Structure of hadron molecule resonance

Tetsuo Hyodo[a],

Daisuke Jido[b], **and Atsushi Hosaka**[c]

Tokyo Institute of Technology[a] *YITP, Kyoto*[b] *RCNP, Osaka*[c]

supported by Global Center of Excellence Program
"Nanoscience and Quantum Physics"

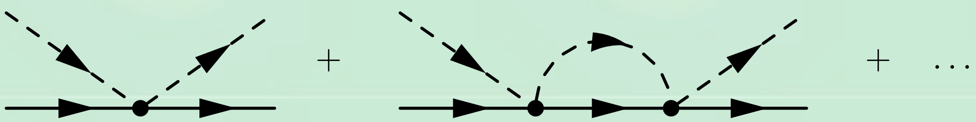
2010, Mar. 23rd

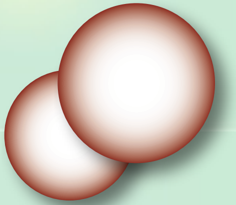
Classification of resonances

Resonances in two-body scattering

- Knowledge of interaction (potential)
- Experimental data (cross section, ...)

Dynamical state: two-body molecule, quasi-bound state, ...

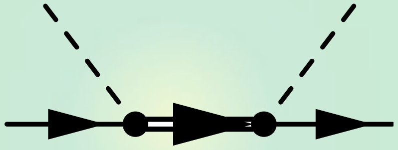


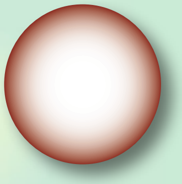


e.g.) Deuteron in NN, positronium in e^+e^-, ...

CDD pole: elementary particle, independent state, ...

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)





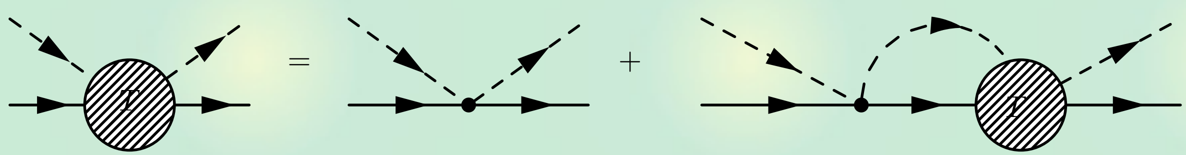
e.g.) J/Ψ in e^+e^-, ...

Chiral unitary approach

Description of meson-baryon scattering, s-wave resonances

- Interaction <-- chiral symmetry
- Amplitude <-- unitarity (coupled channel)

$$T = \frac{1}{V^{-1} - G}$$



V ~ interaction : ChPT at given order

G ~ loop function : subtraction constant (cutoff)

N. Kaiser, P. B. Siegel, W. Weise, Nucl. Phys. A594, 325 (1995),
E. Oset, A. Ramos, Nucl. Phys. A635, 99 (1998),
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001),
M.F.M. Lutz, E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002),
.... many others

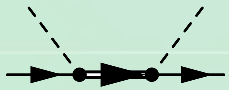
By construction, generated resonances are all dynamical? → **Not always...**

CDD pole in subtraction constant?

Phenomenological (standard) scheme
--> V is given, "a" is determined by data

$$T = \frac{1}{(V^{(1)})^{-1} - G(\underline{a})}$$ leading order

$$T = \frac{1}{(V^{(1)} + V^{(2)})^{-1} - G(\underline{a'})}$$ next to leading order

↑pole  ?

"a" represents the effect which is not included in V.
CDD pole contribution in G?

Natural renormalization scheme
--> fix "a" first, then determine V

to exclude CDD pole contribution from G,
based on theoretical argument.

Natural renormalization condition

Conditions for natural renormalization

- Loop function G should be negative below threshold.
- T matches with V at low energy scale.

