# Hadronic molecule resonances


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supported by Global Center of Excellence Program
"Nanoscience and Quantum Physics"

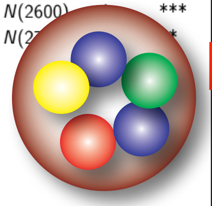

2010, Aug. 19th

# Hadron spectroscopy

## Observed hadronic states (PDG2006)
## All states come from QCD Lagrangian

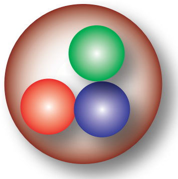
**Constituent quark model systematically reproduces the spectra by qqq and qbar-q.**

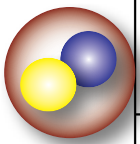
**Exotic hadrons: exceptions to the qqq/qbar-q + more than four quarks**



Θ(1540)+ *

$\sim \frac{1}{300}$!



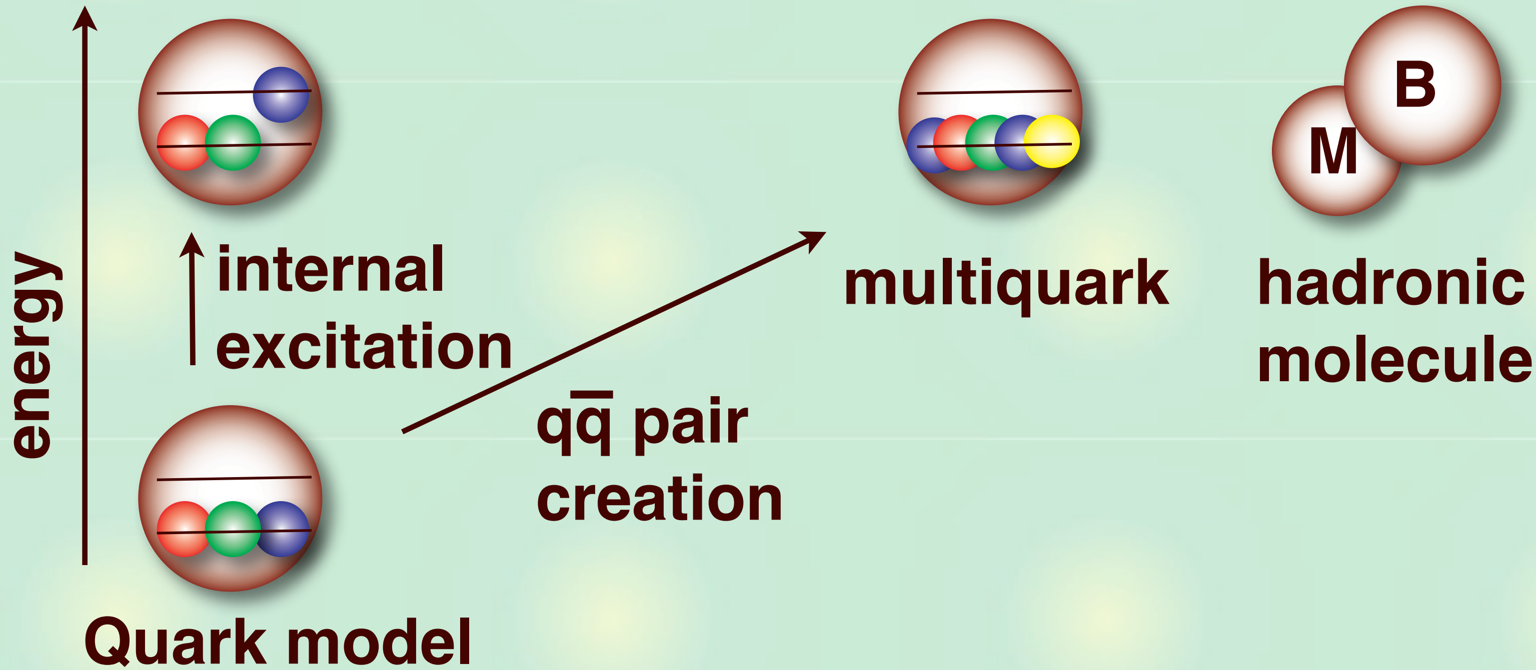


~ 130 baryons

~ 160 mesons

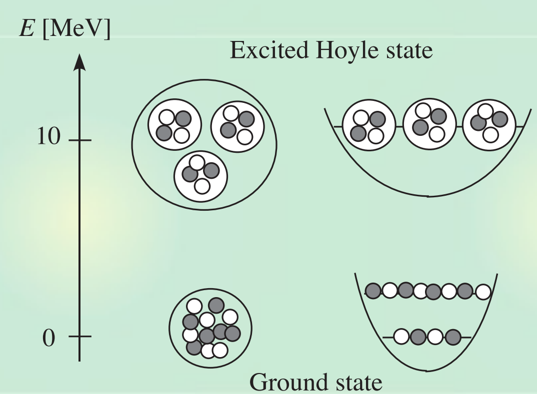
# Structure of hadron resonances

## Example) baryon excited state




**Excited states**
**= resonances in hadron scattering**

**Exotic structure near threshold?**
**c.f. $^{12}C$ Hoyle state**



# Study of the internal structure

**Number of quarks and antiquarks are not conserved.**

$$|\,B\,\rangle = \mathcal{N}_3|\,qqq\,\rangle + \mathcal{N}_5|\,qqq\;q\bar{q}\,\rangle + \mathcal{N}_7|\,qqq\;q\bar{q}\;q\bar{q}\,\rangle + \dots$$

$$\langle\,qqq\,|\,qqq\;q\bar{q}\,\rangle \neq 0$$

**How to investigate the internal structure?**

- Comparison of model calculation v.s. experiments (mass, width, decay properties, etc.)

  : Any model can describe data with appropriate corrections
  : Model-dependent definition

- Extrapolation to the ideal world, change the environment (large Nc, symmetry restoration, etc.)

