Softening of the dynamical sigma meson

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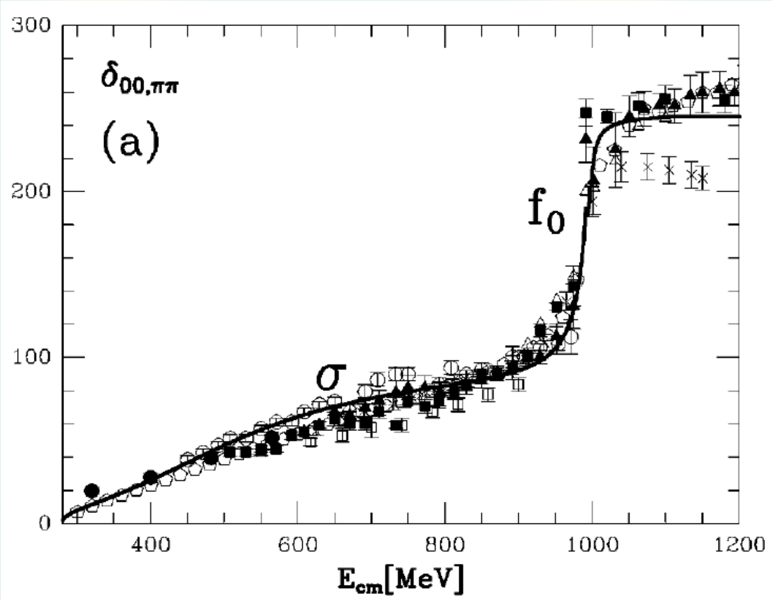
Contents

The sigma meson

$f_0(600)$ or $\sigma : J^P = 0^+, I = 0$

Mass : 400-1200 MeV
Width : 600-1000 MeV
Decay modes : $\sigma \to \pi\pi$ **dominant**
$\sigma \to \gamma\gamma$ **seen**

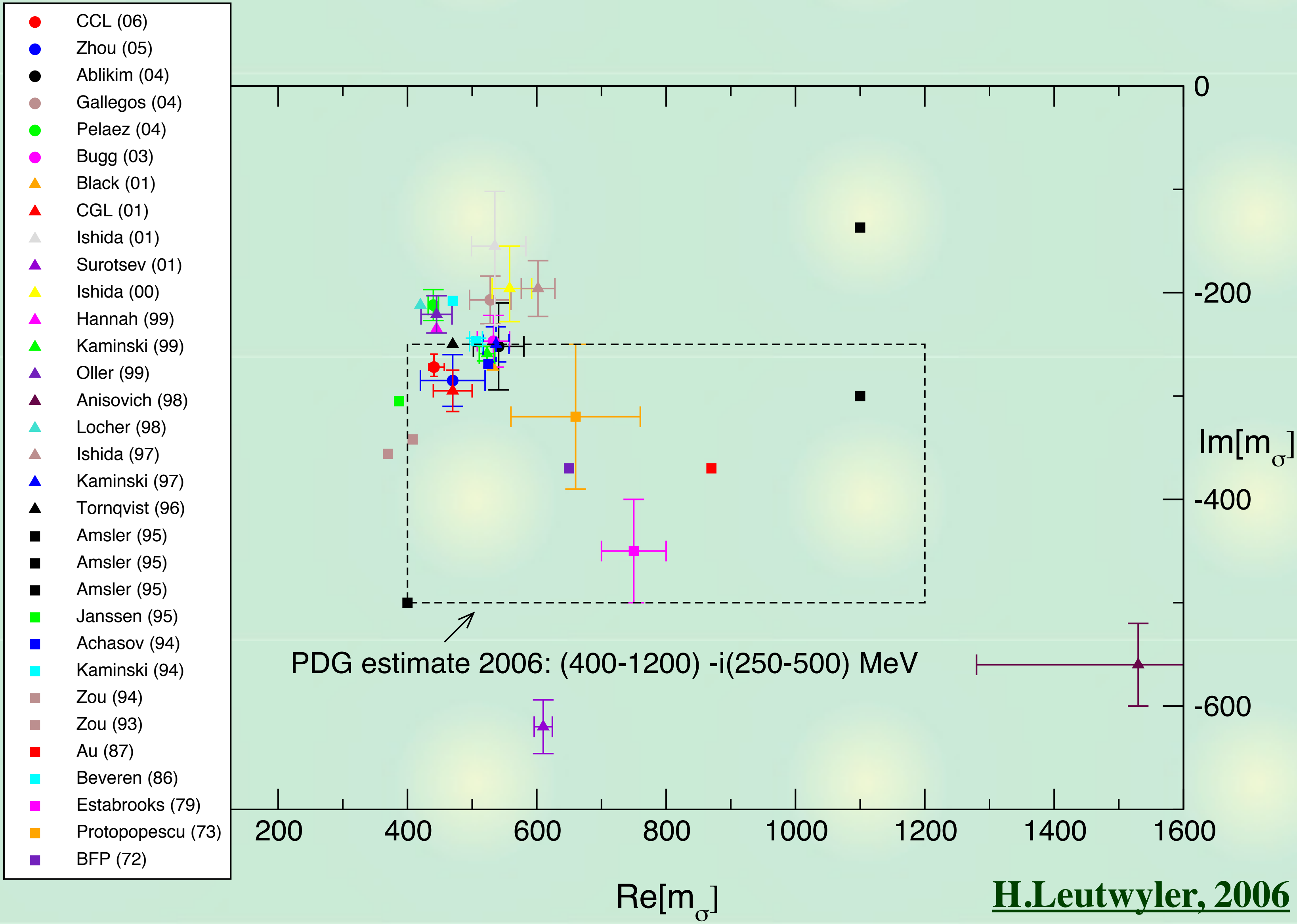


σ meson
- is the lowest resonance in QCD
- plays an important role in hadron mass generation due to spontaneous chiral symmetry breaking
- provides attraction in phenomenological nuclear force

Recent development
: precise pole position is now available.

Existence of the sigma pole

Development of scattering theory + experimental data



H.Leutwyler, 2006

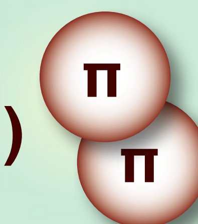
Structure of the sigma meson

Sigma meson in naive constituent quark model ($\sim\bar{q}q$) has some difficulties: light mass (v.s. p-wave excitation), mass ordering of scalar nonet (v.s. $\sigma > \kappa > f_0 \sim a_0$)

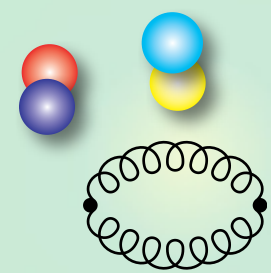
Alternative descriptions of the sigma meson

- Chiral sigma
  (e.g. linear sigma model)
  M. Gell-Mann, M. Levy, Nuovo Cim. 16, 705 (1960), ...

- Dynamical sigma
  (e.g. mesonic molecule generated by π-π attraction)
  J.A. Oller, E. Oset, J.R. Pelaez, Phys. Rev. D59, 074001 (1999), ...



- CDD pole contribution (pre-formed state)
  (e.g. constituent four-quark model, glueball, ...)
  L.R. Jaffe, Phys. Rev. D15, 267 (1977), ...