"a" is uniquely determined such that

$$G(\sqrt{s} = M_T) = 0 \quad \Leftrightarrow \quad T(M_T) = V(M_T)$$

matching with low energy interaction

K. Igi, K. Hikasa, Phys. Rev. D59, 034005 (1999)
U.G. Meissner, J.A. Oller, Nucl. Phys. A673, 311 (2000)

crossing symmetry (matching with u-channel amplitude)

M.F.M. Lutz, E. Kolomeitsev, Nucl. Phys. A700, 193 (2002)

We regard this condition as the exclusion of the CDD pole contribution from G.

Pole in the effective interaction: single channel

Leading order V : Weinberg-Tomozawa term

$$V_{\mathrm{WT}} = -\frac{C}{2f^2}(\sqrt{s} - M_T)$$

C/f² : coupling constant
no s-wave resonance

$$T^{-1} = V_{\mathrm{WT}}^{-1} - G(a_{\mathrm{pheno}}) = V_{\mathrm{natural}}^{-1} - G(a_{\mathrm{natural}})$$

↑ChPT ↑data fit ↑given

Effective interaction in natural scheme

$$V_{\mathrm{natural}} = -\frac{C}{2f^2}(\sqrt{s} - M_T) + \boxed{\frac{C}{2f^2}\frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\mathrm{eff}}}}$$

pole!

$$M_{\mathrm{eff}} = M_T - \frac{16\pi^2 f^2}{C M_T \Delta a}, \quad a_{\mathrm{pheno}} - a_{\mathrm{natural}}$$

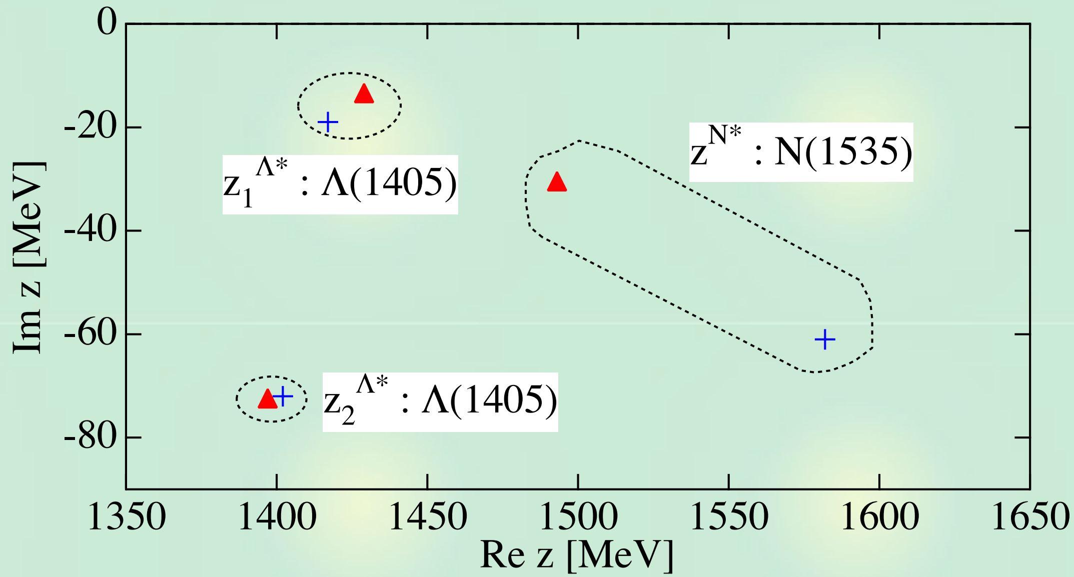
There is always a pole for $a_{\mathrm{pheno}} \neq a_{\mathrm{natural}}$

- **small deviation <=> pole at irrelevant energy scale**
- **large deviation <=> pole at relevant energy scale**