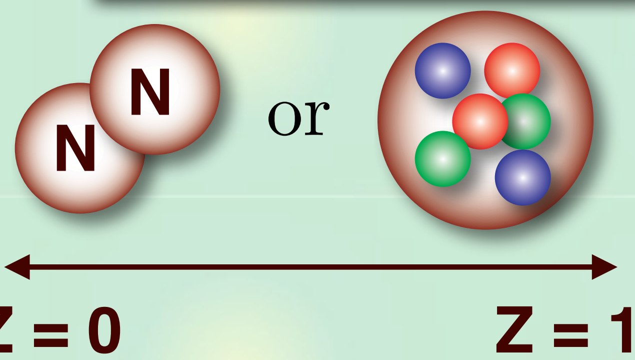
  : Structure may change during the extrapolation
  : Qualitative discussion only

--> model-independent and quantitative distinction?

# Contents

# Main result: theorem

$|\mathrm{deuteron}\rangle =$




$\notin$ NN model space

~ elementary particle

**Z: probability of finding deuteron in a bare elementary state**

**For a bound state with small binding energy, the following equation should be satisfied model independently:**

$$a_s = \left[\frac{2(1-Z)}{2-Z}\right] R + \mathcal{O}(m_\pi^{-1}), \quad r_e = \left[\frac{-Z}{1-Z}\right] R + \mathcal{O}(m_\pi^{-1})$$

**$a_s$: scattering length**

**$r_e$: effective range** <-- Experiments

**R: deuteron radius (binding energy)**

$$a_s = +5.41 \text{ [fm]}, \quad r_e = +1.75 \text{ [fm]}, \quad R \equiv (2\mu B)^{-1/2} = 4.31 \text{ [fm]}$$

$\Rightarrow Z \lesssim 0.2$ --> **deuteron is almost composite!**

# Derivation of the theorem

**The theorem is derived in two steps:**

**Step 1 (Sec. II): Z --> p-n-d coupling constant g**

$$g^2 = \frac{2\sqrt{B}(1-Z)}{\pi\rho} \qquad \rho = 4\pi\sqrt{2\mu^3}$$

**Step 2 (Sec. III): coupling constant g --> $a_s$, $r_e$**

$$a_s = 2R\left[1 + \frac{2\sqrt{B}}{\pi\rho g^2}\right] \qquad r_e = R\left[1 - \frac{2\sqrt{B}}{\pi\rho g^2}\right]$$

**Assumption: B is sufficiently smaller than the typical energy scale of the NN interaction**

$$p \sim m_\pi, \quad B \ll m_\pi^2/2\mu \quad \Leftrightarrow \quad R^2 \gg m_\pi^2$$

