Softening of the dynamical sigma meson

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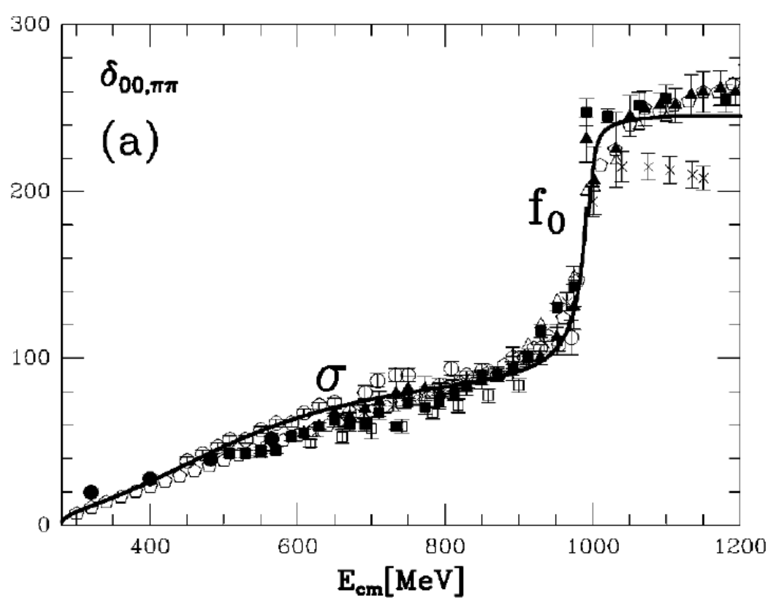
supported by Global Center of Excellence Program
"Nanoscience and Quantum Physics"

2009, Nov. 17th

The sigma meson

$f_0(600)$ or $\sigma : J^P = 0^+, I = 0$

Mass : 400-1200 MeV
Width : 600-1000 MeV
Decay modes : $\sigma \rightarrow \pi\pi$ **dominant**
$\sigma \rightarrow \gamma\gamma$ **seen**



σ meson
- is the lowest resonance in QCD
- plays an important role in hadron mass generation due to spontaneous chiral symmetry breaking
- provides attraction in phenomenological nuclear force

Recent progress in scattering theory + data precession
--> determination of pole position is now possible.

I. Caprini, G. Colangelo, H. Leutwyler, Phys. Rev. Lett. 96, 132001 (2006), ...

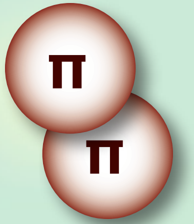
Structure of the sigma meson

Sigma meson in naive constituent quark model (~$\bar{q}q$) has some difficulties: light mass (v.s. p-wave excitation), mass ordering of scalar nonet (v.s. $\sigma > \kappa > f_0 \sim a_0$)

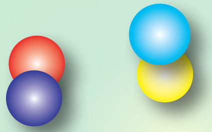
Alternative descriptions of the sigma meson

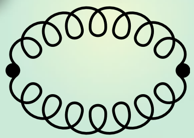
- **Chiral sigma**
  (e.g. linear sigma model)
  M. Gell-Mann, M. Levy, Nuovo Cim. 16, 705 (1960), ...

- **Dynamical sigma**
  (e.g. mesonic molecule generated by π-π attraction)
  J.A. Oller, E. Oset, J.R. Pelaez, Phys. Rev. D59, 074001 (1999), ...



- **CDD pole contribution (pre-formed state)**
  (e.g. constituent four-quark model, glueball, ...)
  L.R. Jaffe, Phys. Rev. D15, 267 (1977), ...



