Compositeness of bound states in chiral dynamics

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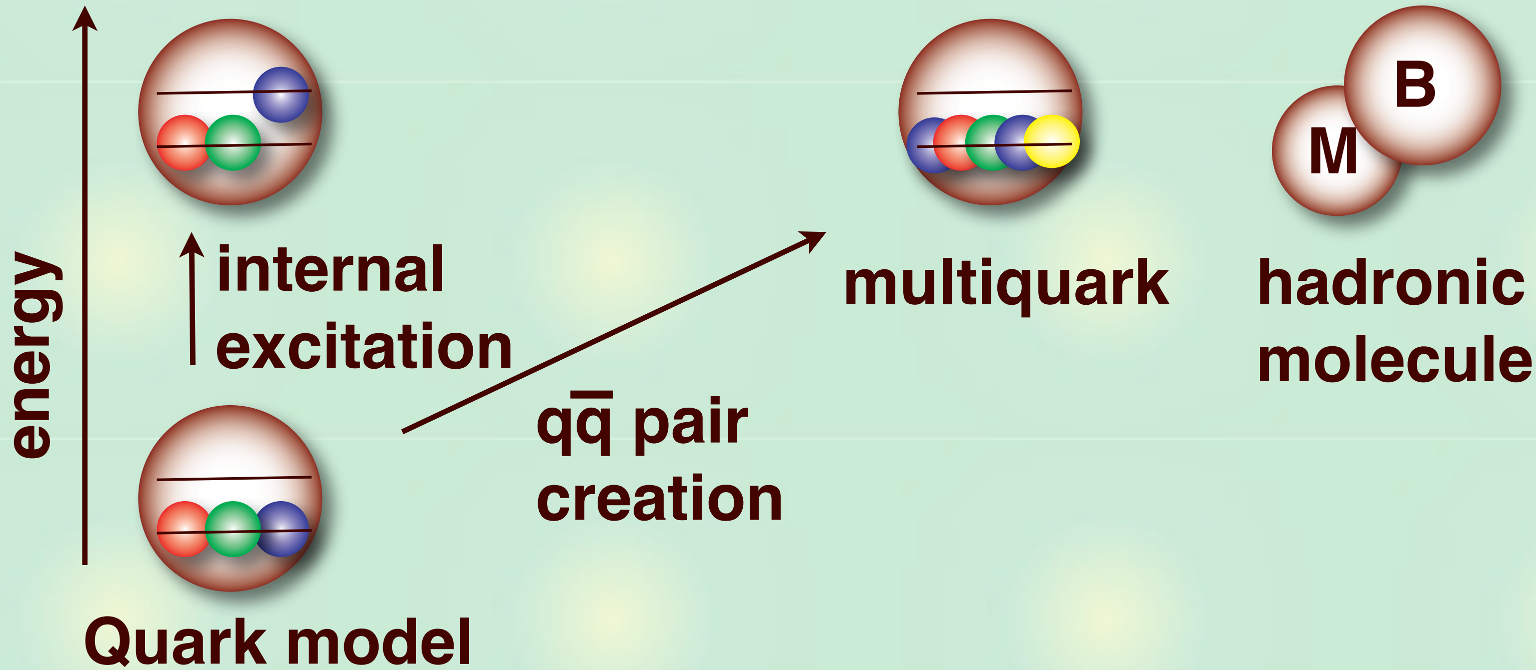
Structure of hadron resonances

Example) baryon excited state



What are 3q state, 5q state, MB state, ...?

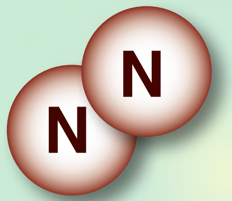
Clear definition of the structure is called for.

