Compositeness of resonances in chiral unitary approach

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Introduction

**Classification of resonances**

Resonances in two-body scattering

- Knowledge of interaction (potential)
- Experimental data (cross section, ...)

**Dynamical state:** two-body molecule, quasi-bound state, ...

![Diagram of dynamical state]

*E.g.* Deuteron in NN, positronium in $e^+e^-$, ...

**CDD pole:** elementary particle, preformed state, ...


![Diagram of CDD pole]

*E.g.* $J/\Psi$ in $e^+e^-$, ...
Description of $S = -1$, $\bar{K}N$ s-wave scattering: $\Lambda(1405)$ in $I=0$

- Interaction $\leftarrow$ chiral symmetry
  

- Amplitude $\leftarrow$ unitarity in coupled channels
  

\[ T = \frac{1}{1 - VG} V \]

chiral

\[ + \]

cutoff
(subtraction constant)


It works successfully, also in $S=0$ sector, meson-meson scattering sectors, systems including heavy quarks, ...
Explicit resonance field in $V$ (interaction)

\[ \begin{array}{c}
\text{Known CDD poles in chiral unitary approach} \\
\end{array} \]


Contracted resonance propagator in higher order $V$


Is that all? subtraction constant?
Chiral unitary approach

### CDD pole in subtraction constant?

#### Phenomenological (standard) scheme

--> $V$ is given, “$a$” is determined by data

1. Leading order
   \[
   T = \frac{1}{(V^{(1)})^{-1} - G(a)}
   \]

2. Next to leading order
   \[
   T = \frac{1}{(V^{(1)} + V^{(2)})^{-1} - G(a')}
   \]

↑ pole ?

“$a$” represents the effect which is not included in $V$.

CDD pole contribution in $G$?

#### Natural renormalization scheme

--> fix “$a$” first, then determine $V$

to exclude CDD pole contribution from $G$, based on theoretical argument.
Natural renormalization condition

Conditions for natural renormalization

- Loop function $G$ should be negative below threshold.
- $T$ matches with $V$ at low energy scale.

“$a$” is uniquely determined such that

\[ G(\sqrt{s} = M_T) = 0 \iff T(M_T) = V(M_T) \]

**subtraction constant:** $a_{\text{natural}}$

**matching with low energy interaction**


**crossing symmetry (matching with u-channel amplitude)**


We regard this condition as the **exclusion of the CDD pole contribution from $G$**.
Pole in the effective interaction: single channel

Leading order $V$: Weinberg-Tomozawa term

$$V_{WT} = -\frac{C}{2f^2}(\sqrt{s} - M_T)$$

$C/f^2$: coupling constant

no s-wave resonance

$$T^{-1} = V_{WT}^{-1} - G(a_{\text{pheno}}) = V_{\text{natural}}^{-1} - G(a_{\text{natural}})$$

$\uparrow$ ChPT  $\uparrow$ data fit  $\uparrow$ given

Effective interaction in natural scheme

$$V_{\text{natural}} = -\frac{C}{2f^2}(\sqrt{s} - M_T) + \frac{C}{2f^2} \frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\text{eff}}}$$

$M_{\text{eff}} = M_T - \frac{16\pi^2 f^2}{CM_T \Delta a}$, \hspace{1cm} a_{\text{pheno}} - a_{\text{natural}}

There is always a pole for \hspace{1cm} a_{\text{pheno}} \neq a_{\text{natural}}

- small deviation $\iff$ pole at irrelevant energy scale
- large deviation $\iff$ pole at relevant energy scale
Pole in the effective interaction

Pole in the effective interaction (\( M_{\text{eff}} \)): pure CDD pole

\[
T^{-1} = V_{\text{WT}}^{-1} - G(a_{\text{pheno}}) = V_{\text{natural}}^{-1} - G(a_{\text{natural}})
\]

For \( \Lambda(1405) \): \( z_{\text{eff}}^\Lambda \approx 7.9 \text{ GeV} \) irrelevant!
For \( N(1535) \): \( z_{\text{eff}}^N = 1693 \pm 37i \text{ MeV} \) relevant?

