Evidence That the Deuteron Is Not an Elementary Particle

S. Weinberg, Phys. Rev. 137 B672-B678 (1965)

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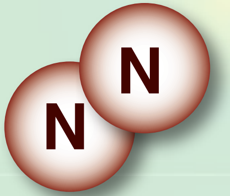
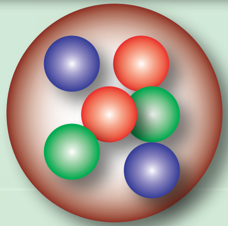
Main result: theorem

$|\text{deuteron}\rangle =$



or



Z = 0 ⟷ Z = 1

Z: probability of finding deuteron in a bare elementary state

For a bound state with small binding energy, the following equation should be satisfied model independently:

$$\boxed{a_s} = \left[\frac{2(1-Z)}{2-Z}\right]\boxed{R} + \mathcal{O}(m_\pi^{-1}), \quad \boxed{r_e} = \left[\frac{-Z}{1-Z}\right]\boxed{R} + \mathcal{O}(m_\pi^{-1})$$

a_s: scattering length
r_e: effective range <-- Experiments (observables)
R: deuteron radius

$$a_s = +5.41[\text{fm}], \quad r_e = +1.75[\text{fm}], \quad R \equiv (2\mu B)^{-1/2} = 4.31[\text{fm}]$$

$\Rightarrow Z \lesssim 0.2$ **--> deuteron is composite!**

Derivation of the theorem

The theorem is derived in two steps:

Step 1 (Sec. II): Z --> p-n-d coupling constant

$$g^2 = \frac{2\sqrt{B}(1-Z)}{\pi\rho} \qquad \rho = \frac{4\pi}{\sqrt{2\mu^3}}$$

Step 2 (Sec. III): coupling constant --> a_s, r_e

$$a_s = 2R\left[1 + \frac{2\sqrt{B}}{\pi\rho g^2}\right] \qquad r_e = R\left[1 - \frac{2\sqrt{B}}{\pi\rho g^2}\right]$$

Assumption: B is sufficiently smaller than the typical energy scale of the NN interaction

$$p \sim m_\pi \qquad B \ll m_\pi^2/2\mu \quad \Leftrightarrow \quad R^2 \gg m_\pi^2$$

--> uncertainty for order R quantity: m_π^{-1}

Definition of the probability Z

Hamiltonian of NN system: free + interaction V

$$\mathcal{H} = \mathcal{H}_0 + V$$

Complete set for free Hamiltonian: bare d (d_0) + continuum

$$1 = |d_0\rangle\langle d_0| + \int d\boldsymbol{k}|\boldsymbol{k}\rangle\langle\boldsymbol{k}|$$

$$\mathcal{H}_0|d_0\rangle = E_0|d_0\rangle, \quad \mathcal{H}_0|\boldsymbol{k}\rangle = E(\boldsymbol{k})|\boldsymbol{k}\rangle$$

(original, d_0: sum of discrete states, p: α)

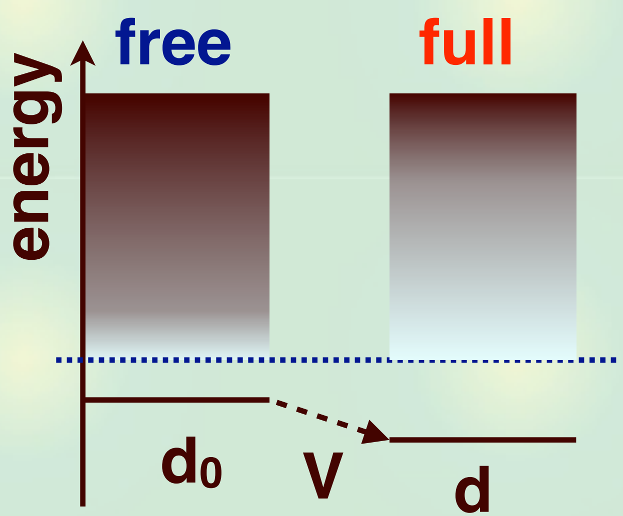
Physical deuteron : eigenstate of full Hamiltonian