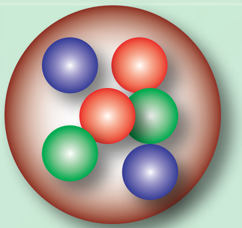
Contents

Weinberg's compositeness and deuteron

Z: probability of finding deuteron in a bare elementary state

S. Weinberg, Phys. Rev. 137, B672 (1965)

$|\mathrm{deuteron}\rangle =$ 

or 

$\notin$ NN model space

~ elementary particle

Z = 0 ⟷ Z = 1

model independent relation for weakly bound state

$$\boxed{a_s} = \left[\frac{2(1-Z)}{2-Z}\right]\boxed{R} + \mathcal{O}(m_\pi^{-1}), \quad \boxed{r_e} = \left[\frac{-Z}{1-Z}\right]\boxed{R} + \mathcal{O}(m_\pi^{-1})$$

a_s: scattering length

r_e: effective range

R: deuteron radius (binding energy)

<-- Experiments

$$a_s = +5.41 \ [\mathrm{fm}], \quad r_e = +1.75 \ [\mathrm{fm}], \quad R \equiv (2\mu B)^{-1/2} = 4.31 \ [\mathrm{fm}]$$

$\Rightarrow Z \lesssim 0.2$ --> deuteron is almost composite!

Derivation in nonrelativistic quantum mechanics

The formula is derived in two steps:

Step 1 (Sec. II): Z --> p-n-d coupling constant g

$$g^2 = \frac{2\sqrt{B}(1-Z)}{\pi\rho} \qquad \rho = 4\pi\sqrt{2\mu^3}$$

Step 2 (Sec. III): coupling constant g --> a_s, r_e

$$a_s = 2R\left[1 + \frac{2\sqrt{B}}{\pi\rho g^2}\right] \qquad r_e = R\left[1 - \frac{2\sqrt{B}}{\pi\rho g^2}\right]$$

Assumption: B is sufficiently smaller than the typical energy scale of the NN interaction

$$p \sim m_\pi, \quad B \ll m_\pi^2/2\mu \quad \Leftrightarrow \quad R^2 \gg m_\pi^2$$

--> uncertainty for order R quantity: m_π^{-1}

Definition of the compositeness 1-Z

Hamiltonian of two-body system: free + interaction V

$$\mathcal{H} = \mathcal{H}_0 + V$$

Complete set for free Hamiltonian: bare $|B_0>$ + continuum

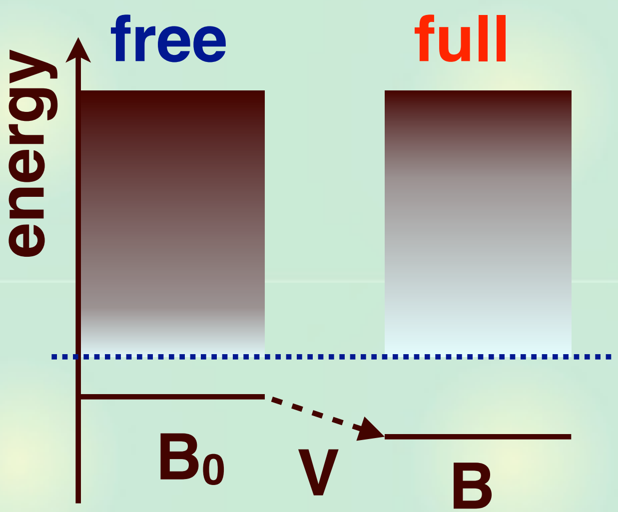
$$1 = |\, B_0 \,\rangle\langle\, B_0 \,| + \int d\boldsymbol{k} |\, \boldsymbol{k} \,\rangle\langle\, \boldsymbol{k} \,|$$

$$\mathcal{H}_0 |\, B_0 \,\rangle = E_0 |\, B_0 \,\rangle, \quad \mathcal{H}_0 |\, \boldsymbol{k} \,\rangle = E(\boldsymbol{k}) |\, \boldsymbol{k} \,\rangle$$

Physical bound state |B> : eigenstate of full Hamiltonian

$$(\mathcal{H}_0 + V)|\, B \,\rangle = -B |\, B \,\rangle$$

B: binding energy



Define Z as the overlap of B and B_0
: probability of finding the physical bound state in the bare state |B>

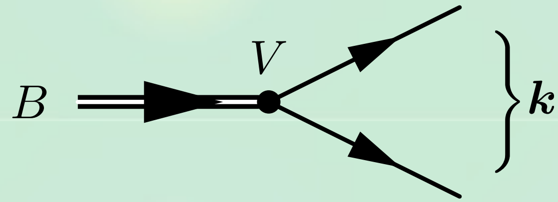
$$Z \equiv |\langle\, B_0 \,|\, B \,\rangle|^2$$

