Λ*N bound state based on chiral dynamics

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- $\bar{K}N$ interaction is strongly attractive $\leftarrow \Lambda(1405)$. Formation of deeply bound state is possible.


- Structure of the $\Lambda(1405)$, kaon condensation, ...

The simplest $\bar{K}$-nucleus: $\bar{K}NN$ three-body system
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- Structure of the $\Lambda(1405)$, kaon condensation, ...

The simplest $\bar{K}$-nucleus: $\bar{K}NN$ three-body system

Theory: rigorous few-body calculations with realistic interactions
- System **bounds**
  - Yamazaki-Akaishi, Shevchenko, et al.,
  - Ikeda-Sato, Doté et al., Wycech-Green,
- Quantitative **difference**: uncertainties in $\bar{K}N$ int. at far below threshold
Introduction

**K in nuclei**

- $\bar{K}N$ interaction is strongly attractive $\leftarrow \Lambda(1405)$. Formation of deeply bound state is possible.


- Structure of the $\Lambda(1405)$, kaon condensation, ...

The simplest $\bar{K}$-nucleus: $\bar{K}NN$ three-body system

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- Quantitative **difference:** uncertainties in $\bar{K}N$ int. at far below threshold

**Experiments:** some “evidences” in $\Lambda N$ mass spectra
-FINUDA, DISTO, OBELIX, etc.
- Interpretation?
Regarding a $\bar{K}N$ pair as the $\Lambda^* = \Lambda(1405)$, construct the "$\Lambda^* N$ potential" with the meson-exchange picture.

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**Λ* hypernuclei model**

Regarding a $\bar{K}N$ pair as the $\Lambda^* = \Lambda(1405)$, construct the “$\Lambda^*N$ potential” with the meson-exchange picture


$\text{Λ}^*$ seems to be surviving in $\bar{K}NN$ system.


$\Lambda^*$ coupling constants: unknown (← FINUDA data).
Regarding a $\bar{K}N$ pair as the $\Lambda^*=\Lambda(1405)$, construct the “$\Lambda^*N$ potential” with the meson-exchange picture

\[ \Lambda^* \] hypernuclei model


\[ \Lambda^* \] seems to be surviving in $\bar{K}NN$ system.


$\Lambda^*$ coupling constants: unknown (\(<--\) FINUDA data).

To determine the coupling and make predictions, we need a framework to describe the $\Lambda^*$ \(<--\) chiral unitary approach
Chiral unitary approach

Description of $S = -1$, $\bar{K}N$ s-wave scattering : $\Lambda(1405)$ in $I=0$

- Interaction $\leftarrow$ chiral symmetry
  

- Amplitude $\leftarrow$ unitarity in coupled channels
  
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\[
T = \frac{1}{1 - VG} V
\]

\[\begin{array}{ccc}
\text{chiral} & = & \text{cutoff} \\
\end{array}\]


It works successfully, also in $S=0$ sector, meson-meson scattering sectors, systems including heavy quarks, ...
Chiral unitary approach

**KN scattering**: comparison with data

**Total cross section of K-p scattering**

**Branching ratio**

<table>
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<tr>
<th></th>
<th>γ</th>
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**πΣ spectrum**

Λ(1405)

**KN scattering: comparison with data**

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**πΣ spectrum**


Good agreement with data above, at, and below KN threshold

$\Lambda(1405)$ mass, width, couplings: prediction of the model
Chiral unitary approach

Two poles for one resonance

Poles of the amplitude in the complex plane: resonance

\[ T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - M_R + i\Gamma_R/2} \]
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Physical \( \Lambda^* \): two poles

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Poles of the amplitude in the complex plane: resonance

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Physical \( \Lambda^* \): two poles

short summary
- \( \Lambda(1405) \): \( \Lambda^*_1 \), \( \Lambda^*_2 \)
- \( \Lambda^*_i \) masses, widths, \( \Lambda^*_i \)-MB couplings predicted

\( \Rightarrow \Lambda^* \) hypernuclei based on chiral dynamics

$\Lambda^* N$ potential

$\Lambda^*_i N$ potential with one boson exchange ($i=1,2$)
Λ*N potential

Λ*N potential with meson-exchange picture

Λ*iN potential with one boson exchange (i=1,2)

○ NNσ, NNω couplings: Jülich (model A) YN potential
\( \Lambda^*N \) potential with meson-exchange picture

\( \Lambda^*_iN \) potential with one boson exchange (\( i=1,2 \))

- NN\( \sigma \), NN\( \omega \) couplings: Jülich (model A) YN potential
- \( \Lambda^*_iKN \) coupling: Chiral unitary approach
Λ*N potential

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- Λ*iΛ*iσ, Λ*iΛ*iω, couplings
  --> estimated by microscopic MB=(KN,πΣ,ηΛ) couplings
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σ decay to ππ

\[ g_{KKσ} = 0 \]
\[ g_{ηησ} = 0 \]
Λ*N potential

Λ*N potential: K exchange

Λ*KN vertex: scalar type ($\Lambda^*=1/2^-$)

$$\mathcal{H}_{\Lambda^* NK} = g_{\Lambda^* KN} (\bar{\Lambda}^* \bar{K} N + \bar{N} K \Lambda^*)$$

Exchange factor
Λ*N potential

Λ*N potential: K exchange

Λ*KN vertex: scalar type (Λ*=1/2⁻)

\[ \mathcal{H}_{\Lambda^* N K} = g_{\Lambda^* K N} (\overline{\Lambda^*} \overline{K} N + \overline{N} K \Lambda^*) \]

Exchange factor

\[ -\mathcal{P}_x \frac{1 + \vec{\sigma}_{\Lambda^*} \cdot \vec{\sigma}_N}{2} \]

spin dependence
(P_x=1 for s-wave)

In total, S=0 (σσ=-3) attractive, S=1 (σσ=1) repulsive.
\( \Lambda^* N \) potential

\( \Lambda^* N \) potential: K exchange

\( \Lambda^* KN \) vertex: scalar type (\( \Lambda^* = 1/2^- \))

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spin dependence

(\( \mathcal{P}_x = 1 \) for s-wave)

In total, \( S=0 \) (\( \sigma \sigma = -3 \)) attractive, \( S=1 \) (\( \sigma \sigma = 1 \)) repulsive.