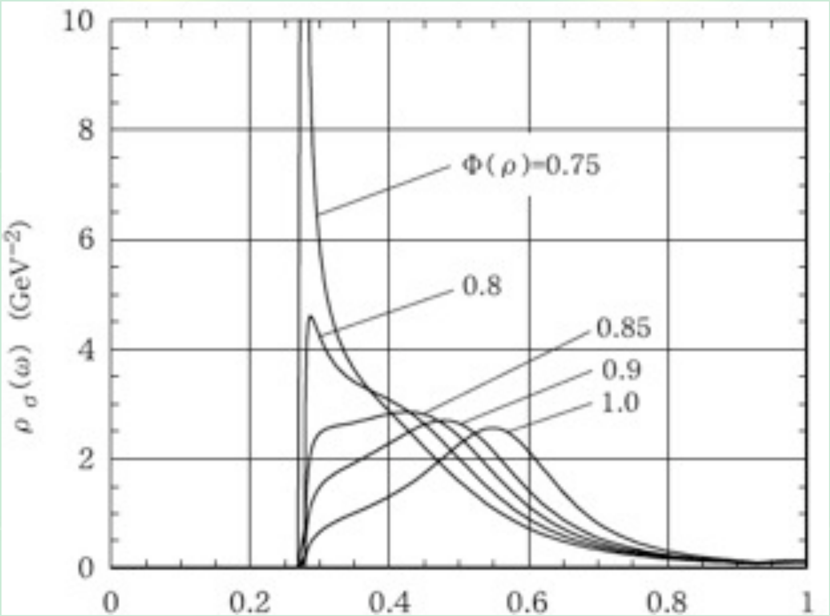


We want to clarify the structure <-- softening

Softening of the sigma meson

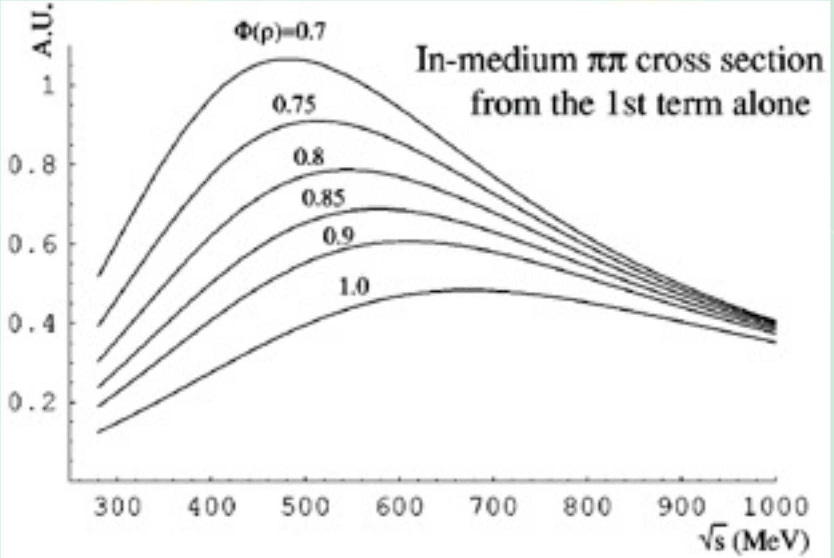
Softening of chiral sigma

T. Hatsuda, T. Kunihiro, H. Shimizu
Phys. Rev. Lett. 82, 2840 (1999)



Partial restoration of chiral sym.
--> Spectral enhancement in
I=J=0 channel near threshold

fluctuation of the order parameter of chiral phase transition



Threshold enhancement of π-π cross section, also for the dynamical sigma meson

D. Jido, T. Hatsuda, T. Kunihiro,
Phys. Rev. D63, 011901 (2001)

Mechanism of the softening (chiral sigma)

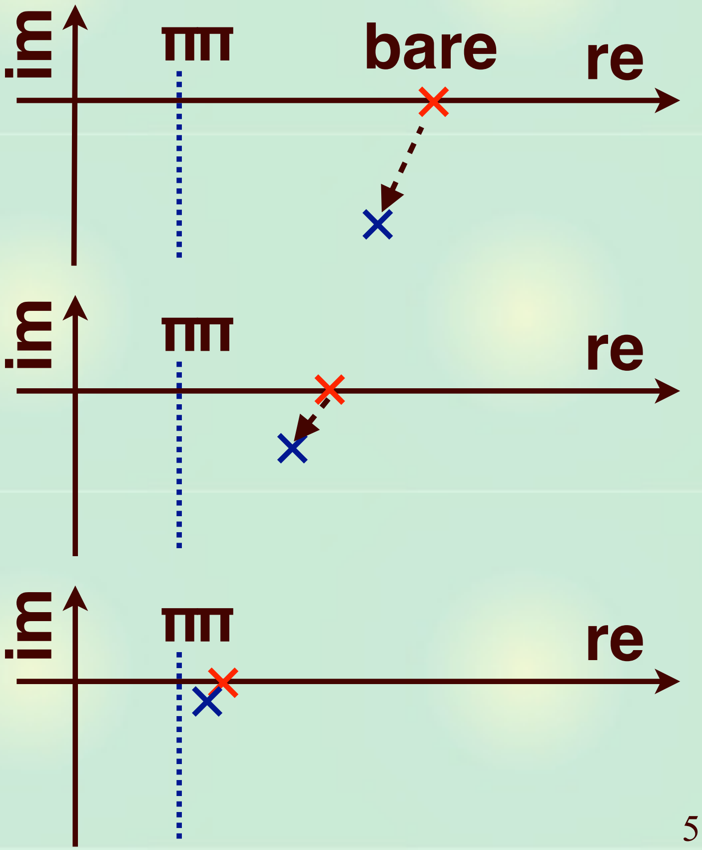
In the previous studies, it seems that the softening takes place, irrespective to the structure of the sigma meson.