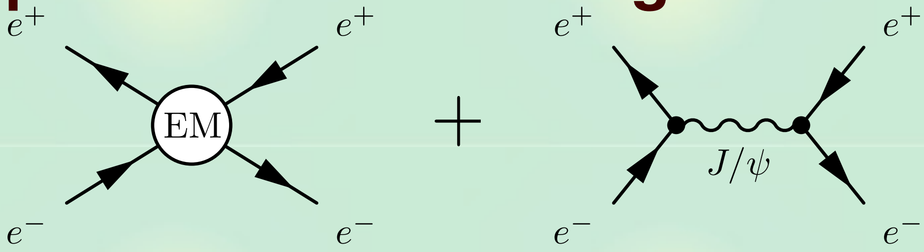
We want to clarify the structure <-- softening

The CDD pole : example

Consider resonant/bound states in e+e- scattering.
- below threshold: positronium
- above 3 GeV: J/ψ

Model (a) : single e+e- channel

potential : electromagnetic interaction (EM)

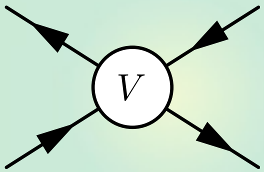


solving Schrödinger equation --> positronium : dynamical

J/ψ pole must be introduced by hand --> CDD pole

Model (b) : e+e- & $c\bar{c}$ coupled channels

potential : EM, $c\bar{c}$, transition



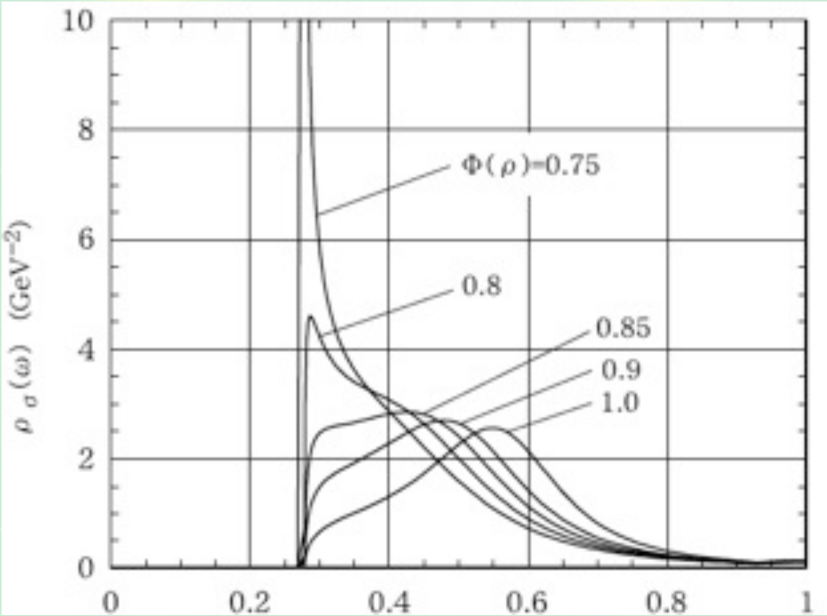
$$V = \begin{pmatrix} V_{\mathrm{EM}} & V_{e^+e^-\leftrightarrow c\bar{c}} \\ V_{e^+e^-\leftrightarrow c\bar{c}} & V_{c\bar{c}} \end{pmatrix}$$

J/ψ is generated by ladder sum --> dynamical

Softening of the sigma meson

Softening of chiral sigma

T. Hatsuda, T. Kunihiro, H. Shimizu
Phys. Rev. Lett. 82, 2840 (1999)

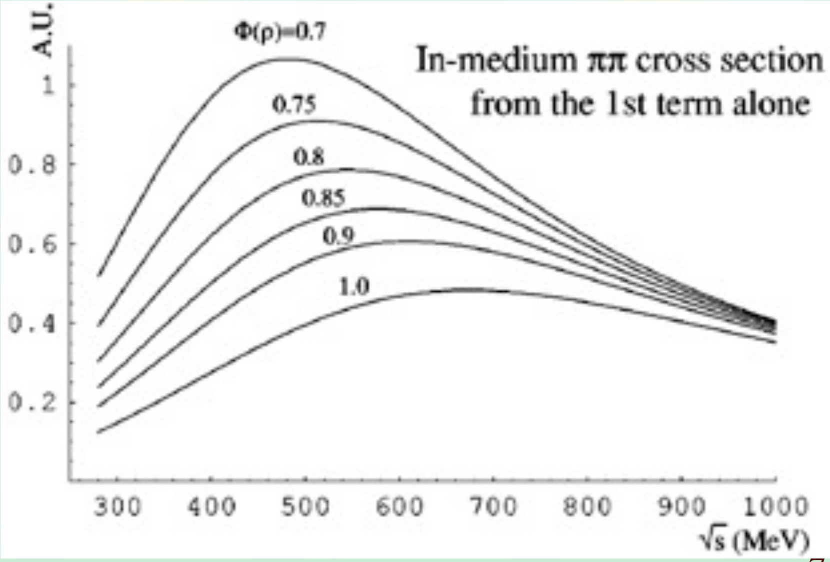


Spectral enhancement in I=J=0 channel near threshold, when the chiral sym. is partially restored.

sigma: fluctuation of the order parameter of chiral phase transition

Threshold enhancement of π-π cross section, also for the dynamical sigma meson

D. Jido, T. Hatsuda, T. Kunihiro,
Phys. Rev. D63, 011901 (2001)



Softening of the sigma meson

Systematic study up to restoration limit.

K. Yokokawa, T. Hatsuda, A. Hayashigaki, T. Kunihiro, Phys. Rev. C66, 022201 (2002)

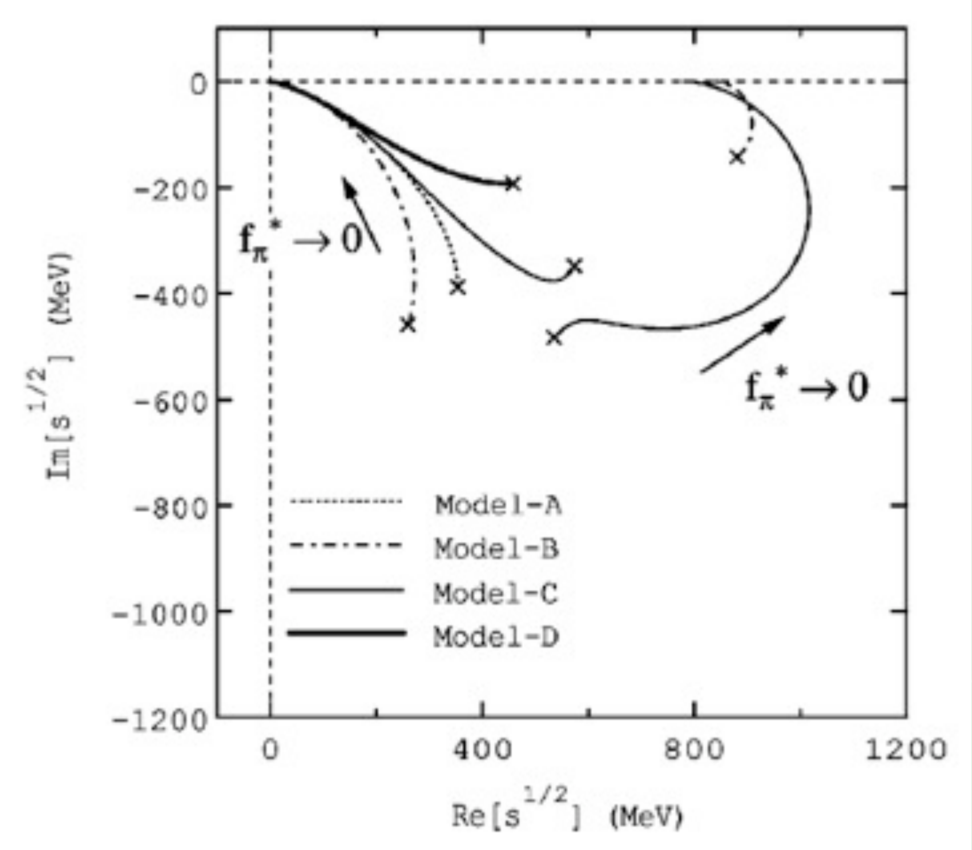
- dynamical model with chiral symmetry, unitarity, analyticity, (crossing)

