

Chiral dynamics and baryon resonances



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Chiral dynamics

- Low energy theorem (chiral symmetry)
- Dispersion theory (unitarity of S-matrix)
- Baryon resonances in meson-baryon scattering



Structure of $\Lambda(1405)$ resonance

- **Dynamical or CDD pole (genuine quark state) ?**
T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008).
- **Nc Behavior and quark structure**
T. Hyodo, D. Jido, L. Roca, Phys. Rev. D77, 056010 (2008).
L. Roca, T. Hyodo, D. Jido, Nucl. Phys. A809, 65-87 (2008).
- **Electromagnetic properties**
T. Sekihara, T. Hyodo, D. Jido, Phys. Lett. B669, 133-138 (2008).

Chiral dynamics : overview

Description of hadron-NG boson scattering and resonance

- Interaction <-- chiral symmetry

Y. Tomozawa, *Nuovo Cim.* 46A, 707 (1966); S. Weinberg, *Phys. Rev. Lett.* 17, 616 (1966)

- Amplitude <-- unitarity (coupled channel)

R.H. Dalitz, T.C. Wong, G. Rajasekaran, *PR*153, 1617 (1967)

$$T = \frac{1}{1 - VG} V$$

chiral

N. Kaiser, P. B. Siegel, W. Weise, *Nucl. Phys.* A594, 325 (1995),

E. Oset, A. Ramos, *Nucl. Phys.* A635, 99 (1998),

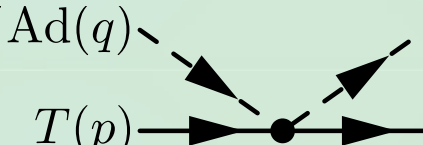
J. A. Oller, U. G. Meissner, *Phys. Lett.* B500, 263 (2001),

M.F.M. Lutz, E. E. Kolomeitsev, *Nucl. Phys.* A700, 193 (2002), many others

works successfully, also in S=0 sector, meson-meson scattering sectors, systems including heavy quarks, ...

Low energy s-wave interaction

Low energy theorem for pion (Ad) scattering with a target (T)

$$\alpha \left[\begin{array}{c} \text{Ad}(q) \\ T(p) \end{array} \right] = \frac{1}{f^2} \frac{p \cdot q}{2M_T} \langle \mathbf{F}_T \cdot \mathbf{F}_{\text{Ad}} \rangle_\alpha + \mathcal{O} \left(\left(\frac{m}{M_T} \right)^2 \right)$$


s-wave : Weinberg-Tomozawa term

Y. Tomozawa, *Nuovo Cim.* **46A**, 707 (1966); S. Weinberg, *Phys. Rev. Lett.* **17**, 616 (1966)

$$V_{ij} = - \frac{C_{ij}}{4f^2} (\omega_i + \omega_j)$$

pion energy
pion decay constant ($g_V=1$)

$$C_{ij} = \sum_{\alpha} C_{\alpha, T} \left(\begin{array}{cc} 8 & T \\ I_{M_i}, Y_{M_i} & I_{T_i}, Y_{T_i} \end{array} \parallel \begin{array}{c} \alpha \\ I, Y \end{array} \right) \left(\begin{array}{cc} 8 & T \\ I_{M_j}, Y_{M_j} & I_{T_j}, Y_{T_j} \end{array} \parallel \begin{array}{c} \alpha \\ I, Y \end{array} \right)$$

$$C_{\alpha, T} = \langle 2\mathbf{F}_T \cdot \mathbf{F}_{\text{Ad}} \rangle = C_2(T) - C_2(\alpha) + 3$$

flavor SU(3) --> sign and strength

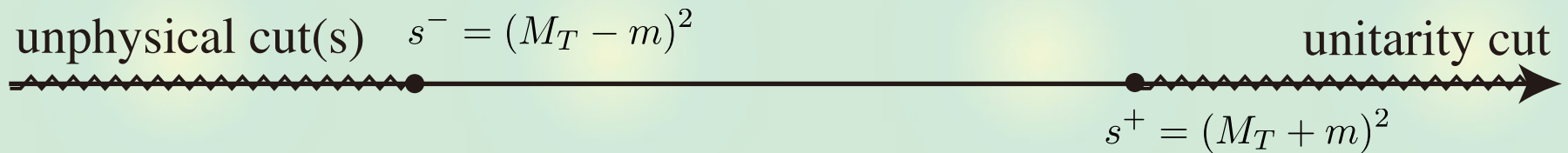
Low energy theorem : leading order term in ChPT