Mechanism of the softening?

Softening of the **chiral sigma** (linear sigma model)

Sigma meson:
bare sigma pole acquires finite width through the coupling to π-π

Chiral symmetry restoration:
--> lowering bare sigma mass
--> reduction of the phase space
--> narrow spectrum



Mechanism of the softening (dynamical sigma)

Softening of the dynamical sigma (ChPT + unitarization)

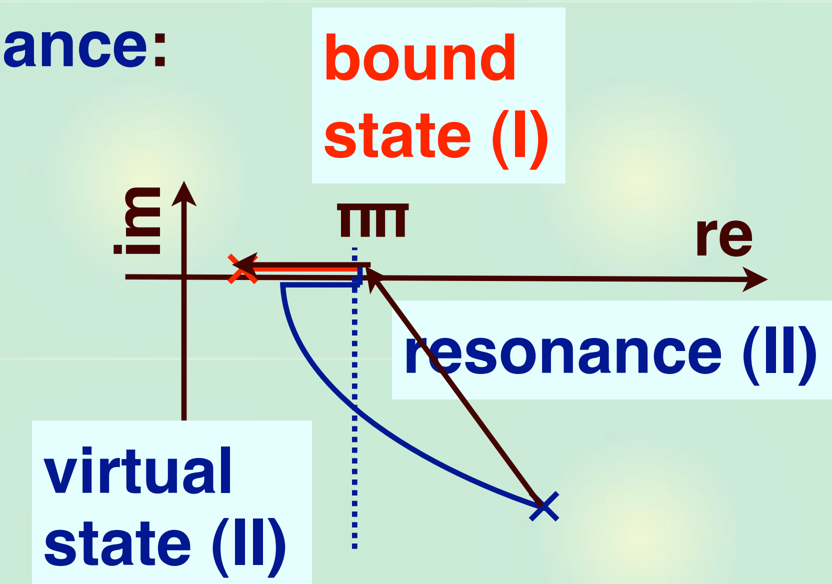
Sigma meson: dynamically generated by π-π attraction

Chiral symmetry restoration:
--> $f_\pi \sim \langle\sigma\rangle$ decreases
--> (attractive) interaction $\sim (f_\pi)^{-2}$ increases
--> resonance turns into bound state, spectrum gets narrow

Special nature of the s-wave resonance:

attractive

- resonance
- virtual state
  pole on the 2nd Riemann sheet below threshold.
  ex.) spin singlet NN
- bound state



--> novel softening pattern?

Tree level interaction

Lagrangian of 2-flavor linear sigma model

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\left[\partial M \partial M^\dagger - \mu^2 M M^\dagger - \frac{2\lambda}{4!}(MM^\dagger)^2 + h(M + M^\dagger)\right], M = \sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}$$

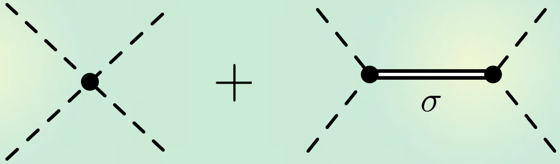
3 parameters <-- m_π, m_σ, $\langle\sigma\rangle$ at mean field level.

