低エネルギーQCDとカイラル有効理論
– 計算ノート –

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注意！

間違いがあるかもしれないので、ご利用は自己責任で。

計算に必要な知識

$$
\begin{aligned}
(\gamma_5)^2 =& 1 \\
\bar{q} =& (q)^\dagger \gamma_0 \\
(\gamma_5)^\dagger =& \gamma_5 \\
\{\gamma_\mu, \gamma_5\} =& 0 \\
\boldsymbol{t} =& \frac{\boldsymbol{\tau}}{2} \\
\tau_a \tau_b =& \delta_{ab} + i\epsilon_{abc}\tau_c
\end{aligned}
$$

$$
\mathrm{tr}\ (t_a t_b) = \frac{1}{2}\delta_{ab}
$$

ここで $\boldsymbol{t}$ は SU(2) の生成子で最後の式のように規格化されている。$\boldsymbol{\tau}$ は通常のパウリ行列。

page 11：射影演算子の性質

$$
\begin{aligned}
P_L + P_R =& \frac{1}{2}(1-\gamma_5) + \frac{1}{2}(1+\gamma_5) \\
=& 1 \\
P_L^2 =& \frac{1}{2}(1-\gamma_5) \times \frac{1}{2}(1-\gamma_5) \\
=& \frac{1}{4}(1-\gamma_5-\gamma_5+(\gamma_5)^2) \\
=& \frac{1}{4}(2-2\gamma_5) \\
=& \frac{1}{2}(1-\gamma_5) \\
=& P_L \\
P_R^2 =& \frac{1}{2}(1+\gamma_5) \times \frac{1}{2}(1+\gamma_5) \\
=& \frac{1}{4}(1+\gamma_5+\gamma_5+(\gamma_5)^2) \\
=& \frac{1}{4}(2+2\gamma_5) \\
=& \frac{1}{2}(1+\gamma_5) \\
=& P_R \\
P_L P_R =& \frac{1}{2}(1-\gamma_5) \times \frac{1}{2}(1+\gamma_5) \\
=& \frac{1}{4}(1-\gamma_5+\gamma_5-(\gamma_5)^2) \\
=& 0 \\
P_R P_L =& \frac{1}{2}(1+\gamma_5) \times \frac{1}{2}(1-\gamma_5) \\
=& \frac{1}{4}(1+\gamma_5-\gamma_5-(\gamma_5)^2) \\
=& 0
\end{aligned}
$$

page 12：反フェルミオン

$$
\begin{aligned}
\bar{q}_L =& (P_L q)^\dagger \gamma_0 \\
=& q^\dagger P_L^\dagger \gamma_0 \\
=& q^\dagger \left(\frac{1+\gamma_5}{2}\right)^\dagger \gamma_0 \\
=& q^\dagger \left(\frac{1+\gamma_5}{2}\right) \gamma_0
\end{aligned}
$$

$$
\begin{aligned}
&= q^\dagger \gamma_0 \left(\frac{1-\gamma_5}{2}\right) \\
&= \bar{q} P_R
\end{aligned}
$$

page 12：カイラリティ、γ_5 の固有値

$$
\begin{aligned}
\gamma_5 q_L =& \gamma_5 \frac{1}{2}(1-\gamma_5)q \\
=& \frac{1}{2}(\gamma_5 - 1)q \\
=& -\frac{1}{2}(1-\gamma_5)q \\
=& -q_L \\
\gamma_5 q_R =& \gamma_5 \frac{1}{2}(1+\gamma_5)q \\
=& \frac{1}{2}(\gamma_5 + 1)q \\
=& q_R
\end{aligned}
$$