Comparison of pole positions

Pole of the full amplitude : physical state ▲

Pole of the V_{WT} + natural : pure dynamical +



==> Λ(1405) is mostly dynamical state

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

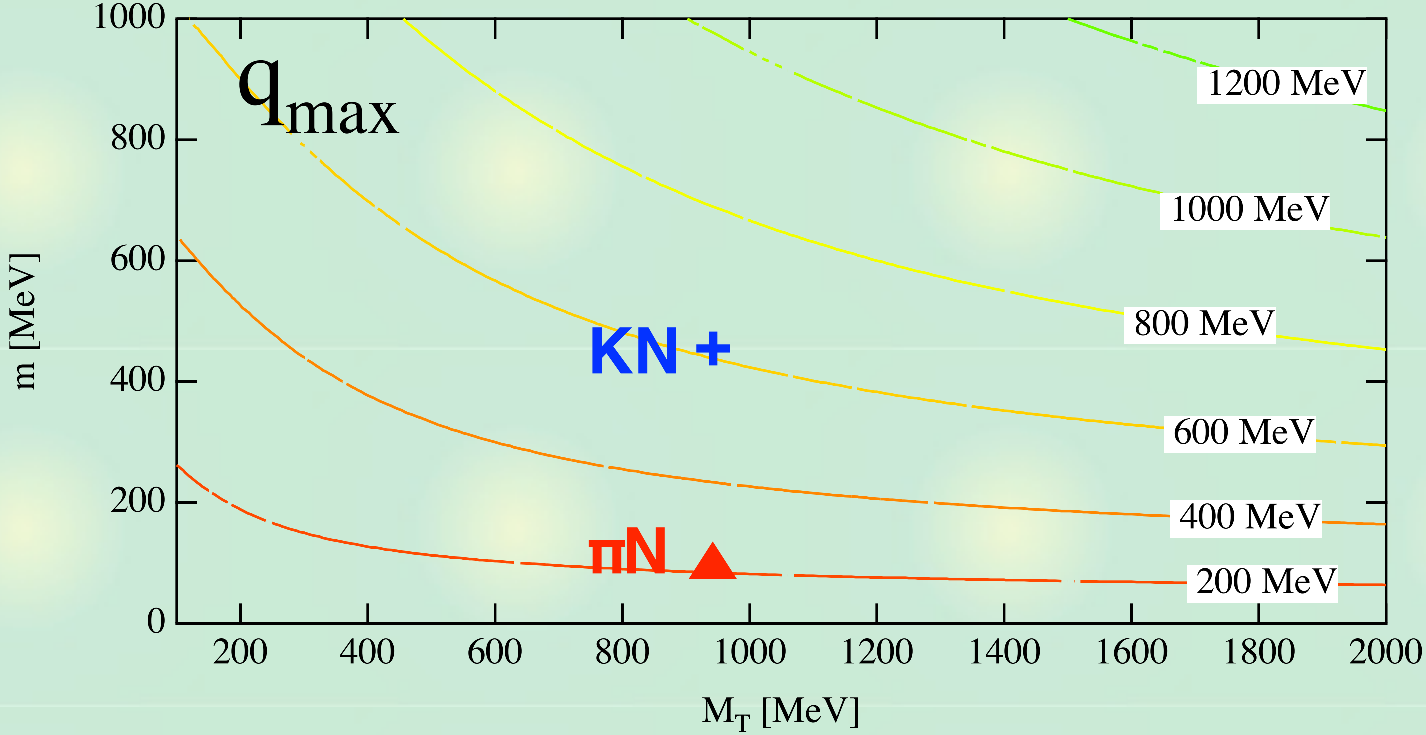
Physical interpretation of the renormalization condition?

- **energy scale**
- **field renormalization constant Z**

Energy scale

Energy scale: matching with cutoff scheme at threshold

$$G^{3d}(M_T + m; q_{\max}) = G^{\dim}(M_T + m; a_{\text{natural}})$$



c.f. "natural value" a ~ -2 for q_{max} ~ 630 MeV

J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001)

For the pion, natural scheme (no CDD pole condition) requires a small cutoff

Weinberg's theorem for deuteron

"Evidence That the Deuteron Is Not an Elementary Particle"

