

# Chiral symmetry in hadron physics



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## Chiral dynamics

- Chiral symmetry and low energy theorem
- Unitarity of S-matrix and dispersion theory
- Baryon resonances in meson-baryon scattering



## Structure of $\Lambda(1405)$ resonance

- Dynamical or CDD pole (genuine quark state) ?

[T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 \(2008\).](#)

- Nc Behavior and quark structure

[T. Hyodo, D. Jido, L. Roca, Phys. Rev. D77, 056010 \(2008\);](#)

[L. Roca, T. Hyodo, D. Jido, Nucl. Phys. A809, 65-87 \(2008\).](#)

- Electromagnetic properties

[T. Sekihara, T. Hyodo, D. Jido, Phys. Lett. B669, 133-138 \(2008\).](#)

# Chiral symmetry

**Chiral symmetry: symmetry of massless fermion**

$$\mathcal{L} = \bar{q}(i\cancel{D} - m)q$$

**Chiral projection operators and Left-(Right-)handed fermions**

$$P_L = \frac{1}{2}(1 - \gamma_5), \quad P_R = \frac{1}{2}(1 + \gamma_5), \quad q_L \equiv P_L q \quad q_R \equiv P_R q$$

**Lagrangian can be decomposed as**

$$\mathcal{L} = \bar{q}_L i\cancel{D} q_L + \bar{q}_R i\cancel{D} q_R - \bar{q}_L m q_R - \bar{q}_R m q_L$$

**If  $m=0$ , L and R fields independently have global symmetries, such as phase transformations and flavor rotations:**

$$q_R \rightarrow \exp\left\{i \sum_{a=0}^{N_F} t^a \theta_R^a\right\} q_R, \quad q_L \rightarrow \exp\left\{i \sum_{a=0}^{N_F} t^a \theta_L^a\right\} q_L$$

$$G = U(N_F)_R \otimes U(N_F)_L$$

$$= U(1)_V \otimes U(1)_A \otimes \boxed{SU(N_F)_R \otimes SU(N_F)_L}$$

**chiral symmetry**

# QCD and chiral symmetry breaking

QCD : up, down, and strange quarks are light.  
 In the limit of vanishing quark masses,  
 QCD Lagrangian has 3-flavor chiral symmetry

$$G = SU(3)_R \otimes SU(3)_L$$

Chiral symmetry is **broken** in two ways:

- spontaneous breaking
- explicit breaking (treated perturbatively)

Spontaneous breaking: **symmetry in the Lagrangian is not manifested in the vacuum** (states).

$$\langle 0 | \bar{q}q | 0 \rangle = v \neq 0$$

not chiral invariant. For u,d quarks,  $v \sim -(250 \text{ MeV})^3$

$$SU(3)_R \otimes SU(3)_L \rightarrow SU(3)_V$$

**flavor symmetry**

## Chiral symmetry breaking in hadron physics

Why important, although it is **broken** anyway?

Consequence of spontaneous breaking:

- appearance of the **Nambu-Goldstone (NG) boson**

$$m_{\pi} \sim 140 \text{ MeV}$$

- **hadron mass generation**

$$m_p \sim 1 \text{ GeV}$$

- **constraint for hadron-NG boson interaction**  
low energy theorem

**PCAC + commutation relation --> current algebra**  
**more systematic low-energy expansion --> ChPT**

**Chiral symmetry and its breaking**

**QCD  $\Leftrightarrow$  hadron phenomena**

# Low energy s-wave interaction

## Low energy theorem for pion (Ad) scattering with a target (T)

$$\alpha \left[ \begin{array}{c} \text{Ad}(q) \\ T(p) \end{array} \right] \begin{array}{c} \text{---} \nearrow \\ \bullet \\ \leftarrow \text{---} \end{array} = \frac{1}{f^2} \frac{p \cdot q}{2M_T} \langle \mathbf{F}_T \cdot \mathbf{F}_{\text{Ad}} \rangle_\alpha + \mathcal{O} \left( \left( \frac{m}{M_T} \right)^2 \right)$$