page 12：ラグランジアン

$$
\begin{aligned}
\mathcal{L}_{\mathrm{kin}} =& \bar{q}\partial\!\!\!/ q \\
=& (\bar{q}_R + \bar{q}_L)\partial\!\!\!/(q_L + q_R) \\
=& \bar{q}_R \partial\!\!\!/ q_L + \bar{q}_R \partial\!\!\!/ q_R + \bar{q}_L \partial\!\!\!/ q_L + \bar{q}_L \partial\!\!\!/ q_R \\
=& \bar{q} P_L \partial\!\!\!/ P_L q + \bar{q}_R \partial\!\!\!/ q_R + \bar{q}_L \partial\!\!\!/ q_L + \bar{q} P_R \partial\!\!\!/ P_R q \\
=& \bar{q} P_L P_R \partial\!\!\!/ q + \bar{q}_R \partial\!\!\!/ q_R + \bar{q}_L \partial\!\!\!/ q_L + \bar{q} P_R P_L \partial\!\!\!/ q \\
=& \bar{q}_L \partial\!\!\!/ q_L + \bar{q}_R \partial\!\!\!/ q_R \\
\mathcal{L}_{\mathrm{mass}} =& \bar{q} m q \\
=& (\bar{q}_R + \bar{q}_L) m (q_L + q_R) \\
=& \bar{q}_R m q_L + \bar{q}_R m q_R + \bar{q}_L m q_L + \bar{q}_L m q_R \\
=& \bar{q}_R m q_L + \bar{q} P_L m P_R q + \bar{q} P_R m P_L q + \bar{q}_L m q_R \\
=& \bar{q}_R m q_L + \bar{q}_L m q_R
\end{aligned}
$$

page 14：極性、軸性変換の右、左成分

$$
\begin{aligned}
q_R + q_L \to & e^{i\theta_V}(q_R + q_L) \\
= & e^{i\theta_V} q_R + e^{i\theta_V} q_L \\
q_R + q_L \to & e^{i\gamma_5\theta_A}(q_R + q_L) \\
= & e^{i\gamma_5\theta_V} q_R + e^{i\gamma_5\theta_V} q_L \\
= & e^{i\theta_V} q_R + e^{-i\theta_V} q_L
\end{aligned}
$$

よって

$$
\begin{aligned}
g_V = & (e^{i\theta_V}, e^{i\theta_V}) \\
g_A = & (e^{i\theta_A}, e^{-i\theta_A})
\end{aligned}
$$

page 15：極性、軸性変換とラグランジアン

反フェルミオン場は、極性変換の下で

$$
\begin{aligned}
\bar{q} = & (q)^\dagger \gamma_0 \\
\to & (q e^{i\theta_V})^\dagger \gamma_0 \\
= & \bar{q} e^{-i\theta_V}
\end{aligned}
$$

となるので（変換パラメーター θ_V は実)、ラグランジアンは極性変換の下で

$$
\begin{aligned}
\mathcal{L} = & \bar{q}\partial\!\!\!/ q + \bar{q} m q \\
\to & \bar{q} e^{-i\theta_V} \partial\!\!\!/ e^{i\theta_V} q + \bar{q} e^{-i\theta_V} m e^{i\theta_V} q \\
= & \bar{q}\partial\!\!\!/ q + \bar{q} m q \\
= & \mathcal{L}
\end{aligned}
$$

と不変である。反フェルミオン場の軸性変換は

$$
\begin{aligned}
\bar{q} = & (q)^\dagger \gamma_0 \\
\to & (q)^\dagger e^{-i\gamma_5\theta_A} \gamma_0 \\
= & (q)^\dagger \gamma_0 e^{+i\gamma_5\theta_A} \\
= & \bar{q} e^{+i\gamma_5\theta_A}
\end{aligned}
$$

