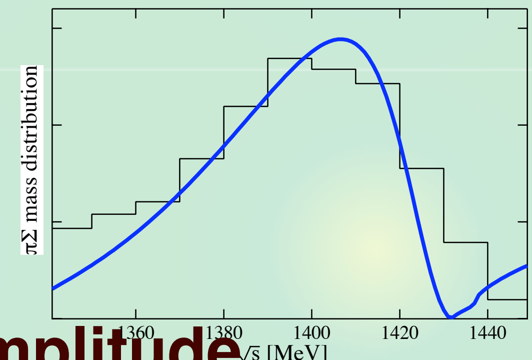
K-p total cross sections



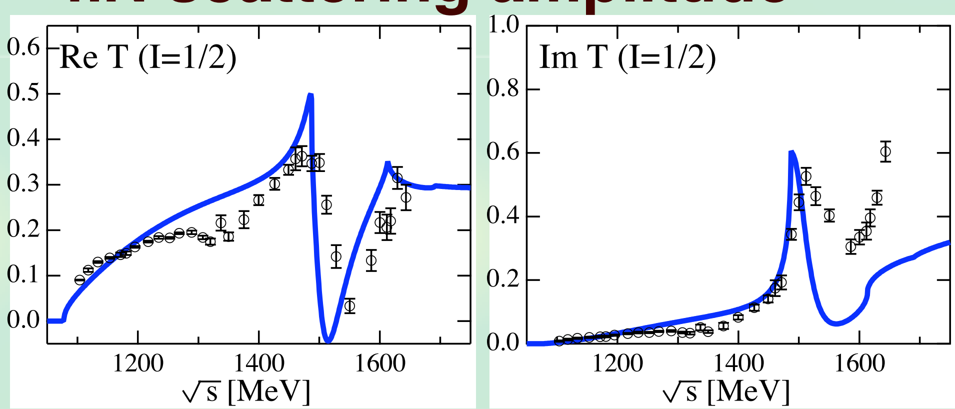
threshold ratios

	γ	R_c	R_n
exp.	2.36	0.664	0.189
theo.	1.80	0.624	0.225

πΣ spectrum



πN scattering amplitude



Comparison of pole positions

Pole of the full amplitude : physical state ▲

$$z_1^{\Lambda^*} = 1429 - 14i \text{ MeV}, \quad z_2^{\Lambda^*} = 1397 - 73i \text{ MeV}$$

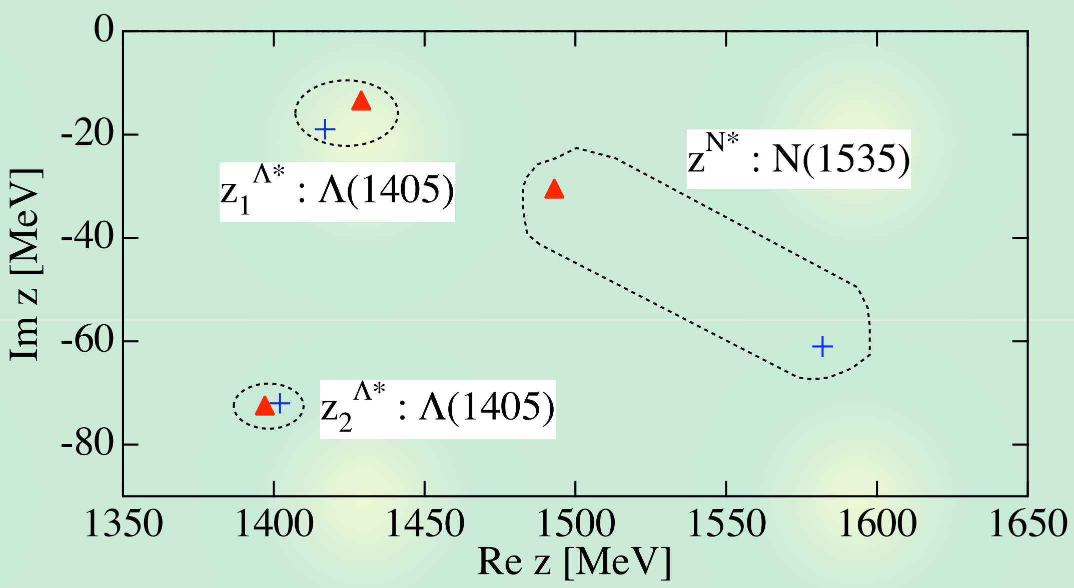
$$z^{N^*} = 1493 - 31i \text{ MeV}$$

two poles for Λ(1405)