$$(\mathcal{H}_0 + V)|d\rangle = -B|d\rangle$$

Z: overlap of d and d_0
(wavefunction renormalization factor)

$$Z \equiv |\langle d_0|d\rangle|^2$$

$$|d\rangle = \sqrt{Z}|d_0\rangle + \sqrt{1-Z}\int d\boldsymbol{k}|\boldsymbol{k}\rangle$$

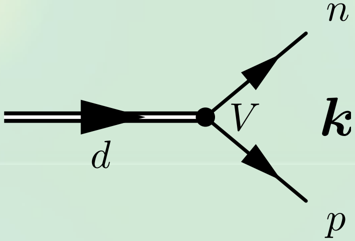


p-n-d coupling constant

After some algebra, we arrive at

$$1 - Z = \int d\boldsymbol{k} \frac{|\langle \boldsymbol{k}|V|d\rangle|^2}{(E(\boldsymbol{k}) + B)^2}$$

$$|\langle \boldsymbol{k}|V|d\rangle| = g(\boldsymbol{k}) :$$



Typical energy scale E_0: below E_0, coupling is constant

$$|\langle \boldsymbol{k}|V|d\rangle| = g(\boldsymbol{k}) \sim g \quad \text{for} \quad |E(\boldsymbol{k})| \leq E_0$$

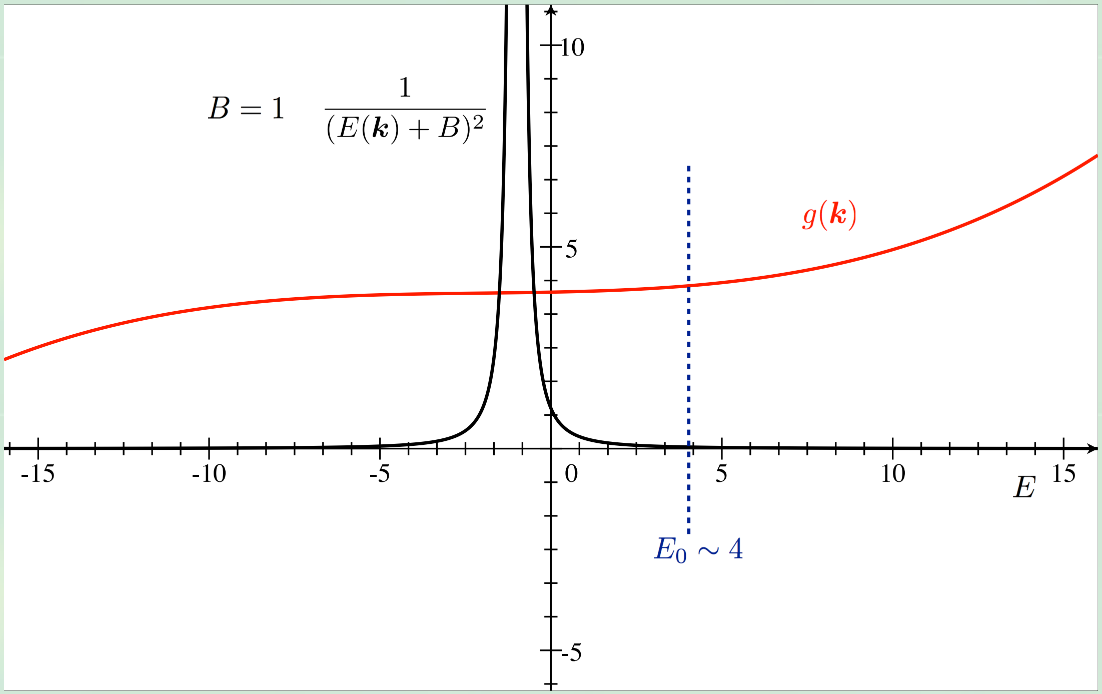
(NN scatt. : $E_0 \approx m_\pi^2/2\mu$ **)**