Difference of interactions \( \Delta V \equiv V_{\text{natural}} - V_{\text{WT}} \)

\[\begin{array}{c}
\Delta V_{11} \\
\Delta V_{22} \\
\Delta V_{33} \\
\Delta V_{44}
\end{array}\]

\[\begin{array}{c}
\Delta V_{11} \\
\Delta V_{22} \\
\Delta V_{33} \\
\Delta V_{44}
\end{array}\]

\( s^{1/2} \text{ [MeV]} \)

===> Important CDD pole contribution in \( N(1535) \)

Next question: quantitative measure for compositeness?
Weinberg’s theorem for deuteron

“Evidence That the Deuteron Is Not an Elementary Particle”
S. Weinberg, Phys. Rev. 137 B672-B678, (1965)

\[ |d\rangle = \sqrt{Z}|d_0\rangle + \sqrt{1-Z} \int dk |k\rangle \]

\[ Z = \text{probability of finding deuteron in a bare elementary state} \]

For a bound state with small binding energy, the following equation should be satisfied model independently:

\[ a_s = \left[ \frac{2(1-Z)}{2-Z} \right] R + \mathcal{O}(m_\pi^{-1}), \quad r_e = \left[ \frac{-Z}{1-Z} \right] R + \mathcal{O}(m_\pi^{-1}) \]

\[ \Rightarrow Z \lesssim 0.2 \quad \text{---> deuteron is composite!} \]

Experiments (observables)

\[ a_s = +5.41[\text{fm}], \quad r_e = +1.75[\text{fm}], \quad R \equiv (2\mu B)^{-1/2} = 4.31[\text{fm}] \]
Derivation of the theorem

The theorem is derived in two steps:

**Step 1 (Sec. II):** $Z \rightarrow p$-n-d coupling constant

\[ g^2 = \frac{2\sqrt{B}(1 - Z)}{\pi \rho} \]

\[ \rho = 4\pi \sqrt{2\mu^3} \]

**Step 2 (Sec. III):** coupling constant $\rightarrow a_s, r_e$

\[ a_s = 2R \left[ 1 + \frac{2\sqrt{B}}{\pi \rho g^2} \right] \]

\[ r_e = R \left[ 1 - \frac{2\sqrt{B}}{\pi \rho g^2} \right] \]

uncertainty for order $R = (2\mu B)^{1/2}$ quantity: $m^{-1}_\pi$

The **coupling constant** $g^2$ can be calculated by the residue of the pole in chiral unitary approach $\Rightarrow Z$?

$\rightarrow$ study $Z$ in natural renormalization scheme
Single-channel problem: $M_T$ and $m$

$$T = \frac{1}{1 - VG(a)} V$$

$$V = -\frac{C}{2f^2}(\sqrt{s} - M_T) = \tilde{C}(\sqrt{s} - M_T)$$

2 parameters: $\tilde{C}, a$

For the system with a **bound state**

$$1 - VG|_{\sqrt{s}=M_B} = 1 - \tilde{C}(M_B - M_T)G(M_B; a) = 0$$

:relation among $\tilde{C}, a, M_B$

--> system can be characterized by $(\tilde{C}, a)$ **or** $(a, M_B)$

Check the Z factor in natural renormalization scheme from the residue of the pole

$$g^2(M_B; a) = \lim_{\sqrt{s} \to M_B} (\sqrt{s} - M_B)T(\sqrt{s})$$
The residue can be calculated analytically:

\[
g^2(M_B; a) = -\frac{M_B - M_T}{G(M_B; a) + (M_B - M_T)G'(M_B)}
\]

\[
1 - Z = \sqrt{\frac{2mM_T}{(M_T + m)(M_T + m - M_B)} \frac{M_T}{8\pi M_B} g^2(M_B; a)} \quad \text{valid for small } M_B
\]

1) \(a = a_{\text{natural}}, \) vary \(M_B\)

2) \(M_B = 10 \text{ MeV, vary } a\)

natural scheme --> \(Z \sim 0\)

large deviation --> \(Z \sim 1\)
We study the origin (dynamical/CDD) of the resonances in the chiral unitary approach.

Natural renormalization scheme
- Exclude CDD pole contribution from the loop function, consistent with N/D.

Comparison with phenomenology
- \( \Lambda(1405) \): predominantly dynamical
- \( N(1535) \): dynamical + CDD pole

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Summary: Compositeness of resonances

We consider a single-channel problem with a bound state to study the compositeness.

Field renormalization constant $Z$: quantitative measure of compositeness

$Z = 0$  $Z = 1$

Residue of the pole --> coupling constant

natural scheme corresponds to $Z \sim 0$