### s-wave : Weinberg-Tomozawa term

Y. Tomozawa, *Nuovo Cim.* **46A**, 707 (1966); S. Weinberg, *Phys. Rev. Lett.* **17**, 616 (1966)

$$V_{ij} = - \frac{C_{ij}}{4f^2} (\omega_i + \omega_j) \text{ pion energy}$$

**pion decay constant ( $g_V=1$ )**

$$C_{ij} = \sum_{\alpha} C_{\alpha,T} \left( \begin{array}{cc|c} 8 & T & \alpha \\ I_{M_i}, Y_{M_i} & I_{T_i}, Y_{T_i} & I, Y \end{array} \right) \left( \begin{array}{cc|c} 8 & T & \alpha \\ I_{M_j}, Y_{M_j} & I_{T_j}, Y_{T_j} & I, Y \end{array} \right)$$

$$C_{\alpha,T} = \langle 2\mathbf{F}_T \cdot \mathbf{F}_{\text{Ad}} \rangle = C_2(T) - C_2(\alpha) + 3$$

**flavor SU(3) --> sign and strength**

The result corresponds to the leading order term in ChPT

# Chiral dynamics : overview

## Description of hadron-NG boson scattering and resonance

### - interaction <-- chiral symmetry

Y. Tomozawa, *Nuovo Cim.* 46A, 707 (1966); S. Weinberg, *Phys. Rev. Lett.* 17, 616 (1966)

### - scattering amplitude <-- unitarity in coupled channels chiral interaction is strong, especially for 3-flavor case

R.H. Dalitz, T.C. Wong, G. Rajasekaran, *Phys. Rev.* 153, 1617 (1967)

$$T = \frac{1}{1 - VG} V$$

**chiral**

N. Kaiser, P. B. Siegel, W. Weise, *Nucl. Phys.* A594, 325 (1995),

E. Oset, A. Ramos, *Nucl. Phys.* A635, 99 (1998),

J. A. Oller, U. G. Meissner, *Phys. Lett.* B500, 263 (2001),

M.F.M. Lutz, E. E. Kolomeitsev, *Nucl. Phys.* A700, 193 (2002), .... many others

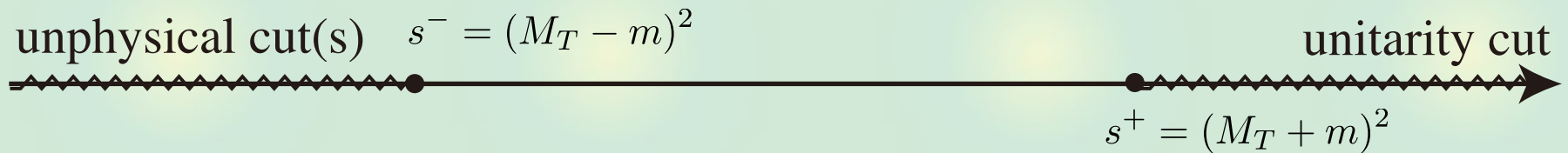
works successfully, also in  $S=0$  sector, meson-meson scattering sectors, systems including heavy quarks, ...

# Scattering theory : N/D method

Single-channel scattering, masses:  $M_T$  and  $m$

G.F. Chew, S. Mandelstam, Phys. Rev. 119, 467 (1960)

$$s = W^2$$



**N/D method:**

Divide  $T$  into  $N$ (umerator) and  $D$ (enominator)

unitarity cut  $\rightarrow D$ , unphysical cut(s)  $\rightarrow N$

$$T(s) = N(s)/D(s) \quad \text{phase space (optical theorem)}$$

$$\text{Im}D(s) = \text{Im}[T^{-1}(s)]N(s) = \rho(s)N(s)/2 \quad \text{for } s > s^+$$

$$\text{Im}N(s) = \text{Im}[T(s)]D(s) \quad \text{for } s < s^-$$

**Dispersion relation for  $N$  and  $D$**

$\rightarrow$  set of integral equations, input :  $\text{Im}[T(s)]$  for  $s < s^-$



