Compositeness of bound states and resonances in chiral dynamics

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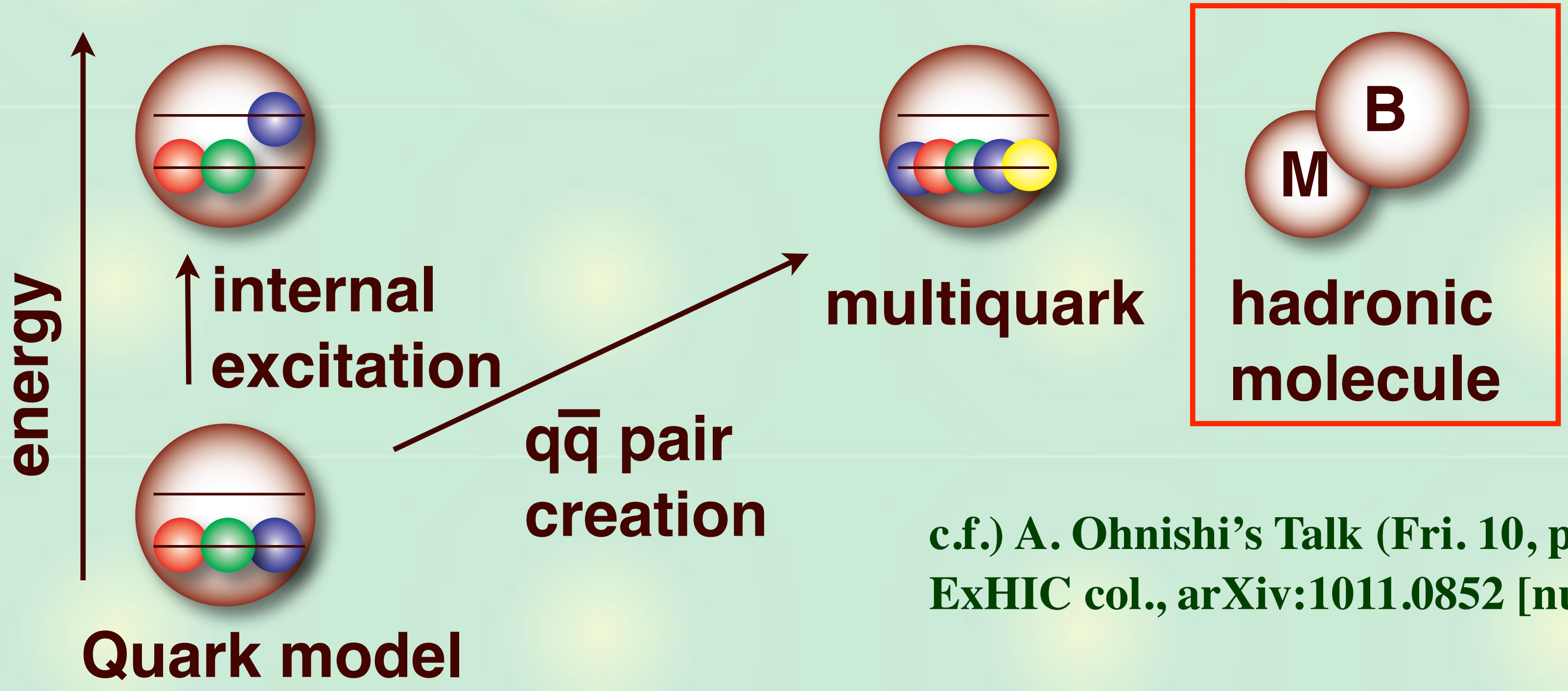
Structure of hadron resonances

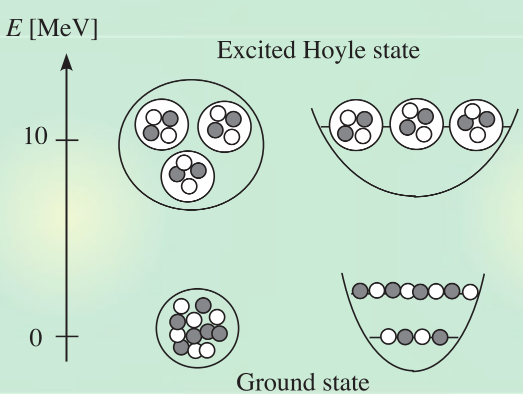
Example) baryon excited state



c.f.) A. Ohnishi's Talk (Fri. 10, pm);
ExHIC col., arXiv:1011.0852 [nucl-th]

Excited states
= resonances in hadron scattering

Exotic structure near threshold?
c.f. ^{12}C Hoyle state



Study of the internal structure

How to investigate the internal structure?