π-π scattering amplitude in general (crossing symmetry)

$$T_{\mathrm{tree}}(s,t,u) = A(s,t,u)\delta_{ab}\delta_{cd} + A(t,s,u)\delta_{ac}\delta_{bd} + A(u,t,s)\delta_{ad}\delta_{bc}$$

Tree-level π-π scattering amplitude

$$A(s,t,u) = -\frac{m_\sigma^2 - m_\pi^2}{\langle\sigma\rangle^2} - \frac{(m_\sigma^2 - m_\pi^2)^2}{\langle\sigma\rangle^2}\frac{1}{s - m_\sigma^2}$$



$$= \boxed{\frac{s - m_\pi^2}{\langle\sigma\rangle^2}} - \frac{(s - m_\pi^2)^2}{\langle\sigma\rangle^2}\frac{1}{s - m_\sigma^2}$$

leading order term of ChPT

- **consistent with low energy expansion**
- **1st and 2nd terms are chiral invariant**

Tree level interaction

Introduce a parameter using chiral invariant decomposition

$$A(s;x) = \frac{s - m_\pi^2}{\langle\sigma\rangle^2} - \boxed{x}\frac{(s - m_\pi^2)^2}{\langle\sigma\rangle^2}\frac{1}{s - m_\sigma^2}$$

$x \to 1$: **linear sigma model (chiral sigma)**

$x \to 0$: **leading order term in ChPT (dynamical sigma)**

$x \to 1/2$: **model C in Yokokawa et al. (σ-ρ degeneracy, KSRF relation, and duality)**

Parameter x is useful to extrapolate models.
The origin of the resonance can be investigated.

Projecting the amplitude onto I=J=0, we obtain

$$T_{\text{tree}}(s;x) = \frac{m_\sigma^2 - m_\pi^2}{\langle\sigma\rangle^2}\left[\frac{2s - m_\pi^2}{m_\sigma^2 - m_\pi^2}(1-x) - 5x \right.$$
$$\left. -3x\frac{m_\sigma^2 - m_\pi^2}{s - m_\sigma^2} - 2x\frac{m_\sigma^2 - m_\pi^2}{s - 4m_\pi^2}\ln\left(\frac{m_\sigma^2}{m_\sigma^2 + s - 4m_\pi^2}\right)\right]$$

Unitarization

Unitarity of S-matrix: conservation of probability.
Tree-level amplitude violates unitarity at certain energy.

Optical theorem:

$$\mathrm{Im}\ T^{-1}(s) = -\frac{\Theta(s)}{2} \quad \text{for} \quad s > 4m_\pi^2 \qquad \Theta(s) = (16\pi)^{-1}\sqrt{1-\frac{4m_\pi^2}{s}}$$

Scattering amplitude (N/D method + matching with T_{tree})

J. A. Oller, E. Oset, Phys. Rev. D60, 074023 (1999)

$$T(s;x) = \frac{1}{T^{-1}_{\mathrm{tree}}(s;x) + G(s)}$$

$$G(s) = \frac{1}{2}\frac{1}{(4\pi)^2}\left\{\boxed{a(\mu)} + \ln\frac{m_\pi^2}{\mu^2} + \sqrt{1-\frac{4m_\pi^2}{s}}\left[\ln\frac{\sqrt{1-\frac{4m_\pi^2}{s}}+1}{\sqrt{1-\frac{4m_\pi^2}{s}}-1}\right]\right\}$$

Subtraction constant: to exclude the CDD pole

$$G(s) = 0 \quad \text{at} \quad s = m_\pi^2 \qquad\qquad a(m_\pi) = -\frac{\pi}{\sqrt{3}}$$

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

Amplitude in vacuum

Numerical result in vacuum

Input: $m_\pi = 140$ MeV, $m_\sigma = 550$ MeV, $\langle\sigma\rangle = 93$ MeV

	x	sigma origin	scattering length $(m_\pi)^{-1}$	pole position [MeV]
model A	1	chiral	0.244	423 - 126 i
model B	0	dynamical	0.174	364 - 356 i
(experiment)			0.216 [1]	441 - 272 i [2]

[1] S. Pislak *et al.*, Phys. Rev. D67, 072004 (2003)
[2] I. Caprini, G. Colangelo, H. Leutwyler, Phys. Rev. Lett. 96, 132001 (2006)

(we do not try to fine-tune the parameters)

Prescription for symmetry restoration

We introduce the effect of chiral symmetry restoration from the outside of the model, by modifying m_π, m_σ, $\langle\sigma\rangle$.