# General form of the (s-wave) amplitude

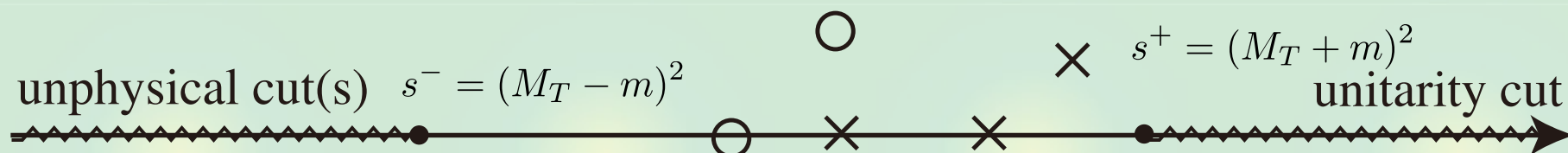
Neglect unphysical cut (crossed diagrams), set  $N=1$

U. G. Meissner, J. A. Oller, Nucl. Phys. A673, 311 (2000)

$$T^{-1}(\sqrt{s}) = \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)}$$

- pole (and zero) of the amplitude

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)



**CDD pole(s),  $R_i, W_i$  : not known in advance**

$$T^{-1}(\sqrt{s}) = \boxed{\sum_i \frac{R_i}{\sqrt{s} - W_i}} + \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)}$$

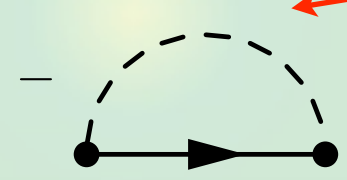
**CDD pole contribution --> independent particle**

G.F. Chew, S.C. Frautschi, Phys. Rev. 124, 264 (1961)

# Order by order matching with ChPT

Identify loop function  $G$ , the rest contribution  $\rightarrow V^{-1}$

$$T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s} - W_i} + \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s_+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)}$$



$$= -i \int \frac{d^4q}{(2\pi)^4} \frac{2M_T}{(P - q)^2 - M_T^2 + i\epsilon} \frac{1}{q^2 - m^2 + i\epsilon} \Big|_{\text{dim.reg.}}$$

$$= -\frac{2M_T}{(4\pi)^2} \left\{ \boxed{a} + \frac{m^2 - M_T^2 + s}{2s} \ln \frac{m^2}{M_T^2} + \frac{\bar{q}}{\sqrt{s}} \ln \frac{\phi_{++}(s)\phi_{+-}(s)}{\phi_{-+}(s)\phi_{--}(s)} \right\}$$

**subtraction constant (cutoff)**

$$= -G(\sqrt{s}; a)$$

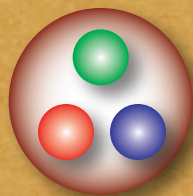
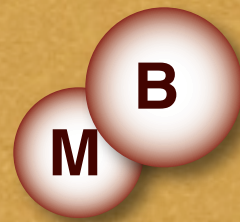
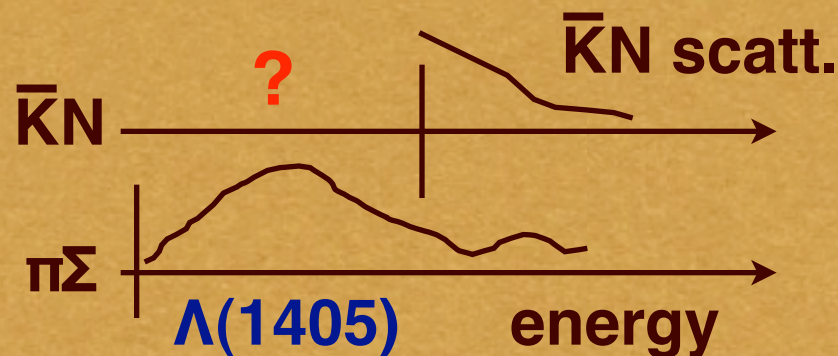
$T(\sqrt{s}) = [V^{-1}(\sqrt{s}) - G(\sqrt{s}; a)]^{-1}$