  K. Igi, K. Hikasa, Phys. Rev. D59, 034005 (1999)

- 4 cases: σ pole on/off ⊗ ρ pole on/off

- roughly corresponds to dynamical sigma and/or CDD pole

- "universal softening" at $f_\pi^*/f_\pi \ll 1$



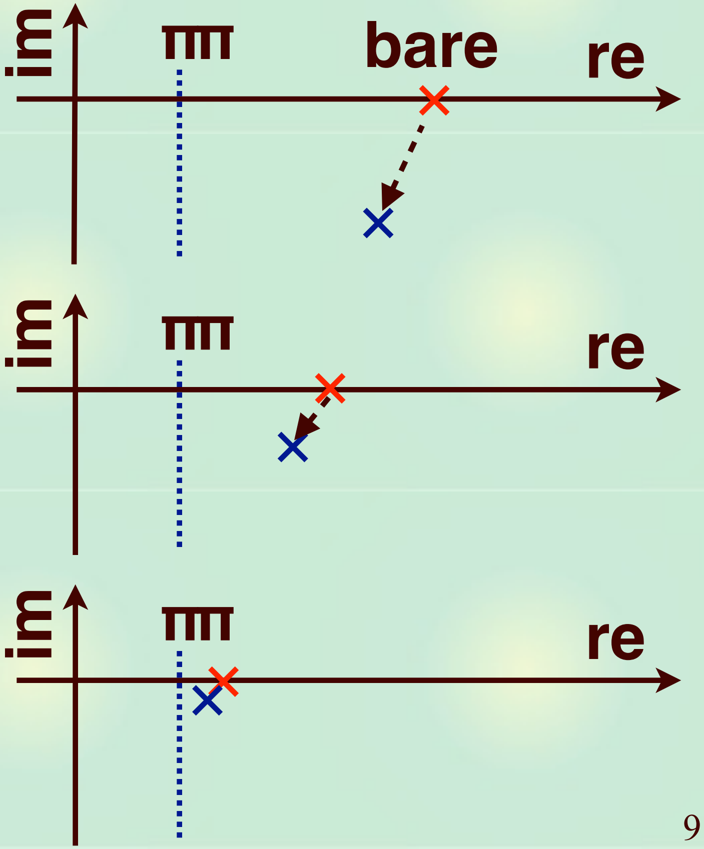
Mechanism of the softening

In the previous studies, it seems that the softening takes place, irrespective to the structure of the sigma meson.

Mechanism of the softening?

Softening of the chiral sigma (linear sigma model)

Sigma meson:
bare sigma pole acquires finite width through the coupling to π-π



Chiral symmetry restoration:
--> lowering bare sigma mass
--> reduction of the phase space
--> narrow spectrum

Mechanism of the softening

Softening of the dynamical sigma (ChPT + unitarization)

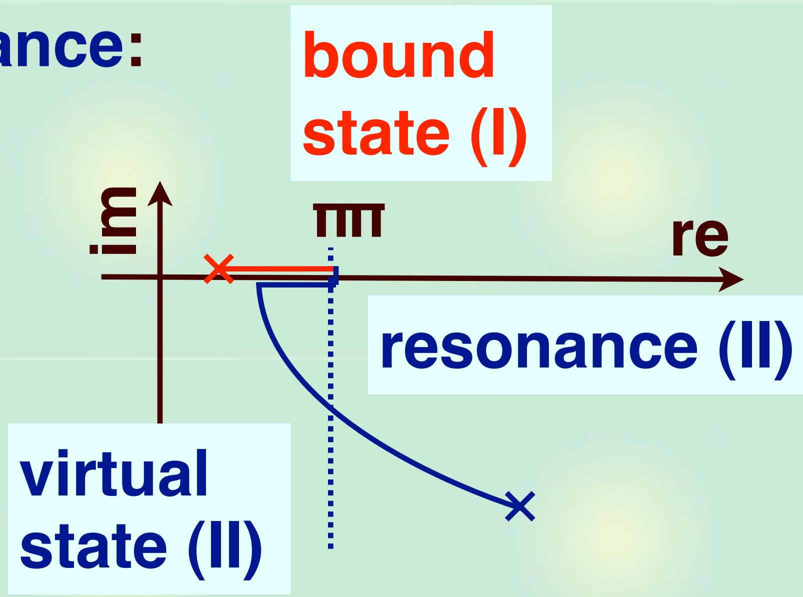
Sigma meson: dynamically generated by π-π attraction

Chiral symmetry restoration:
--> $f_\pi \sim \langle\sigma\rangle$ decreases
--> (attractive) interaction $\sim (f_\pi)^{-2}$ increases
--> resonance turns into bound state, spectrum gets narrow

Special nature of the s-wave resonance:

attractive

- resonance
- virtual state
  - pole on the 2nd Riemann sheet below threshold.
  - ex) spin singlet deuteron
- bound state



--> novel softening pattern?

The aim of this study

We want to study the structure of the "sigma meson" through the behavior in the softening phenomena.

For this, we use a schematic model of chiral dynamics.

Comparison with previous studies

	m_π	chiral restoration	sigma meson
Jido, Hatsuda, Kunihiro	finite	phi -> 0.7	chiral, dynamical
Yokokawa, et al.	0	phi -> 0	dynamical, CDD, mixture
This work	finite	phi -> 0	chiral, dynamical, CDD, mixture

It is important to keep m_π finite and to take restoration limit.

Tree level interaction

Lagrangian of 2-flavor linear sigma model

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\left[\partial M \partial M^\dagger - \mu^2 M M^\dagger - \frac{2\lambda}{4!}(MM^\dagger)^2 + h(M + M^\dagger)\right], M = \sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}$$

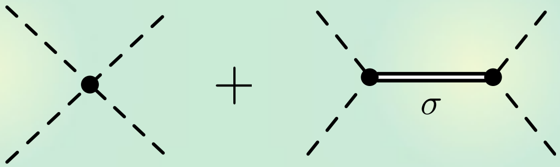
3 parameters <-- m_π, m_σ, <σ> at mean field level.

π-π scattering amplitude in general (crossing symmetry)

$$T_{\mathrm{tree}}(s,t,u) = A(s,t,u)\delta_{ab}\delta_{cd} + A(t,s,u)\delta_{ac}\delta_{bd} + A(u,t,s)\delta_{ad}\delta_{bc}$$

Tree-level π-π scattering amplitude

$$A(s,t,u) = -\frac{m_\sigma^2 - m_\pi^2}{\langle\sigma\rangle} - \frac{(m_\sigma^2 - m_\pi^2)^2}{\langle\sigma\rangle}\frac{1}{s - m_\sigma^2}$$



$$= \boxed{\frac{s - m_\pi^2}{\langle\sigma\rangle^2}} - \frac{(s - m_\pi^2)^2}{\langle\sigma\rangle^2}\frac{1}{s - m_\sigma^2}$$

leading order term of ChPT

- consistent with low energy expansion
- 1st and 2nd terms are chiral invariant

Tree level interaction

Introduce a parameter using chiral invariant decomposition

$$A(s;x) = \frac{s-m_\pi^2}{\langle\sigma\rangle^2} - \boxed{x}\frac{(s-m_\pi^2)^2}{\langle\sigma\rangle^2}\frac{1}{s-m_\sigma^2}$$

$x \to 1$ **: linear sigma model**

$x \to 0$ **: leading order term in ChPT**

$x \to 1/2$ **: model C in Yokokawa et al. (σ-ρ degeneracy, KSRF relation, and duality)**

Parameter x is useful to extrapolate models.
The origin of the resonance can be investigated (later).

Projecting the amplitude onto I=J=0, we obtain

$$T_{\rm tree}(s;x) = \frac{m_\sigma^2-m_\pi^2}{\langle\sigma\rangle^2}\left[\frac{2s-m_\pi^2}{m_\sigma^2-m_\pi^2}(1-x) - 5x \right.$$
$$\left. -3x\frac{m_\sigma^2-m_\pi^2}{s-m_\sigma^2} - 2x\frac{m_\sigma^2-m_\pi^2}{s-4m_\pi^2}\ln\left(\frac{m_\sigma^2}{m_\sigma^2+s-4m_\pi^2}\right)\right]$$

Unitarization

Unitarity of S-matrix : conservation of probability.
Tree-level amplitude violates unitarity at certain energy.