**--> uncertainty for order R quantity: $m_\pi^{-1}$**

# Definition of the probability Z

**Hamiltonian of NN system: free + interaction V**

$$\mathcal{H} = \mathcal{H}_0 + V$$

**Complete set for free Hamiltonian: bare |$d_0$> + continuum**

$$1 = |\, d_0 \,\rangle\langle\, d_0 \,| + \int d\boldsymbol{k} |\, \boldsymbol{k} \,\rangle\langle\, \boldsymbol{k} \,|$$

$$\mathcal{H}_0 |\, d_0 \,\rangle = E_0 |\, d_0 \,\rangle, \quad \mathcal{H}_0 |\, \boldsymbol{k} \,\rangle = E(\boldsymbol{k}) |\, \boldsymbol{k} \,\rangle$$

**(original, $d_0$: sum of discrete states, k: α)**

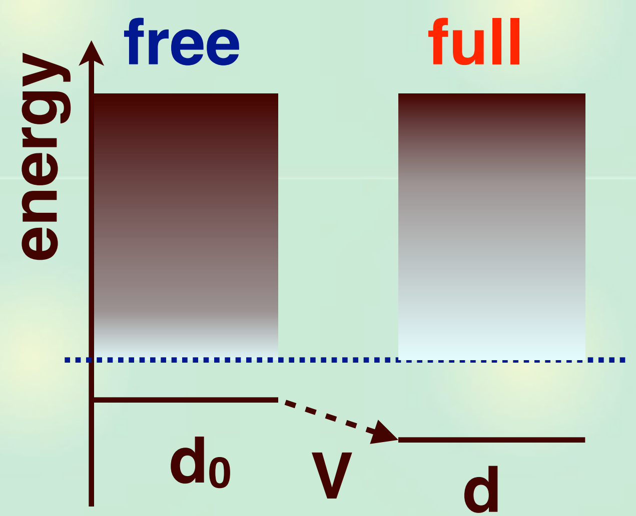
**Physical deuteron |d> : eigenstate of full Hamiltonian**

$$(\mathcal{H}_0 + V) |\, d \,\rangle = -B |\, d \,\rangle$$

**Z: overlap of d and $d_0$**
**(wavefunction renormalization factor)**

$$Z \equiv |\langle\, d_0 \,|\, d \,\rangle|^2$$

$$|\, d \,\rangle = \sqrt{Z} |\, d_0 \,\rangle + \sqrt{1 - Z} \int d\boldsymbol{k} |\, \boldsymbol{k} \,\rangle$$

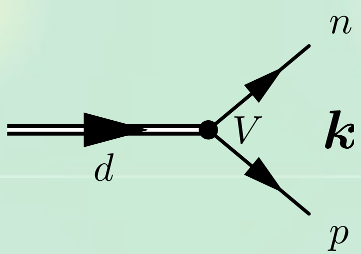


# p-n-d coupling constant

## After some algebra, we arrive at

$$1 - Z = \int d\boldsymbol{k} \frac{|\langle \boldsymbol{k} | V | d \rangle|^2}{[E(\boldsymbol{k}) + B]^2}$$

$$\langle \boldsymbol{k} | V | d \rangle = g(\boldsymbol{k}) :$$




## Typical energy scale $E_0$: below $E_0$, coupling is constant

$$\langle \boldsymbol{k} | V | d \rangle = g(\boldsymbol{k}) \sim g \quad \text{for} \quad |E(\boldsymbol{k})| \leq E_0$$