scattering amplitude

**V? chiral expansion of T, (conceptual) matching with ChPT**

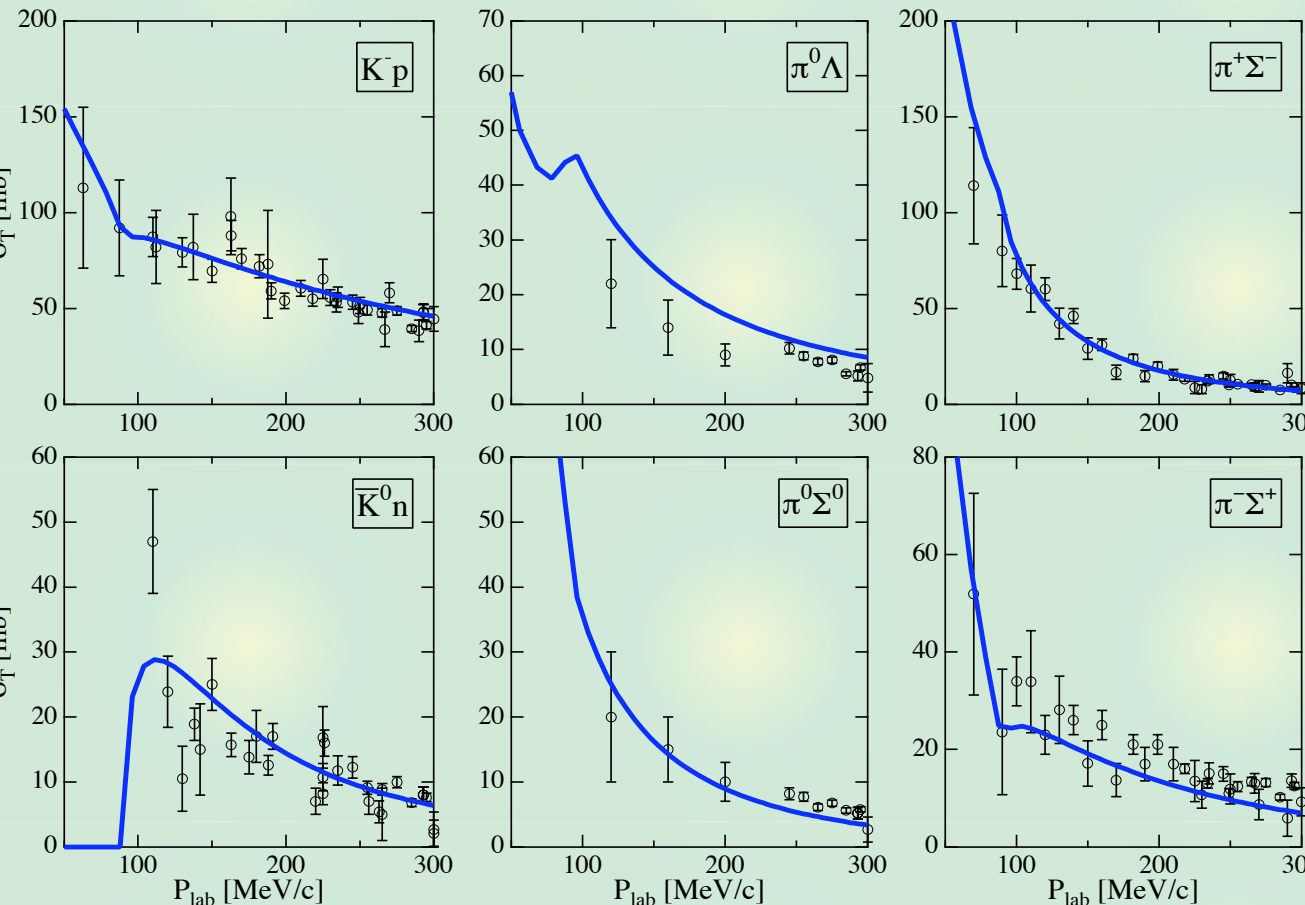
**J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001)**

$$T^{(1)} = V^{(1)}, \quad T^{(2)} = V^{(2)}, \quad T^{(3)} = V^{(3)} - V^{(1)}GV^{(1)}, \quad \dots$$