であるので、ラグランジアンは

$$
\begin{aligned}
\mathcal{L} = & \bar{q}\partial\!\!\!/ q + \bar{q} m q \\
\to & \bar{q} e^{i\gamma_5\theta_A} \partial\!\!\!/ e^{i\gamma_5\theta_A} q + \bar{q} e^{i\gamma_5\theta_A} m e^{i\gamma_5\theta_A} q \\
= & \bar{q}\partial\!\!\!/ e^{-i\gamma_5\theta_A} e^{i\gamma_5\theta_A} q + \bar{q} e^{2i\gamma_5\theta_A} m q \\
= & \bar{q}\partial\!\!\!/ q + \bar{q} e^{2i\gamma_5\theta_A} m q \\
\neq & \mathcal{L}
\end{aligned}
$$

となる。つまり質量項がある場合は不変でない。

page 18：右、左生成子の交換関係

$$
\begin{aligned}
[t_R^a, t_R^b] =&([t^a, t^b], 0)\\
=&(if^{ab}{}_c t^c, 0)\\
=&if^{ab}{}_c (t^c, 0)\\
=&if^{ab}{}_c t_R^c\\
[t_L^a, t_L^b] =&(0, [t^a, t^b])\\
=&(0, if^{ab}{}_c t^c)\\
=&if^{ab}{}_c (0, t^c)\\
=&if^{ab}{}_c t_L^c\\
[t_R^a, t_L^b] =&(0, 0)\\
=&0
\end{aligned}
$$

page 18：極性、軸性生成子の交換関係

$$
\begin{aligned}
[t_V^a, t_V^b] =&([t^a, t^b], [t^a, t^b])\\
=&(if^{ab}{}_c t^c, if^{ab}{}_c t^c)\\
=&if^{ab}{}_c (t^c, t^c)\\
=&if^{ab}{}_c t_V^c\\
[t_A^a, t_A^b] =&([t^a, t^b], [-t^a, -t^b])\\
=&(if^{ab}{}_c t^c, if^{ab}{}_c t^c)\\
=&if^{ab}{}_c (t^c, t^c)\\
=&if^{ab}{}_c t_V^c\\
[t_A^a, t_V^b] =&([t^a, t^b], [-t^a, t^b])\\
=&(if^{ab}{}_c t^c, -if^{ab}{}_c t^c)\\
=&if^{ab}{}_c (t^c, -t^c)\\
=&if^{ab}{}_c t_A^c
\end{aligned}
$$

page 28：線形シグマ模型の場のカイラル変換

$\sigma \sim \bar{q}q$、$\boldsymbol{\pi} \sim \bar{q}i\boldsymbol{\tau}\gamma_5 q$。極性変換の下で

$$
\begin{aligned}
\bar{q}q \to& \bar{q}(1 - i\boldsymbol{\theta}^V \cdot \boldsymbol{\tau}/2)(1 + i\boldsymbol{\theta}^V \cdot \boldsymbol{\tau}/2)q + \mathcal{O}(\theta^2) \\
=& \bar{q}q + \mathcal{O}(\theta^2) \\
\bar{q}i\tau_a\gamma_5 q \to& \bar{q}(1 - i\theta_b^V \tau_b/2)i\tau_a\gamma_5(1 + i\theta_b^V \tau_b/2)q + \mathcal{O}(\theta^2) \\
=& \bar{q}i\tau_a\gamma_5 q + \bar{q}(-i)\theta_b^V(\tau_b/2)i\tau_a\gamma_5 q + \bar{q}i\tau_a\gamma_5 i\theta_b^V(\tau_b/2)q + \mathcal{O}(\theta^2) \\
=& \bar{q}i\tau_a\gamma_5 q + \frac{\theta_b^V}{2}[\bar{q}\tau_b\tau_a\gamma_5 q - \bar{q}\tau_a\tau_b\gamma_5 q] + \mathcal{O}(\theta^2) \\
=& \bar{q}i\tau_a\gamma_5 q + \frac{\theta_b^V}{2}[\bar{q}(\delta_{ba} + i\epsilon_{bac}\tau_c)\gamma_5 q - \bar{q}(\delta_{ab} + i\epsilon_{abc}\tau_c)\gamma_5 q] + \mathcal{O}(\theta^2) \\
=& \bar{q}i\tau_a\gamma_5 q + \frac{\theta_b^V}{2}[\bar{q}i(-\epsilon_{abc})\tau_c\gamma_5 q - \bar{q}i\epsilon_{abc}\tau_c\gamma_5 q] + \mathcal{O}(\theta^2) \\
=& \bar{q}i\tau_a\gamma_5 q - \epsilon_{abc}\theta_b^V\bar{q}i\tau_c\gamma_5 q + \mathcal{O}(\theta^2)
\end{aligned}
$$