1 - Z : Compositeness of the bound state

Model-independent but approximated method

With the Schrödinger equation, we obtain

$$1 - Z = \int d\boldsymbol{k} \frac{|\langle \boldsymbol{k} | V | B \rangle|^2}{[E(\boldsymbol{k}) + B]^2} \qquad \langle \boldsymbol{k} | V | B \rangle :$$



$$= 4\pi \sqrt{2\mu^3} \int_0^\infty dE \frac{\sqrt{E} |G_W(E)|^2}{(E+B)^2} \qquad \langle \boldsymbol{k} | V | B \rangle \equiv G_W[E(\boldsymbol{k})]$$ **for s-wave**

Approximation: For small binding energy B<<1, the vertex $G_W(E)$ can be regarded as a constant: $G_W(E) \sim g_W$

Then the integration can be done analytically, leading to

$$1 - Z = 2\pi^2 \sqrt{2\mu^3} \frac{g_W^2}{\sqrt{B}}$$

Compositeness <-- coupling g and binding energy B

S. Weinberg, Phys. Rev. 137 B672-B678 (1965)

- **Model-independent: no information of V**
- **Approximated: valid only for small B**

Derivation in quantum field theory

Field theory with Yukawa coupling (ψ,ϕ,B_0)

D. Lurie and A. J. Macfarlane, Phys. Rev. 136, B816 (1963)

$$\mathcal{L}_0 = \bar{\psi}(i\partial\!\!\!/ - M)\psi + \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2) + \bar{B}_0(i\partial\!\!\!/ - M_{B_0})B_0$$

$$\mathcal{L}_{\mathrm{int}} = g_0\bar{\psi}\phi B_0 + (\mathrm{h.c.})$$

Physical bound state B at total energy $W=M_B$

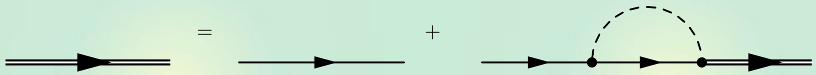
Free (full) propagators of B_0 (B) field (positive energy part)

$$\Delta_0(W) = \frac{1}{W - M_{B_0}}, \quad \Delta(W) = \frac{\boxed{Z}}{W - M_B}$$

Z: residue of the full propagator

Dyson equation: relation between full and free propagators

$$\Delta(W) = \Delta_0(W) + \Delta_0(W)g_0G(W)g_0\Delta(W)$$



$$G(W) = i\int\frac{d^4q}{(2\pi)^4}\frac{2M}{(P-q)^2 - M^2 + i\epsilon}\frac{1}{q^2 - m^2 + i\epsilon}$$

Derivation in quantum field theory

Solution of Dyson equation

$$\Rightarrow \Delta(W) = \frac{1}{W - M_{B_0} - g_0^2 G(W)}$$

G(W) diverges: renormalization parameter ``a''

$$\Delta(W) = \frac{1}{W - g_0^2 G(W;a)}$$

Renormalization condition, pole at W=M_B : $M_B = g_0^2 G(M_B;a)$

The field renormalization constant: residue of the propagator

$$Z = \lim_{W \to M_B} \frac{W - M_B}{W - g_0^2 G(W;a)} = \frac{1}{1 - g_0^2 G'(M_B)}$$

Vertex renormalization $g^2 = g_0^2 Z$

Compositeness in Yukawa theory

$$\boxed{1 - Z = -g^2 G'(M_B)}$$

Compositeness: summary

We have defined the compositeness of the bound state 1-Z.

Method 1: nonrelativistic quantum mechanics

$$1 - Z_{NR} = g^2 \frac{M|\lambda^{1/2}(M_B^2, M^2, m^2)|}{16\pi M_B^2(M + m - M_B)} \quad \text{for } M_B \to M + m$$

model independent, but valid only for weak binding

Method 2: field theory with Yukawa coupling

$$1 - Z = -g^2 G'(M_B)$$

exact (no approximation), but Lagrangian dependent

Application?

For a bound state in model calculations or experiments, compositeness can be evaluated by the mass of the bound state ``$M_B$" and the coupling constant ``g".

