Toward a realistic $\bar{K}N$-$\pi\Sigma$ interaction

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Contents

Chiral symmetry breaking in hadron physics

Chiral symmetry: QCD with massless quarks

Consequence of chiral symmetry breaking in hadron physics

- **appearance of the Nambu-Goldstone (NG) boson**

  $m_\pi \sim 140 \text{ MeV}$

- **dynamical generation of hadron masses**

  $M_p \sim 1 \text{ GeV} \sim 3M_q, \quad M_q \sim 300 \text{ MeV} \quad v.s. \quad 3-7 \text{ MeV}$

- **constraints on the interaction of NG boson and a hadron**

  low energy theorems <-- current algebra

  systematic low energy ($m, p/4\pi f_\pi$) expansion: ChPT

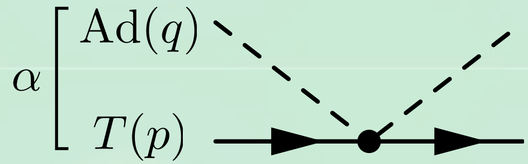
Chiral symmetry and its breaking

$$SU(3)_R \otimes SU(3)_L \to SU(3)_V$$

Underlying QCD <==> observed hadron phenomena

s-wave low energy interaction

Low energy NG boson (Ad) + target hadron (T) scattering



$$= \frac{1}{f^2}\frac{p\cdot q}{2M_T}\langle \boldsymbol{F}_T\cdot\boldsymbol{F}_{\mathrm{Ad}}\rangle_\alpha + \mathcal{O}\left(\left(\frac{m}{M_T}\right)^2\right)$$

Projection onto s-wave: Weinberg-Tomozawa (WT) term

Y. Tomozawa, Nuovo Cim. 46A, 707 (1966); S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

$$V_{ij} = -\frac{C_{ij}}{4\boxed{f}^2}\boxed{(\omega_i+\omega_j)}$$ **energy dependence (derivative coupling)**

decay constant of π (g_V=1)

$$C_{ij} = \sum_\alpha C_{\alpha,T}\begin{pmatrix} 8 & T & \| & \alpha \\ I_{M_i},Y_{M_i} & I_{T_i},Y_{T_i} & \| & I,Y\end{pmatrix}\begin{pmatrix} 8 & T & \| & \alpha \\ I_{M_j},Y_{M_j} & I_{T_j},Y_{T_j} & \| & I,Y\end{pmatrix}$$

$$C_{\alpha,T} = \langle 2\boldsymbol{F}_T\cdot\boldsymbol{F}_{\mathrm{Ad}}\rangle_\alpha = C_2(T) - C_2(\alpha) + 3$$

Group theoretical structure and flavor SU(3) symmetry determines the sign and the strength of the interaction

Low energy theorem: leading order term in ChPT

Scattering amplitude and unitarity

Unitarity of S-matrix: Optical theorem

$$\mathrm{Im}[T^{-1}(s)] = \boxed{\frac{\rho(s)}{2}}$$

phase space of two-body state

General amplitude by dispersion relation

$$T^{-1}(\sqrt{s}) = \boxed{\sum_i \frac{R_i}{\sqrt{s}-W_i} + \tilde{a}(s_0)} + \frac{s-s_0}{2\pi}\int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s'-s)(s'-s_0)}$$

R_i, W_i, a: to be determined by chiral interaction

Identify dispersion integral = loop function G, the rest = V^{-1}

$$\boxed{T(\sqrt{s}) = \frac{1}{V^{-1}(\sqrt{s}) - G(\sqrt{s};a)}}$$

Scattering amplitude

V? chiral expansion of T, (conceptual) matching with ChPT

$$T^{(1)} = V^{(1)}, \quad T^{(2)} = V^{(2)}, \quad T^{(3)} = V^{(3)} - V^{(1)}GV^{(1)}, \quad \dots$$