S. Weinberg, Phys. Rev. 137 B672-B678, (1965)

Z: probability of finding deuteron in a bare elementary state

$$|\,d\,\rangle = \sqrt{Z}|\,d_0\,\rangle + \sqrt{1-Z}\int d\boldsymbol{k}|\,\boldsymbol{k}\,\rangle$$

$$1 = |d_0\rangle\langle d_0| + \int d\boldsymbol{k}|\boldsymbol{k}\rangle\langle\boldsymbol{k}|$$

For a bound state with small binding energy, the following equation should be satisfied model independently:

$$a_s = \left[\frac{2(1-Z)}{2-Z}\right]R + \mathcal{O}(m_\pi^{-1}), \quad r_e = \left[\frac{-Z}{1-Z}\right]R + \mathcal{O}(m_\pi^{-1})$$

<-- Experiments (observables)

$$a_s = +5.41[\mathrm{fm}], \quad r_e = +1.75[\mathrm{fm}], \quad R \equiv (2\mu B)^{-1/2} = 4.31[\mathrm{fm}]$$

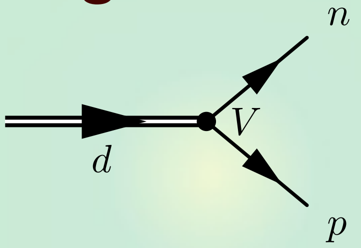
$\Rightarrow Z \lesssim 0.2$ **--> deuteron is composite!**

Derivation of the theorem

The theorem is derived in two steps:

Step 1 (Sec. II): Z --> p-n-d coupling constant

$$g^2 = \frac{2\sqrt{B}(1-Z)}{\pi\rho} \qquad \rho = 4\pi\sqrt{2\mu^3}$$



Step 2 (Sec. III): coupling constant --> a_s, r_e

$$a_s = 2R\left[1 + \frac{2\sqrt{B}}{\pi\rho g^2}\right] \qquad r_e = R\left[1 - \frac{2\sqrt{B}}{\pi\rho g^2}\right]$$

uncertainty for order $R=(2\mu B)^{1/2}$ quantity: m_π^{-1}

The coupling constant g^2 can be calculated in the chiral unitary approach ==> Z?

--> Consider single-channel problem with one bound state.