- Comparison of model calculation with experiments
  (mass, width, decay properties, etc.)

  : Any model can describe data with appropriate corrections
  : Model-dependent result

- Extrapolation to the ideal world, change the environment
  (large Nc, symmetry restoration, etc.)

  : Structure may change during the extrapolation
  : Qualitative discussion only

--> model-independent and quantitative study?

c.f.) T. Sekihara's Talk (Fri. 10, am); T. Sekihara, T. Hyodo, D. Jido, in preparation

Contents

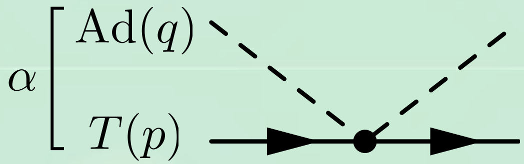
- Introduction
- Chiral SU(3) dynamics for baryon resonances
- Origin of resonances in chiral dynamics
  - Natural renormalization condition

    T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)
- Compositeness of bound states
  - Field renormalization constant Z

    T. Hyodo, D. Jido, A. Hosaka, arXiv:1009.5754 [nucl-th]; in preparation
- Summary

s-wave low energy interaction

Low energy NG boson (Ad) + target hadron (T) scattering



$$= \frac{1}{f^2}\frac{p\cdot q}{2M_T}\langle \boldsymbol{F}_T \cdot \boldsymbol{F}_{\mathrm{Ad}}\rangle_\alpha + \mathcal{O}\left(\left(\frac{m}{M_T}\right)^2\right)$$

Projection onto s-wave: Weinberg-Tomozawa (WT) term

Y. Tomozawa, Nuovo Cim. 46A, 707 (1966); S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

$$V_{ij} = -\frac{C_{ij}}{4f^2}(\omega_i + \omega_j)$$

energy dependence (derivative coupling)

decay constant of π (g_V=1)

$$C_{ij} = \sum_\alpha C_{\alpha,T}\begin{pmatrix} 8 & T & \| & \alpha \\ I_{M_i}, Y_{M_i} & I_{T_i}, Y_{T_i} & \| & I, Y \end{pmatrix}\begin{pmatrix} 8 & T & \| & \alpha \\ I_{M_j}, Y_{M_j} & I_{T_j}, Y_{T_j} & \| & I, Y \end{pmatrix}$$

$$C_{\alpha,T} = \langle 2\boldsymbol{F}_T \cdot \boldsymbol{F}_{\mathrm{Ad}}\rangle_\alpha = C_2(T) - C_2(\alpha) + 3$$

Group theoretical structure and flavor SU(3) symmetry determines the sign and the strength of the interaction

Low energy theorem: leading order term in ChPT

Scattering amplitude and unitarity

Unitarity of S-matrix: Optical theorem

$$\mathrm{Im}[T^{-1}(s)] = \frac{\rho(s)}{2}$$ phase space of two-body state

General amplitude by dispersion relation

$$T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s} - W_i} + \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)}$$

R_i, W_i, a: to be determined by chiral interaction

Identify dispersion integral = loop function G, the rest = V^{-1}

$$T(\sqrt{s}) = \frac{1}{V^{-1}(\sqrt{s}) - G(\sqrt{s}; a)}$$ Scattering amplitude

V? chiral expansion of T, (conceptual) matching with ChPT

$$T^{(1)} = V^{(1)}, \quad T^{(2)} = V^{(2)}, \quad T^{(3)} = V^{(3)} - V^{(1)} G V^{(1)}, \quad \ldots$$