**$\bar{K}N$  scattering and  $\Lambda(1405)$**  **$\Lambda(1405) : J^P = 1/2^-, I = 0$** **(PDG)****mass:  $1406.5 \pm 4.0$  MeV, width:  $50 \pm 2$  MeV****Decay mode:  $\Lambda(1405) \rightarrow (\pi\Sigma)_{I=0}$  100%****“naive” quark model  
: p-wave  $\sim 1600$  MeV?****N. Isgur, G. Karl, PRD18, 4187 (1978)****Coupled channel  
multi-scattering****R.H. Dalitz, T.C. Wong,  
G. Rajasekaran, PR153, 1617 (1967)** **$\bar{K}N$  interaction below threshold****T. Hyodo, W. Weise, PRC 77, 035204 (2008)****-->  $\bar{K}N$  potential, kaonic nuclei****A. Dote, T. Hyodo, W. Weise,  
NPA804, 197 (2008); PRC 79, 014003 (2009)**

# How it works? vs experimental data

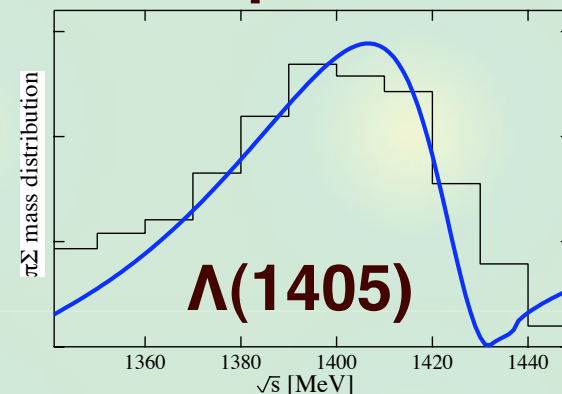
## Total cross sections



## threshold ratios

	$\gamma$	$R_c$	$R_n$
exp.	2.36	0.664	0.189
theo.	1.80	0.624	0.225

## $\pi\Sigma$ spectrum



T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Phys. Rev. C68, 018201 (2003),

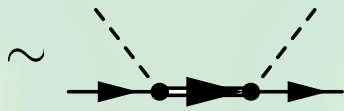
T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Prog. Theor. Phys. 112, 73 (2004)