Assumption: $B \ll E_0$

$$\Rightarrow 1 - Z \sim g^2 \int \frac{d\boldsymbol{k}}{(E(\boldsymbol{k}) + B)^2}$$

$$= g^2 \rho \int_0^\infty \frac{\sqrt{E}\, dE}{(E + B)^2}$$

$$\rho = 4\pi/\sqrt{2\mu^3}$$



Integrate analytically

$$g^2 = \frac{2\sqrt{B}(1 - Z)}{\pi \rho}$$

Scattering equations

The Lippmann-Schwinger equation

$$T(W) = V + V\frac{1}{W - \underline{\mathcal{H}_0}}T(W)$$

$$\Leftrightarrow T(W) = V + V\frac{1}{W - \underline{\mathcal{H}}}V \quad \text{(Chew-Goldberger solution)}$$

Complete set for full Hamiltonian (asymptotic completeness)

$$1 = |d\rangle\langle d| + \int d\boldsymbol{k}|\boldsymbol{k}, \mathrm{in}\rangle\langle \boldsymbol{k}, \mathrm{in}| \qquad V|\,\boldsymbol{k}, \mathrm{in}\,\rangle = T|\,\boldsymbol{k}\,\rangle$$

$$T_{\boldsymbol{k}'\boldsymbol{k}}(W) = V_{\boldsymbol{k}'\boldsymbol{k}} + \frac{\langle\,\boldsymbol{k}'\,|V|\,d\,\rangle\langle\,d\,|V|\,\boldsymbol{k}\,\rangle}{W+B} + \int d\boldsymbol{k}''\frac{\langle\,\boldsymbol{k}'\,|V|\,\boldsymbol{k}'', \mathrm{in}\,\rangle\langle\,\boldsymbol{k}'', \mathrm{in}\,|V|\,\boldsymbol{k}\,\rangle}{W - E(\boldsymbol{k}'')}$$

Setting $W = E(\boldsymbol{k}) + i\epsilon$, we obtain the Low equation

$$T_{\boldsymbol{k}'\boldsymbol{k}} = V_{\boldsymbol{k}'\boldsymbol{k}} + \frac{\langle\,\boldsymbol{k}'\,|V|\,d\,\rangle\langle\,d\,|V|\,\boldsymbol{k}\,\rangle}{E(\boldsymbol{k})+B} + \int d\boldsymbol{k}''\frac{T_{\boldsymbol{k}'\boldsymbol{k}''}T_{\boldsymbol{k}''\boldsymbol{k}}}{E(\boldsymbol{k}) - E(\boldsymbol{k}'') + i\epsilon}$$

So far no approximations.

Solution for the scattering equation

The same assumption: $B \ll E_0$, external energy $E \ll E_0$

$$\frac{\langle \boldsymbol{k}' |V| d \rangle \langle d |V| \boldsymbol{k} \rangle}{E(\boldsymbol{k}) + B} \sim \frac{g^2}{E(\boldsymbol{k}) + B} \propto \frac{1}{\sqrt{B}} \gg V_{\boldsymbol{k}'\boldsymbol{k}}$$

We neglect the 1st term (information of V is lost!!).

$$T_{\boldsymbol{k}'\boldsymbol{k}} = \frac{g^2}{E(\boldsymbol{k}) + B} + \int d\boldsymbol{k}'' \frac{T_{\boldsymbol{k}'\boldsymbol{k}''} T_{\boldsymbol{k}''\boldsymbol{k}}}{E(\boldsymbol{k}) - E(\boldsymbol{k}'') + i\epsilon}$$