1) chiral condensate (pion decay constant): decreases

$$\langle\sigma\rangle = \Phi\langle\sigma\rangle_0, \quad 0 \le \Phi \le 1$$

2) mass of pion: no change

$$\frac{\partial m_\pi}{\partial \Phi} = 0$$

3) mass of chiral sigma for model A: decreases

$$m_\sigma\big|_{\Phi\to 0} = m_\pi \quad \text{(case I)} \qquad m_\sigma = \sqrt{\lambda\frac{\langle\sigma\rangle^2}{3} + m_\pi^2}$$

with λ and m_π being fixed.

The effect of symmetry restoration is modeled by the change of Φ.

Restoration limit and chiral partner

Properties of the chiral partner in the restoration limit

1) mass degeneracy with pion
2) coupling to π-π scattering state vanishes

Model for chiral sigma (model A ~ linear sigma model)

- pole term in the tree-level interaction

$$T_{\text{tree}}(s;1) = -\frac{\lambda^2\langle\sigma\rangle^2}{3}\frac{1}{s-m_\pi^2-\frac{\lambda}{3}\langle\sigma\rangle^2}+\dots$$

- renormalization condition

$$G(s)=0 \quad \text{at} \quad s=m_\pi^2$$

$$T(s;1)|_{s\to m_\pi,\Phi\to 0} = T_{\text{tree}}(s;1)|_{s\to m_\pi,\Phi\to 0} = \frac{0}{s-m_\pi^2} \equiv \frac{g^2}{s-M_{\text{pole}}^2}$$

Thus, the pole in the π-π amplitude behaves as

$g\to 0, \quad M_{\text{pole}}\to m_\pi \quad \text{for} \quad \Phi\to 0$ **like the chiral partner**

Restoration limit and chiral partner

Model for dynamical sigma (model B)

$$T_{\text{tree}}(s;x) \propto \frac{1}{\langle\sigma\rangle^2}$$

The amplitude in the restoration limit is solely determined by the loop function G, irrespective to x:

$$T(s;x) = \frac{1}{T^{-1}_{\text{tree}}(s,x) + G(s)} \to \frac{1}{G(s)} \quad \text{for} \quad \Phi \to 0$$

- renormalization condition requires a pole at m_π

$$G(s) = 0 \quad \text{at} \quad s = m_\pi^2$$

- coupling can be calculated: proportional to m_π

$$g^2|_{\Phi\to 0} = (s - m_\pi^2)T(s)\Big|_{s\to m_\pi^2,\Phi\to 0}$$

$$= \frac{s - m_\pi^2}{G(s)}\Bigg|_{s\to m_\pi^2} = (4\pi)^2\left(\frac{1}{2} - \frac{\pi}{3\sqrt{3}}\right)^{-1} m_\pi^2$$

How to interpret this result?

Restoration limit and chiral partner

Properties of chiral sigma

$$g \to 0, \quad M_{\text{pole}} \to m_\pi \quad \text{for} \quad \Phi \to 0$$

Properties of dynamical sigma

$$g^2 \to (4\pi)^2 \left(\frac{1}{2} - \frac{\pi}{3\sqrt{3}}\right)^{-1} m_\pi^2, \quad M_{\text{pole}} \to m_\pi \quad \text{for} \quad \Phi \to 0$$

- mass degeneracy with pion
- coupling to π-π: vanishes in the chiral limit

Dynamical sigma as chiral partner of pion?

- Renormalization condition plays an important role.
  <-- consistency with chiral theorem. Generally,