**Good agreement with data above, at, and below threshold**

# Two poles for one resonance

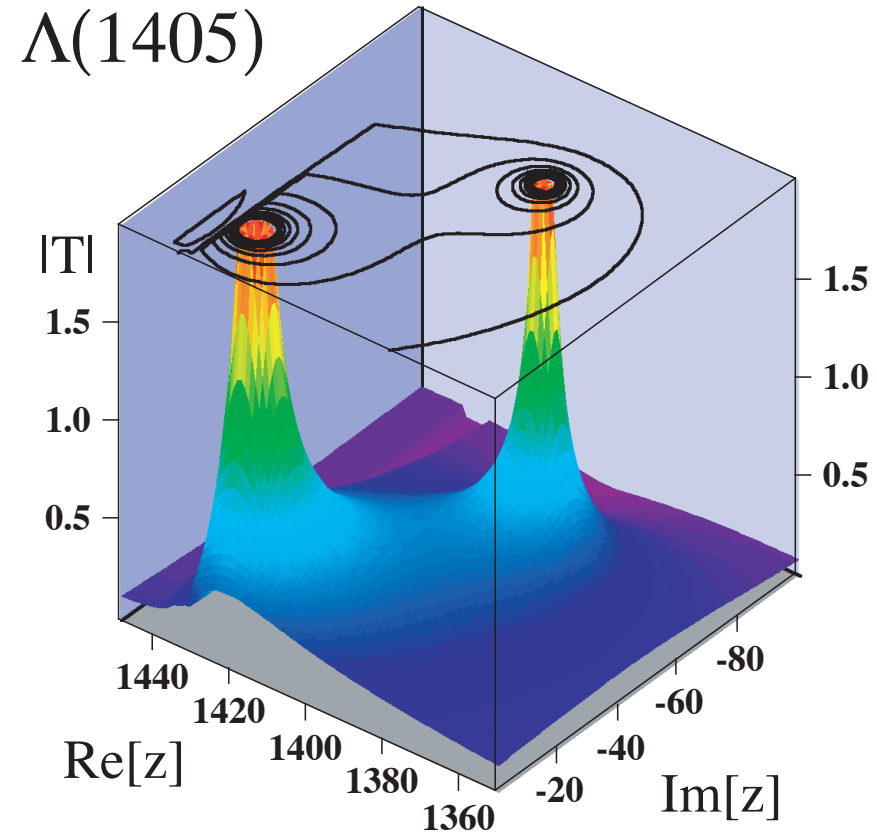
Poles of the amplitude in the complex plane : resonance

$$T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - M_R + i\Gamma_R/2}$$



Real part	Mass
Imaginary part	Width/2
Residues	Couplings

Physical “ $\Lambda(1405)$ ”  
: superposition of two states



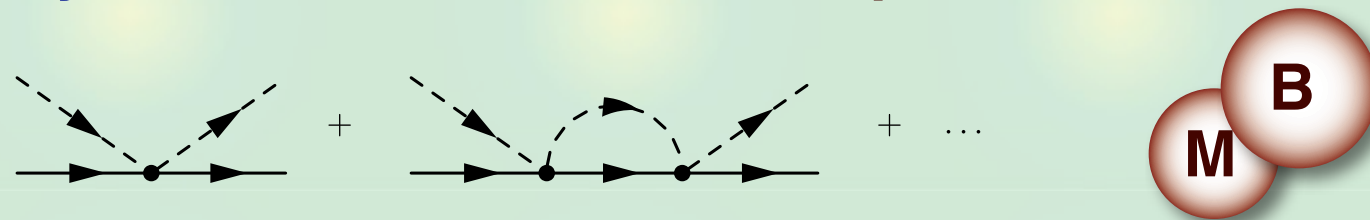
D. Jido, J.A. Oller, E. Oset, A. Ramos, U.G. Meissner, Nucl. Phys. A 723, 205 (2003);  
T. Hyodo, W. Weise, Phys. Rev. C 77, 035204 (2008)

## Dynamical state and CDD pole

### Resonances in two-body scattering

- Knowledge of interaction (potential)
- Experimental data (cross section, phase shift,...)

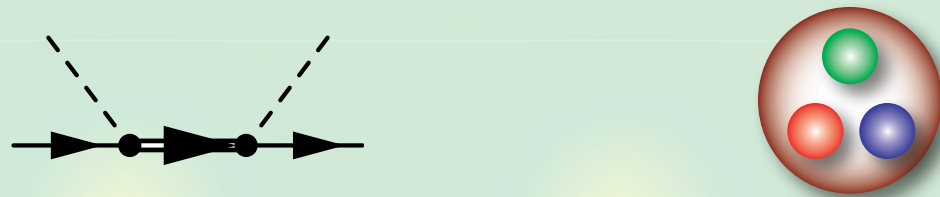
(a) **dynamical** state: molecule, quasi-bound, ...



... in the present case : meson-baryon molecule

(b) **CDD** pole: elementary, independent, ...

L. Castillejo, R.H. Dalitz, F.J. Dyson, *Phys. Rev.* 101, 453 (1956)



... in the present case : three-quark state

Resonances in chiral dynamics  $\rightarrow$  (a) **dynamical**?