Amplitude T: consistent with chiral symmetry + unitarity

Chiral unitary approach

Description of S = -1, $\bar{K}N$ s-wave scattering: Λ(1405) in I=0

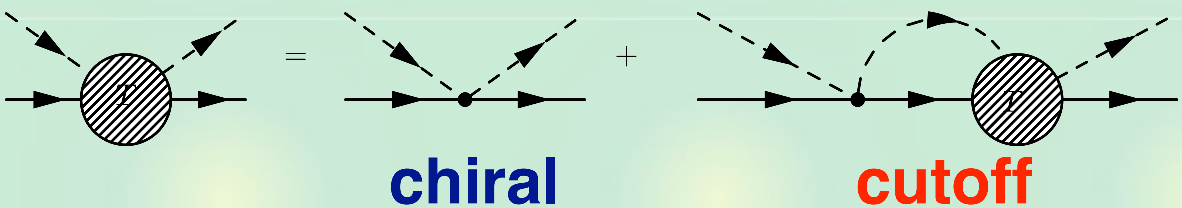
- Interaction <-- chiral symmetry

Y. Tomozawa, Nuovo Cim. 46A, 707 (1966); S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

- Amplitude <-- unitarity in coupled channels

R.H. Dalitz, T.C. Wong, G. Rajasekaran, Phys. Rev. 153, 1617 (1967)

$$T = \frac{1}{1 - VG} V$$

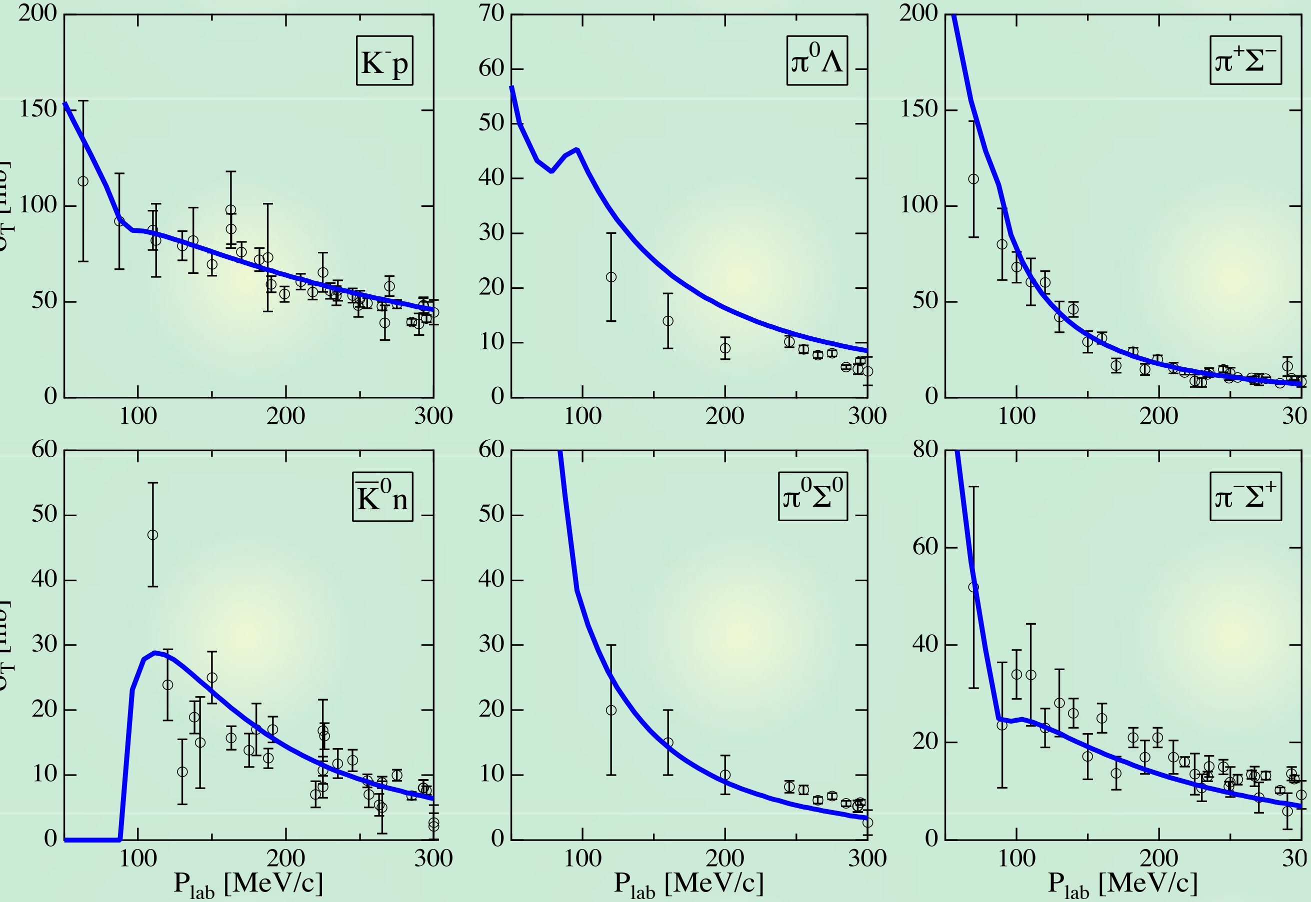


N. Kaiser, P. B. Siegel, W. Weise, Nucl. Phys. A594, 325 (1995),
E. Oset, A. Ramos, Nucl. Phys. A635, 99 (1998),
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001),
M.F.M. Lutz, E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002), ... many others