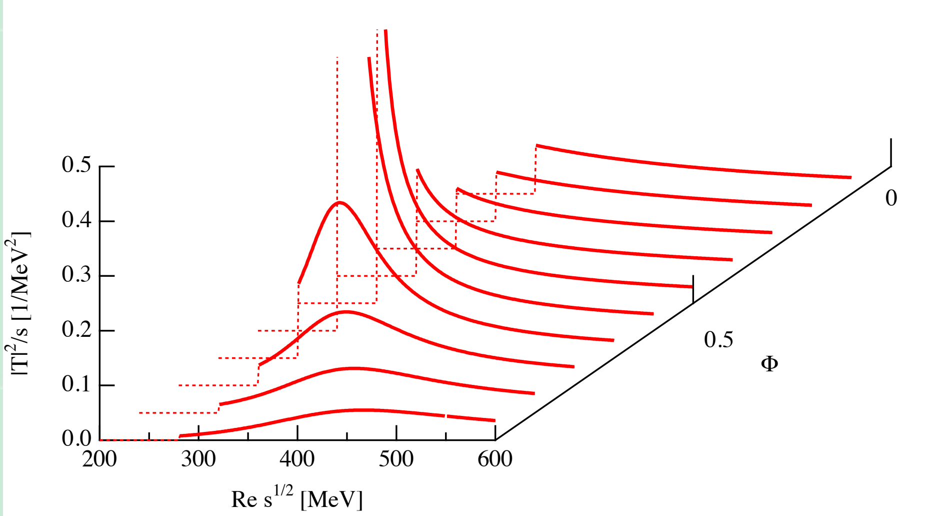
$G(\mu^2) = 0 \quad \text{at} \quad 0 \le \mu \le 2m_\pi$ --> deviation ~ m_π

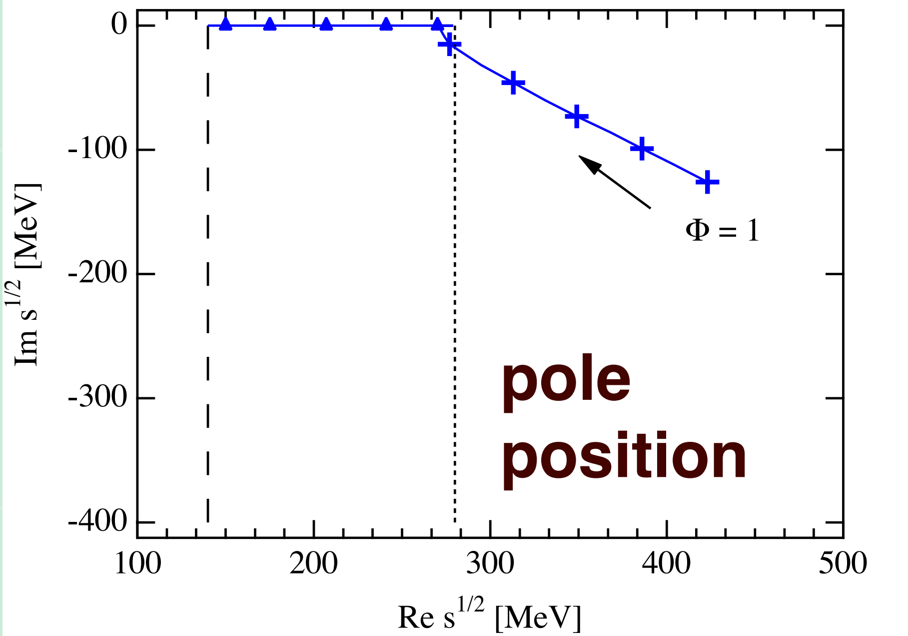
- dynamical resonance as chiral partner (mass degeneracy)

J.A. Oller, hep-ph/0007349
S. Leupold, M.F.M. Lutz, M. Wagner, 0811.2398 [nucl-th]

