Compositeness of bound states and resonances in chiral unitary approach

Tetsuo Hyodo[a],

Daisuke Jido[b], and Atsushi Hosaka[c]

Tokyo Institute of Technology[a] *YITP, Kyoto[b]* *RCNP, Osaka[c]*

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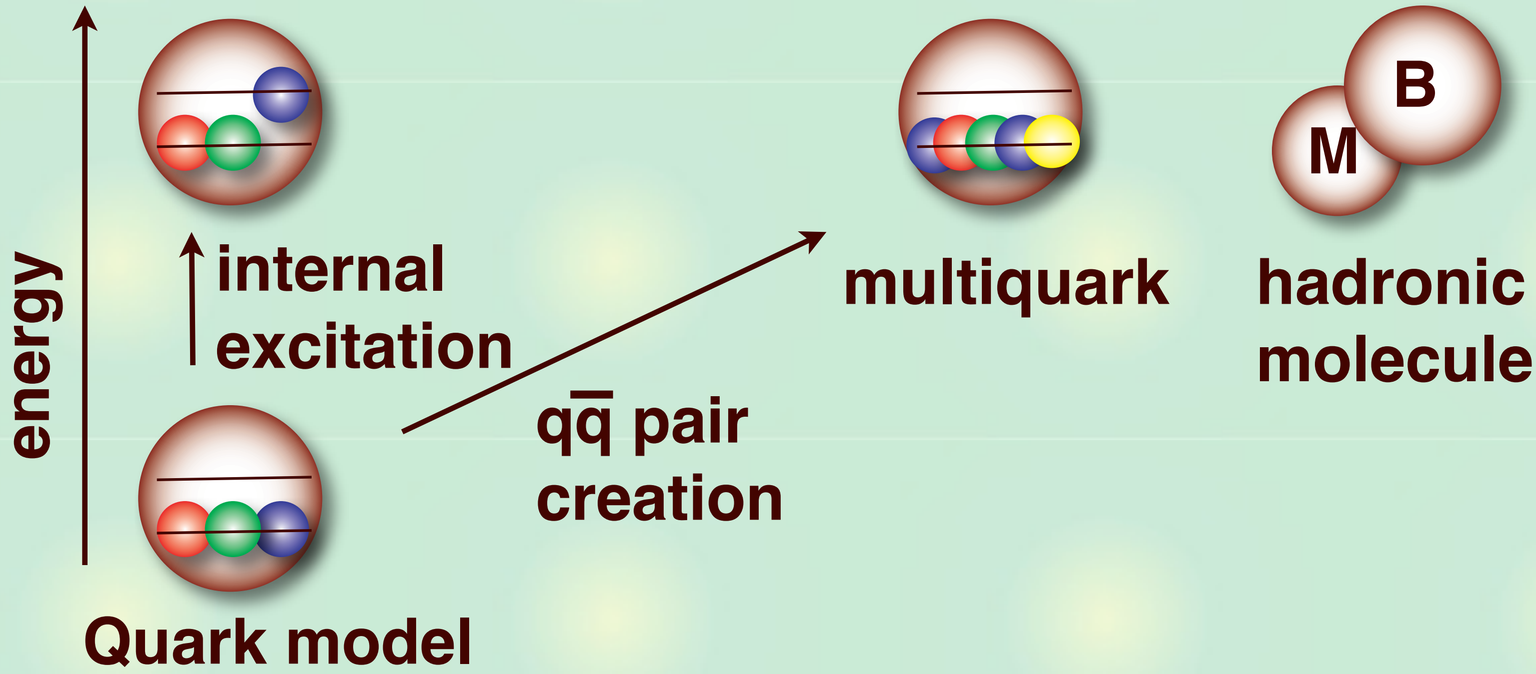
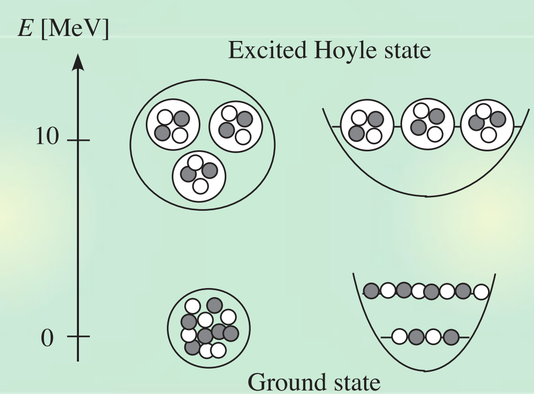
Structure of hadron resonances

Example) baryon excited state



Excited states
= resonances in hadron scattering

Exotic structure near threshold?
c.f. ^{12}C Hoyle state, X,Y,Z charmonia,...