Amplitude T: consistent with chiral symmetry + unitarity

Chiral unitary approach

Meson-baryon scattering amplitude

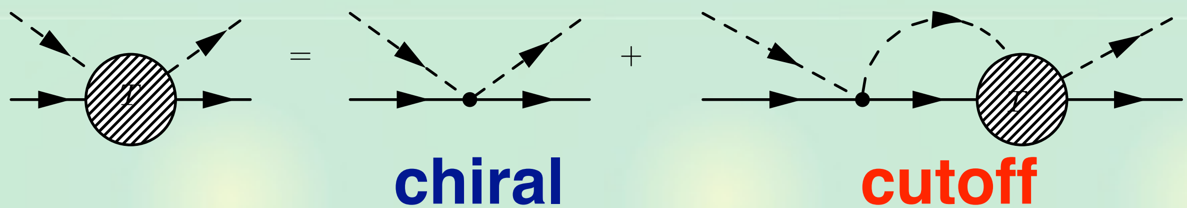
- **Interaction <-- chiral symmetry**

  Y. Tomozawa, Nuovo Cim. 46A, 707 (1966); S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

- **Amplitude <-- unitarity in coupled channels**

  R.H. Dalitz, T.C. Wong, G. Rajasekaran, Phys. Rev. 153, 1617 (1967)

$$T = \frac{1}{1 - VG} V$$



N. Kaiser, P. B. Siegel, W. Weise, Nucl. Phys. A594, 325 (1995),
E. Oset, A. Ramos, Nucl. Phys. A635, 99 (1998),
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001),
M.F.M. Lutz, E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002), ... many others

It works successfully, also in S=0 sector, meson-meson scattering sectors, systems including heavy quarks, ...

Hadron excited states

Resonances are “dynamically generated”

light baryon	$J^P = 1/2^-$	$\Lambda(1405)$ $\Lambda(1670)$ $\Sigma(1670)$
		$N(1535)$ $\Xi(1620)$ $\Xi(1690)$
	$J^P = 3/2^-$	$\Lambda(1520)$ $\Xi(1820)$ $\Sigma(1670)$
heavy		$\Lambda_c(2880)$ $\Lambda_c(2593)$ $D_s(2317)$
light meson	$J^P = 1^+$	$b_1(1235)$ $h_1(1170)$ $h_1(1380)$ $a_1(1260)$
		$f_1(1285)$ $K_1(1270)$ $K_1(1440)$
	$J^P = 0^+$	$\sigma(600)$ $\kappa(900)$ $f_0(980)$ $a_0(980)$

No states with exotic quantum number

- No attraction in exotic channel

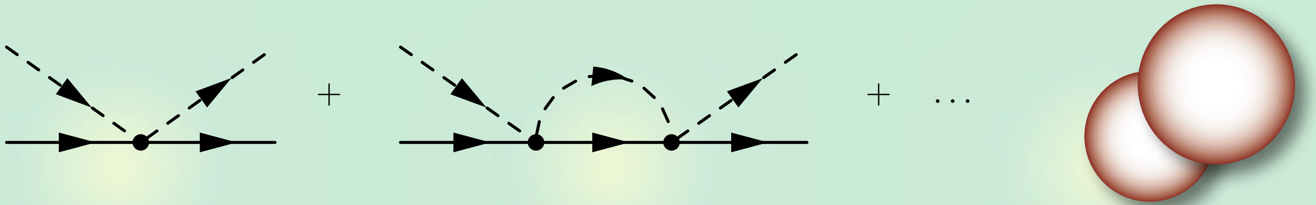
T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. Lett. 97, 192002 (2006);
Phys. Rev. D75, 034002 (2007)

--> Structure of these resonances?

Classification of resonances

Resonances in two-body scattering

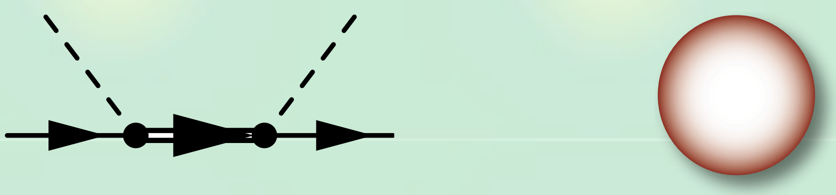
- **Dynamical state**: composite particle, two-body molecule, ...



e.g.) Deuteron in NN, positronium in e^+e^-, ...