**(NN scattering : $E_0 \approx m_\pi^2/2\mu$ )**

## Assumption: $B \ll E_0$

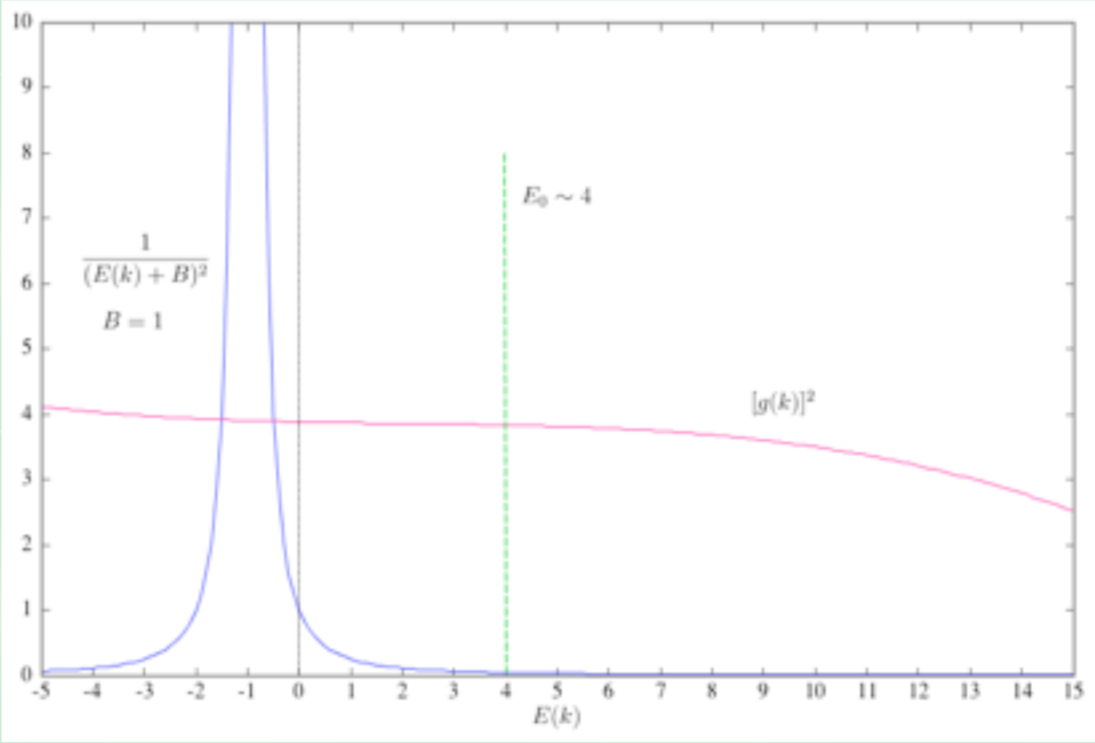
$$\Rightarrow 1 - Z \sim g^2 \int \frac{d\boldsymbol{k}}{[E(\boldsymbol{k}) + B]^2}$$

$$= g^2 \rho \int_0^\infty \frac{\sqrt{E}\, dE}{[E + B]^2}$$

$$\rho = 4\pi\sqrt{2\mu^3}$$

## Integrate analytically

$$g^2 = \frac{2\sqrt{B}(1 - Z)}{\pi\rho}$$



# Scattering equations

## The Lippmann-Schwinger equation

$$T(W) = V + V\frac{1}{W - \mathcal{H}_0}T(W)$$

$$\Rightarrow T(W) = V + V\frac{1}{W - \mathcal{H}}V$$ **(Chew-Goldberger solution)**

## Complete set for full Hamiltonian (asymptotic completeness)

$$1 = |\,d\,\rangle\langle\,d\,| + \int d\boldsymbol{k}|\,\boldsymbol{k}, \mathrm{in}\,\rangle\langle\,\boldsymbol{k}, \mathrm{in}\,| \qquad V|\,\boldsymbol{k}, \mathrm{in}\,\rangle = T|\,\boldsymbol{k}\,\rangle$$

$$T_{\boldsymbol{k}'\boldsymbol{k}}(W) = V_{\boldsymbol{k}'\boldsymbol{k}} + \frac{\langle\,\boldsymbol{k}'\,|V|\,d\,\rangle\langle\,d\,|V|\,\boldsymbol{k}\,\rangle}{W + B} + \int d\boldsymbol{k}''\frac{\langle\,\boldsymbol{k}'\,|V|\,\boldsymbol{k}'', \mathrm{in}\,\rangle\langle\,\boldsymbol{k}'', \mathrm{in}\,|V|\,\boldsymbol{k}\,\rangle}{W - E(\boldsymbol{k}'')}$$

## Setting $W = E(\boldsymbol{k}) + i\epsilon$, we obtain the Low equation

$$T_{\boldsymbol{k}'\boldsymbol{k}} = V_{\boldsymbol{k}'\boldsymbol{k}} + \frac{\langle\,\boldsymbol{k}'\,|V|\,d\,\rangle\langle\,d\,|V|\,\boldsymbol{k}\,\rangle}{E(\boldsymbol{k}) + B} + \int d\boldsymbol{k}''\frac{T_{\boldsymbol{k}'\boldsymbol{k}''}T_{\boldsymbol{k}''\boldsymbol{k}}}{E(\boldsymbol{k}) - E(\boldsymbol{k}'') + i\epsilon}$$