# CDD pole contribution in chiral unitary approach

## Amplitude in chiral unitary model

$$T = \frac{1}{\boxed{V^{-1}} - \boxed{G}}$$

**V** : interaction kernel (potential)  
**G** : loop integral (Green's function)

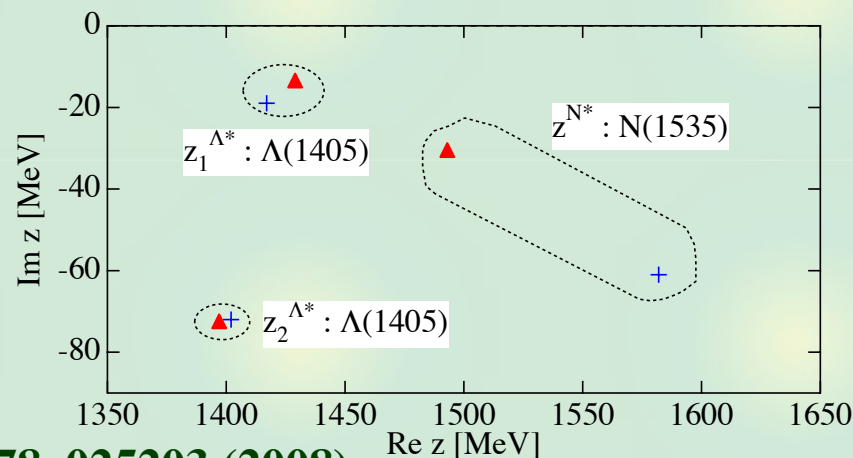
## Known CDD pole contribution

- (1) Explicit resonance field in **V**
- (2) Contracted resonance propagator in **V**

Defining “natural renormalization scheme”,  
we find **CDD pole contribution in G** (subtraction constant).

**N(1535)** in  $\pi N$  scattering  
 --> dynamical + CDD pole

**$\Lambda(1405)$**  in  $\bar{K} N$  scattering  
 --> **mostly dynamical**



## Nc scaling in the model

**Nc** : number of color in QCD

Hadron effective theory / quark structure

The Nc behavior is known from the general argument.

←-- introducing Nc dependence in the model,  
analyze the resonance properties with respect to Nc

J.R. Pelaez, *Phys. Rev. Lett.* **92**, 102001 (2004)