Optical theorem :

$$\mathrm{Im}\, T^{-1}(s) = -\frac{\Theta(s)}{2} \quad \text{for} \quad s > 4m_\pi^2 \qquad \Theta(s) = (16\pi)^{-1}\sqrt{1-\frac{4m_\pi^2}{s}}$$

Scattering amplitude (N/D method + matching with T_{tree})

J. A. Oller, E. Oset, Phys. Rev. D60, 074023 (1999)

$$T(s;x) = \frac{1}{T^{-1}_{\text{tree}}(s;x) + G(s)}$$

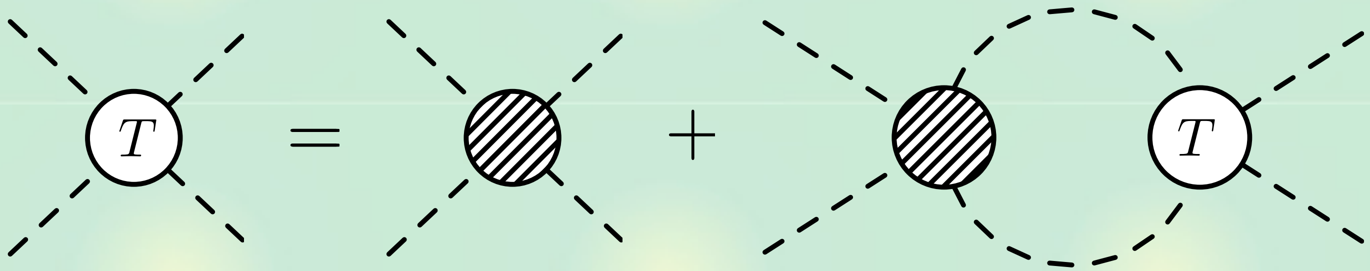
$$G(s) = \frac{1}{2}\frac{1}{(4\pi)^2}\left\{ \boxed{a(\mu)} + \ln\frac{m_\pi^2}{\mu^2} + \sqrt{1-\frac{4m_\pi^2}{s}}\left[\ln\frac{\sqrt{1-\frac{4m_\pi^2}{s}}+1}{\sqrt{1-\frac{4m_\pi^2}{s}}-1}\right]\right\}$$

- **Left hand cut (crossed diagrams) is neglected.**
- **zeroth N/D iteration (N=1); c.f. single iteration (N=T_{tree}).**

K. Igi, K. Hikasa, Phys. Rev. D59, 034005 (1999)

Renormalization

Dynamical model with chiral symmetry.



With sufficient attraction, a resonance can be generated.

Single-subtraction <=> log divergence of loop function
We determine the cutoff degree of freedom as

$$G(s) = 0 \quad \text{at} \quad s = m_\pi^2 \qquad\qquad a(m_\pi) = -\frac{\pi}{\sqrt{3}}$$

Exclude the CDD pole contribution from the loop function

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

Consistency of the amplitude with chiral low energy theorem

K. Igi, K. Hikasa, Phys. Rev. D59, 034005 (1999)

Crossing symmetry (matching with u-channel amplitude)

M.F.M. Lutz, E. Kolomeitsev, Nucl. Phys. A700, 193 (2002)

Prescription for symmetry restoration

We introduce the effect of chiral symmetry restoration from the outside of the model, by modifying m_π, m_σ, $\langle\sigma\rangle$.

1) chiral condensate (pion decay constant) : decreases

$$\langle\sigma\rangle = \Phi\langle\sigma\rangle_0, \quad 0 \le \Phi \le 1$$

2) mass of pion : no change

$$\frac{\partial m_\pi}{\partial \Phi} = 0$$

3) mass of sigma -- two possibilities

- case I (chiral sigma : decreases)

$$m_\sigma\big|_{\Phi\to 0} = m_\pi \quad \text{(case I)} \qquad m_\sigma = \sqrt{\lambda\frac{\langle\sigma\rangle^2}{3} + m_\pi^2}$$

- case II (CDD pole : no change)

$$\frac{\partial m_\sigma}{\partial \Phi} = 0 \quad \text{(case II)}$$

Restoration limit and chiral partner

Properties of the chiral partner in the restoration limit

- **mass degeneracy with pion**
- **coupling to π-π scattering state vanishes**

Model for chiral sigma (x=1, case I ~ linear sigma model)

- **pole term in the tree-level interaction**

$$T_{\text{tree}}(s;1) = -\frac{\lambda^2 \langle\sigma\rangle^2}{3} \frac{1}{s - m_\pi^2 - \frac{\lambda}{3}\langle\sigma\rangle^2} + \dots$$

- **renormalization condition**

$$G(s) = 0 \quad \text{at} \quad s = m_\pi^2$$

$$T(m_\pi^2;1)|_{\Phi\to 0} = T_{\text{tree}}(m_\pi^2;1)|_{\Phi\to 0} \equiv -\frac{g^2}{s - M_{\text{pole}}^2}$$

Thus, the pole in the π-π amplitude behaves as

$g \to 0, \quad M_{\text{pole}} \to m_\pi \quad \text{for} \quad \Phi \to 0$ **like the chiral partner**

Restoration limit and chiral partner

Model for dynamical sigma and/or CDD pole (case II)

$$T_{\rm tree}(s;x) \propto \frac{1}{\langle\sigma\rangle^2}$$

The amplitude in the restoration limit is solely determined by the loop function G, irrespective to x:

$$T(s;x) = \frac{1}{T^{-1}_{\rm tree}(s,x) + G(s)} \to \frac{1}{G(s)} \quad \text{for} \quad \Phi \to 0$$

- renormalization condition requires a pole at m_π

$$G(s) = 0 \quad \text{at} \quad s = m_\pi^2$$

- coupling can be calculated : proportional to m_π

$$g^2\big|_{\Phi\to 0} = -(s-m_\pi^2)T(s)\big|_{s\to m_\pi^2,\Phi\to 0}$$

$$= -\left.\frac{s-m_\pi^2}{G(s)}\right|_{s\to m_\pi^2} = (4\pi)^2\left(\frac{\pi}{3\sqrt{3}} - \frac{1}{2}\right)^{-1} m_\pi^2$$

How to interpret this result?

Restoration limit and chiral partner

Properties of chiral sigma

$$g \to 0, \quad M_{\text{pole}} \to m_\pi \quad \text{for} \quad \Phi \to 0$$

Properties of dynamical sigma

$$g^2 \to (4\pi)^2 \left(\frac{\pi}{3\sqrt{3}} - \frac{1}{2} \right)^{-1} m_\pi^2, \quad M_{\text{pole}} \to m_\pi \quad \text{for} \quad \Phi \to 0$$

- mass degenerates with pion
- coupling to π-π : vanishes in the chiral limit

Dynamical sigma as chiral partner of pion?