Scattering theory : N/D method

Single-channel scattering, masses: M_T and m

G.F. Chew, S. Mandelstam, Phys. Rev. 119, 467 (1960)

$$s = W^2$$



Divide T into N (umerator) and D (inominator)

unitarity cut $\rightarrow D$, unphysical cut(s) $\rightarrow N$

$$T(s) = N(s)/D(s) \quad \text{phase space (optical theorem)}$$

$$\text{Im}D(s) = \text{Im}[T^{-1}(s)]N(s) = \boxed{\rho(s)} N(s)/2 \quad \text{for } s > s^+$$

$$\text{Im}N(s) = \text{Im}[T(s)]D(s) \quad \text{for } s < s^-$$

Dispersion relation for N and D

\rightarrow set of integral equations, input : $\text{Im}[T(s)]$ for $s < s^-$

General form of the (s-wave) amplitude

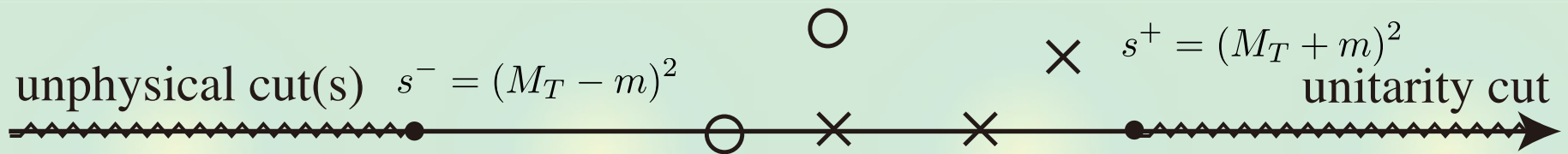
Neglect unphysical cut (crossed diagrams), set $N=1$

U. G. Meissner, J. A. Oller, Nucl. Phys. A673, 311 (2000)

$$T^{-1}(\sqrt{s}) = \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)}$$

• pole (and zero) of the amplitude

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)



CDD pole(s), R_i, W_i : not known in advance

$$T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s} - \sqrt{s_i}} + \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)}$$

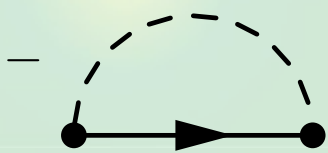
CDD pole contribution --> independent particle

G.F. Chew, S.C. Frautschi, Phys. Rev. 124, 264 (1961)

Order by order matching with ChPT

Identify loop function G , the rest contribution $\rightarrow V^{-1}$

$$T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s} - \sqrt{s_i}} + \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s_+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)}$$



$$\begin{aligned}
 &= -i \int \frac{d^4q}{(2\pi)^4} \frac{2M_T}{(P - q)^2 - M_T^2 + i\epsilon} \frac{1}{q^2 - m^2 + i\epsilon} \Big|_{\text{dim.reg.}} \\
 &= -\frac{2M_T}{(4\pi)^2} \left\{ \boxed{a} + \frac{m^2 - M_T^2 + s}{2s} \ln \frac{m^2}{M_T^2} + \frac{\bar{q}}{\sqrt{s}} \ln \frac{\phi_{++}(s)\phi_{+-}(s)}{\phi_{-+}(s)\phi_{--}(s)} \right\} \\
 &= -G(\sqrt{s}; a) \quad \text{subtraction constant (cutoff)}
 \end{aligned}$$

$T(\sqrt{s}) = [V^{-1}(\sqrt{s}) - G(\sqrt{s}; a)]^{-1}$

scattering amplitude

V? chiral expansion of T, (conceptual) matching with ChPT

J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001)

$$T^{(1)} = V^{(1)}, \quad T^{(2)} = V^{(2)}, \quad T^{(3)} = V^{(3)} - V^{(1)}GV^{(1)}, \dots$$

$\bar{K}N$ scattering and $\Lambda(1405)$

$\Lambda(1405) : J^P = 1/2^-, I = 0$

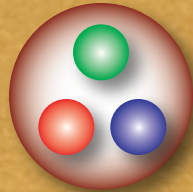
PDG

Mass : 1406.5 ± 4.0 MeV

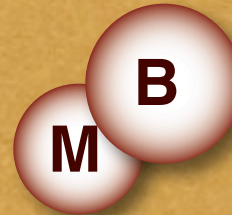
Width : 50 ± 2 MeV

Decay mode : $\Lambda(1405) \rightarrow (\pi\Sigma)_{I=0}$ **100%**

“naive” quark model
: p-wave
~1600 MeV?