It works successfully, also in S=0 sector, meson-meson scattering sectors, systems including heavy quarks, ...

The simplest model (1 parameter) v.s. experimental data

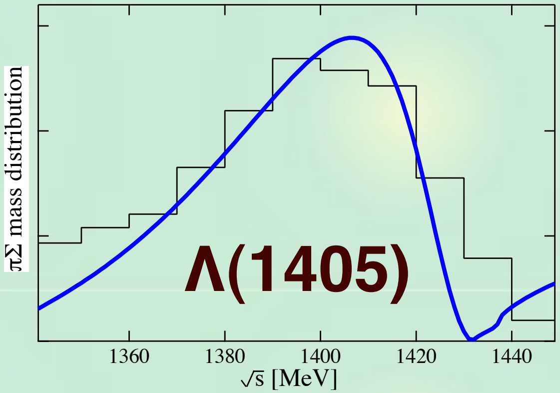
Total cross section of K-p scattering



Branching ratio

	γ	R_c	R_n
exp.	2.36	0.664	0.189
theo.	1.80	0.624	0.225

πΣ spectrum



T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, PRC68, 018201 (2003); PTP 112, 73 (2004)

Good agreement with data above, at, and below $\bar{K}N$ threshold

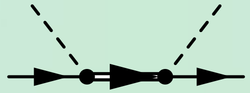
Λ(1405) mass, width, couplings: prediction of the model