Definition of the compositeness 1-Z

Hamiltonian of two-body system: free + interaction V

$$\mathcal{H} = \mathcal{H}_0 + V$$

Complete set for free Hamiltonian: bare $|B_0>$ + continuum

$$1 = |\, B_0 \,\rangle\langle\, B_0 \,| + \int d\boldsymbol{k} |\, \boldsymbol{k} \,\rangle\langle\, \boldsymbol{k} \,|$$

$$\mathcal{H}_0 |\, B_0 \,\rangle = E_0 |\, B_0 \,\rangle, \quad \mathcal{H}_0 |\, \boldsymbol{k} \,\rangle = E(\boldsymbol{k}) |\, \boldsymbol{k} \,\rangle$$

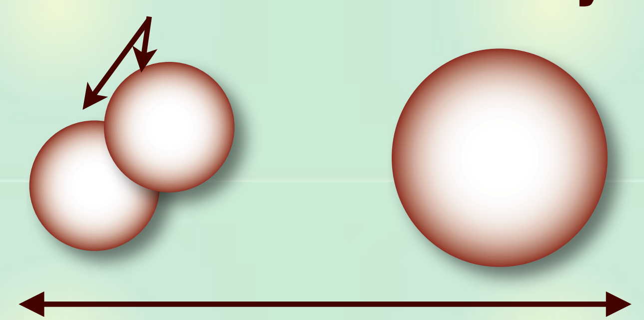
Physical bound state |B> : eigenstate of full Hamiltonian

$$(\mathcal{H}_0 + V) |\, B \,\rangle = -B |\, B \,\rangle$$

B: binding energy

Define Z as the overlap of B and B_0 : probability of finding the physical bound state in the bare state |B>

$$Z \equiv |\langle\, B_0 \,|\, B \,\rangle|^2$$



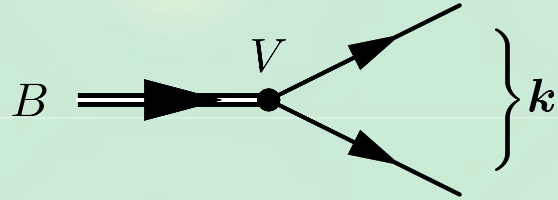
1 - Z : Compositeness of the bound state

Model-independent but approximated method

With the Schrödinger equation, we obtain

$$1 - Z = \int d\boldsymbol{k} \frac{|\langle \boldsymbol{k} | V | B \rangle|^2}{[E(\boldsymbol{k}) + B]^2}$$

$\langle \boldsymbol{k} | V | B \rangle :$



$$= 4\pi\sqrt{2\mu^3} \int_0^\infty dE \frac{\sqrt{E}|G_W(E)|^2}{(E+B)^2}$$

$\langle \boldsymbol{k} | V | B \rangle \equiv G_W[E(\boldsymbol{k})]$ **for s-wave**

Approximation: For small binding energy B<<1, the vertex $G_W(E)$ can be regarded as a constant: $G_W(E) \sim g_W$

Then the integration can be done analytically, leading to

$$1 - Z = 2\pi^2\sqrt{2\mu^3}\frac{g_W^2}{\sqrt{B}}$$

Compositeness <-- coupling g and binding energy B

S. Weinberg, Phys. Rev. 137 B672-B678 (1965)

- **Model-independent: no information of V**
- **Approximated: valid only for small B**

Exact but model-dependent method

Formal solution of the Lippmann-Schwinger equation

$$T(E) = V + V\frac{1}{E-\mathcal{H}}V$$

Insert complete set for full Hamiltonian (Low's equation)

$$1 = |\,B\,\rangle\langle\,B\,| + \int d\boldsymbol{k}|\,\boldsymbol{k},\mathrm{in}\,\rangle\langle\,\boldsymbol{k},\mathrm{in}\,| \qquad V|\,\boldsymbol{k},\mathrm{in}\,\rangle = T|\,\boldsymbol{k}\,\rangle$$

$$t(E) = v(E) + \boxed{\frac{|G_W(E)|^2}{E+B}} + 4\pi\sqrt{2\mu^3}\int_0^\infty dE'\frac{\sqrt{E'}|t(E)|^2}{E-E'+i\epsilon}$$

(for s-wave)

--> integrand of the formula for 1-Z !