- Renormalization condition plays an important role.
  <-- consistency with chiral theorem. Generally,

$G(\mu^2) = 0 \quad \text{at} \quad 0 \le \mu \le 2m_\pi$ --> deviation ~ m_π

- dynamical resonance as chiral partner (mass degeneracy)

J.A. Oller, hep-ph/0007349
S. Leupold, M.F.M. Lutz, M. Wagner, 0811.2398 [nucl-th]

Model setup

We numerically analyze four models:

	x	m_σ	sigma origin
model A	1	case I	chiral
model B	0	-	dynamical
model C	1	case II	CDD
model D	1/2	case II	CDD + dynamical

model C : same with model A, but m_σ unchanged.
pole term + repulsion (c.f. linear sigma model)

$$T_{\rm tree}(s;x) \equiv T^{\rm (contact)}_{\rm tree}(s;x) + T^{\rm (pole)}_{\rm tree}(s;x)$$

model D : pole term + attraction

existence of dynamical state <--> sign of the contact term

Results in vacuum

Input: $m_\pi = 140$ MeV, $m_\sigma = 550$ MeV, $\langle\sigma\rangle = 93$ MeV

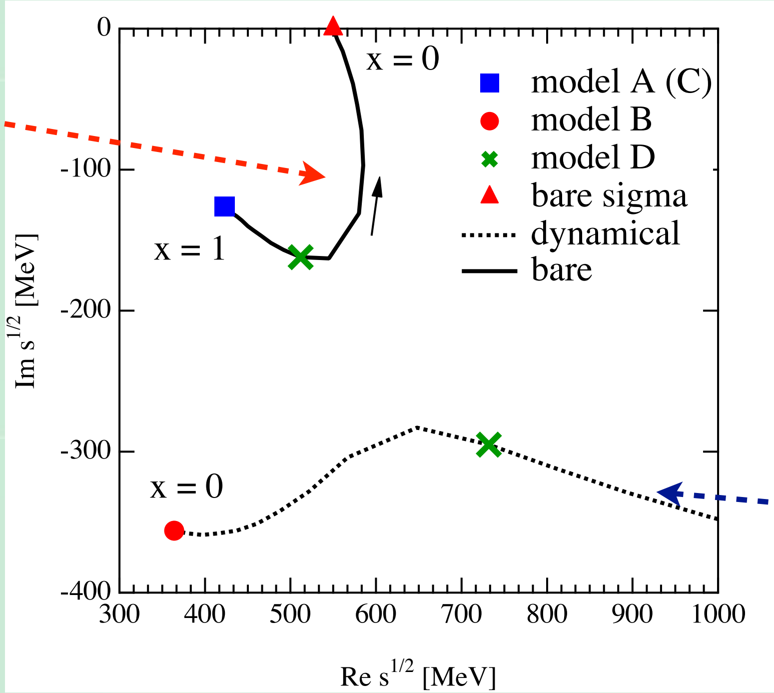
	scattering length $(m_\pi)^{-1}$	pole position [MeV]
model A, C	0.244	423 - 126 i
model B	0.174	364 - 356 i
model D	0.208	512 - 162 i, 732 - 295 i
(experiment)	0.216 [1]	441 - 272 i [2]

[1] S. Pislak *et al.*, Phys. Rev. D67, 072004 (2003)
[2] I. Caprini, G. Colangelo, H. Leutwyler, Phys. Rev. Lett. 96, 132001 (2006)

Model extrapolations and origin of the pole

Poles in the complex energy plane

bare pole



dynamical

Trace pole position of model A (x=1) with x -> 0

: approaches to bare pole, through one pole in model D

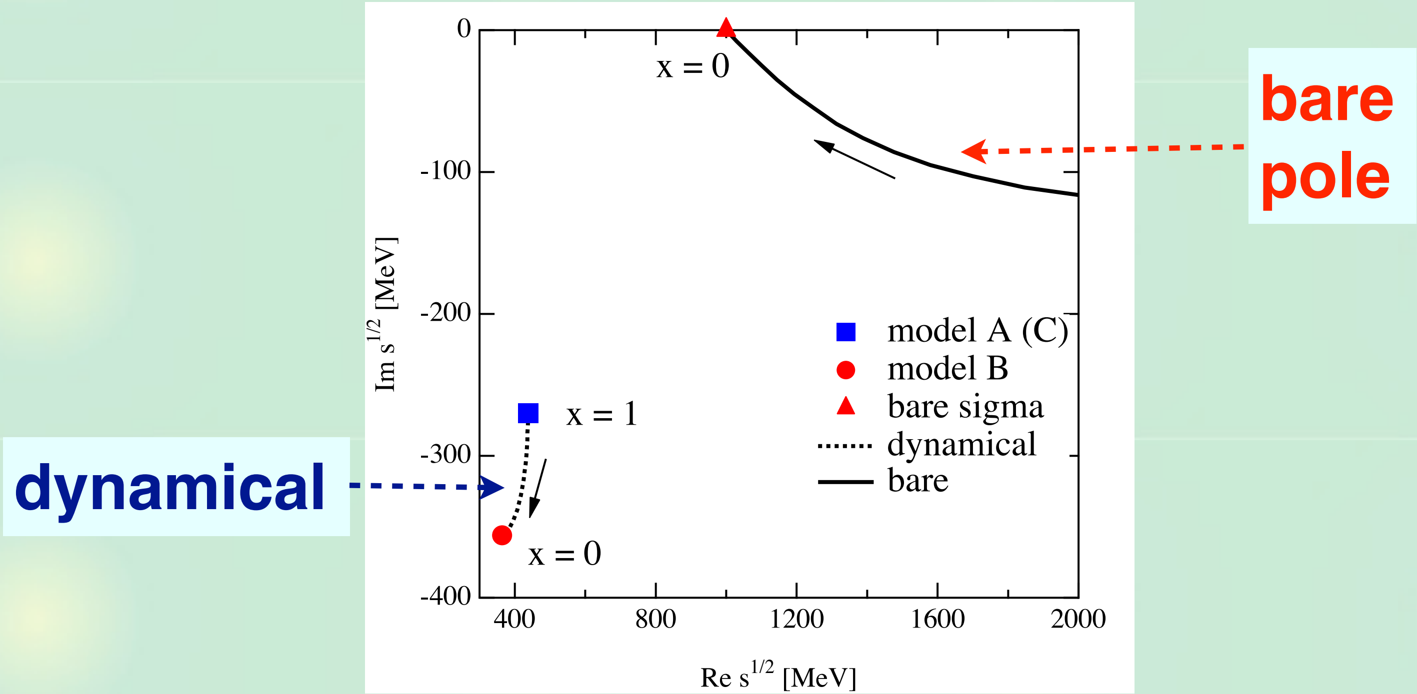
Trace pole position of model B (x=0) with x -> 1

: dissolves into continuum, through one pole in model D

Poles in model D: one bare pole origin, one dynamical.