**Nc scaling of (excited)  
qqq baryon**

$$M_R \sim \mathcal{O}(N_c), \quad \Gamma_R \sim \mathcal{O}(1)$$

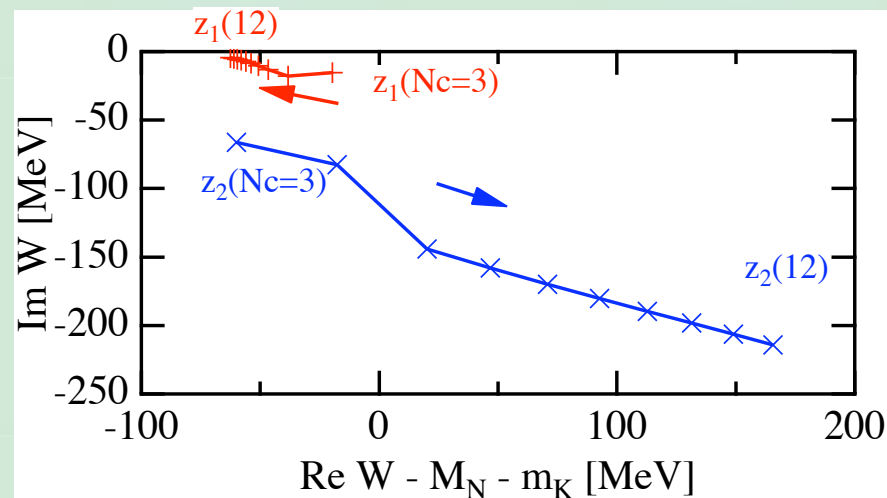
**Result of chiral dynamics**

$$\Gamma_R \neq \mathcal{O}(1)$$

→ non-qqq (i.e. dynamical) structure of the  $\Lambda(1405)$

T. Hyodo, D. Jido, L. Roca, *Phys. Rev.* **D77**, 056010 (2008).

L. Roca, T. Hyodo, D. Jido, *Nucl. Phys.* **A809**, 65-87 (2008).

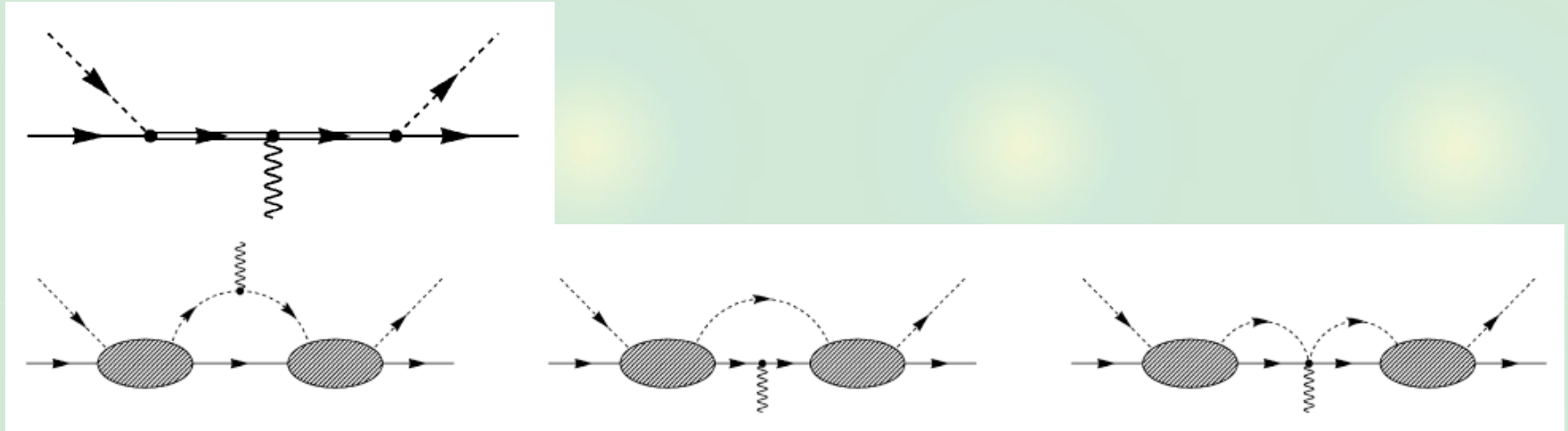




## Electromagnetic properties

Attaching photon to resonance

--> em properties : rms, form factors,...



result of mean squared radii :

$$|\langle r^2 \rangle_E| = 0.33 \text{ [fm}^2\text{]}$$

large (em) size of the  $\Lambda(1405)$  : c.f.  $-0.12 \text{ [fm}^2\text{]}$  for neutron

--> meson-baryon picture

T. Sekihara, T. Hyodo, D. Jido, Phys. Lett. B669, 133-138 (2008);

T. Sekihara, T. Hyodo, D. Jido, in preparation.

## Summary : Chiral dynamics

Chiral symmetry in QCD, its spontaneous breaking, and dynamical scattering model are reviewed.

Chiral symmetry enables us to connect **hadron phenomena** with underlying theory of **QCD**.

Interaction constrained by **chiral symmetry** + coupled-channel **unitarity condition**

=> successful description of hadron scattering and resonances:  
e.g.  $\Lambda(1405)$  in  $\bar{K}N$  scattering.

**Internal structure** of resonances can be investigated in several ways.

## Summary : Structure of $\Lambda(1405)$

The structure of the  $\Lambda(1405)$  is studied:



Dynamical or CDD?

=> dominance of the MB components



Analysis of Nc scaling

=> non-qqq structure



Electromagnetic properties

=> large e.m. size



Independent analyses consistently support the **meson-baryon molecule picture** for the  $\Lambda(1405)$