## So far no approximations.

# Solution for the scattering equation

**The same assumption: B << $E_0$, external energy E << $E_0$**

$$\frac{\langle \boldsymbol{k}' | V | d \rangle \langle d | V | \boldsymbol{k} \rangle}{E(\boldsymbol{k}) + B} \sim \frac{g^2}{E(\boldsymbol{k}) + B} \propto \frac{1}{\sqrt{B}} \gg V_{\boldsymbol{k}'\boldsymbol{k}}$$

**We neglect the 1st term (information of V is lost!!).**

$$T_{\boldsymbol{k}'\boldsymbol{k}} = \frac{g^2}{E(\boldsymbol{k}) + B} + \int d\boldsymbol{k}'' \frac{T_{\boldsymbol{k}'\boldsymbol{k}''} T_{\boldsymbol{k}''\boldsymbol{k}}}{E(\boldsymbol{k}) - E(\boldsymbol{k}'') + i\epsilon}$$

**S-wave scattering (no angular dependence)**

$$T_{\boldsymbol{k}'\boldsymbol{k}} \to t\left[E(\boldsymbol{k})\right] \delta_{\boldsymbol{k}'\boldsymbol{k}}$$

$$t(E) = \frac{g^2}{E + B} + \rho \int_0^\infty dE'' \frac{\sqrt{E''} |t(E)|^2}{E - E'' + i\epsilon}$$

**The solution of the integral equation (well-known? We should solve $t^{-1}(E)$ using optical theorem and analyticity)**

$$t(E) = \left[\frac{E + B}{g^2} + \frac{\pi\rho(B - E)}{2\sqrt{B}} + i\pi\rho\sqrt{E}\right]^{-1}$$

# Amplitude, phase shift, and scattering length

## The result of low-energy scattering amplitude

$$t(E) = \left[\frac{E+B}{g^2} + \frac{\pi\rho(B-E)}{2\sqrt{B}} + i\pi\rho\sqrt{E}\right]^{-1}$$

## S-wave phase shift

$$e^{2i\delta(E)} = 1 - 2i\pi\rho\sqrt{E}t(E)$$

$$\cot\delta = -\frac{1}{\pi\rho\sqrt{E}}\left[\frac{E+B}{g^2} + \frac{\pi\rho(B-E)}{2\sqrt{B}}\right]$$

## Scattering length $a_s$, effective range $r_e$

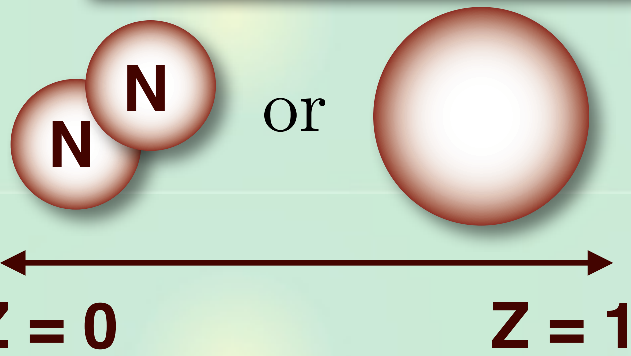
$$k\cot\delta = -\frac{1}{a_s} + r_e\frac{k^2}{2}, \quad E = \frac{k^2}{2\mu}, \quad R = \frac{1}{\sqrt{2\mu B}}$$

## We obtain the final result (no expansion needed)

$$a_s = 2R\left[1 + \frac{2\sqrt{B}}{\pi\rho g^2}\right] \qquad r_e = R\left[1 - \frac{2\sqrt{B}}{\pi\rho g^2}\right]$$

# Main result: theorem

$$|\mathrm{deuteron}\rangle =$$




**Z: probability of finding deuteron in a bare elementary state**

**For a bound state with small binding energy, the following equation should be satisfied model independently:**

**- coupling constant <--> Z**

$$g^2 = \frac{2\sqrt{B}(1-Z)}{\pi\rho}$$

**- scattering length, effective range <--> Z**

$$a_s = \left[\frac{2(1-Z)}{2-Z}\right] R + \mathcal{O}(m_\pi^{-1}), \quad r_e = \left[\frac{-Z}{1-Z}\right] R + \mathcal{O}(m_\pi^{-1})$$