Exact expression of the compositeness 1-Z

$$1 - Z = 4\pi\sqrt{2\mu^3}\int_0^\infty dE\frac{\sqrt{E}|G_W(E)|^2}{(E+B)^2}$$

$$= 4\pi\sqrt{2\mu^3}\int_0^\infty dE\frac{\sqrt{E}}{E+B}\left[t(E) - v(E) - 4\pi\sqrt{2\mu^3}\int_0^\infty dE'\frac{\sqrt{E'}|t(E')|^2}{E-E'+i\epsilon}\right]$$

T. Hyodo, D. Jido, A. Hosaka, in preparation

Short summary

We have defined the compositeness of the bound state 1-Z.

$$1 - Z = 1 - |\langle B_0 | B \rangle|^2 = \int d\boldsymbol{k} \frac{|\langle \boldsymbol{k} | V | B \rangle|^2}{[E(\boldsymbol{k}) + B]^2}$$

Method 1: model independent but approximated

$$1 - Z = 2\pi^2 \sqrt{2\mu^3} \frac{g_W^2}{\sqrt{B}}$$

Method 2: exact but model dependent

$$1 - Z = 4\pi\sqrt{2\mu^3} \int_0^\infty dE \frac{\sqrt{E}}{E + B} \left[t(E) - v(E) - 4\pi\sqrt{2\mu^3} \int_0^\infty dE' \frac{\sqrt{E'}|t(E')|^2}{E - E' + i\epsilon} \right]$$

- Exact: valid for any B
- Model dependent: interaction V has to be specified (c.f. potential + wave function --> observable)
- Imaginary part vanishes by the optical theorem
- RHS can be calculated by model (chiral unitary approach)

Single-channel chiral unitary approach

Single-channel scattering amplitude: masses M and m

$$T(W)=\frac{1}{1-V(W)G(W;a)}V(W)$$

$$V(W)=C(W-M)$$

Change of subtraction constant
<--> introduction of a pole term in the interaction

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

$$a\to a-\Delta a,$$

$$V(W)\to\tilde{V}(W)=C(W-M)-C\frac{(W-M)^2}{(W-M_{\rm eff})},\quad M_{\rm eff}=M+\frac{(4\pi)^2}{2MC\Delta a}$$

We should define the benchmark of the subtraction constant
--> Natural renormalization constant $a_{natural}$

If $a_{natural}$ corresponds to purely composite case,
then M_{eff} corresponds to the mass of the bare state M_{B0}

Single-channel chiral unitary approach

We use the model-independent formula

$$1 - Z = 2\pi^2\sqrt{2\mu^3}\frac{g_W^2}{\sqrt{B}}$$

We need to calculate the coupling g and binding energy B

- condition for the bound state: $M_B = M + m - B$

$$1 - C(M_B - M)G(M_B; a) = 0$$

--> system can be characterized by (M_B, a) or (M_B, M_{B0})

- coupling constant: residue of the pole at M_B

$$[g(M_B; a)]^2 = \lim_{W\to M_B}(W - M_B)T(W) = -\frac{M_B - M}{G(M_B; a) + (M_B - M)G'(M_B)}$$