N. Isgur, G. Karl, PRD18, 4187 (1978)



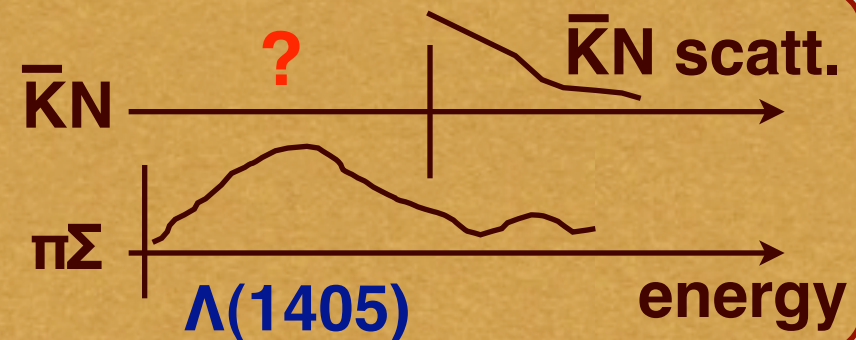
Coupled channel
multi-scattering

R.H. Dalitz, T.C. Wong,
G. Rajasekaran, PR153, 1617 (1967)

$\bar{K}N$ int. below threshold

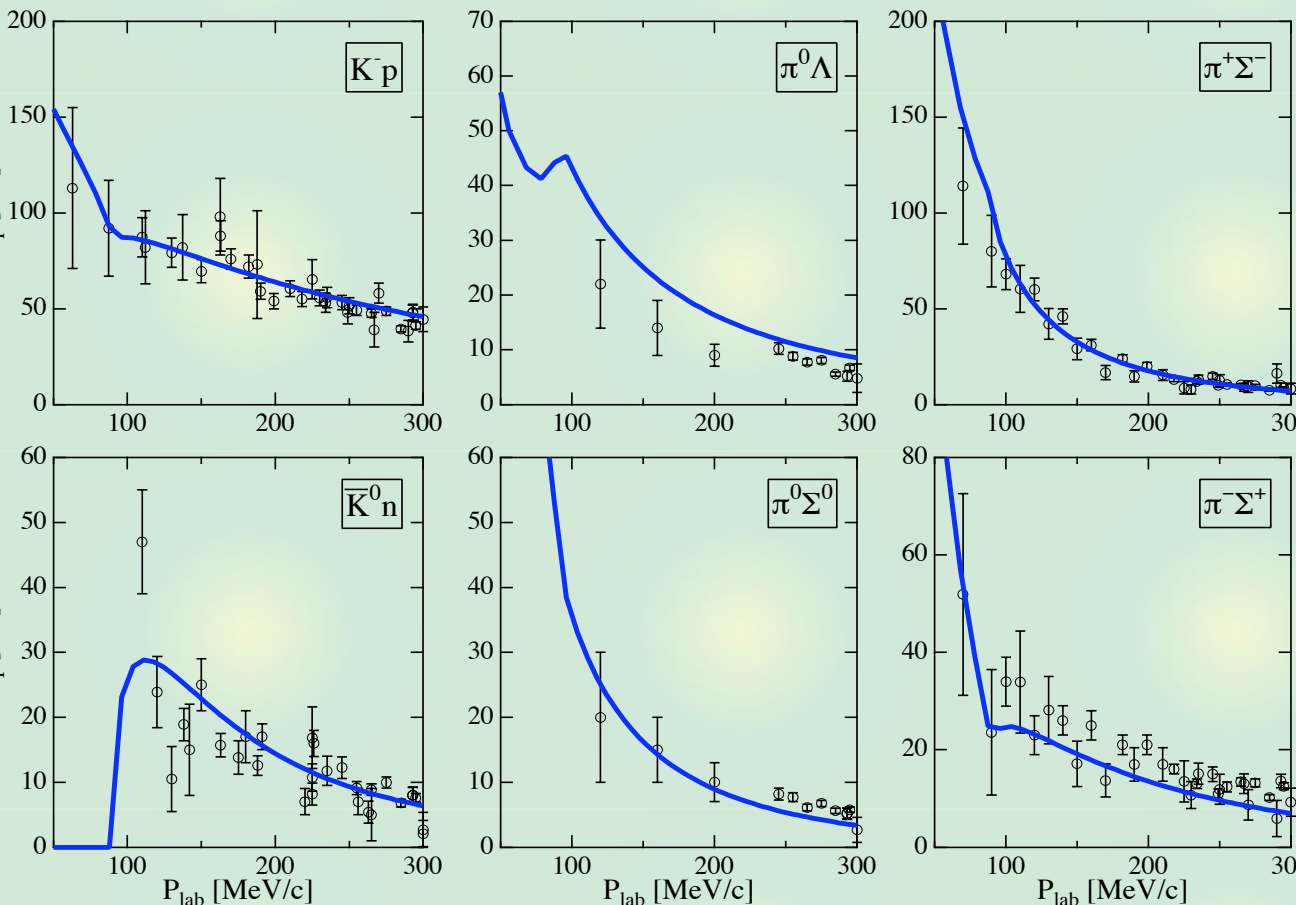
T. Hyodo, W. Weise, PRC 77, 035204 (2008)

