Chiral dynamics and baryon resonances

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Chiral dynamics

- Low energy theorem (chiral symmetry)
- Dispersion theory (unitarity of S-matrix)
- Baryon resonances in meson-baryon scattering

Structure of $\Lambda(1405)$ resonance

- **Dynamical or CDD pole (genuine quark state)?**

- **Nc Behavior and quark structure**

- **Electromagnetic properties**
Chiral dynamics

Description of hadron-NG boson scattering and resonance

- Interaction <-- chiral symmetry


- Amplitude <-- unitarity (coupled channel)

R.H. Dalitz, T.C. Wong, G. Rajasekaran, PR153, 1617 (1967)

\[
T = \frac{1}{1 - VG} V
\]


works successfully, also in S=0 sector, meson-meson scattering sectors, systems including heavy quarks, ...
Low energy s-wave interaction

Low energy theorem for pion (Ad) scattering with a target (T)

\[ \alpha \left[ \frac{\text{Ad}(q)}{T(p)} \right] = \frac{1}{f^2} \frac{p \cdot q}{2M_T} \langle F_T \cdot F_{\text{Ad}} \rangle_\alpha + O\left(\left(\frac{m}{M_T}\right)^2\right) \]

s-wave : Weinberg-Tomozawa term


\[ V_{ij} = -\frac{C_{ij}}{4f^2}(\omega_i + \omega_j) \]

pion energy

\[ C_{ij} = \sum_\alpha C_{\alpha,T} \left( \begin{array}{c} 8 \\ I_{M_i}, Y_{M_i} \\ I_{T_i}, Y_{T_i} \end{array} \right) \left( \begin{array}{c} \alpha \\ 8 \\ I, Y \end{array} \right) \left( \begin{array}{c} 8 \\ I_{M_j}, Y_{M_j} \\ I_{T_j}, Y_{T_j} \end{array} \right) \left( \begin{array}{c} \alpha \\ \alpha \\ I, Y \end{array} \right) \]

pion decay constant \((g_V=1)\)

flavor SU(3) --> sign and strength

Low energy theorem : leading order term in ChPT
Scattering theory: N/D method

Single-channel scattering, masses: $M_T$ and $m$

G.F. Chew, S. Mandelstam, Phys. Rev. 119, 467 (1960)

\[ s = W^2 \]

unphysical cut(s) \[ s^- = (M_T - m)^2 \]

unitarity cut \[ s^+ = (M_T + m)^2 \]

Divide $T$ into $N$ (umerator) and $D$ (inominator)

\[ T(s) = N(s)/D(s) \]

phase space (optical theorem)

\[ \text{Im}D(s) = \text{Im}[T^{-1}(s)] N(s) = \frac{\rho(s) N(s)}{2} \text{ for } s > s^+ \]

\[ \text{Im}N(s) = \text{Im}[T(s)] D(s) \text{ for } s < s^- \]

Dispersion relation for $N$ and $D$

\[ \rightarrow \text{set of integral equations, input: } \text{Im}[T(s)] \text{ for } s < s^- \]
General form of the (s-wave) amplitude

Neglect unphysical cut (crossed diagrams), set N=1


\[ T^{-1}(\sqrt{s}) = \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)} \]

- pole (and zero) of the amplitude


unphysical cut(s) \( s^- = (M_T - m)^2 \)

\[ T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s} - \sqrt{s_i}} + \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)} \]

CDD pole(s), \( R_i, W_i \): not known in advance

CDD pole contribution --> independent particle

Chiral dynamics

Order by order matching with ChPT

Identify loop function $G$, the rest contribution $\rightarrow V^{-1}$

\[
T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s} - \sqrt{s}_i} + \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)}
\]

\[
- = -i \int \frac{d^4q}{(2\pi)^4} \frac{2M_T}{(P - q)^2 - M_T^2 + i\epsilon} q^2 - m^2 + i\epsilon \bigg|_{\text{dim.reg.}}
\]

\[
= -\frac{2M_T}{(4\pi)^2} \left\{ a + \frac{m^2 - M_T^2 + s}{2s} \ln \frac{m^2}{M_T^2} + \frac{\bar{q}}{\sqrt{s}} \ln \frac{\phi_{++}(s) \phi_{+-}(s)}{\phi_{--}(s) \phi_{-+}(s)} \right\}
\]