Model extrapolations and origin of the pole

If $m_\sigma = 1$ GeV, then

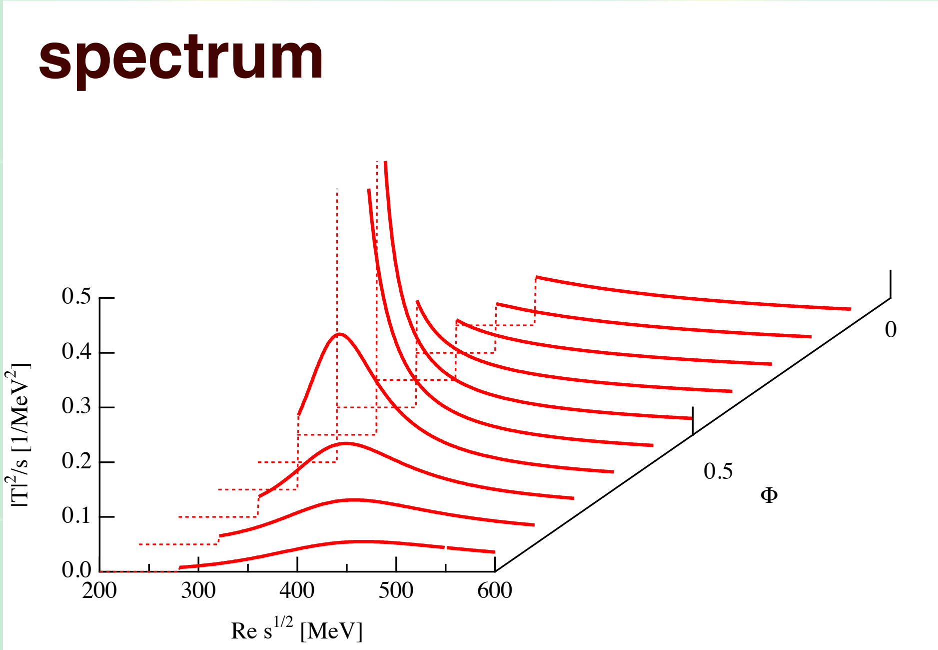


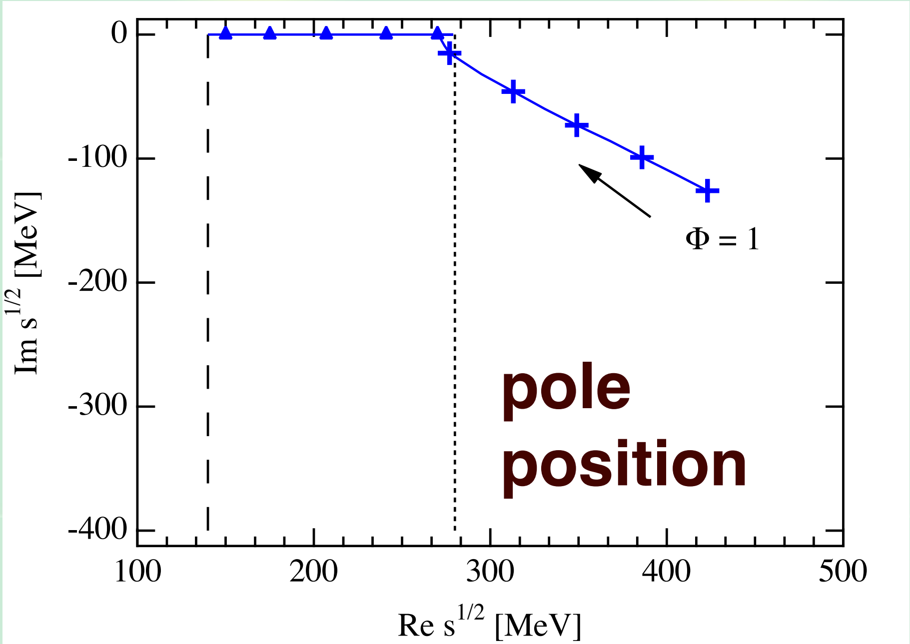
A bare pole at sufficiently high energy than the energy region under consideration ==> effective attraction

$$-G^2 \frac{1}{s-m_\sigma^2} = G^2 \frac{1}{m_\sigma^2}\left(1+\frac{s}{m_\sigma^2}+\cdots\right) \quad \text{for} \quad s \ll m_\sigma^2.$$

In this case the origin of the pole in model A(C) is dynamical

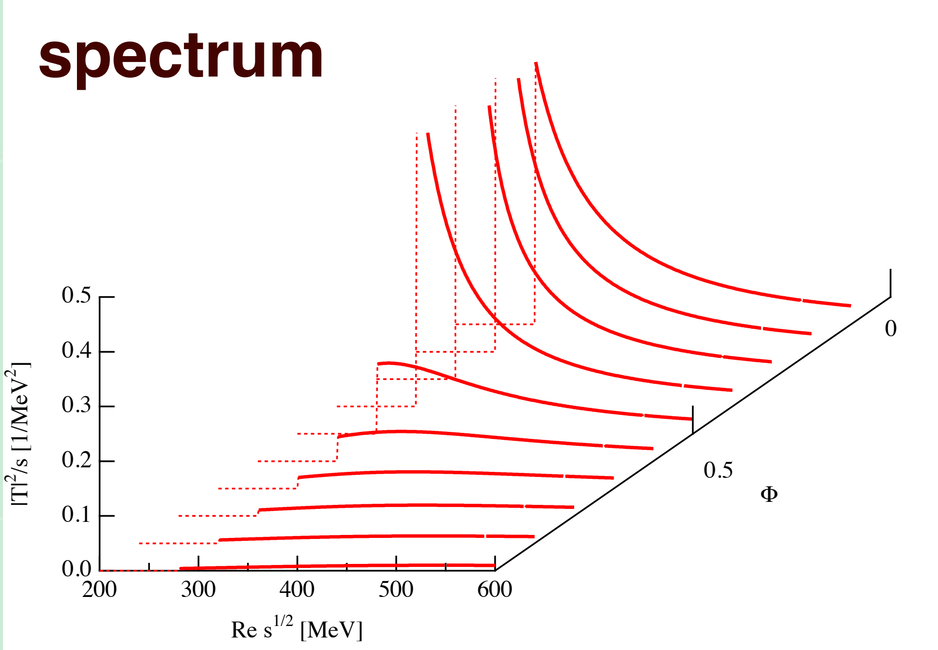
Results in model A

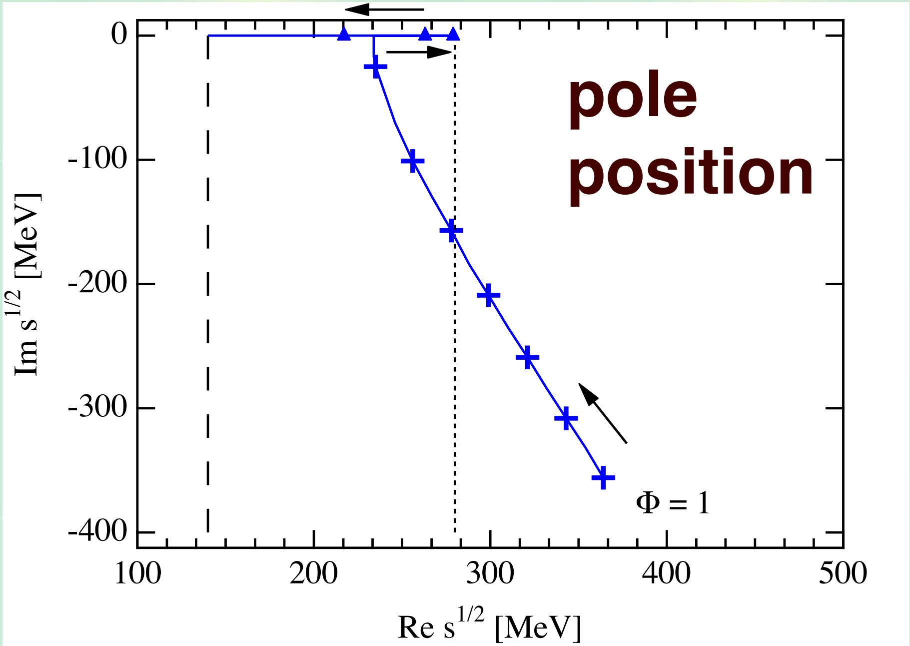




- Linear sigma model + unitarization : chiral sigma
- Softening takes place, as expected.
- peak at threshold : $\Phi \sim 0.6$
  <=> bare sigma pole moves below the threshold
- M_{pole} -> m_π as Φ -> 0