よって無限少変換（$\theta_{V,A}$ の１次）では

$$
\begin{aligned}
\sigma \to& \sigma \\
\boldsymbol{\pi} \to& \boldsymbol{\pi} - \boldsymbol{\theta}^V \times \boldsymbol{\pi}
\end{aligned}
$$

軸性変換の下で

$$
\begin{aligned}
\bar{q}q \to& \bar{q}(1 + i\gamma_5\boldsymbol{\theta}^A \cdot \boldsymbol{\tau}/2)(1 + i\gamma_5\boldsymbol{\theta}^A \cdot \boldsymbol{\tau}/2)q + \mathcal{O}(\theta^2) \\
=& \bar{q}q + \boldsymbol{\theta}^A \cdot [\bar{q}i\gamma_5\boldsymbol{\tau}q] + \mathcal{O}(\theta^2) \\
\bar{q}i\tau_a\gamma_5 q \to& \bar{q}(1 + i\gamma_5\theta_b^A\tau_b/2)i\tau_a\gamma_5(1 + i\gamma_5\theta_b^A\tau_b/2)q + \mathcal{O}(\theta^2) \\
=& \bar{q}i\tau_a\gamma_5 q + \frac{\theta_b^V}{2}[-\bar{q}\tau_b\tau_a\gamma_5\gamma_5 q - \bar{q}\tau_a\tau_b\gamma_5\gamma_5 q] + \mathcal{O}(\theta^2) \\
=& \bar{q}i\tau_a\gamma_5 q - \frac{\theta_b^V}{2}[\bar{q}(\delta_{ba} + i\epsilon_{bac}\tau_c)q + \bar{q}(\delta_{ab} + i\epsilon_{abc}\tau_c)q] + \mathcal{O}(\theta^2) \\
=& \bar{q}i\tau_a\gamma_5 q - \frac{\theta_a^V}{2}[\bar{q}\gamma_5 q] - \frac{\theta_b^V}{2}[\bar{q}i(-\epsilon_{abc})\tau_c\gamma_5 q + \bar{q}i\epsilon_{abc}\tau_c q] + \mathcal{O}(\theta^2) \\
=& \bar{q}i\tau_a q - \frac{\theta_a^V}{2}[\bar{q}q]
\end{aligned}
$$

よって無限少変換では

$$
\begin{aligned}
\sigma \to& \sigma + \boldsymbol{\theta}^A \cdot \boldsymbol{\pi} \\
\boldsymbol{\pi} \to& \boldsymbol{\pi} - \boldsymbol{\theta}^A \sigma
\end{aligned}
$$

page 29：カイラル不変量

極性変換の下で

$$
\begin{aligned}
\sigma^2 + \boldsymbol{\pi}\cdot\boldsymbol{\pi} \to &\sigma^2 + (\boldsymbol{\pi} - \boldsymbol{\theta}^V \times \boldsymbol{\pi})\cdot(\boldsymbol{\pi} - \boldsymbol{\theta}^V \times \boldsymbol{\pi}) \\
=&\sigma^2 + \boldsymbol{\pi}\cdot\boldsymbol{\pi} - \boldsymbol{\pi}\cdot\boldsymbol{\theta}^V \times \boldsymbol{\pi} - \boldsymbol{\theta}^V \times \boldsymbol{\pi}\cdot\boldsymbol{\pi} + \mathcal{O}(\theta^2) \\
=&\sigma^2 + \boldsymbol{\pi}\cdot\boldsymbol{\pi} - \boldsymbol{\theta}^V \cdot(\boldsymbol{\pi}\times\boldsymbol{\pi}) - (\boldsymbol{\pi}\times\boldsymbol{\pi})\cdot\boldsymbol{\theta}^V + \mathcal{O}(\theta^2) \\
=&\sigma^2 + \boldsymbol{\pi}\cdot\boldsymbol{\pi} + \mathcal{O}(\theta^2)
\end{aligned}
$$