Softening in model A

spectrum



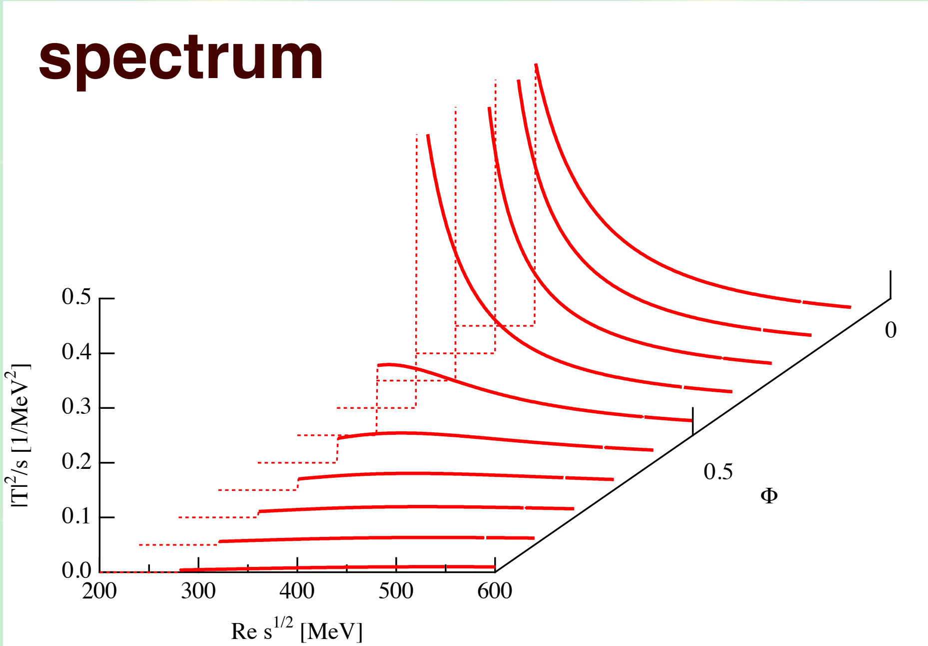


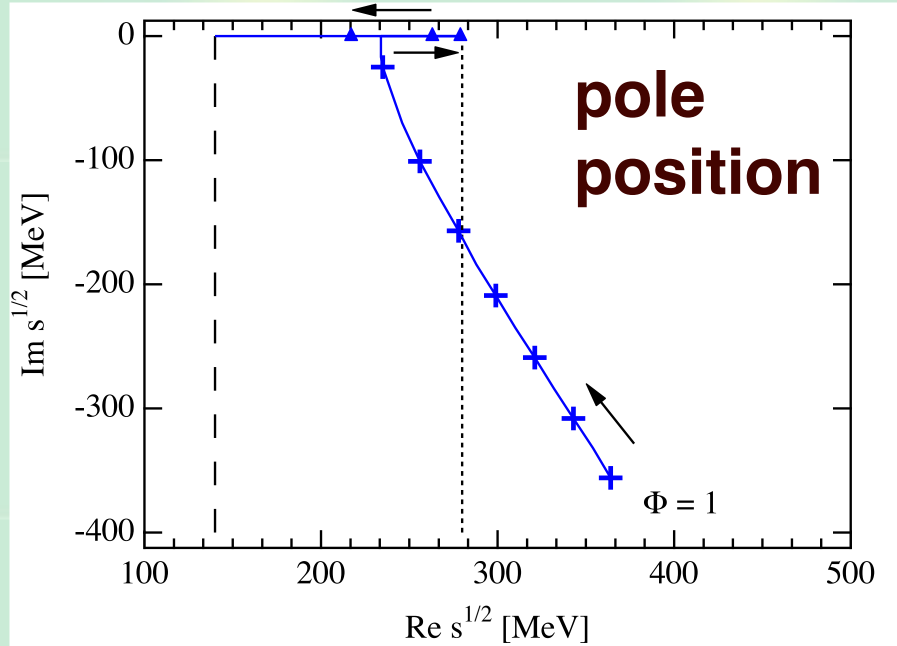
- Linear sigma model + unitarization : chiral sigma

- Softening takes place, as expected.

- peak at threshold : $\Phi \sim 0.6$
  <=> bare sigma pole moves below the threshold