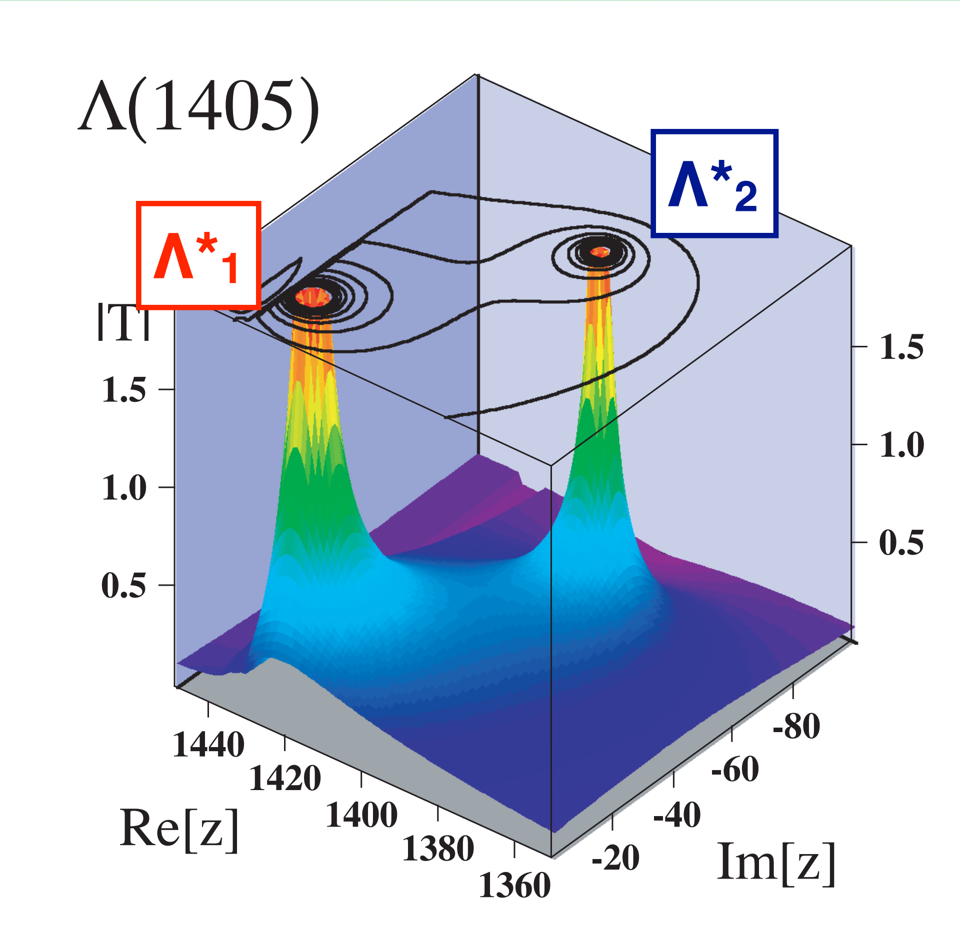
Two poles for one resonance

Poles of the amplitude in the complex plane: resonance

D. Jido, J.A. Oller, E. Oset, A. Ramos, U.G. Meissner, Nucl. Phys. A 723, 205 (2003);
T. Hyodo, W. Weise, Phys. Rev. C 77, 035204 (2008)

$$T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - M_R + i\Gamma_R/2}$$

$\sim$ 



Physical Λ(1405) is a superposition of two states
$\bar{K}$N bound state
+ πΣ resonance

Relevant pole for
$\bar{K}$N interaction ~ 1420 MeV

$\bar{K}$NN-πΣN ~ Λ^*_1N-Λ^*_2N --> T. Uchino's talk

Origin of the two-pole structure

Leading order chiral interaction for $\bar{K}N$-πΣ channel

T. Hyodo, W. Weise, Phys. Rev. C 77, 035204 (2008)

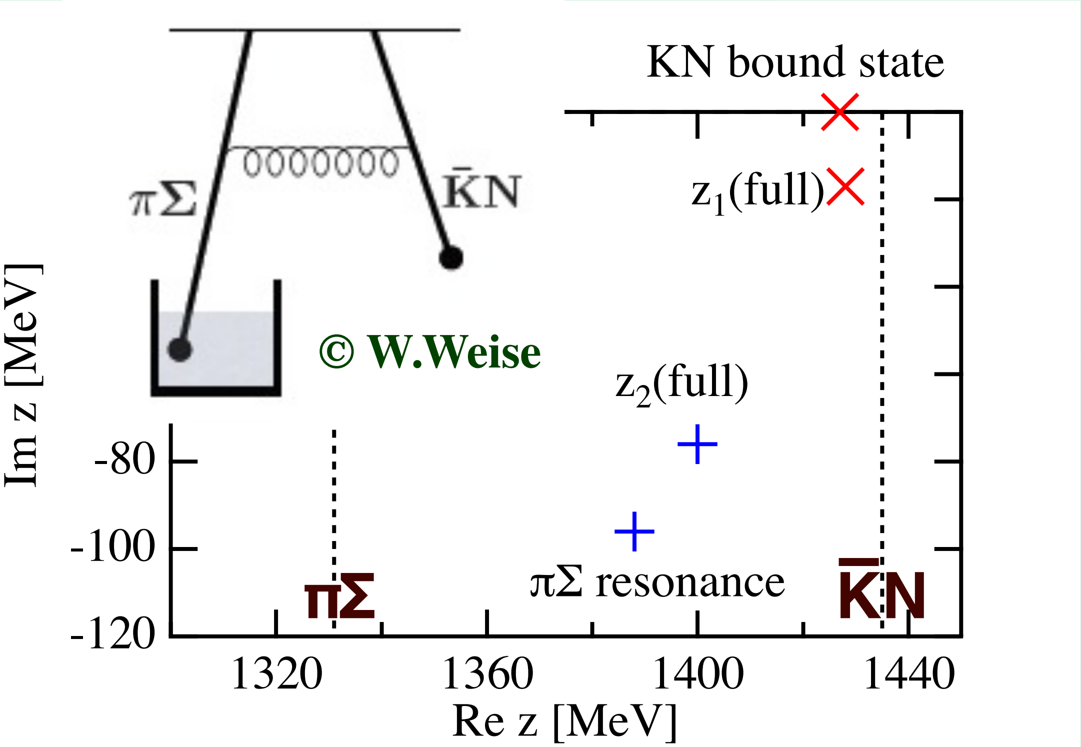
$$V_{ij} = -C_{ij}\frac{\omega_i + \omega_j}{4f^2}$$

$$C_{ij} = \begin{array}{cc} \bar{K}N & \pi\Sigma \\ \end{array}$$
$$C_{ij} = \begin{pmatrix} 3 & -\sqrt{\frac{3}{2}} \\ -\sqrt{\frac{3}{2}} & 4 \end{pmatrix}$$

at threshold

$$\omega_i \sim m_i, \quad 3.3m_\pi \sim m_K$$

$$\Rightarrow V_{\bar{K}N} \sim 2.5V_{\pi\Sigma}$$



Very strong attraction in $\bar{K}$N (higher energy) --> bound state
Strong attraction in πΣ (lower energy) --> resonance

Two poles also emerge with NLO contributions.