軸性変換の下で

$$
\begin{aligned}
\sigma^2 + \boldsymbol{\pi}\cdot\boldsymbol{\pi} \to &(\sigma + \boldsymbol{\theta}^A\cdot\boldsymbol{\pi})^2 + (\boldsymbol{\pi} - \boldsymbol{\theta}^A\sigma)\cdot(\boldsymbol{\pi} - \boldsymbol{\theta}^A\sigma) \\
=&\sigma^2 + 2\sigma\boldsymbol{\theta}^A\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{\pi} - \boldsymbol{\pi}\cdot\boldsymbol{\theta}^A\sigma - \boldsymbol{\theta}^A\sigma\cdot\boldsymbol{\pi} + \mathcal{O}(\theta^2) \\
=&\sigma^2 + \boldsymbol{\pi}\cdot\boldsymbol{\pi} + \mathcal{O}(\theta^2)
\end{aligned}
$$

page 30：線形シグマ模型の真空

$$
\begin{aligned}
\frac{\partial V(\phi)}{\partial\phi} =&\mu^2\phi + \frac{\lambda}{6}\phi^3 \\
=&\phi(\mu^2 + \frac{\lambda}{6}\phi^2) \\
=&0
\end{aligned}
$$

これを満たすのは

$$
\begin{cases}
\phi = 0 \\
\phi = \sqrt{-\frac{6\mu^2}{\lambda}}
\end{cases}
$$

page 31：自発的に対称性の破れた線形シグマ模型（スライドの「場の一次は落とす」は間違い）

$\sigma = \tilde{\sigma} + \langle\sigma\rangle$ と書きなおして整理：

$$
\begin{aligned}
\mathcal{L}_{\mathrm{kin}} =&\frac{1}{2}\partial_\mu(\tilde{\sigma} + \langle\sigma\rangle)\partial^\mu(\tilde{\sigma} + \langle\sigma\rangle) + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} \\
=&\frac{1}{2}\partial_\mu\tilde{\sigma}\partial^\mu\tilde{\sigma} + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} \\
\mathcal{L}_{\mathrm{mass}} =&-\frac{\mu^2}{2}[(\tilde{\sigma} + \langle\sigma\rangle)(\tilde{\sigma} + \langle\sigma\rangle) + \boldsymbol{\pi}\cdot\boldsymbol{\pi}] \\
=&-\frac{\mu^2}{2}\tilde{\sigma}^2 - \frac{\mu^2}{2}\boldsymbol{\pi}\cdot\boldsymbol{\pi} - \mu^2\langle\sigma\rangle\tilde{\sigma} + (\mathrm{const}) \\
\mathcal{L}_{\mathrm{int}} =&-\frac{\lambda}{4!}[(\tilde{\sigma} + \langle\sigma\rangle)(\tilde{\sigma} + \langle\sigma\rangle) + \boldsymbol{\pi}\cdot\boldsymbol{\pi}]^2
\end{aligned}
$$