- **CDD pole**: elementary particle, preformed state, ...

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)



~ pole term in V

e.g.) J/Ψ in e^+e^-, ...

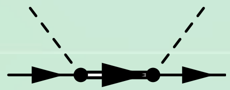
Asymptotic fields: hadrons (no quark structure).
Hadrons are ``elementary'' in this study.

CDD pole in subtraction constant?

Phenomenological (standard) scheme
--> V is given, "a" is determined by data

$$T = \frac{1}{(V^{(1)})^{-1} - G(\underline{a})}$$ **leading order**

$$T = \frac{1}{(V^{(1)} + V^{(2)})^{-1} - G(\underline{a'})}$$ **next to leading order**

↑pole  **?**

"a" represents the effect which is not included in V.
CDD pole contribution in G?

Natural renormalization scheme
--> fix "a" first, then determine V

to exclude CDD pole contribution from G,
based on theoretical argument.

Natural renormalization condition

Conditions for the subtraction constant

1) Loop function G should be **negative** below threshold.
<--> no states below threshold

$$G(\sqrt{s}) \sim \sum_n \frac{|\langle \dots \rangle|^2}{\sqrt{s} - E_n} \leq 0 \quad \text{for} \quad \sqrt{s} \leq E_0$$ --> upper limit for "a"

2) T matches with the chiral interaction V at **low energy**.

$$T(\mu_m; a) = V(\mu_m) \quad \text{for} \quad M_T \leq \mu_m \leq M_T + m$$ --> lower limit for "a"

To satisfy 1) and 2), "a" is uniquely determined as

$$G(\sqrt{s} = M_T) = 0 \quad \Leftrightarrow \quad T(M_T) = V(M_T)$$

- subtraction constant: a_{natural}

We regard this condition as the **exclusion of the CDD pole contribution from G**.

Pole in the effective interaction: single channel

Leading order V: Weinberg-Tomozawa term

$$V_{\mathrm{WT}} = -\frac{C}{2f^2}(\sqrt{s} - M_T) \qquad G(\sqrt{s};a) = \frac{2M_T}{(4\pi)^2}\Big\{a + \dots$$

$$T^{-1} = V_{\mathrm{WT}}^{-1} - G(a_{\mathrm{pheno}}) = V_{\mathrm{natural}}^{-1} - G(a_{\mathrm{natural}})$$

↑ChPT ↑data fit ↑given

Effective interaction in natural scheme

$$V_{\mathrm{natural}} = -\frac{C}{2f^2}(\sqrt{s} - M_T) + \boxed{\frac{C}{2f^2}\frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\mathrm{eff}}}}$$

pole!
a seed of resonance?

$$M_{\mathrm{eff}} = M_T - \frac{16\pi^2 f^2}{C M_T \Delta a}, \qquad \Delta a = a_{\mathrm{pheno}} - a_{\mathrm{natural}}$$

There is always a pole for $a_{\mathrm{pheno}} \neq a_{\mathrm{natural}}$

- small deviation <=> pole at irrelevant energy scale
- large deviation <=> pole at relevant energy scale