\[
= -G(\sqrt{s}; a) \quad \text{subtraction constant (cutoff)}
\]

\[
T(\sqrt{s}) = [V^{-1}(\sqrt{s}) - G(\sqrt{s}; a)]^{-1}
\]

scattering amplitude

$V$? chiral expansion of $T$, (conceptual) matching with ChPT


\[
T^{(1)} = V^{(1)}, \quad T^{(2)} = V^{(2)}, \quad T^{(3)} = V^{(3)} - V^{(1)} GV^{(1)}, \ldots
\]
**KN scattering and $\Lambda(1405)$**

$\Lambda(1405) : J^P = 1/2^-, I = 0$

**PDG**

- Mass : $1406.5 \pm 4.0$ MeV
- Width : $50 \pm 2$ MeV
- Decay mode : $\Lambda(1405) \rightarrow (\pi \Sigma)_{I=0} \quad 100\%$

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**“naive” quark model**

- $p$-wave
- $\sim 1600$ MeV?

N. Isgur, G. Karl, PRD18, 4187 (1978)

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**Coupled channel multi-scattering**

R.H. Dalitz, T.C. Wong, G. Rajasekaran, PR153, 1617 (1967)

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**$\bar{K}N$ int. below threshold**


$\rightarrow$ Kaonic nuclei

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$\Lambda(1405)$
Chiral dynamics

How it works? vs experimental data

Total cross sections

threshold ratios

<table>
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<tr>
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<th>$\gamma$</th>
<th>$R_c$</th>
<th>$R_n$</th>
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<tr>
<td>theo.</td>
<td>1.80</td>
<td>0.624</td>
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πΣ spectrum

$\Lambda(1405)$


Good agreement with data above, at, and below threshold
Chiral dynamics

Two poles for one resonance

Poles of the amplitude in the complex plane: resonance

\[ T_{ij}(\sqrt{s}) \sim \frac{g_ig_j}{\sqrt{s} - M_R + i\Gamma_R/2} \]

Physical "\( \Lambda(1405) \)"

: superposition of two states

Dynamical state and CDD pole

Resonances in two-body scattering

- Knowledge of interaction (potential)
- Experimental data (cross section, phase shift, ...)

(a) dynamical state: molecule, quasi-bound, ...

... in the present case: meson-baryon molecule

(b) CDD pole: elementary, independent, ...


... in the present case: three-quark state

Resonances in chiral dynamics $\Rightarrow$ (a) dynamical?
CDD pole contribution in chiral unitary approach

Amplitude in chiral unitary model

\[ T = \frac{1}{V^{-1}G} \]

\( V \) : interaction kernel (potential)
\( G \) : loop integral (Green’s function)

Known CDD pole contribution

1. Explicit resonance field in \( V \)

2. Contracted resonance propagator in \( V \)

Is that all? subtraction constant?
Subtraction constant

Phenomenological (standard) scheme

--> V is given, “a” is determined by data

\[ T = \frac{1}{(V^{(1)})^{-1} - G(a)} \]

leading order

\[ T = \frac{1}{(V^{(1)} + V^{(2)})^{-1} - G(a')} \]

next to leading order

↑ pole ?

“a” represents the effect which is not included in V.

CDD pole contribution in G?

Natural renormalization scheme

--> fix “a” first, then determine V

exclude CDD pole contribution from G, based on theoretical argument.
Two renormalization schemes

Phenomenological scheme

V is given by ChPT (for instance, leading order term), fit cutoff in G to data

Natural renormalization scheme

determine G to exclude CDD pole contribution, V is to be determined

Same physics (scattering amplitude T)

\[ T = \frac{1}{V_{\text{ChPT}}^{-1} - G(a_{\text{pheno}})} = \frac{1}{(V_{\text{natural}})^{-1} - G(a_{\text{natural}})} \]

↑ Effective interaction
Origin of the resonance
Pole in the effective interaction