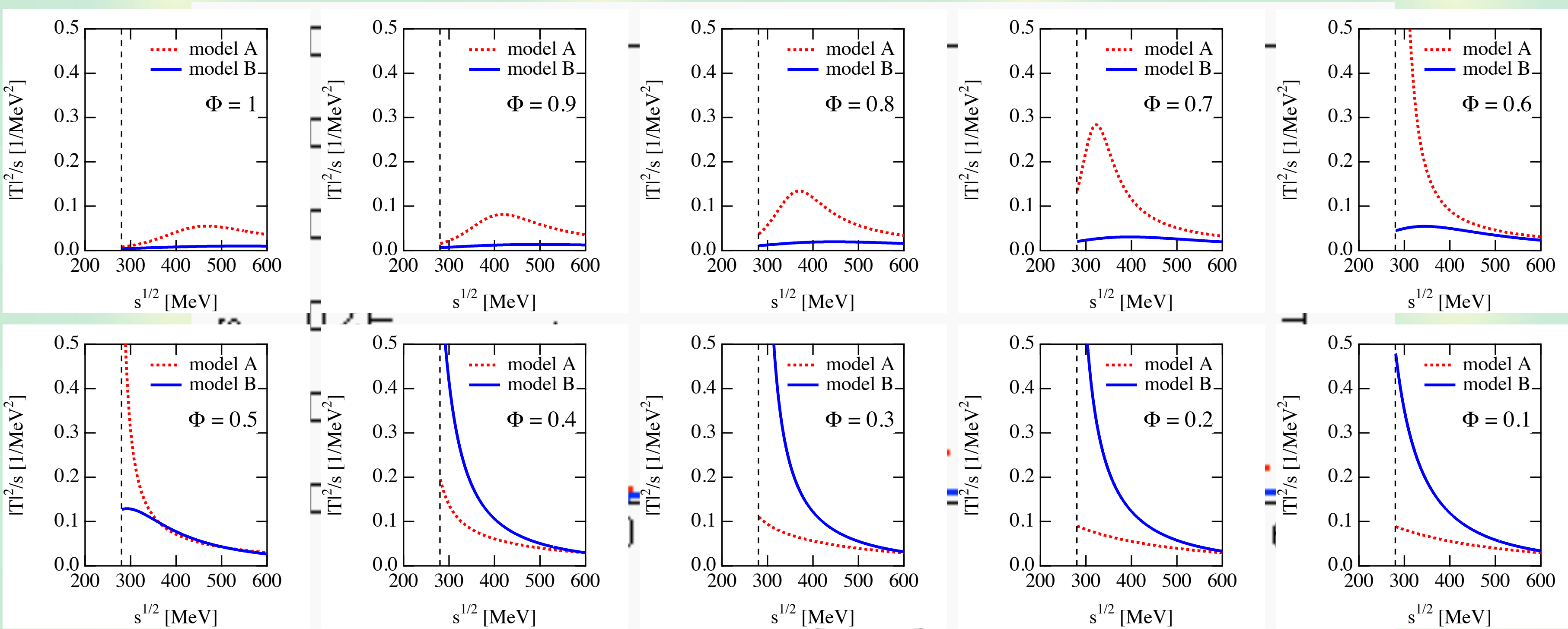
Results in model B



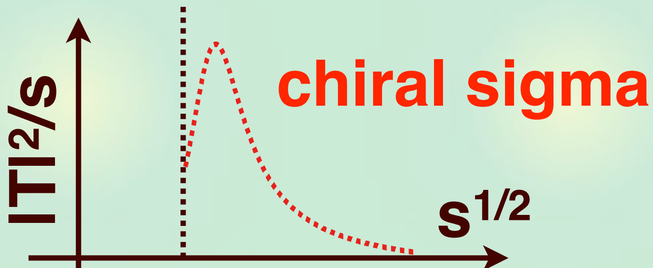


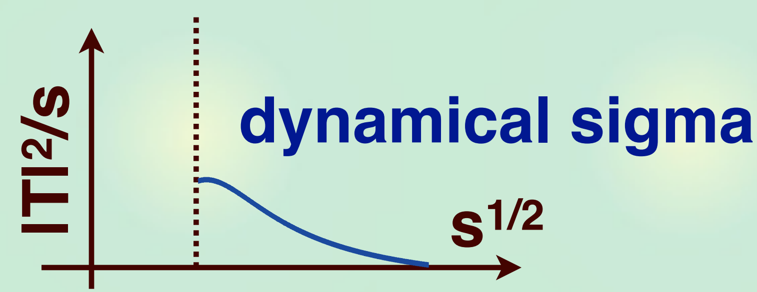
- ChPT + unitarization : dynamical sigma
- Softening takes place, but virtual state appears.
- at Re[M_{pole}] = $2m_\pi$ ($\Phi \sim 0.6$), due to finite width, spectrum does not show the peak structure
- peak at threshold : $\Phi \sim 0.3$ <=> formation of bound state
- M_{pole} -> m_π as Φ -> 0

Comparison of model A and model B

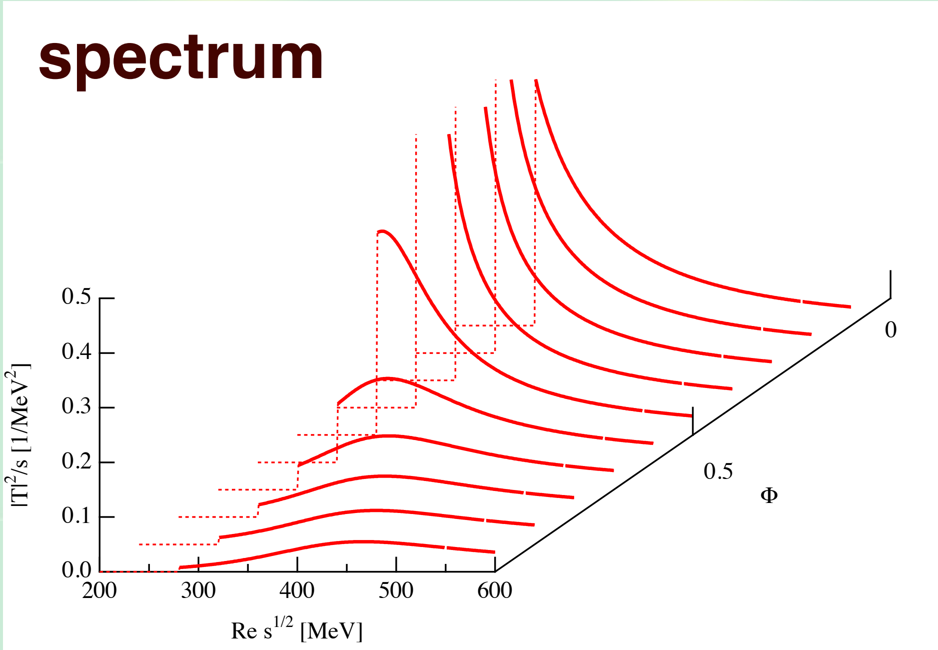


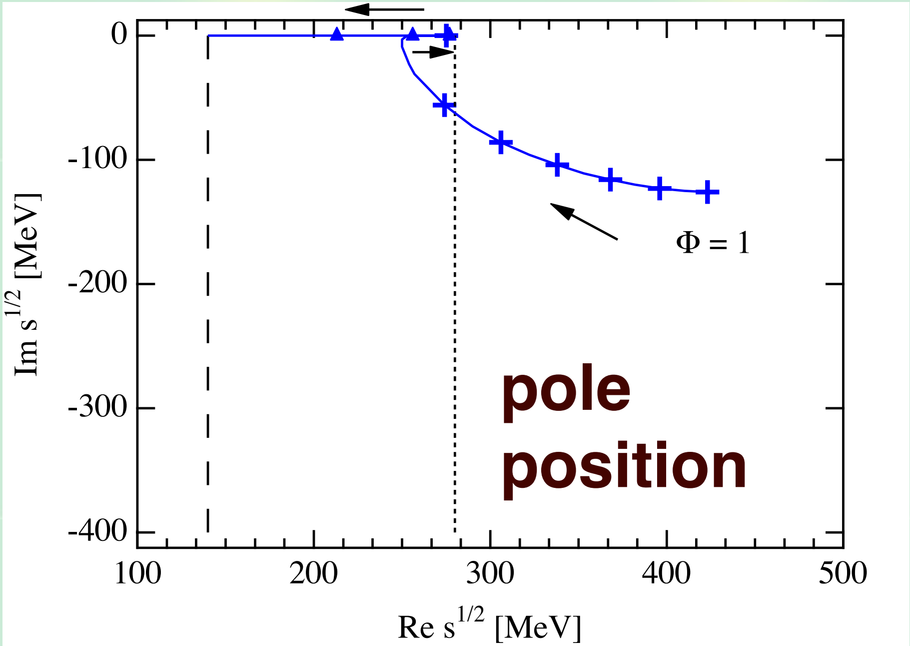
- Strong threshold enhancement : different from each other.
- Shape of the spectrum?