Pole in the effective interaction

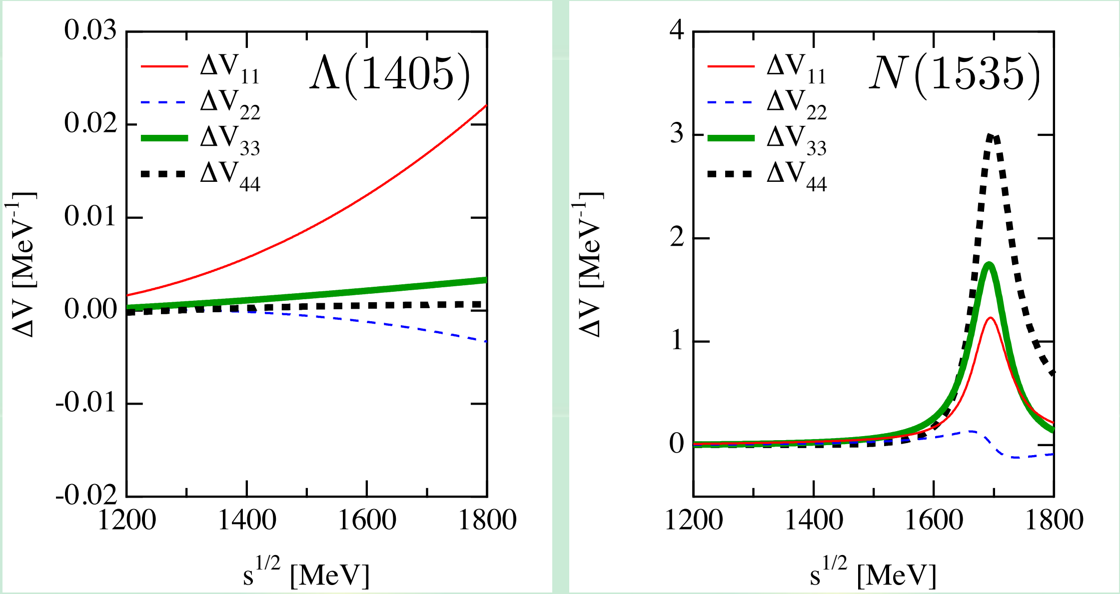
Pole in the effective interaction (M_{eff}) : pure CDD pole

$$T^{-1} = V_{\rm WT}^{-1} - G(a_{\rm pheno}) = \boxed{V_{\rm natural}^{-1}} - G(a_{\rm natural})$$

For Λ(1405): $z_{\rm eff}^{\Lambda^*} \sim 7.9$ GeV **irrelevant!**

For N(1535): $z_{\rm eff}^{N^*} = 1693 \pm 37i$ MeV **relevant?**

Difference of interactions $\Delta V \equiv V_{\rm natural} - V_{\rm WT}$



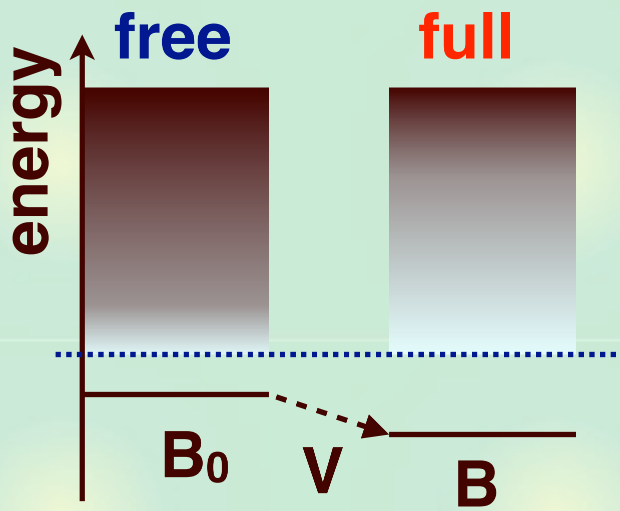
==> Important CDD pole contribution in N(1535)

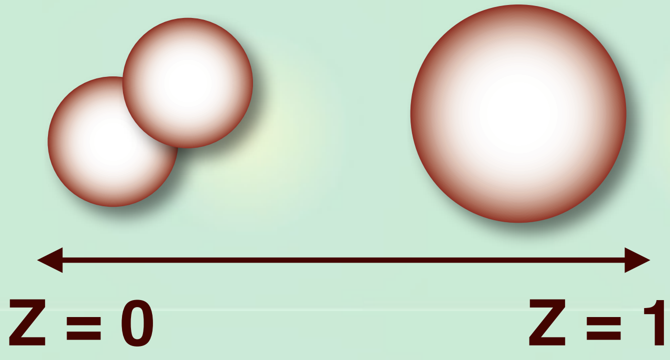
Next question: quantitative measure for compositeness?