Chiral dynamics: overview

Description of S = -1, $\bar{K}N$ s-wave scattering: Λ(1405) in I=0

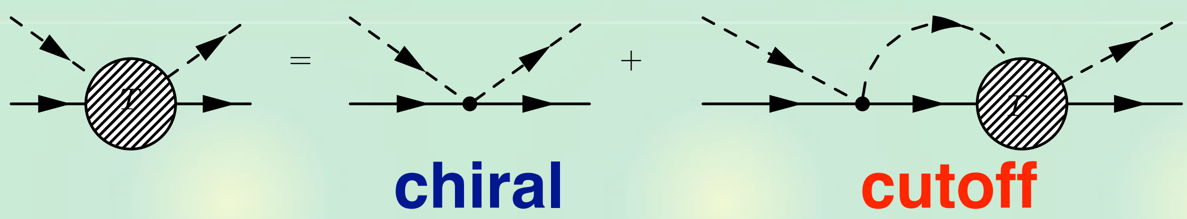
- Interaction <-- chiral symmetry

Y. Tomozawa, Nuovo Cim. 46A, 707 (1966); S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

- Amplitude <-- unitarity in coupled channels

R.H. Dalitz, T.C. Wong, G. Rajasekaran, Phys. Rev. 153, 1617 (1967)

$$T = \frac{1}{1 - VG} V$$



N. Kaiser, P. B. Siegel, W. Weise, Nucl. Phys. A594, 325 (1995),
E. Oset, A. Ramos, Nucl. Phys. A635, 99 (1998),
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001),
M.F.M. Lutz, E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002), ... many others

It works successfully in various hadron scatterings.

A review: T. Hyodo, D. Jido, to appear in Prog. Part. Nucl. Phys. (2011)

Natural renormalization condition

Single-channel scattering of meson m and baryon M.

$$T(W)=\frac{1}{1-V(W)G(W;a)}V(W)$$

Interaction V: energy-independent and energy-dependent

$$V(W)=\begin{cases} V^{(\mathrm{const})}=Cm & \text{constant interaction} \\ V^{(\mathrm{WT})}(W)=C(W-M) & \text{WT interaction} \end{cases}$$

Bound state condition: pole at W=M_B

$$1-V(M_B)G(M_B;a)=0$$

Coupling constant: residue of the pole

$$g^2=\lim_{W\to M_B}(W-M_B)T(W)=\begin{cases} -[G'(M_B)]^{-1} & \text{constant interaction} \\ -\left[G'(M_B)+\dfrac{G(M_B;a)}{M_B-M}\right]^{-1} & \text{WT interaction} \end{cases}$$

We determine mass and coupling of the bound state

Compositeness of bound states

Compositeness in Yukawa theory

$$1 - Z = -g^2 G'(M_B) = \begin{cases} 1 & \text{constant interaction} \\ \left[1 + \dfrac{G(M_B; a)}{(M_B - M) G'(M_B)} \right]^{-1} & \text{WT interaction} \end{cases}$$

- constant interaction --> purely composite bound state

- WT interaction --> mixture of composite and elementary

- Purely composite bound state for WT interaction:

$$G'(M_B) = -\infty \quad \text{or} \quad G(M_B; a) = 0$$

$$M_B = M + m \quad \text{or} \quad C \to -\infty$$

1) zero energy bound state
2) infinitely strong two-body attraction

Relation with natural renormalization scheme?

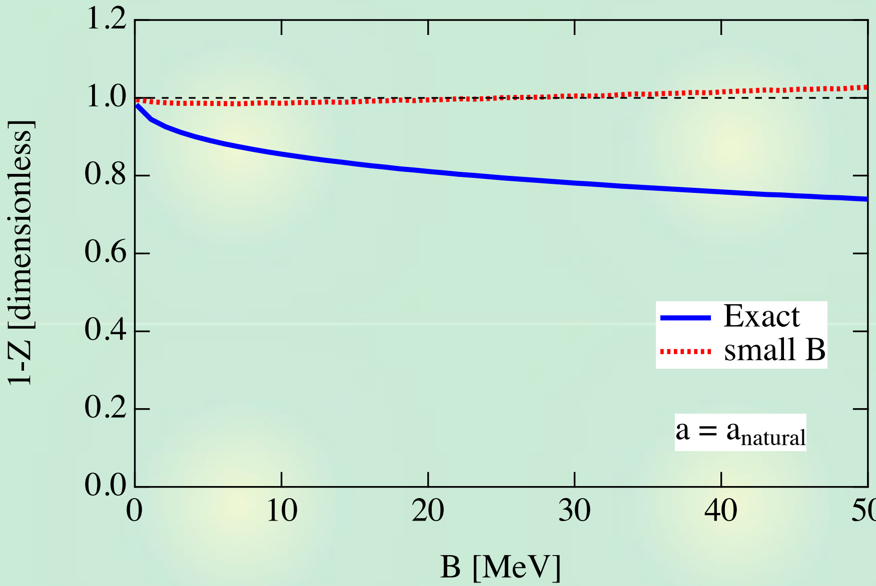
Consistency check of the natural renormalization scheme