Quantitative study with $\bar{K}N$ threshold data

Calibration of the amplitude by the $\bar{K}N$ threshold data

- Threshold branching ratio

$$\gamma = \frac{\Gamma(K^- p \to \pi^+ \Sigma^-)}{\Gamma(K^- p \to \pi^- \Sigma^+)} = 2.36 \pm 0.04$$

$$R_c = \frac{\Gamma(K^- p \to \pi^+ \Sigma^-, \pi^- \Sigma^+)}{\Gamma(K^- p \to \text{all inelastic channels})} = 0.664 \pm 0.011$$

$$R_n = \frac{\Gamma(K^- p \to \pi^0 \Lambda)}{\Gamma(K^- p \to \text{neutral states})} = 0.189 \pm 0.015$$

R.J. Nowak *et al.*, Nucl. Phys. B139, 61 (1978); D.N. Tovee *et al.*, *ibid*, B33, 493 (1971)

- Shift and width of the kaonic hydrogen <-- SIDDHARTA

$$\Delta E - \frac{i}{2}\Gamma = -2\alpha^3 \mu_c^2 a_{K^-p}[1 - 2\alpha\mu_c(\ln\alpha - 1)a_{K^-p}]$$

U.-G. Meissner, U. Raha, A. Rusetsky, Eur. Phys. J. C35, 349 (2004)

A systematic χ^2 study with LO and NLO interactions.

Y. Ikeda, T. Hyodo, W. Weise, work in progress

- Preliminary results with LO --> W. Weise's talk

Why is the πΣ interaction relevant?

A result of three-body coupled-channel calculation

Y. Ikeda and T. Sato, Phys. Rev. C76, 035203 (2007)

Model	$\bar{K}N$ (MeV)	$\pi\Sigma$ (MeV)	Scattering length (fm)	Resonance energy (MeV)	Model (A)
(a)	1095	1450	$\boxed{-1.70 + i0.68}$	$1419.8 - i29.4$	$-79.3 - i37.1$
(f)	1160	1100	$-1.72 + i0.44$	$\boxed{1405.8 - i25.2}$	$-63.3 - i22.2$

form factor **Λ(1405) pole** **dibaryon pole**

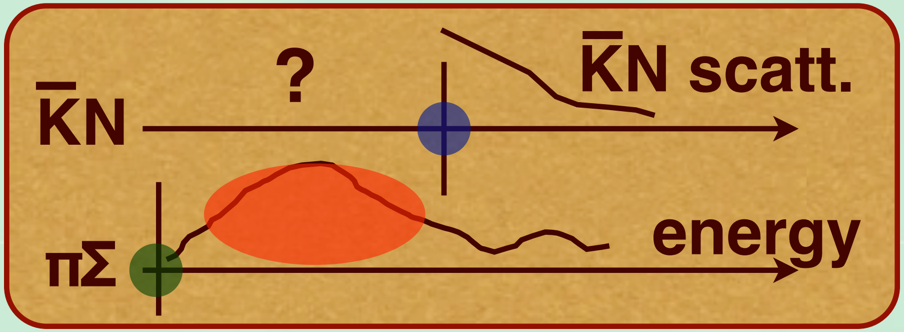
(a): shallow Λ*, deep dibaryon, strong πΣ interaction
(f): deep Λ*, shallow dibaryon, weak πΣ interaction

No simple correspondence in Λ* mass and dibaryon mass.
Strength of the πΣ interaction is important for ``deep” state?

What kind of πΣ information?

Precise data at $\bar{K}N$ threshold

- threshold branching ratio
- K-p (possibly with K-n) scattering length <-- SIDDHARTA



More constraints in πΣ channel

- Precise data of πΣ spectrum
  exp.) CLAS, LEPS, HADES,...
  theory) reaction study for each experiment

- Any information at πΣ threshold
  scattering length, effective range,...