S-wave scattering (no angular dependence)

$$T_{\boldsymbol{k}'\boldsymbol{k}} \to t[E(|\boldsymbol{k}|)]\delta_{\boldsymbol{k}'\boldsymbol{k}}$$

$$t(E) = \frac{g^2}{E + B} + \rho \int_0^\infty dE'' \frac{\sqrt{E''}|t(E'')|^2}{E - E'' + i\epsilon}$$

The solution of the integral equation (well-known? We should solve $t^{-1}(E)$ using optical theorem and analyticity)

$$t(E) = \left[\frac{E + B}{g^2} + \frac{\pi\rho(B - E)}{2\sqrt{B}} + i\pi\rho\sqrt{E} \right]^{-1}$$

Amplitude, phase shift, and scattering length

The result of low-energy scattering amplitude

$$t(E) = \left[\frac{E+B}{g^2} + \frac{\pi\rho(B-E)}{2\sqrt{B}} + i\pi\rho\sqrt{E}\right]^{-1}$$

S-wave phase shift

$$e^{2i\delta(E)} = 1 - 2i\pi\rho\sqrt{E}t(E)$$

$$\cot\delta = -\frac{1}{\pi\rho\sqrt{E}}\left[\frac{E+B}{g^2} + \frac{\pi\rho(B-E)}{2\sqrt{B}}\right]$$

Scattering length a_s, effective range r_e

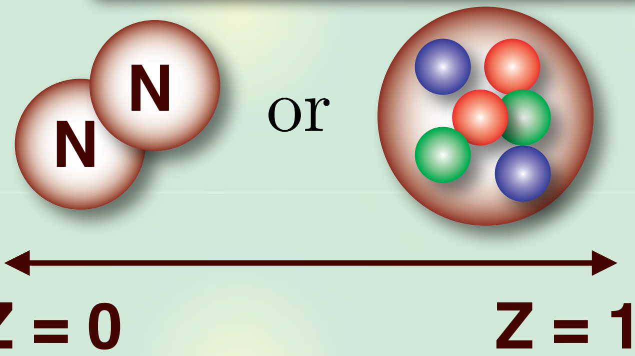
$$k\cot\delta = -\frac{1}{a_s} + r_e\frac{k^2}{2} \qquad E = \frac{k^2}{2\mu}, \quad R = \frac{1}{\sqrt{2\mu B}}$$

We obtain the final result (no expansion needed)

$$a_s = 2R\left[1 + \frac{2\sqrt{B}}{\pi\rho g^2}\right] \qquad r_e = R\left[1 - \frac{2\sqrt{B}}{\pi\rho g^2}\right]$$

Main result: theorem

$$|\text{deuteron}\rangle =$$



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Small B
--> dominance of pole contribution
--> interaction V is only reflected in the coupling g

Applicability of this method

Pro: model independence

No explicit form of Hamiltonian is needed.

Contra: assumptions in the analysis

(i) The particle must be stable.

(ii) The particle must couple to a two-particle channel with threshold not too much above the particle mass (and the absence of nearby coupled channels).

(iii) The particle must be in s-wave scattering.

No other example than deuteron is found in Nature.

One begins to suspect that Nature is doing her best to keep us from learning whether the "elementary" particles deserve that title.

What can we do?

After 40 years, application to $a_0(980)$ and $f_0(980)$ mesons

V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, A. Kudryavtsev, PLB586, 53 (2004)
"Evidence that the $a_0(980)$ and $f_0(980)$ are not elementary particles"

The method was extended to narrow resonances.

My personal interest:

1) Relation with natural renormalization scheme

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

--> For a bound state solution, Z~0 is confirmed.

2) Extension to hadron resonances, Λ(1405), σ meson, ...

large width, coupled-channel effect, ... ?

--> Complex scaling method provides the complete set decomposition including resonances?

T. Hyodo, D. Jido, in preparation

"Evidence that the Λ(1405) is not an elementary particle"...