Compositeness and field renormalization constant

Spectra of free Hamiltonian and full Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + V$$





$|B_0\rangle$: bare state (CDD pole) **$|B\rangle$: physical state.**

Field renormalization constant Z

: overlap of bare state $|B_0\rangle$ and physical state $|B\rangle$

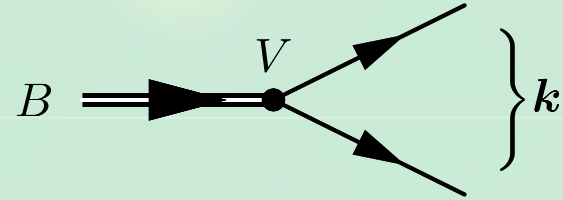
$$Z \equiv |\langle B_0 | B \rangle|^2$$

1 - Z : Compositeness of the bound state

Expression for the compositeness

With the Schrödinger equation, we obtain

$$1 - Z = \int d\boldsymbol{k} \frac{|\langle \boldsymbol{k}|V|B\rangle|^2}{[E(\boldsymbol{k}) + B]^2} \qquad \langle \boldsymbol{k}|V|B\rangle :$$



$$= 4\pi\sqrt{2\mu^3}\int_0^\infty dE \frac{\sqrt{E}}{E+B}\left[t(E) - v(E) - 4\pi\sqrt{2\mu^3}\int_0^\infty dE' \frac{\sqrt{E'}|t(E')|^2}{E - E' + i\epsilon}\right]$$

T. Hyodo, D. Jido, A. Hosaka, arXiv:1009.5754 [nucl-th]

Approximation: For small binding energy B<<1, the vertex <kIVIB> can be regarded as a constant: $\langle \boldsymbol{k}|V|B\rangle \sim g_W$

Then the integration can be done analytically, leading to

$$1 - Z = 2\pi^2\sqrt{2\mu^3}\frac{g_W^2}{\sqrt{B}}$$

Compositeness <-- coupling g and binding energy B

S. Weinberg, Phys. Rev. 137 B672-B678 (1965)

- Model-independent: no information of V

Single-channel chiral unitary approach

To apply the argument on Z, we study the **bound state** with mass M_B in the **single channel** chiral unitary approach.

- particle masses: M and m, bound state M_B
- Weinberg-Tomozawa interaction
- parameters: subtraction "a" and M_B (or coupling)

We use the **model-independent** formula

$$1 - Z = 2\pi^2 \sqrt{2\mu^3} \frac{g_W^2}{\sqrt{B}}$$

- coupling constant: residue of the pole at M_B

$$[g(M_B; a)]^2 = \lim_{W \to M_B} (W - M_B) T(W) = -\frac{M_B - M}{G(M_B; a) + (M_B - M) G'(M_B)}$$