Threshold behavior of πΣ scattering

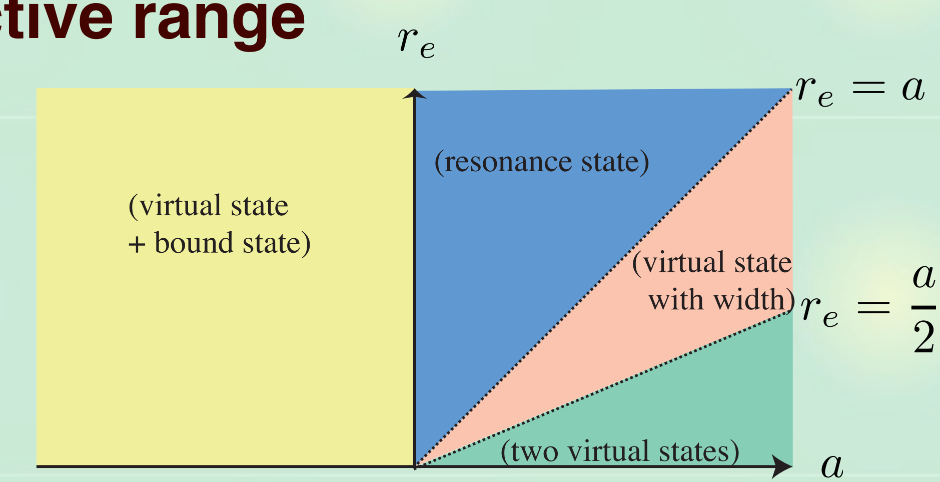
πΣ threshold information and K̄N-πΣ amplitude

Y. Ikeda, T. Hyodo, D. Jido, H. Kamano, T. Sato, K. Yazaki, in preparation;
D. Jido, T. Sekihara, Y. Ikeda, T. Hyodo, Y. Kanada-En'yo, E. Oset, NPA835, 59 (2010)

Fix the KN(I=0) scattering length
--> various solutions for the sub-threshold amplitude

Model	A1a	A1b	B1 E-dep	B1 E-indep
parameter ($\pi\Sigma$)	$d_{\pi\Sigma} = -1.67$	$d_{\pi\Sigma} = -2.85$	$\Lambda_{\pi\Sigma} = 1005$ MeV	$\Lambda_{\pi\Sigma} = 1465$ MeV
parameter ($\bar{K}N$)	$d_{\bar{K}N} = -1.79$	$d_{\bar{K}N} = -2.05$	$\Lambda_{\bar{K}N} = 1188$ MeV	$\Lambda_{\bar{K}N} = 1086$ MeV
pole 1 [MeV]	$1422 - 16i$	$1425 - 11i$	$1422 - 22i$	$1423 - 29i$
pole 2 [MeV]	$1375 - 72i$ (R)	1321 (B)	$1349 - 54i$ (R)	1325 (V)
$a_{\pi\Sigma}$ [fm]	0.934	-2.30	1.44	5.50
r_e [fm]	5.02	5.89	3.96	0.458
$a_{\bar{K}N}$ [fm] (input)	$-1.70 + 0.68i$	$-1.70 + 0.68i$	$-1.70 + 0.68i$	$-1.70 + 0.68i$

- scattering length + effective range
--> nature of the pole
(resonance, virtual, or bound)