--> Kaonic nuclei



How it works? vs experimental data

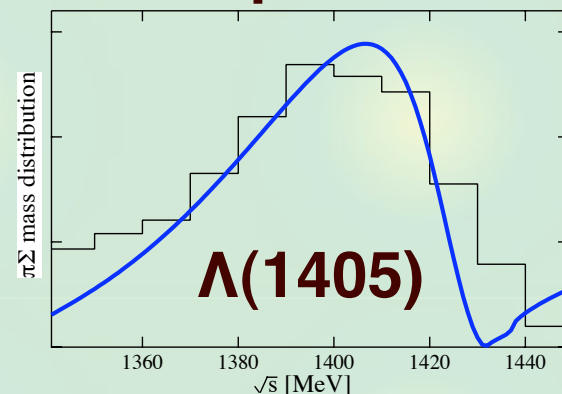
Total cross sections



threshold ratios

	γ	R_c	R_n
exp.	2.36	0.664	0.189
theo.	1.80	0.624	0.225

$\pi\Sigma$ spectrum



T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Phys. Rev. C68, 018201 (2003),

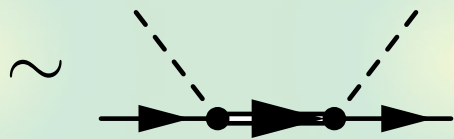
T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Prog. Theor. Phys. 112, 73 (2004)

Good agreement with data above, at, and below threshold

Two poles for one resonance

Poles of the amplitude in the complex plane : resonance

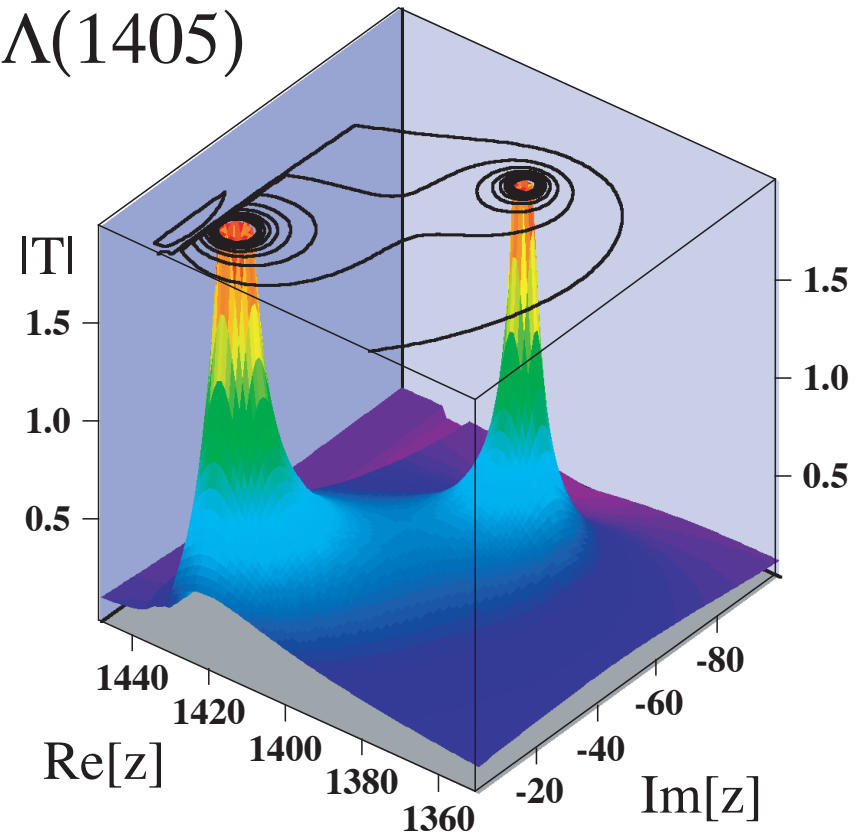
$$T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - M_R + i\Gamma_R/2}$$