Mass difference of \( \Lambda^* \) and \( N \)

--> effective K mass

\[
\tilde{m}_K = \sqrt{m_K^2 - (M_{\Lambda^*} - M_N)^2}
\]

\( \Lambda^* N \) potential

**\( \Lambda^* N \) potential: mixing interaction**

Chiral unitary approach --> two \( \Lambda^* \) states : \( \Lambda^*_1, \Lambda^*_2 \)

With sufficient attraction, two \( \Lambda^* N \) bound states in \( B=2 \) system : \( \Lambda^*_1 N, \Lambda^*_2 N \)

There can be the mixing of \( \Lambda^*_1 N \) \( \leftrightarrow \) \( \Lambda^*_2 N \)
\( \Lambda^*N \) potential

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Λ* potential

Λ*N potential: mixing interaction

Chiral unitary approach --> two Λ* states: Λ*₁, Λ*₂

With sufficient attraction, two Λ*N bound states in B=2 system: Λ*₁N, Λ*₂N

There can be the mixing of Λ*₁N <--→ Λ*₂N

No mixing  With mixing

\[ \bar{K}NN \]

\[ \Lambda^*_2N \]

Binding  Mixing

\[ \Lambda^*_1N \]

Binding  Mixing
\( \Lambda^* N \) potential: mixing interaction

Chiral unitary approach \( \rightarrow \) two \( \Lambda^* \) states : \( \Lambda^*_1, \Lambda^*_2 \)

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There can be the mixing of \( \Lambda^*_1N \leftrightarrow \Lambda^*_2N \)
Numerical results for the $\Lambda^* N$ bound states

$\Lambda^* N$ potential

Diagonal potentials for $\Lambda^* 2N$ in $S=0$ and $S=1$, s-wave

\begin{align*}
S &= 0, \Lambda(1426) \\
S &= 1, \Lambda(1426)
\end{align*}
Numerical results for the $\Lambda\ast N$ bound states

$\Lambda\ast N$ potential

Diagonal potentials for $\Lambda\ast_2 N$ in $S=0$ and $S=1$, s-wave

$S=0$: K exchange is attractive
--> attractive pocket at intermediate range
Numerical results for the Λ*N bound states

**Λ*N potential**

Diagonal potentials for Λ*_2*N in S=0 and S=1, s-wave

**S=0:** K exchange is **attractive**

→ **attractive pocket** at intermediate range

**S=1:** K exchange is **repulsive**

→ no intermediate attraction.

(Short range dip: artificial, not physical)
Numerical results for the $\Lambda*N$ bound states

$\Lambda*N$ bound states without mixing

Solve the schrödinger equation for the s-wave $\Lambda^*iN$ potential \textbf{without} the mixing interaction.
Numerical results for the Λ*N bound states

Λ*N bound states without mixing

Solve the schrödinger equation for the s-wave Λ*\_i\_N potential without the mixing interaction.

- no physical bound states in S=1 channels
Numerical results for the $Λ^*N$ bound states

**$Λ^*N$ bound states without mixing**

Solve the schrödinger equation for the s-wave $Λ^*_iN$ potential **without** the mixing interaction.

- no physical bound states in $S=1$ channels
- for $S=0$ we obtain the bound states in both $Λ^*_i$

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Λ*N bound states without mixing

Solve the schrödinger equation for the s-wave Λ*iN potential without the mixing interaction.

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Two Λ*N states in spin S=0 channel
Numerical results for the Λ*N bound states

**Λ*N bound states with mixing**

With mixing, the higher state becomes a resonance. Real scaling method ≈ changing the box size.

λ: strength of the mixing interaction, physical for λ=1
Numerical results for the $\Lambda^*N$ bound states

$\Lambda^*N$ bound states with mixing

With mixing, the higher state becomes a resonance.
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**Λ*N bound states with mixing**

With mixing, the higher state becomes a *resonance*. Real scaling method ≈ changing the box size.

λ: strength of the mixing interaction, physical for λ=1

The lower energy state bounds more. The higher energy state disappears (above Λ*2N threshold?)
We study the $\Lambda^*N$ two-body system based on the $\Lambda^*N$ potential with chiral dynamics.

Chiral unitary model: \textit{two states} $\Lambda^*_1, \Lambda^*_2$

Both $\Lambda^*_i$ generate bound states with $N$ in spin $S=0$ channels, \textit{<-- K exchange}.

With the mixing, \textit{lower states bounds more, and higher states dissolves.}

B.E.(from $\bar{KNN}$) = 52-58 MeV
\textit{<-- strong mixing between $\Lambda^*_1N - \Lambda^*_2N$}

\textbf{T. Uchino, T. Hyodo, M. Oka, in preparation}
Summary

taken from T. Uchino, Master thesis

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Diagram showing the transition between No Mixing and With Mixing states. The transitions between states are indicated with energies: 13 MeV and 58 MeV. The states are labeled with symbols: $\bar{K}NN$, $A_2^*N$, and $A_1^*N$. The diagram also highlights transitions between states with energies of 0.3 MeV and 13 MeV.