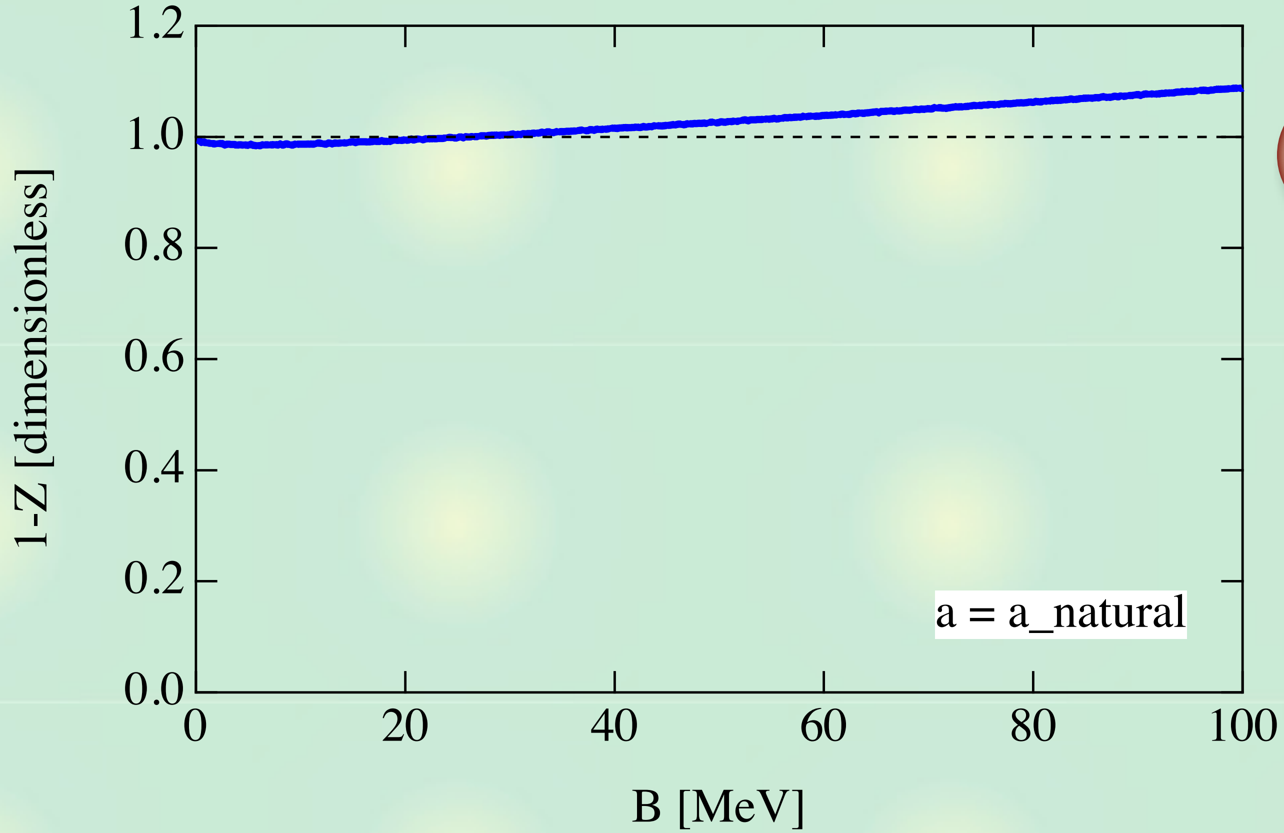
- normalization of the amplitude (a kinematical factor)

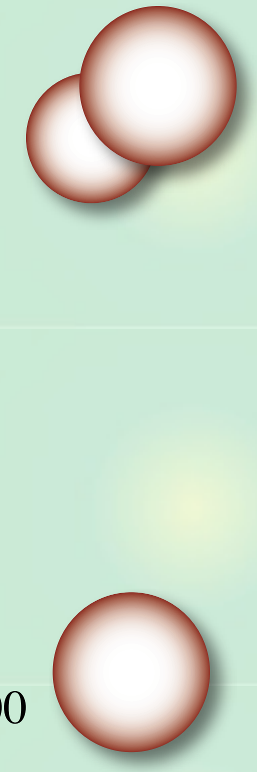
$$1 - Z = \frac{M |\bar{q}(M_B)|}{8\pi M_B (M + m - M_B)} [g(M_B; a)]^2$$ (for small $B = M + m - M_B$)