$$
\begin{aligned}
=&-\frac{\lambda}{4!}[\tilde{\sigma}^2+2\langle\sigma\rangle\tilde{\sigma}+\langle\sigma\rangle^2+\boldsymbol{\pi}\cdot\boldsymbol{\pi}]^2\\
=&-\frac{\lambda}{4!}[\tilde{\sigma}^4+4\langle\sigma\rangle^2\tilde{\sigma}^2+\langle\sigma\rangle^4+(\boldsymbol{\pi}\cdot\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\boldsymbol{\pi})\\
&+4\tilde{\sigma}^2\langle\sigma\rangle\tilde{\sigma}+2\tilde{\sigma}^2\langle\sigma\rangle^2+2\tilde{\sigma}^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}\\
&+4\langle\sigma\rangle\tilde{\sigma}\langle\sigma\rangle^2+4\langle\sigma\rangle\tilde{\sigma}\boldsymbol{\pi}\cdot\boldsymbol{\pi}+2\langle\sigma\rangle^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}]\\
=&-\frac{\lambda}{4!}[4\langle\sigma\rangle^3\tilde{\sigma}+6\langle\sigma\rangle^2\tilde{\sigma}^2+2\langle\sigma\rangle^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}\\
&+4\langle\sigma\rangle\tilde{\sigma}^3+\tilde{\sigma}^4+4\langle\sigma\rangle\tilde{\sigma}\boldsymbol{\pi}\cdot\boldsymbol{\pi}\\
&+2\tilde{\sigma}^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}+(\boldsymbol{\pi}\cdot\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\boldsymbol{\pi})]+(\mathrm{const})\\
=&-\frac{\lambda}{6}\langle\sigma\rangle^3\tilde{\sigma}-\frac{\lambda}{4}\langle\sigma\rangle^2\tilde{\sigma}^2-\frac{\lambda}{12}\langle\sigma\rangle^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}\\
&-\frac{\lambda}{6}\langle\sigma\rangle\tilde{\sigma}^3-\frac{\lambda}{24}\tilde{\sigma}^4-\frac{\lambda}{6}\langle\sigma\rangle\tilde{\sigma}\boldsymbol{\pi}\cdot\boldsymbol{\pi}\\
&-\frac{\lambda}{12}\tilde{\sigma}^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}-\frac{\lambda}{24}(\boldsymbol{\pi}\cdot\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\boldsymbol{\pi})+(\mathrm{const})
\end{aligned}
$$

全て加えると

$$
\begin{aligned}
\mathcal{L}=&\frac{1}{2}\partial_\mu\tilde{\sigma}\partial^\mu\tilde{\sigma}+\frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}-\frac{\mu^2}{2}\tilde{\sigma}^2-\frac{\mu^2}{2}\boldsymbol{\pi}\cdot\boldsymbol{\pi}-\mu^2\langle\sigma\rangle\tilde{\sigma}\\
&-\frac{\lambda}{6}\langle\sigma\rangle^3\tilde{\sigma}-\frac{\lambda}{4}\langle\sigma\rangle^2\tilde{\sigma}^2-\frac{\lambda}{12}\langle\sigma\rangle^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}\\
&-\frac{\lambda}{6}\langle\sigma\rangle\tilde{\sigma}^3-\frac{\lambda}{24}\tilde{\sigma}^4-\frac{\lambda}{6}\langle\sigma\rangle\tilde{\sigma}\boldsymbol{\pi}\cdot\boldsymbol{\pi}\\
&-\frac{\lambda}{12}\tilde{\sigma}^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}-\frac{\lambda}{24}(\boldsymbol{\pi}\cdot\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\boldsymbol{\pi})+(\mathrm{const})\\
=&\frac{1}{2}\partial_\mu\tilde{\sigma}\partial^\mu\tilde{\sigma}+\frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}-\left(\frac{\mu^2}{2}+\frac{\lambda}{4}\langle\sigma\rangle^2\right)\tilde{\sigma}^2\\
&-\left(\mu^2+\frac{\lambda}{6}\langle\sigma\rangle^2\right)\boldsymbol{\pi}\cdot\boldsymbol{\pi}-\left(\mu^2+\frac{\lambda}{6}\langle\sigma\rangle^2\right)\langle\sigma\rangle\tilde{\sigma}\\
&-\frac{\lambda}{6}\langle\sigma\rangle\tilde{\sigma}^3-\frac{\lambda}{24}\tilde{\sigma}^4-\frac{\lambda}{6}\langle\sigma\rangle\tilde{\sigma}\boldsymbol{\pi}\cdot\boldsymbol{\pi}\\
&-\frac{\lambda}{12}\tilde{\sigma}^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}-\frac{\lambda}{24}(\boldsymbol{\pi}\cdot\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\boldsymbol{\pi})+(\mathrm{const})
\end{aligned}
$$