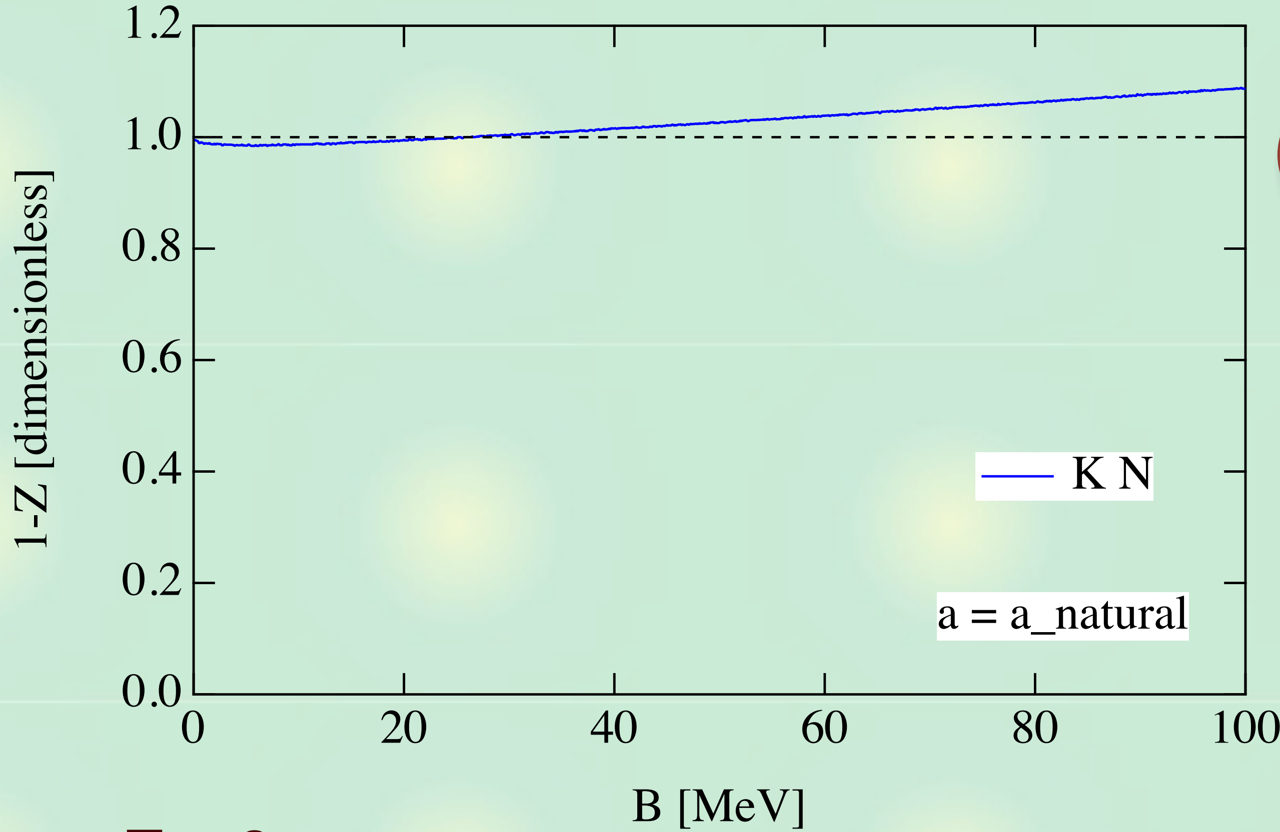
- normalization of the amplitude (just a kinematical factor)

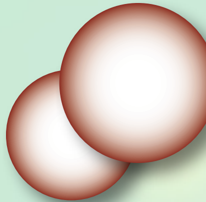
$$1 - Z = \frac{M|\lambda^{1/2}(M_B^2, M^2, m^2)|}{16\pi M_B^2(M + m - M_B)}g^2(M_B; a)$$ **(for small $B = M + m - M_B$)**

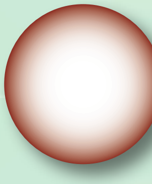
Numerical analysis

Compositeness of the bound state in chiral unitary approach

1) B dependence with M_{B0} -> ∞ (a = $a_{natural}$)