Results in model C

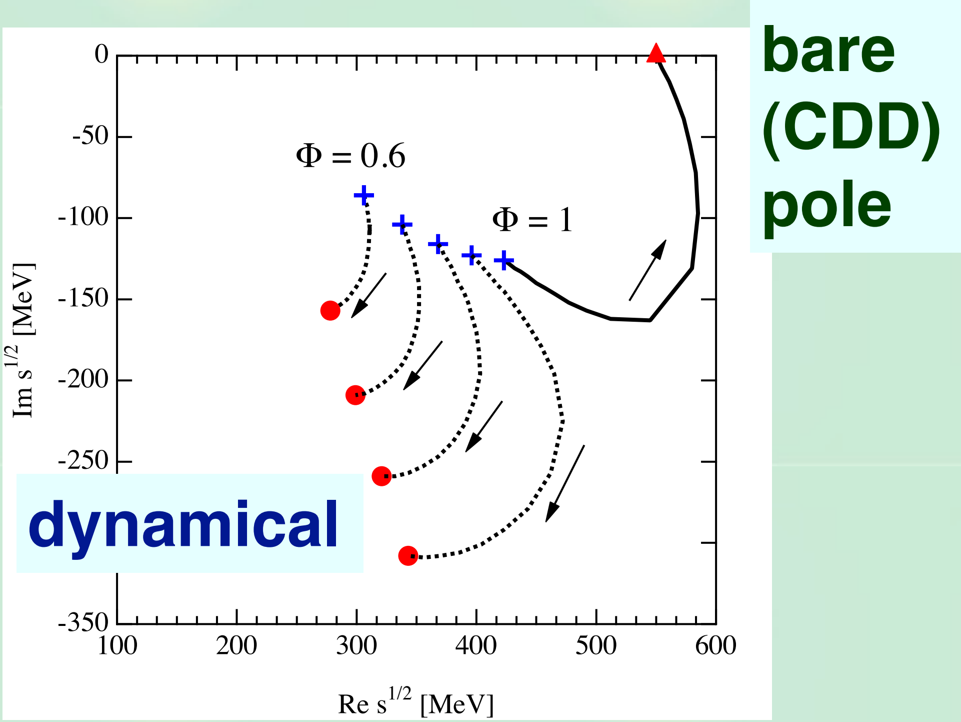




- Bare pole + unitarization : CDD pole
- Softening : similar to dynamical sigma (virtual state).
  <--> bare pole origin ??
- peak at threshold : $\Phi \sim 0.3$ <=> formation of bound state
- M_{pole} -> m_π as Φ -> 0

Property change of the pole

Extrapolation to x=0 for 1 ≧ Φ ≧ 0.6

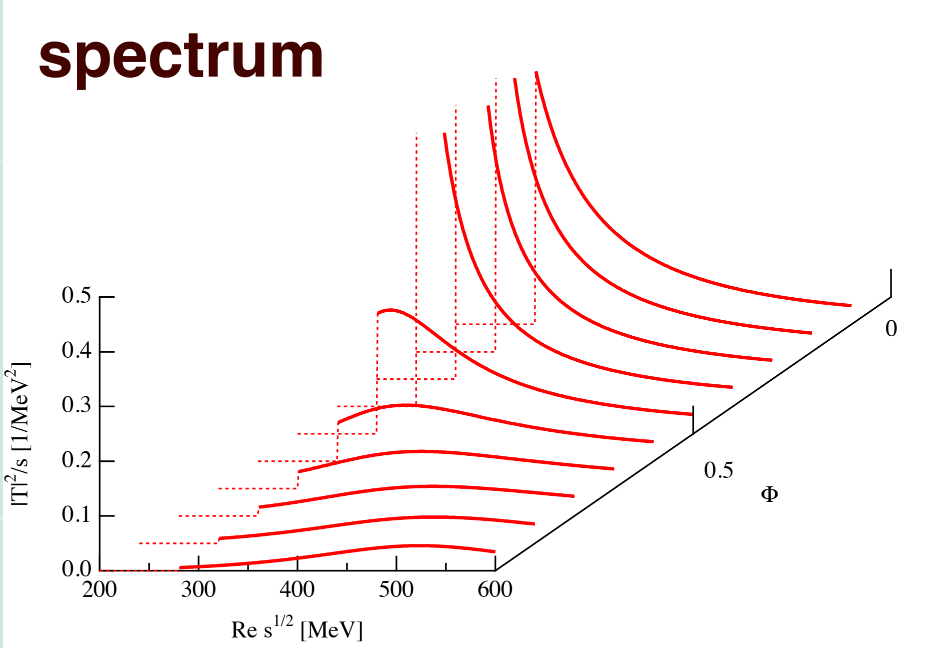


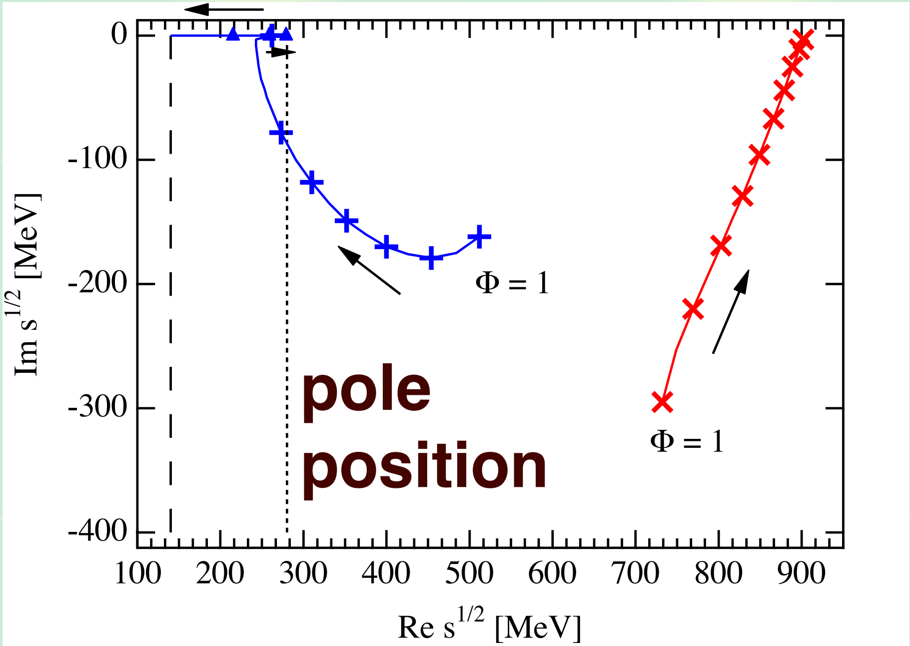
Symmetry restoration --> pole moves to lower energy

Bare pole unchanged, so it behaves as an effective attraction.

Origin of the pole : CDD pole --> dynamical sigma.

Results in model D





- Pole + attraction + unitarization : CDD pole + dynamical
- Softening : similar to dynamical sigma
- the other pole : goes to a kinematical singularity, reducing residue (irrelevant for spectrum).
- M_{pole} -> m_{π} as Φ -> 0

Summary : chiral dynamics and sigma meson

We study the structure of the sigma meson with chiral symmetry restoration.

- We classify the possible structure of the sigma meson into three classes:
  - (i) chiral sigma
  - (ii) dynamical sigma
  - (iii) CDD pole contribution
- We construct two-flavor dynamical chiral models which account for them.
- Dynamical sigma (s-wave resonance) is expected to behave differently.

Summary : results

- **In the chiral restoration limit:**

  Mass and coupling of the dynamical sigma behave similarly with chiral sigma, in the chiral limit. Chiral partner?

- **Softening phenomena:**

  Dynamical sigma softens qualitatively differently from chiral sigma.
  <-- virtual state (s-wave resonance)

  CDD pole case reduces to dynamical case around threshold. Universality?