Leading order $V$ : Weinberg-Tomozawa term

$$V_{WT} = -\frac{C}{2f^2} (\sqrt{s} - M_T)$$

$C/f^2$ : coupling constant

no s-wave resonance

$$T^{-1} = V_{WT}^{-1} - G(a_{\text{pheno}}) = (V_{\text{natural}})^{-1} - G(a_{\text{natural}})$$

$\uparrow$ ChPT $\uparrow$ data fit $\uparrow$ given

Effective interaction in natural scheme

$$V_{\text{natural}} = -\frac{C}{2f^2} (\sqrt{s} - M_T) + \frac{C}{2f^2} \frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\text{eff}}}$$

$$M_{\text{eff}} = M_T - \frac{16\pi^2 f^2}{CM_T\Delta a}, \quad \Delta a = a_{\text{pheno}} - a_{\text{natural}}$$

Physically meaningful pole : $C > 0, \quad \Delta a < 0$

There is always a pole for $a_{\text{pheno}} \neq a_{\text{natural}}$

$\rightarrow$ energy scale of the effective pole is relevant.
Pole of the full amplitude: physical state

\[ z_1^{\Lambda^*} = 1429 - 14i \text{ MeV}, \quad z_2^{\Lambda^*} = 1397 - 73i \text{ MeV} \]

\[ z^{N^*} = 1493 - 31i \text{ MeV} \]

Pole of the \( V_{WT} + \) natural: pure dynamical

\[ z_1^{\Lambda^*} = 1417 - 19i \text{ MeV}, \quad z_2^{\Lambda^*} = 1402 - 72i \text{ MeV} \]

\[ z^{N^*} = 1582 - 61i \text{ MeV} \]

\[ \Rightarrow \Lambda(1405) \text{ is mostly dynamical state} \]
Structure of $\Lambda(1405)$ resonance

Pole in the effective interaction

$$T^{-1} = V_{ WT}^{-1} - G(a_{pheno}) = \left( V_{natural} \right)^{-1} - G(a_{natural})$$

Pole of the effective interaction (Meff) : pure CDD pole

$z_{\text{eff}}^{\Lambda^*} \sim 7.9$ GeV irrelevant!

$z_{\text{eff}}^{N^*} = 1693 \pm 37i$ MeV relevant?

Difference of interactions $\Delta V \equiv V_{\text{natural}} - V_{WT}$

$\Rightarrow$ Important CDD pole contribution in $N(1535)$
Nc scaling in the model

Nc : number of color in QCD
Hadron effective theory / quark structure

The Nc behavior is known from the general argument.
<-- introducing Nc dependence in the model,
analyze the resonance properties with respect to Nc


Nc scaling of (excited)
qqq baryon

\[ M_R \sim \mathcal{O}(N_c), \quad \Gamma_R \sim \mathcal{O}(1) \]

Result : \( \Gamma_R \neq \mathcal{O}(1) \)
~ non-qqq (i.e. dynamical) structure

Electromagnetic properties

Attaching photon to resonance
--> em properties : rms, form factors,...

Structure of Λ(1405) resonance
large (em) size of the Λ(1405) : c.f. -0.12 [fm$^2$] for neutron
--> meson-baryon picture

result of mean squared radii :

$$|\langle r^2 \rangle_E| = 0.33 \ [fm^2]$$

Structure of Λ(1405) resonance

Summary: Chiral dynamics

Framework of chiral coupled-channel approach is reviewed.

Interaction given by chiral symmetry + coupled-channel unitarity condition

=> successful description of meson-baryon scattering and resonances.

On top of the successful reproduction of scattering data, the internal structure of resonances can be investigated in several ways.
The structure of the Λ(1405) is:

Dynamical or CDD?
=> dominance of the MB components

Analysis of Nc scaling
=> non-qqq structure

Electromagnetic properties
=> large e.m. size
The structure of the $\Lambda(1405)$ is:

- Dynamical or CDD?
  - $\Rightarrow$ dominance of the MB components

- Analysis of Nc scaling
  - $\Rightarrow$ non-qqq structure

- Electromagnetic properties
  - $\Rightarrow$ large e.m. size

Independent analyses consistently support the meson-baryon molecule picture of the $\Lambda(1405)$