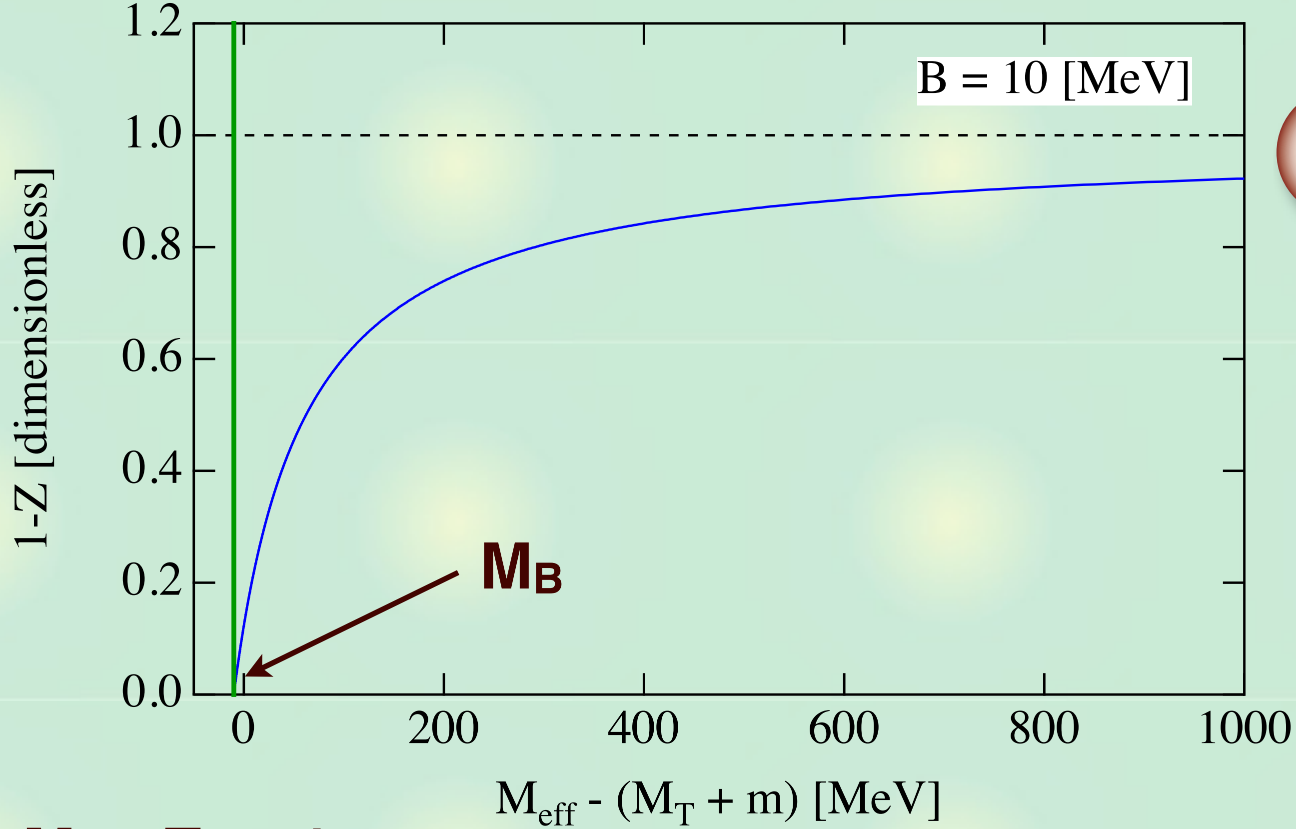


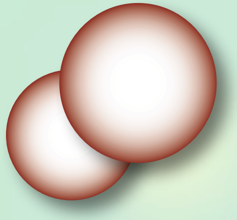
- M_{B0} -> ∞ : Z ~ 0
- Z = 0 at B = 0
- large B behavior is not justified by the approximation

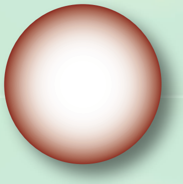
Numerical analysis

Compositeness of the bound state in chiral unitary approach

2) M_{B0} dependence with B = 10 MeV







- M_{B0} -> M_B : Z -> 1
- Mass difference of M_{B0} and M_B : self-energy of bare state
  --> large if the composite component is large

Summary

We study the compositeness of the particles

- We derive the exact form of the compositeness of a bound state in terms of the scattering amplitude

- We apply to the bound state in chiral unitary model to check the natural renormalization condition

  M_{B0} -> ∞ ($a=a_{natural}$) : Z ~ 0, composite

  M_{B0} -> M_B : Z ~ 1, elementary

  : $a_{natural}$ is a good benchmark

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