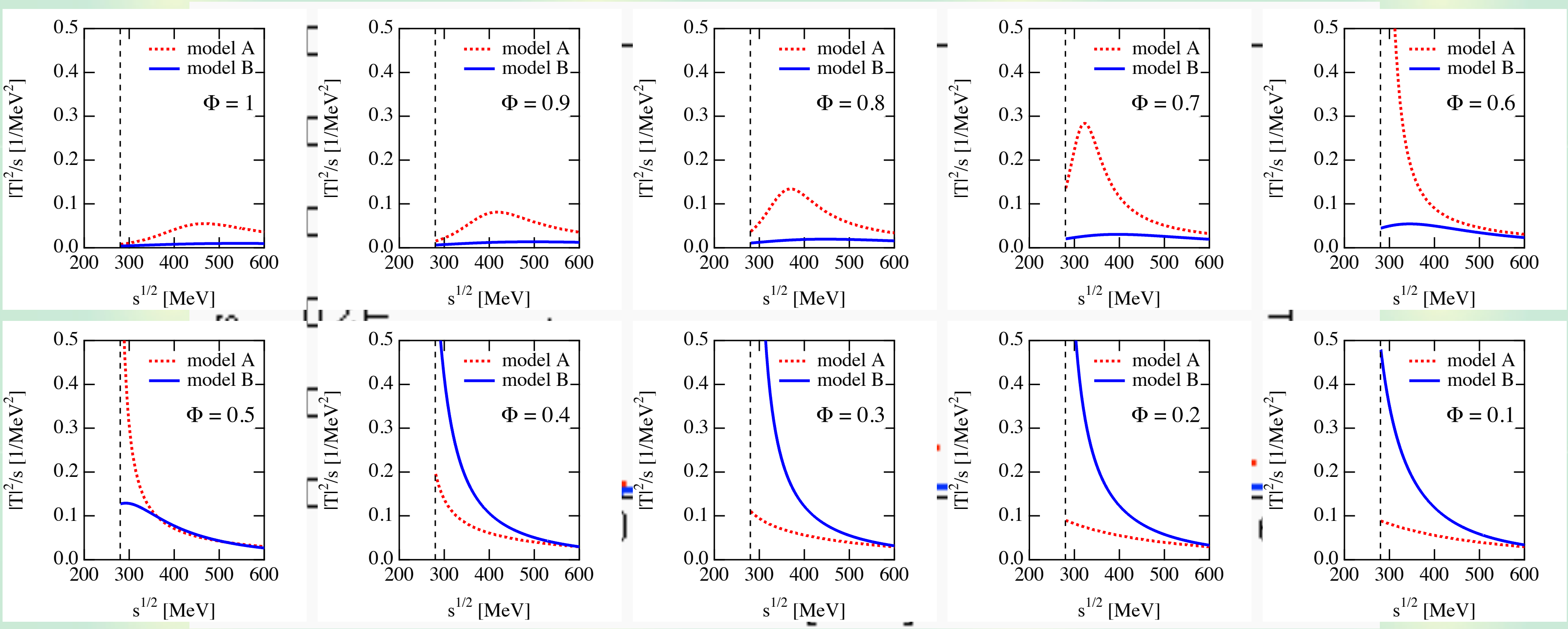
Softening in model B



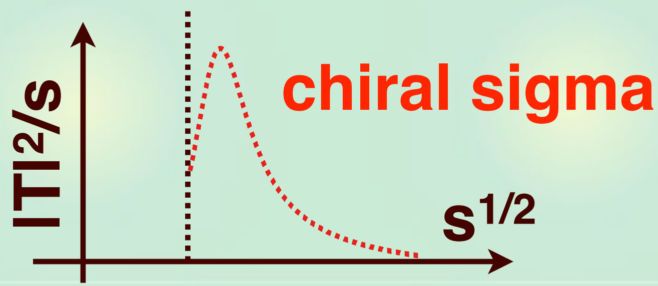


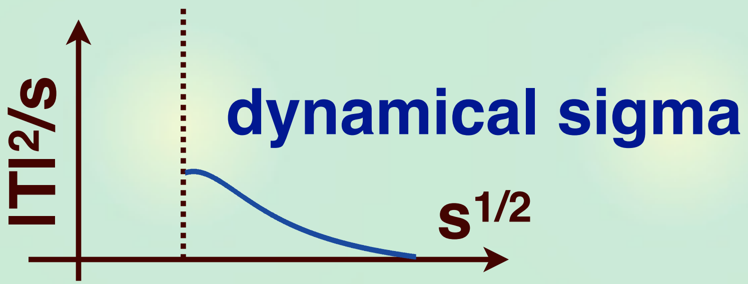
- ChPT + unitarization : dynamical sigma
- Softening takes place, but virtual state appears.
- at Re[M_{pole}] = $2m_\pi$ ($\Phi \sim 0.6$), due to finite width, spectrum does not show the peak structure
- peak at threshold : $\Phi \sim 0.3$ <=> formation of bound state

Comparison of model A and model B



- Strong threshold enhancement : different from each other.
- Shape of the spectrum?





Summary

We study the structure of the sigma meson using chiral symmetry restoration.

- **Dynamical chiral models with**
  - **(i) chiral sigma**
  - **(ii) dynamical sigma**
- **In the restoration limit, both behave similarly: dynamical chiral partner?**
- **Dynamical sigma softens qualitatively differently from chiral sigma.**

  <-- virtual state (s-wave resonance)