Real part	Mass
Imaginary part	Width/2
Residues	Couplings

Physical “ $\Lambda(1405)$ ”
: superposition of two states

$\Lambda(1405)$



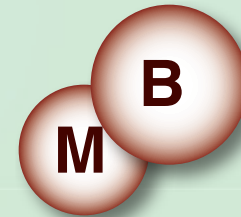
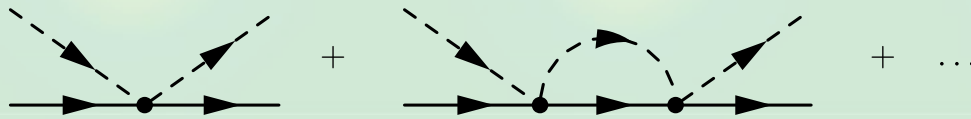
D. Jido, J.A. Oller, E. Oset, A. Ramos, U.G. Meissner, Nucl. Phys. A 723, 205 (2003);
T. Hyodo, W. Weise, Phys. Rev. C 77, 035204 (2008)

Dynamical state and CDD pole

Resonances in two-body scattering

- Knowledge of interaction (potential)
- Experimental data (cross section, phase shift,...)

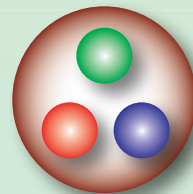
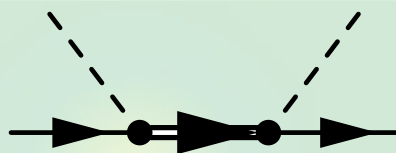
(a) dynamical state: molecule, quasi-bound, ...



... in the present case : meson-baryon molecule

(b) CDD pole: elementary, independent, ...

L. Castillejo, R.H. Dalitz, F.J. Dyson, *Phys. Rev.* 101, 453 (1956)



... in the present case : three-quark state

Resonances in chiral dynamics \rightarrow **(a) dynamical?**

CDD pole contribution in chiral unitary approach

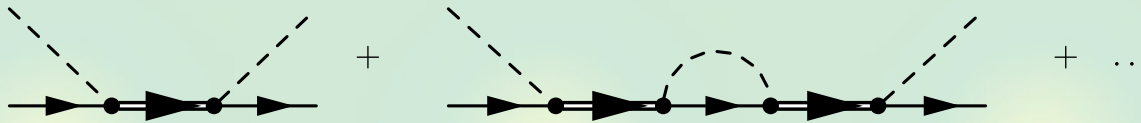
Amplitude in chiral unitary model

$$T = \frac{1}{\boxed{V^{-1}} - \boxed{G}}$$

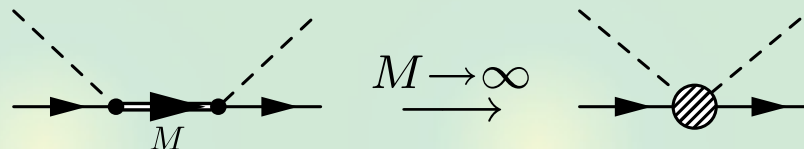
V : interaction kernel (potential)
G : loop integral (Green's function)

Known CDD pole contribution

(1) Explicit resonance field in **V**



(2) Contracted resonance propagator in **V**



Is that all? subtraction constant?

Subtraction constant

Phenomenological (standard) scheme

--> V is given, “ a ” is determined by data

$$T = \frac{1}{(V^{(1)})^{-1} - \underline{G(a)}}$$

leading order

$$T = \frac{1}{(V^{(1)} + V^{(2)})^{-1} - \underline{G(a')}}$$

next to leading order



“ a ” represents the effect which is not included in V .

CDD pole contribution in G ?

Natural renormalization scheme

--> fix “ a ” first, then determine V

exclude CDD pole contribution from G ,
based on theoretical argument.