Natural renormalization condition
<-- to exclude elementary contribution from the loop function

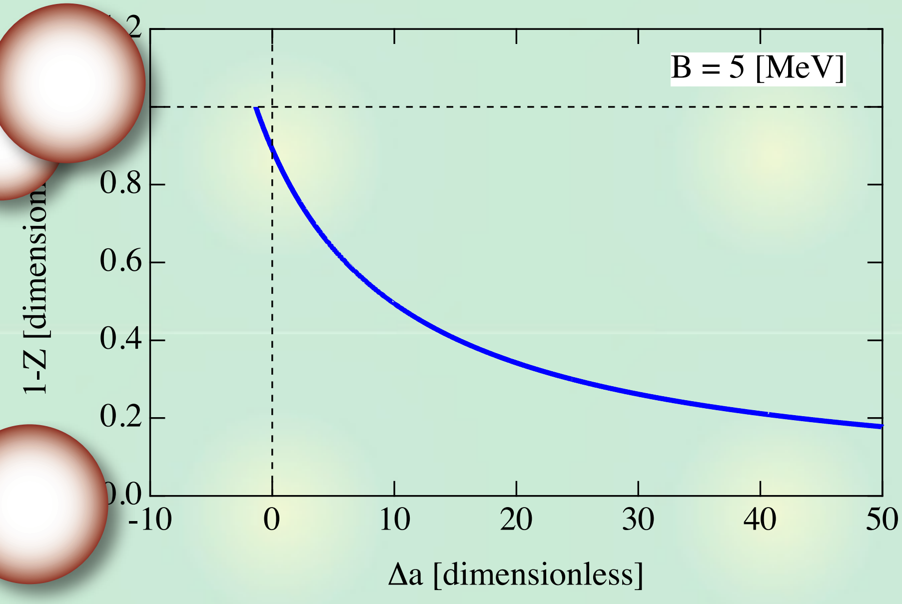
T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

$$G(W=M;a_{\rm natural})=0$$

1) a = $a_{\rm natural}$, vary B



2) B = 5 MeV, vary a



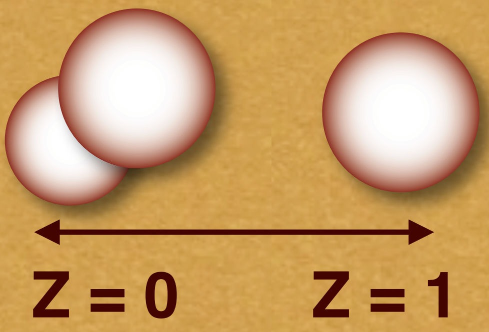
natural scheme --> Z ~ 0

large deviation --> Z ~ 1

Summary 1

Compositeness of the bound state

- Field renormalization constant Z: compositeness



- Model independent formula

$$1 - Z_{NR} = g^2 \frac{M|\lambda^{1/2}(M_B^2, M^2, m^2)|}{16\pi M_B^2 (M + m - M_B)} \quad \text{for } M_B \to M + m$$

S. Weinberg, Phys. Rev. 137 B672 (1965)

- Exact formula in field theory

$$1 - Z = -g^2 G'(M_B)$$

D. Lurie and A. J. Macfarlane, Phys. Rev. 136, B816 (1963)

- Both agrees at small binding

Summary 2

Application to chiral unitary approach

- Bound state in chiral dynamics

  Energy independent interaction
  --> purely composite bound state

  Energy-dependent chiral interaction
  --> mixture of composite and elementary

- Natural scheme corresponds to Z ~ 0

  --> composite particle is generated

T. Hyodo, D. Jido, A. Hosaka, AIP Conf. Proc. 1322, 374 (2010);
T. Hyodo, D. Jido, A. Hosaka, in preparation