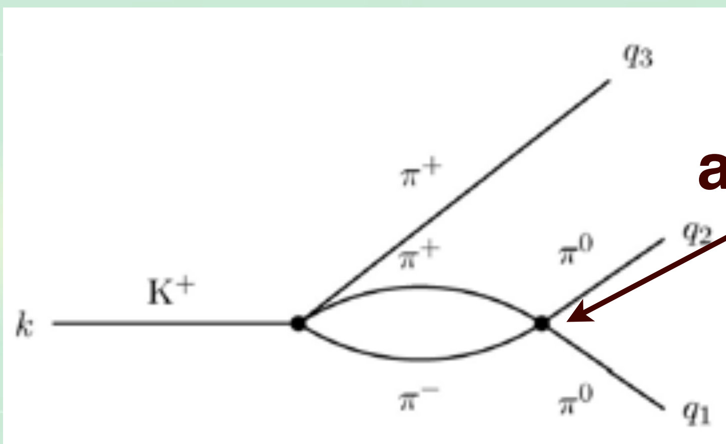
Determination of the scattering length

Extraction of hadron scattering length

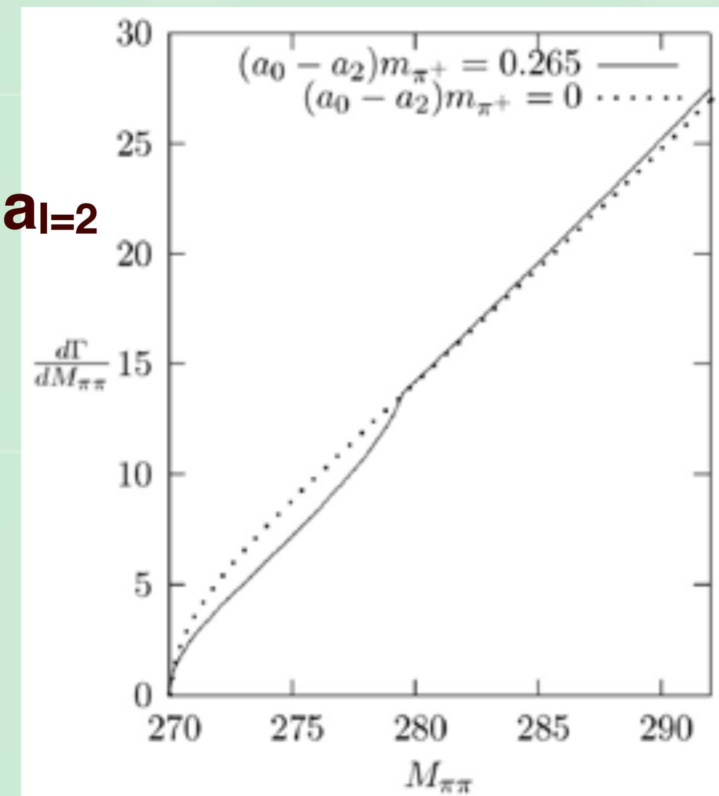
- shift and width of atomic state (c.f. Kaonic hydrogen)
- extrapolation of low energy phase shift
- final state interaction from heavy particle's decay

Cabibbo's method for π-π scattering length

N. Cabibbo, Phys. Rev. Lett. 93, 121801 (2004)



$a_{I=0} - a_{I=2}$

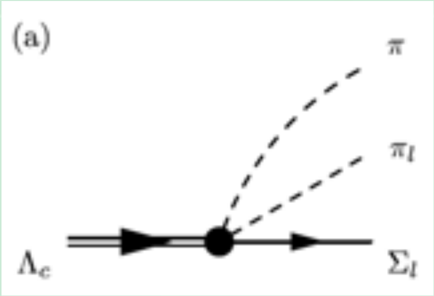


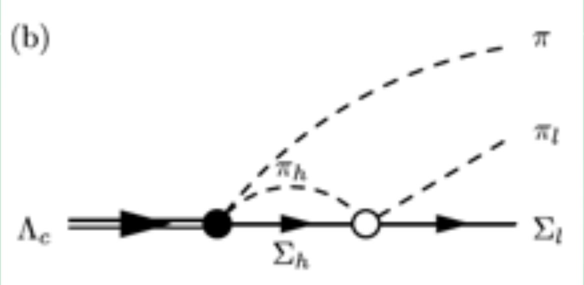
isospin violation
+ threshold cusp
+ amplitude interference
--> extraction of $a_{I=0} - a_{I=2}$