Two renormalization schemes

Phenomenological scheme

V is given by ChPT (for instance, leading order term),
fit cutoff in G to data

Natural renormalization scheme

determine G to exclude CDD pole contribution,
 V is to be determined

Same physics (scattering amplitude T)

$$T = \frac{1}{V_{\text{ChPT}}^{-1} - G(a_{\text{pheno}})} = \frac{1}{(V_{\text{natural}})^{-1} - G(a_{\text{natural}})}$$

↑ Effective interaction
Origin of the resonance

Pole in the effective interaction

Leading order V : Weinberg-Tomozawa term

$$V_{\text{WT}} = -\frac{C}{2f^2}(\sqrt{s} - M_T) \quad \mathbf{C/f^2 : coupling constant}$$

no s-wave resonance

$$T^{-1} = V_{\text{WT}}^{-1} - G(a_{\text{pheno}}) = (V_{\text{natural}})^{-1} - G(a_{\text{natural}})$$

↑ ChPT

↑ data fit

↑ given

Effective interaction in natural scheme

$$V_{\text{natural}} = -\frac{C}{2f^2}(\sqrt{s} - M_T) + \frac{C}{2f^2} \frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\text{eff}}} \quad \mathbf{pole!}$$

$$M_{\text{eff}} = M_T - \frac{16\pi^2 f^2}{CM_T \Delta a}, \quad \Delta a = a_{\text{pheno}} - a_{\text{natural}}$$

Physically meaningful pole : $C > 0, \quad \Delta a < 0$

There is always a pole for $a_{\text{pheno}} \neq a_{\text{natural}}$

--> energy scale of the effective pole is relevant.

Comparison of pole positions

Pole of the full amplitude : physical state ▲

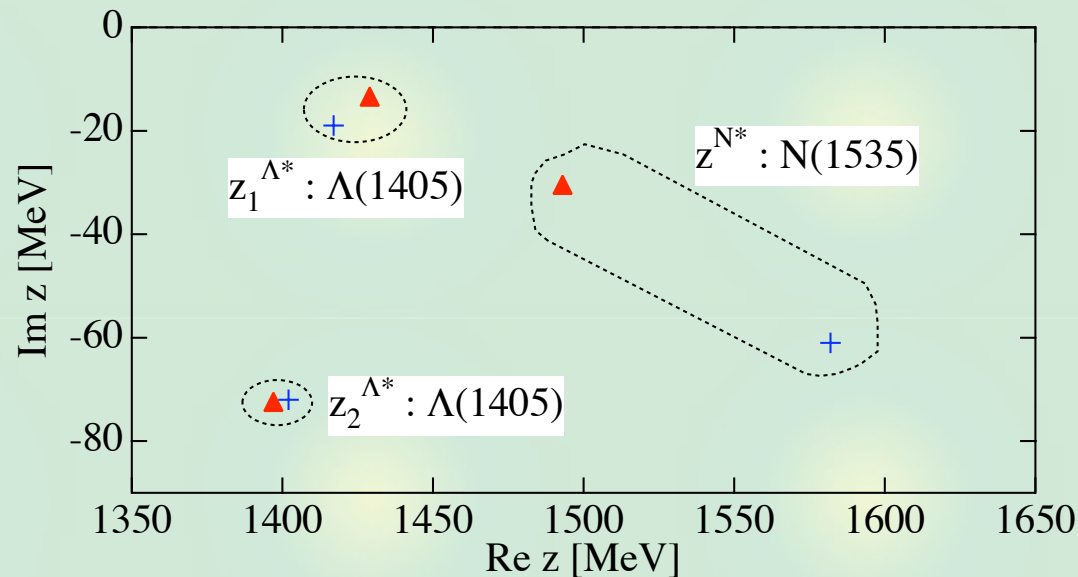
$$z_1^{\Lambda^*} = 1429 - 14i \text{ MeV}, \quad z_2^{\Lambda^*} = 1397 - 31i \text{ MeV} \quad \text{two poles for } \Lambda(1405)$$

$$z^{N^*} = 1493 - 31i \text{ MeV}$$

Pole of the V_{WT} + natural : pure dynamical +

$$z_1^{\Lambda^*} = 1417 - 19i \text{ MeV}, \quad z_2^{\Lambda^*} = 1402 - 72i \text{ MeV}$$