ここで $\langle\sigma\rangle=\sqrt{-6\mu^2/\lambda}$ を代入すると

$$
\begin{aligned}
\mathcal{L}=&\frac{1}{2}\partial_\mu\tilde{\sigma}\partial^\mu\tilde{\sigma}+\frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}-\left(\frac{\mu^2}{2}-\frac{3\mu^2}{2}\right)\tilde{\sigma}^2\\
&-\frac{\lambda}{6}\sqrt{-\frac{6\mu^2}{\lambda}}\tilde{\sigma}^3-\frac{\lambda}{24}\tilde{\sigma}^4-\frac{\lambda}{6}\sqrt{-\frac{6\mu^2}{\lambda}}\tilde{\sigma}\boldsymbol{\pi}\cdot\boldsymbol{\pi}\\
&-\frac{\lambda}{12}\tilde{\sigma}^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}-\frac{\lambda}{24}(\boldsymbol{\pi}\cdot\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\boldsymbol{\pi})+(\mathrm{const})\\
=&\frac{1}{2}\partial_\mu\tilde{\sigma}\partial^\mu\tilde{\sigma}+\frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}+\mu^2\tilde{\sigma}^2\\
&-\sqrt{-\frac{\lambda\mu^2}{6}}\tilde{\sigma}^3-\frac{\lambda}{24}\tilde{\sigma}^4-\sqrt{-\frac{\lambda\mu^2}{6}}\tilde{\sigma}\boldsymbol{\pi}\cdot\boldsymbol{\pi}
\end{aligned}
$$

$$-\frac{\lambda}{12}\tilde{\sigma}^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}-\frac{\lambda}{24}(\boldsymbol{\pi}\cdot\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\boldsymbol{\pi})+(\mathrm{const})$$

注）場の一次の項はキャンセルして消えるので、スライドの「場の一次は落とす」は間違い。一方で、ラグランジアンの定数項は物理に影響がないので落とす。

page 35：カイラルラグランジアンの運動項

$$\begin{aligned}
\mathcal{L}=&a\mathrm{Tr}\ (\partial_\mu U^\dagger\partial^\mu U)\\
=&a\mathrm{Tr}\ [\partial_\mu(1-i\sqrt{2}\pi/f+\cdots)\partial^\mu(1+i\sqrt{2}\pi/f+\cdots)]\\
=&a\frac{2}{f^2}\mathrm{Tr}\ [\partial_\mu\pi\partial^\mu\pi]\\
=&a\frac{2}{f^2}\mathrm{Tr}\ [\partial_\mu\pi_a\sqrt{2}t_a\partial^\mu\pi_b\sqrt{2}t_b]\\
=&a\frac{4}{f^2}\partial_\mu\pi_a\partial^\mu\pi_b\mathrm{Tr}\ [t_at_b]\\
=&a\frac{4}{f^2}\partial_\mu\pi_a\partial^\mu\pi_b\frac{\delta_{ab}}{2}\\
=&a\frac{2}{f^2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}
\end{aligned}$$

この係数が 1/2 になるためには

$$\begin{aligned}
a\frac{2}{f^2}=&\frac{1}{2}\\
a=&\frac{f^2}{4}
\end{aligned}$$