Numerical analysis

Compositeness of the bound state in chiral unitary approach

1) B dependence with $a = a_{natural}$



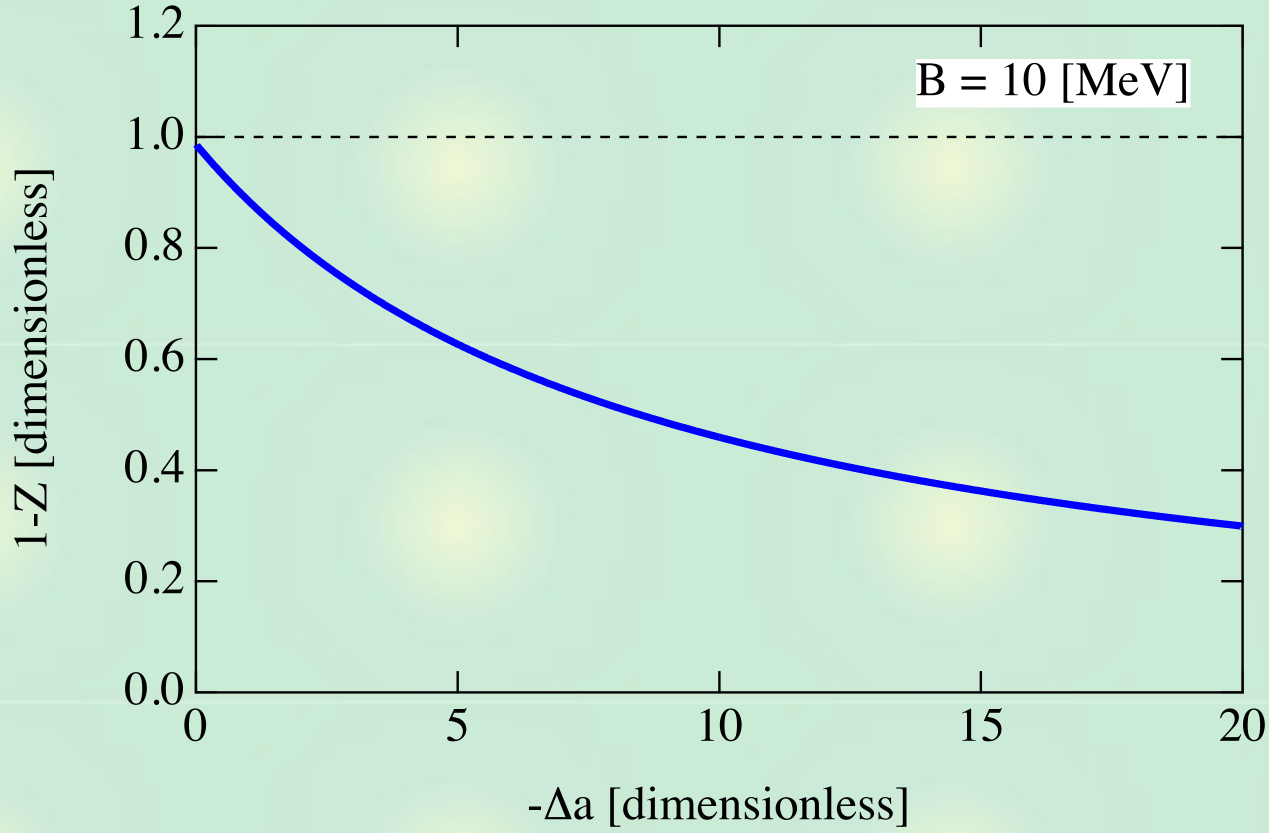


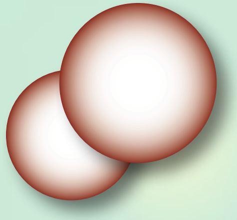
- Z ~ 0: **composite particle in natural renormalization**
- large B behavior is not justified by the approximation

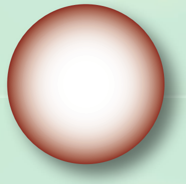
Numerical analysis

Compositeness of the bound state in chiral unitary approach

2) Δa (=a-$a_{natural}$) dependence with B = 10 MeV







- **deviation** from the natural value: **bare state** contribution
- **large deviation: large bare state contribution**

Summary

Structure of resonances/bound states

- **Natural renormalization scheme**
  exclude CDD pole contribution from the loop function to generate **purely molecule resonance**
  T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)
- **Field renormalization constant Z:**
  quantitative measure of **compositeness**
- **Natural scheme corresponds to Z ~ 0**
  --> generated bound state: composite
  T. Hyodo, D. Jido, A. Hosaka, arXiv:1009.5754 [nucl-th]

Future plan

To apply to hadron resonances, we should ...

- extend to the coupled-channel problem

  This may be straightforward,
  but technically complicated.

- extend to resonances

  Define Z in relativistic field theory
  (comparison with Yukawa theory)
  The composite condition seems to be
  $G(M_B)=0$
  c.f. natural scheme $G(M)=0$

T. Hyodo, D. Jido, A. Hosaka, in preparation