$$z^{N^*} = 1582 - 61i \text{ MeV}$$



==> $\Lambda(1405)$ is mostly dynamical state

Pole in the effective interaction

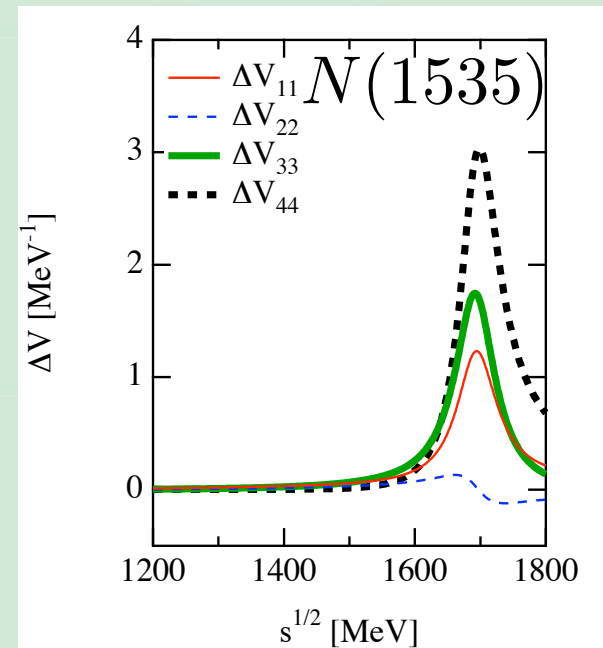
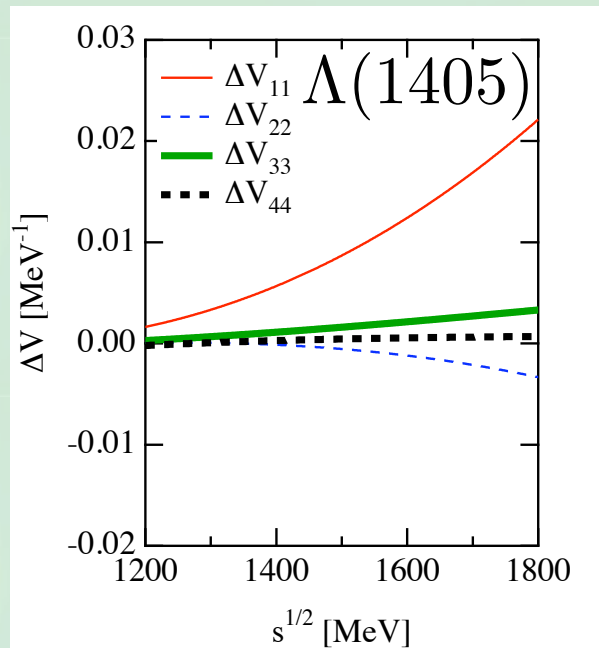
$$T^{-1} = V_{\text{WT}}^{-1} - G(a_{\text{pheno}}) = \boxed{V_{\text{natural}}}^{-1} - G(a_{\text{natural}})$$

Pole of the effective interaction (M_{eff}) : pure **CDD pole**

$$z_{\text{eff}}^{\Lambda^*} \sim 7.9 \text{ GeV} \quad \text{irrelevant!}$$

$$z_{\text{eff}}^{N^*} = 1693 \pm 37i \text{ MeV} \quad \text{relevant?}$$

Difference of interactions $\Delta V \equiv V_{\text{natural}} - V_{\text{WT}}$



==> Important **CDD pole contribution in N(1535)**

Nc scaling in the model

Nc : number of color in QCD

Hadron effective theory / quark structure

The Nc behavior is known from the general argument.

← introducing Nc dependence in the model,
analyze the resonance properties with respect to Nc

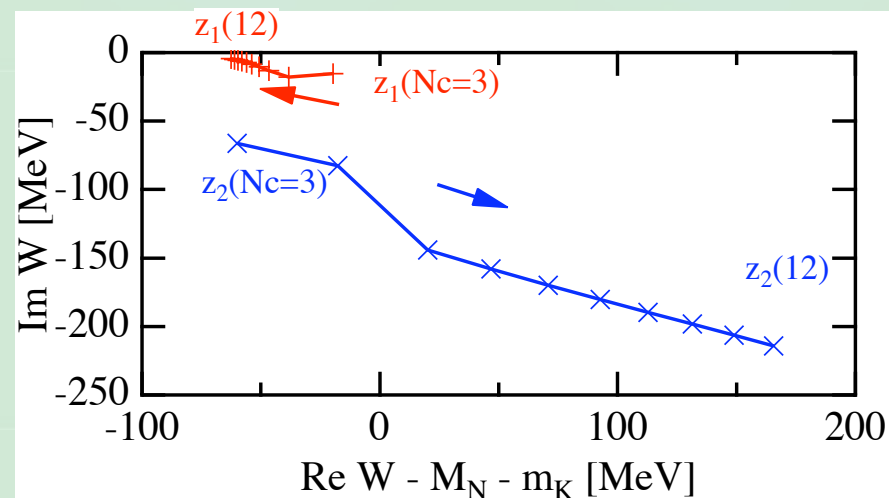
J.R. Pelaez, *Phys. Rev. Lett.* **92**, 102001 (2004)