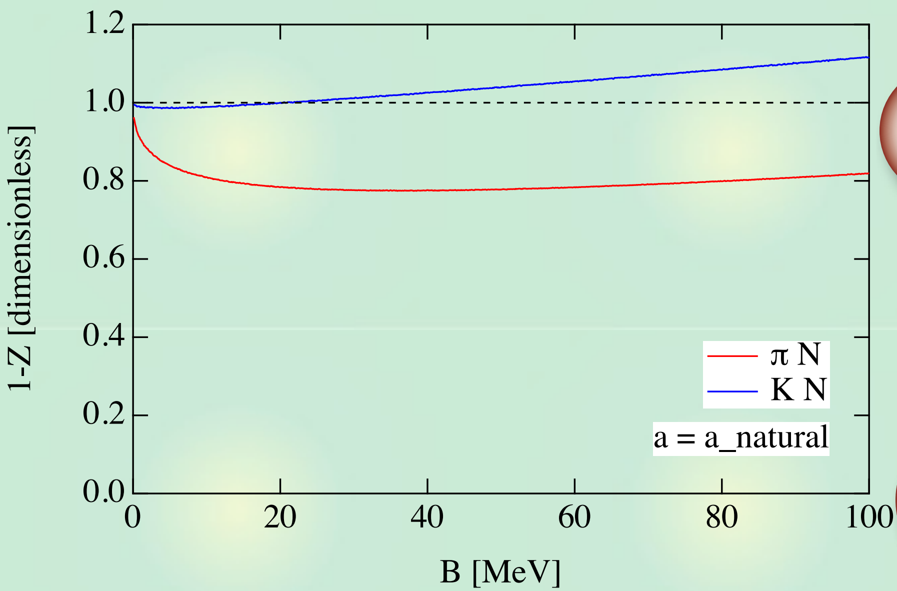
Field renormalization constant

WT int., single channel, one bound state <-- M_B, a

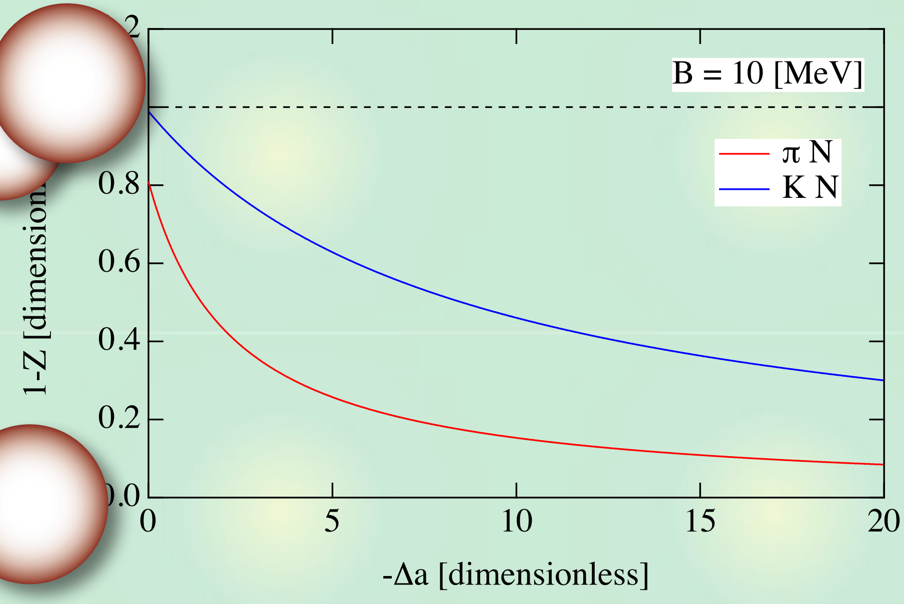
$$g^2(M_B;a) = -\frac{M_B - M_T}{G(M_B;a) + (M_B - M_T)G'(M_B)}$$

$$1 - Z = \sqrt{\frac{2mM_T}{(M_T+m)(M_T+m-M_B)}}\frac{M_T}{8\pi M_B}g^2(M_B;a)$$

1) a = $a_{natural}$, vary M_B



2) M_B = 10 MeV, vary a



natural scheme --> Z ~ 0

large deviation --> Z ~ 1

Summary: Origin of resonances

We study the origin (dynamical/CDD) of the resonances in the chiral unitary approach

- Natural renormalization scheme

  Exclude CDD pole contribution from the loop function, consistent with N/D.

- Comparison with phenomenology
  --> Pole in the effective interaction

  Λ(1405) : predominantly dynamical
  N(1535) : dynamical + CDD pole

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

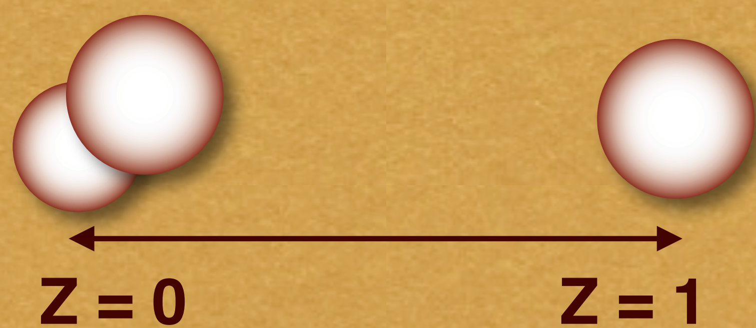
Summary: theoretical foundation of the natural scheme

Meaning of the natural scheme

- **Energy scale in 3d cutoff**

  for the light meson, natural scheme corresponds to a small 3d cutoff

- **Field renormalization constant Z: quantitative measure of "compositeness"**



natural scheme corresponds to Z ~ 0

T. Hyodo, D. Jido, A. Hosaka, in preparation