# Chiral unitary approach

## Description of S = -1, $\bar{K}N$ s-wave scattering: Λ(1405) in I=0

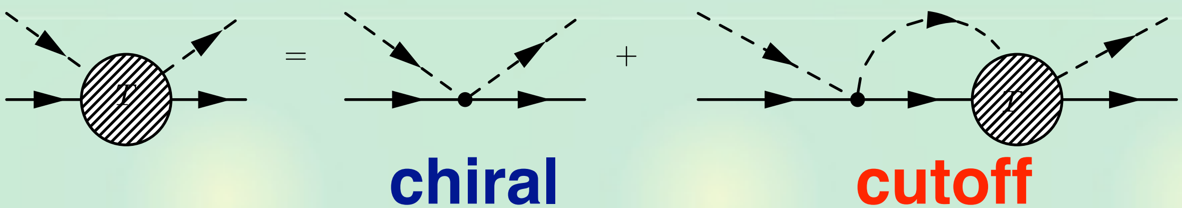
- Interaction <-- chiral symmetry

Y. Tomozawa, Nuovo Cim. 46A, 707 (1966); S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

- Amplitude <-- unitarity in coupled channels

R.H. Dalitz, T.C. Wong, G. Rajasekaran, Phys. Rev. 153, 1617 (1967)

$$T = \frac{1}{1 - VG} V$$




N. Kaiser, P. B. Siegel, W. Weise, Nucl. Phys. A594, 325 (1995),
E. Oset, A. Ramos, Nucl. Phys. A635, 99 (1998),
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001),
M.F.M. Lutz, E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002), .... many others

**It works successfully, also in S=0 sector, meson-meson scattering sectors, systems including heavy quarks, ...**

# Natural renormalization condition

Conditions for natural renormalization

- Loop function G should be negative below threshold.
- T matches with V at low energy scale.

"a" is uniquely determined such that

$$G(\sqrt{s} = M_T) = 0 \quad \Leftrightarrow \quad T(M_T) = V(M_T)$$

subtraction constant: $a_{\text{natural}}$

We regard this condition as the exclusion of the CDD pole contribution from G.

- Λ(1405) is dominated by meson-baryon structure
- N(1535) requires some additional component

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

We expect that Z should be zero or small for $a_{\text{natural}}$

How to check? --> calculate the coupling constant g

# Field renormalization constant

## Single-channel problem: $M_T$ and m

$$T = \frac{1}{1 - VG(a)} V$$

$$V = -\frac{C}{2f^2}(\sqrt{s} - M_T) = \tilde{C}(\sqrt{s} - M_T)$$

**2 parameters:** $(\tilde{C}, a)$

## For the system with a bound state

$$1 - VG|_{\sqrt{s}=M_B} = 1 - \tilde{C}(M_B - M_T)G(M_B; a) = 0$$

**:relation among** $\tilde{C}, a, M_B$

**--> bound state can be characterized by** $(\tilde{C}, a)$ **or** $(a, M_B)$

## Check the Z factor in natural renormalization scheme from the residue of the pole

$$g^2(M_B; a) = \lim_{\sqrt{s} \to M_B} (\sqrt{s} - M_B) T(\sqrt{s})$$

# Field renormalization constant

## The residue can be calculated analytically:

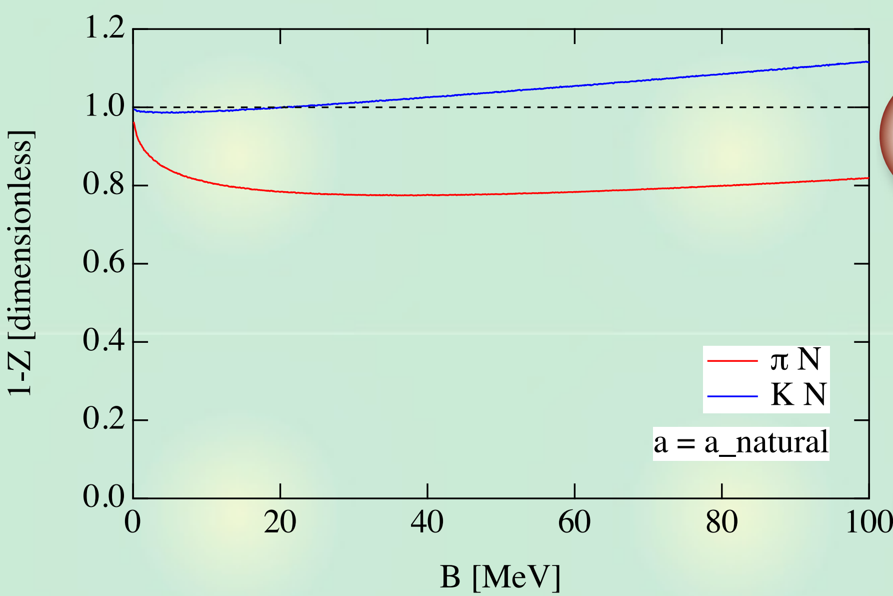
$$g^2(M_B;a) = -\frac{M_B - M_T}{G(M_B;a) + (M_B - M_T)G'(M_B)} \longleftarrow (a, M_B)$$