**Nc scaling of (excited)
qqq baryon**

$$M_R \sim \mathcal{O}(N_c), \quad \Gamma_R \sim \mathcal{O}(1)$$

Result : $\Gamma_R \neq \mathcal{O}(1)$

~ non-qqq (i.e. dynamical) structure



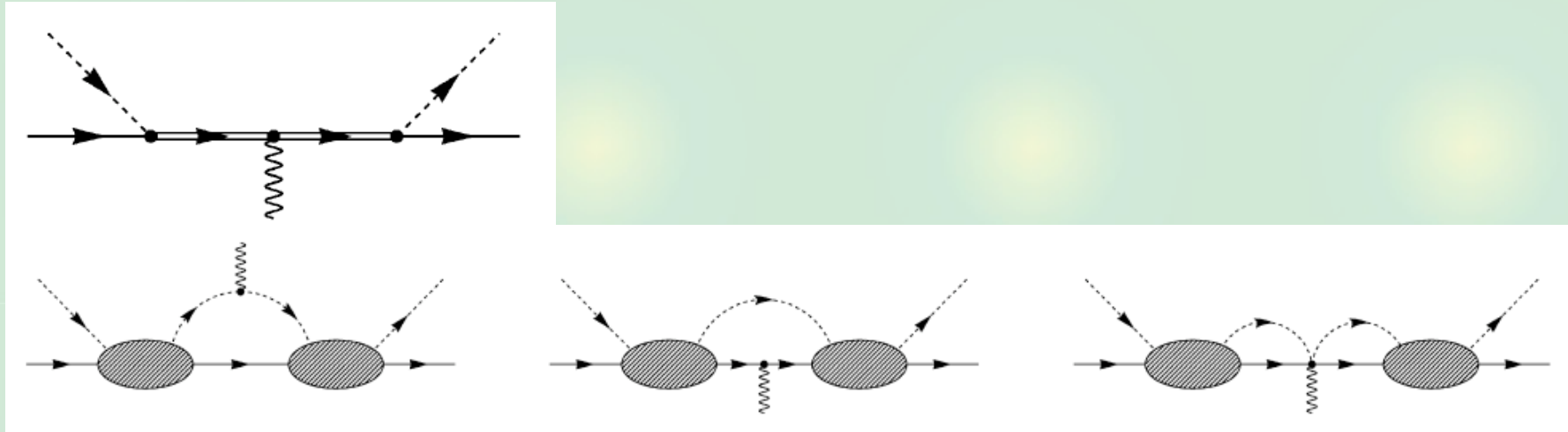
T. Hyodo, D. Jido, L. Roca, *Phys. Rev. D* **77**, 056010 (2008).

L. Roca, T. Hyodo, D. Jido, *Nucl. Phys. A* **809**, 65-87 (2008).

Electromagnetic properties

Attaching photon to resonance

--> em properties : rms, form factors,...



result of mean squared radii :


$$|\langle r^2 \rangle_E| = 0.33 \text{ [fm}^2\text{]}$$

large (em) size of the $\Lambda(1405)$: c.f. $-0.12 \text{ [fm}^2\text{]}$ for neutron


--> meson-baryon picture

Summary : Chiral dynamics

Framework of chiral coupled-channel approach is reviewed.

 Interaction given by **chiral symmetry** + coupled-channel **unitarity condition**

=> successful description of meson -baryon scattering and resonances.

 On top of the successful reproduction of scattering data, the **internal structure** of resonances can be investigated in **several ways.**

Summary : Structure of $\Lambda(1405)$

The structure of the $\Lambda(1405)$ is:



Dynamical or CDD?

=> dominance of the MB components



Analysis of N_c scaling

=> non-qqq structure







Electromagnetic properties

=> large e.m. size

Summary : Structure of $\Lambda(1405)$

The structure of the $\Lambda(1405)$ is:

-  Dynamical or CDD?
=> dominance of the MB components
-  Analysis of N_c scaling
=> non-qqq structure
-  Electromagnetic properties
=> large e.m. size
-  Independent analyses consistently support the **meson-baryon molecule picture** of the $\Lambda(1405)$