Pole of the V_{WT} + natural : pure dynamical +

$$z_1^{\Lambda^*} = 1417 - 19i \text{ MeV}, \quad z_2^{\Lambda^*} = 1402 - 72i \text{ MeV}$$

$$z^{N^*} = 1582 - 61i \text{ MeV}$$



==> Λ(1405) is mostly dynamical state

Pole in the effective interaction

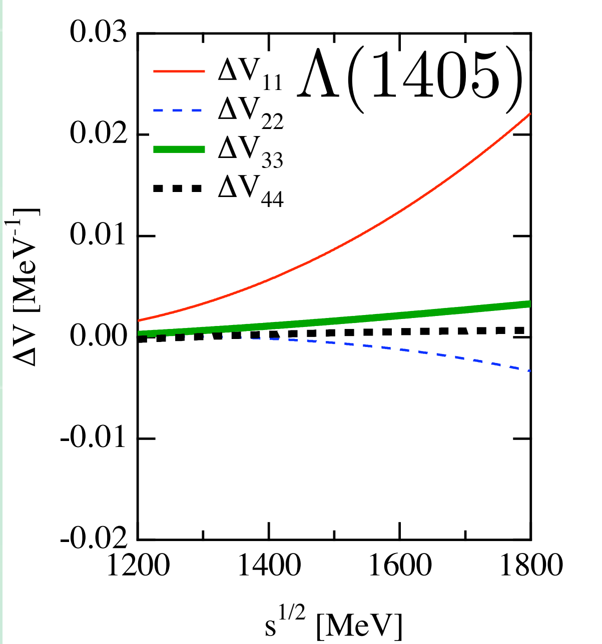
$$T^{-1} = V_{\rm WT}^{-1} - G(a_{\rm pheno}) = (V_{\rm natural})^{-1} - G(a_{\rm natural})$$

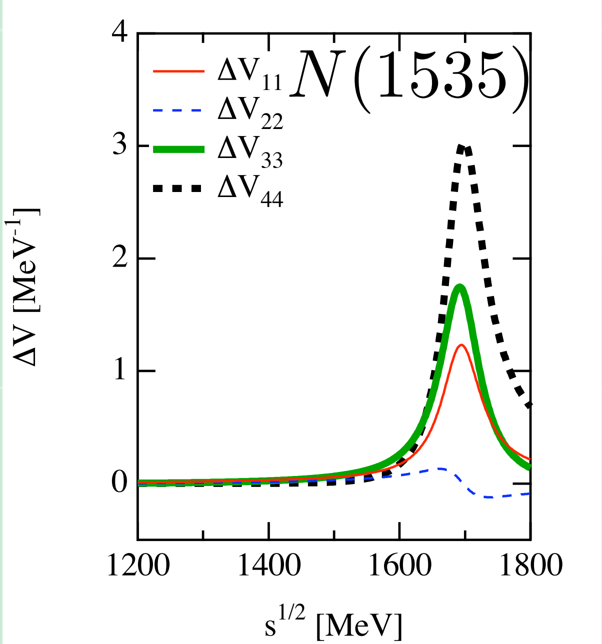
Pole of the effective interaction (Meff) : pure CDD pole

$z_{\rm eff}^{\Lambda^*} \sim 7.9 \text{ GeV}$ **irrelevant!**

$z_{\rm eff}^{N^*} = 1693 \pm 37i \text{ MeV}$ **relevant?**

Difference of interactions $\Delta V \equiv V_{\rm natural} - V_{\rm WT}$





==> Important CDD pole contribution in N(1535)

N(1535) coupling strengths

Residues of the pole --> coupling strengths

$$T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - M_R + i\Gamma_R/2}$$

pole in	property	πN	ηN	KΛ	KΣ
full T	physical	0.949	1.64	1.45	2.96
$V_{natural}$	CDD	4.67	2.15	5.71	7.44
WT+natural	Dynamical	0.353	2.11	1.71	2.93

Coupling properties of the physical pole is similar with those of dynamical pole.

Dynamical nature (on top of CDD pole) is also important?

Summary: formulation

We study the origin (dynamical/CDD) of the resonances in the chiral unitary approach

- Natural renormalization scheme

  Exclude CDD pole contribution from the loop function, consistent with N/D.

- Comparison with phenomenology

  --> Pole in the effective interaction

  We extract the CDD pole contribution hidden in the subtraction constant into effective interaction V.

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

Summary: application1 Λ(1405)

The origin of the Λ(1405) is dominated by dynamical component.

- **Nc scaling analysis**

  T. Hyodo, D. Jido, R. Loca, Phys. Rev. D77, 056010 (2008)
  R. Loca, T. Hyodo, D. Jido, Nucl. Phys. A809, 65 (2008)

  --> Λ(1405) is non-qqq dominant

- **Electromagnetic property**

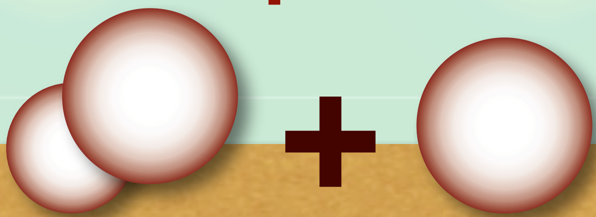
  T. Sekihara, T. Hyodo, D. Jido, Phys. Lett. B669, 133 (2008)

  --> relatively large charge radius

- **= consistent with present analysis**

Summary: application2 N(1535)

The N(1535) consists of both CDD pole and dynamical component.



- **Comparison of pole position**

  --> large effect of the CDD pole
  --> 3-quark state?
  Chiral partner of the nucleon?

- **Residues (coupling strengths)**

  --> important role of the dynamical component