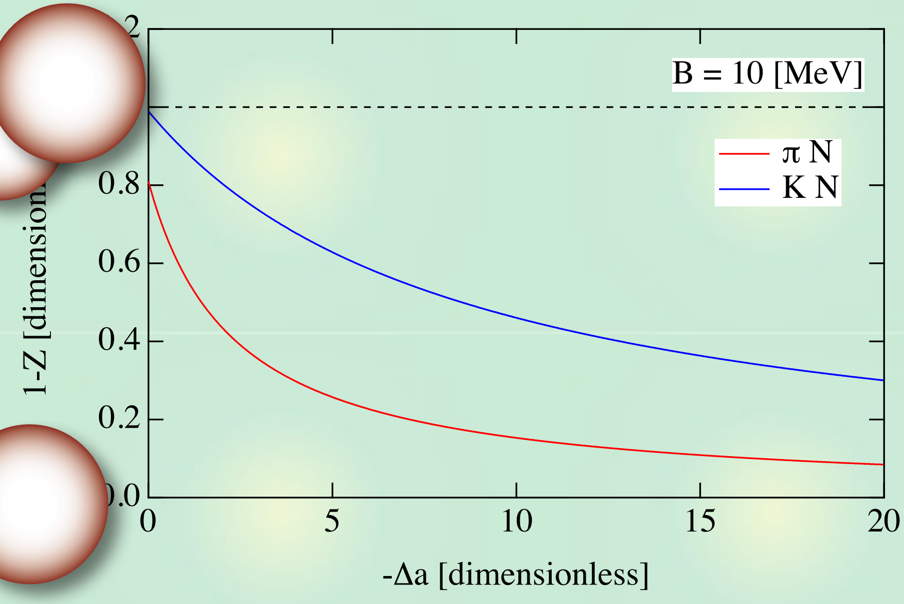
$$1 - Z = \sqrt{\frac{2mM_T}{(M_T+m)(M_T+m-M_B)}}\frac{M_T}{8\pi M_B}g^2(M_B;a)$$

**valid for small B = (M_T + m) - M_B**

**1) a = a_natural, vary B**

**2) B = 10 MeV, vary a**



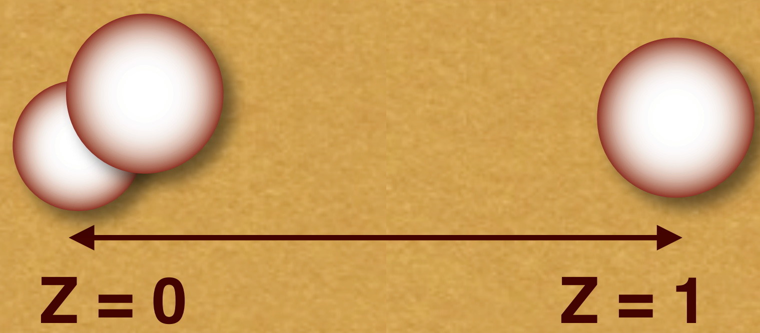

**natural scheme --> Z ~ 0**

**large deviation --> Z ~ 1**

# Summary 1

## Weinberg's study of the deuteron structure

- **Field renormalization constant Z: quantitative measure of compositeness**




- **For small B, Z is related to the coupling constant and scattering observables model independently.**

S. Weinberg, Phys. Rev. 137 B672-B678 (1965)

# Summary 2

## Application to chiral unitary approach

- **Natural renormalization scheme**

  exclude CDD pole contribution from the loop function to generate **purely molecule resonance**

  T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

- **Residue of the pole --> coupling constant**

  natural scheme corresponds to **Z ~ 0**
  --> composite particle is generated

  T. Hyodo, D. Jido, A. Hosaka, in preparation