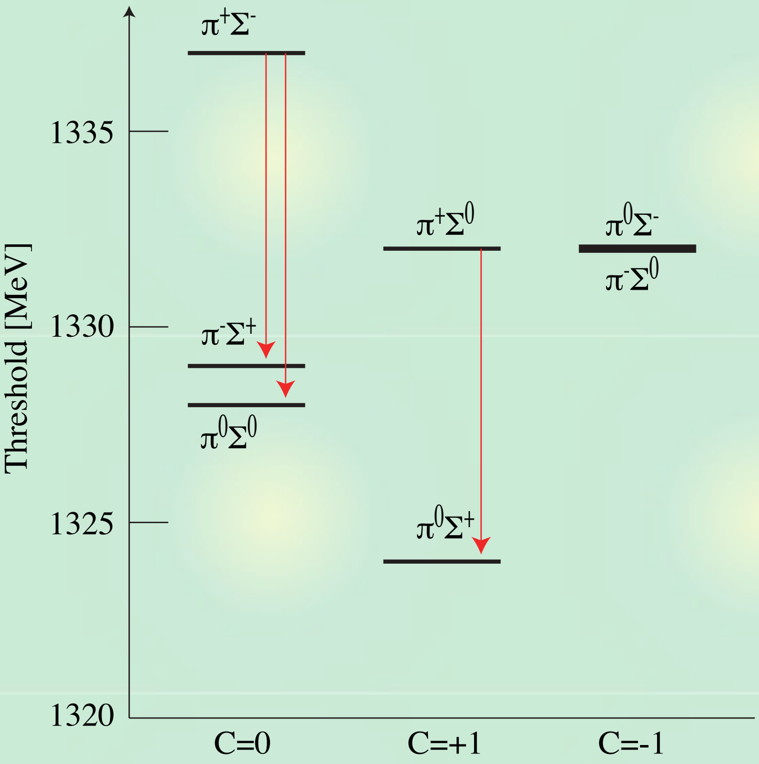
Determination of πΣ scattering length

Similar approach to πΣ spectrum in $\Lambda_c \to \pi\,(\pi\Sigma)$

T. Hyodo, M. Oka, work in progress







Σ^+(~uus) < Σ^0(~uds) < Σ^-(~dds)
--> complicated spectrum

To utilize threshold cusp, appreciable mass difference between $(\pi\Sigma)_h$ and $(\pi\Sigma)_l$ is necessary.

$$\pi^+\Sigma^- \to \pi^-\Sigma^+, \quad \pi^+\Sigma^- \to \pi^0\Sigma^0, \quad \pi^+\Sigma^0 \to \pi^0\Sigma^+,$$

Determination of πΣ scattering length

Three decay channels

$$\langle \pi^-\Sigma^+ |T| \pi^+\Sigma^- \rangle|_{\text{threshold}} = \frac{1}{3}a^0 - \frac{1}{2}a^1 + \frac{1}{6}a^2 = a^{xx}$$

$$\langle \pi^0\Sigma^0 |T| \pi^+\Sigma^- \rangle|_{\text{threshold}} = \frac{1}{3}a^0 - \frac{1}{3}a^2 = a^{00}$$

$$\langle \pi^0\Sigma^+ |T| \pi^+\Sigma^0 \rangle|_{\text{threshold}} = -\frac{1}{2}a^1 + \frac{1}{2}a^2 = a^{x}$$

mode	$\Lambda_c \to \pi(\pi\Sigma)_h$	$\Lambda_c \to \pi(\pi\Sigma)_l$
a^{xx}	1.7 %	3.6 %
a^{00}	1.7 %	1.8 %
a^{x}	1.8 %	not known

A lot of Λ_c in B decay (Belle, Babar) --> feasible?

Structure around the cusp in $(\pi\Sigma)_l$ + spectrum in $(\pi\Sigma)_h$ --> extraction of the scattering length

Three unknown scattering lengths, two constraints

$$a^{xx} - a^{00} = a^{x}$$

I=2 scattering length: lattice QCD --> Y. Ikeda's talk

Summary

We study the $\bar{K}N$-$\pi\Sigma$ system and $\Lambda(1405)$ based on chiral SU(3) symmetry and unitarity

- Chiral low energy theorem: constrains for the NG boson dynamics
- Two poles for the $\Lambda(1405)$
  <-- attractive $\bar{K}N$ and $\pi\Sigma$ interactions

  T. Hyodo and W. Weise, Phys. Rev. C 77, 035204 (2008)
- Systematic study for the $\bar{K}N$-$\pi\Sigma$ system
  <-- precise data of $\bar{K}N$ scattering length is highly called for.

  Y. Ikeda, T. Hyodo, W. Weise, in preparation

Summary

We emphasize the importance of the $\pi\Sigma$ interaction for the $\bar{K}N$-$\pi\Sigma$ physics

- No simple connection between $\Lambda(1405)$ mass and strange dibaryon mass.

- $\pi\Sigma$ threshold data is important for the amplitude at ``deep" region.

  Y. Ikeda, T. Hyodo, D. Jido, H. Kamano, T. Sato, K. Yazaki, in preparation

- $\pi\Sigma$ scattering length can be extracted from the Λ_c decay. Lattice QCD may help to complete the constraints.

  T. Hyodo, M. Oka, in preparation;
  Y. Ikeda, HAL QCD collaboration, in preparation.