Structure of hadron molecule resonance

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supported by Global Center of Excellence Program
"Nanoscience and Quantum Physics"

2010, Mar. 23rd
Introduction

Classification of resonances

Resonances in two-body scattering

- Knowledge of interaction (potential)
- Experimental data (cross section, ...)

**Dynamical state**: two-body molecule, quasi-bound state, ...

![Diagram of two-body scattering]

- e.g.) Deuteron in NN, positronium in e^+e^-, ...

**CDD pole**: elementary particle, independent state, ...

![Diagram of CDD pole]

- e.g.) J/Ψ in e^+e^-, ...

Chiral unitary approach

Description of meson-baryon scattering, s-wave resonances

- Interaction <- chiral symmetry
- Amplitude <- unitarity (coupled channel)

\[ T = \frac{1}{V^{-1} - G} \]

\[ V \sim \text{interaction} : \text{ChPT at given order} \]
\[ G \sim \text{loop function} : \text{subtraction constant (cutoff)} \]

.... many others

By construction, generated resonances are all dynamical?  Not always...
CDD pole in subtraction constant?

Phenomenological (standard) scheme

--> V is given, “a” is determined by data

\[
T = \frac{1}{(V^{(1)})^{-1} - G(a)}
\]

leading order

\[
T = \frac{1}{(V^{(1)} + V^{(2)})^{-1} - G(a')}
\]

next to leading order

↑ pole ?

“a” represents the effect which is not included in V.

CDD pole contribution in G?

Natural renormalization scheme

--> fix “a” first, then determine V

to exclude CDD pole contribution from G, based on theoretical argument.
Natural renormalization scheme

Natural renormalization condition

Conditions for natural renormalization

- Loop function $G$ should be negative below threshold.
- $T$ matches with $V$ at low energy scale.

“$a$” is uniquely determined such that

$$G(\sqrt{s} = M_T) = 0 \iff T(M_T) = V(M_T)$$

Matching with low energy interaction


Crossing symmetry (matching with u-channel amplitude)


We regard this condition as the exclusion of the CDD pole contribution from $G$. 
Pole in the effective interaction: single channel

Leading order $V$: Weinberg-Tomozawa term

$$V_{WT} = -\frac{C}{2f^2}(\sqrt{s} - M_T)$$

$C/f^2$: coupling constant

no $s$-wave resonance

$$T^{-1} = V_{WT}^{-1} - G(a_{\text{pheno}}) = V_{\text{natural}}^{-1} - G(a_{\text{natural}})$$

$\uparrow$ ChPT  $\uparrow$ data fit  $\uparrow$ given

Effective interaction in natural scheme

$$V_{\text{natural}} = -\frac{C}{2f^2}(\sqrt{s} - M_T) + \frac{C}{2f^2} \frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\text{eff}}}$$

$M_{\text{eff}} = M_T - \frac{16\pi^2 f^2}{CM_T \Delta a}$,  $a_{\text{pheno}} - a_{\text{natural}}$

There is always a pole for $a_{\text{pheno}} \neq a_{\text{natural}}$

- small deviation $\iff$ pole at irrelevant energy scale
- large deviation $\iff$ pole at relevant energy scale
Comparison of pole positions

Pole of the full amplitude: **physical state** ▲
Pole of the $V_{WT} +$ natural: pure **dynamical** +

$\Rightarrow$ $\Lambda(1405)$ is mostly dynamical state

Physical interpretation of the renormalization condition?
- energy scale
- field renormalization constant $Z$

More about natural renormalization

Energy scale: matching with cutoff scheme at threshold

\[ G^{3d}(M_T + m; q_{\text{max}}) = G^{\text{dim}}(M_T + m; a_{\text{natural}}) \]

For the pion, natural scheme (no CDD pole condition) requires a small cutoff

More about natural renormalization

Weinberg’s theorem for deuteron

“Evidence That the Deuteron Is Not an Elementary Particle”
S. Weinberg, Phys. Rev. 137 B672-B678, (1965)

Z: probability of finding deuteron in a bare elementary state

\[
|d\rangle = \sqrt{Z} |d_0\rangle + \sqrt{1-Z} \int dk |k\rangle
\]

\[
1 = |d_0\rangle \langle d_0| + \int dk |k\rangle \langle k|
\]

For a bound state with \textbf{small binding energy}, the following equation should be satisfied \textbf{model independently}:

\[
a_s = \left[ \frac{2(1-Z)}{2-Z} \right] R + \mathcal{O}(m_\pi^{-1}), \quad r_e = \left[ \frac{-Z}{1-Z} \right] R + \mathcal{O}(m_\pi^{-1})
\]

\(--\) Experiments (observables)

\[
a_s = +5.41[\text{fm}], \quad r_e = +1.75[\text{fm}], \quad R \equiv (2\mu_B)^{-1/2} = 4.31[\text{fm}]
\]

\[Z \lesssim 0.2 \quad \Rightarrow \text{deuteron is composite!}\]
Derivation of the theorem

The theorem is derived in two steps:

**Step 1 (Sec. II):** $Z \rightarrow p$-n-d coupling constant

\[ g^2 = \frac{2\sqrt{B}(1 - Z)}{\pi \rho} \]

\[ \rho = 4\pi \sqrt{2\mu^3} \]

**Step 2 (Sec. III):** coupling constant $\rightarrow a_s, r_e$

\[ a_s = 2R \left[ 1 + \frac{2\sqrt{B}}{\pi \rho g^2} \right] \]

\[ r_e = R \left[ 1 - \frac{2\sqrt{B}}{\pi \rho g^2} \right] \]

uncertainty for order $R = (2\mu B)^{1/2}$ quantity: $m_\pi^{-1}$

The coupling constant $g^2$ can be calculated in the chiral unitary approach $\implies Z$?

$\rightarrow$ Consider single-channel problem with one bound state.
More about natural renormalization

**Field renormalization constant**

WT int., single channel, one bound state $\leftarrow M_B, a$

$$g^2(M_B; a) = -\frac{M_B - M_T}{G(M_B; a) + (M_B - M_T)G'(M_B)}$$

$$1 - Z = \sqrt{\frac{2mM_T}{(M_T + m)(M_T + m - M_B)}} \frac{M_T}{8\pi M_B} g^2(M_B; a)$$

1) $a = a_{\text{natural}}$, vary $M_B$

2) $M_B = 10$ MeV, vary $a$

natural scheme $\rightarrow Z \sim 0$

large deviation $\rightarrow Z \sim 1$
We study the origin (dynamical/CDD) of the resonances in the chiral unitary approach.

Natural renormalization scheme

Exclude CDD pole contribution from the loop function, consistent with N/D.

Comparison with phenomenology

--> **Pole** in the effective interaction

Λ(1405): predominantly dynamical
N(1535): dynamical + CDD pole

Summary: theoretical foundation of the natural scheme

Meaning of the natural scheme

Energy scale in 3d cutoff

for the light meson, natural scheme corresponds to a small 3d cutoff

Field renormalization constant $Z$: quantitative measure of “compositeness”

natural scheme corresponds to $Z \sim 0$